Mensuration

Area Of A Trapezium

Namita has a garden in front of her house. The garden is in the shape of a trapezium. The lengths of the parallel sides of the garden are 15 m and 20 m and the distance between these parallel sides is 12 m. She wants to spread fertilizer in the garden, which costs Rs 5 per square metre. Namita is wondering what it will cost her to spread the fertilizer in the entire garden. Can we help her out?

Let us now discuss some examples based on the areas of trapeziums.

Example 1:

What is the area of a trapezium (in m^2) whose parallel sides are 125 cm and 7.5 dm and the perpendicular distance between them is 90 cm.

Solution:

The parallel sides are $a = 125 \text{ cm} = \frac{125}{100} = 1.25 \text{ m}$ and

$$b = 7.5 \text{ dm} = \frac{7.5}{10} = 0.75 \text{ m}$$

$$\left(\text{As } 1 \text{ cm} = \frac{1}{100} \text{ m and } 1 \text{ dm} = \frac{1}{10} \text{ m}\right)$$

Perpendicular distance between the parallel sides = h = 90 cm $= \frac{90}{100} = 0.9 \text{ m}$

But we know that area of a trapezium $= \frac{1}{2} h(a+b)$

$$= \left[\frac{1}{2} \times 0.9(1.25 + 0.75)\right] \text{m}^2$$

$$= \left(\frac{1}{2} \times 0.9 \times 2\right) \text{m}^2$$

= 0.9 m^2

Hence, the area of the given trapezium is 0.9 m^2.

Example 2:

Area of a trapezium is 285 dm^2. If length of one of the parallel sides is 24 dm and its height is 15 dm, then what is the length of the other parallel side?

Solution:

Given that area of trapezium = 285 dm^2

Length of one of the parallel sides = a = 24 dm

Height = h = 15 dm

We have to find another parallel side i.e. 'b'.

But we know that area of a trapezium $= \frac{1}{2}h(a+b)$

$$285 = \frac{1}{2} \times 15(24+b)$$
$$24+b = \frac{285 \times 2}{15}$$

$24 + b = 38$

$b = 38 - 24$

b = 14 dm

Hence, the length of the other parallel side of the trapezium is 14 dm.

Example 3:

The two parallel sides of a trapezium are in the ratio 3:5 and its height is 9 cm. What are the dimensions of the parallel sides of the trapezium if its area is 90 cm^2?

Solution:

Let the two parallel sides of the trapezium be $a = 3x$ and $b = 5x$.

Given that, height = h = 9 cm

And area of trapezium = 90 cm^2

But we know that area of trapezium $= \frac{1}{2}h(a+b)$

$$90 \text{ cm}^2 = \left[\frac{1}{2}\times 9(3x+5x)\right] \text{ cm}$$

$$90 \text{ cm}^2 = \left[\frac{1}{2}\times 9\times 8x\right] \text{ cm}$$

$$x = \frac{90\times 2}{9\times 8} \text{ cm}$$

$$x = \frac{5}{2} = 2.5 \text{ cm}$$

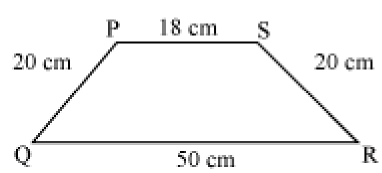
But, $a = 3x = 3 \times 2.5$ cm = 7.5 cm

$b = 5x = 5 \times 2.5$ cm = 12.5 cm

Hence, the dimensions of the parallel sides of the given trapezium are 7.5 cm and 12.5 cm.

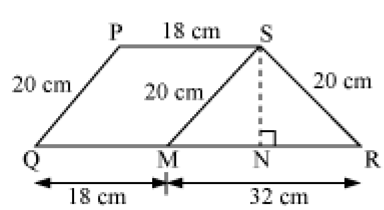
Example 4:

The following figure PQRS is a trapezium, where PS || QR. If PS = 18 cm, QR = 50 cm, PQ = RS = 20 cm, then what is the area of trapezium PQRS?



Solution:

In the trapezium PQRS, let us draw SM || PQ which intersects QR at M.



Let us also draw SN ⊥ MR.

As PS || QR

Therefore, PS || QM……………… (1)

From our construction, SM || PQ………… (2)

Thus, from equation (1) and equation (2), PQMS is a parallelogram.

(Opposite sides of a parallelogram are parallel)

Hence, SM = PQ (opposite sides of a parallelogram are equal)

and QM = PS

SM = 20 cm and QM = 18 cm

But MR = QR − QM = 50 − 18 = 32 cm

Triangle SMR is an isosceles triangle (as SM and SR = 20 cm each)

Now, $NR = \frac{1}{2} MR = \frac{1}{2} \times 32 = 16 \text{ cm}$

Now, the triangle SNR is a right-angled triangle.

Hence, $SR^2 = SN^2 + NR^2$ (Using Pythagoras theorem)

$20^2 = SN^2 + 16^2$

$SN^2 = 400 - 256 = 144$

SN = $\sqrt{144}$ = 12 cm

This is the height of trapezium.

Hence, we obtain a = 18 cm, b = 50 cm and h = 12 cm.

Area of the trapezium

$$= \frac{1}{2} h(a+b)$$
$$= \left[\frac{1}{2} \times 12(18+50)\right] \text{ cm}^2$$

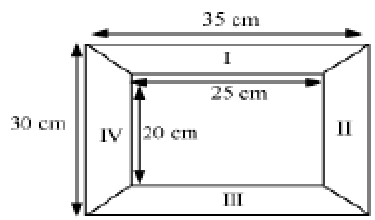
$= (6 \times 68)\ cm^2$

$= 408\ cm^2$

Hence, the area of the given trapezium is $408\ cm^2$.

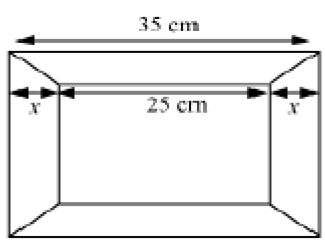
Example 5:

Diagram of the following picture frame has outer dimensions of 35 cm × 30 cm and inner dimensions of 25 cm × 20 cm. If the width of each section is the same, then find the area of each outer section of the frame.



Solution:

Let the width of the given picture frame be *x* cm.



From the figure, it is clear that

$x + 25 + x = 35$

$2x = 35 - 25 = 10$

$x = 5$ cm

Hence, the width of the frame is 5 cm.

This is the height for the outer sections I, II, III and IV which are in the shape of a trapezium.

Thus, area of the trapezium I = $\frac{1}{2}h(a+b)$

$$=\left[\frac{1}{2}\times 5(35+25)\right]\text{ cm}^2$$
$$=\left(\frac{1}{2}\times 5\times 60\right)\text{ cm}^2$$
$$=150\text{ cm}^2$$

Thus, area of the trapezium II = $\frac{1}{2}h(a+b)$

$$=\left[\frac{1}{2}\times 5(30+20)\right]\text{ cm}^2$$
$$=\left(\frac{1}{2}\times 5\times 50\right)\text{ cm}^2$$
$$=125\text{ cm}^2$$

Thus, area of the trapezium III = $\frac{1}{2}h(a+b)$

$$=\left[\frac{1}{2}\times 5(35+25)\right]\text{ cm}^2$$
$$=\left(\frac{1}{2}\times 5\times 60\right)\text{ cm}^2$$
$$=150\text{ cm}^2$$

Thus, area of the trapezium IV = $\frac{1}{2}h(a+b)$

$$=\left[\frac{1}{2}\times 5(30+20)\right]\text{ cm}^2$$
$$=\left(\frac{1}{2}\times 5\times 50\right)\text{ cm}^2$$
$$=125\text{ cm}^2$$

Total area of the outer section = sum of the areas of all the trapeziums

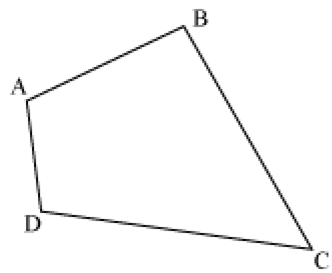
= 150 + 125 + 150 + 125

= 550 cm^2.

Hence, the total area of the outer section is 550 cm^2.

Area Of A General Quadrilateral

Look at the following quadrilateral ABCD.



Now, we have to find its area. Note that we cannot classify it as a rectangle, parallelogram or trapezium. Had we been able to do so, we could have easily applied the corresponding formula for the area. Now what do we do?

Example 1:

The length of the diagonal and the lengths of the perpendiculars from the opposite vertices to that diagonal of a quadrilateral are in the ratio 8: 3: 4. If the area of this quadrilateral is 448 m^2, then find the dimension of the diagonal and the perpendiculars.

Solution:

Let the length of the diagonal (d) of the quadrilateral be $8x$.

Thus, the lengths of the perpendiculars from opposite vertices are $h_1 = 3x$ and $h_2 = 4x$.

It is given that area of the quadrilateral = 448 m^2

$$\therefore \frac{1}{2} d\left(h_1 + h_2\right) = 448$$

$$\frac{1}{2} \times 8x\left(3x + 4x\right) = 448$$

$$4x \times 7x = 448$$

$$28x^2 = 448$$

$$x^2 = 16$$

$$x = \sqrt{16} = 4$$

$\therefore d = 8x = (8 \times 4)$ m = 32 m

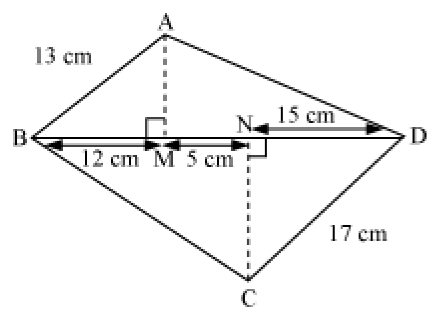
$h_1 = 3x = (3 \times 4)$ m = 12 m

$h_2 = 4x = (4 \times 4)$ m = 16 m

Thus, the length of the diagonal of the quadrilateral is 32 m, while the lengths of the perpendiculars on the diagonal from the opposite vertices of the quadrilateral are 12 m and 16 m.

Example 2:

Find the area of quadrilateral ABCD in the following figure.



Solution:

ΔABM is a right-angled triangle.

Applying Pythagoras theorem in ΔABM,

$AM^2 + BM^2 = AB^2$

$AM^2 + 12^2 = 13^2$

$AM^2 = 169 - 144 = 25$

$AM = \sqrt{25} = 5$ cm

$\therefore h_1 = 5$ cm

Similarly, ΔCND is also a right-angled triangle.

Applying Pythagoras theorem in ΔCND,

$CN^2 + DN^2 = CD^2$

$CN^2 + 15^2 = 17^2$

$CN^2 = 289 - 225 = 64$

$$CN = \sqrt{64} = 8 \text{ cm}$$

$\therefore h_2 = 8$ cm

Length of diagonal BD = d = BM + MN + ND = (12 + 5 + 15) cm = 32 cm

Hence, area of the quadrilateral ABCD = $\frac{1}{2}d(h_1 + h_2)$

$$= \left[\frac{1}{2} \times 32 \times (5+8)\right] \text{ cm}^2$$
$$= \left(\frac{1}{2} \times 32 \times 13\right) \text{ cm}^2$$

= 208 cm^2

Area Of A Rhombus

Example 1:

A floor of a building consists of 5000 rhombus shaped tiles. The length of diagonals of each tile is 6 dm and 80 cm. If the cost of polishing the floor is Rs 70 per 3m^2, then find the total cost of polishing the floor.

Solution:

The diagonals of rhombus shaped marble tiles are

$$d_1 = 6\,\text{dm} = \frac{6}{10} = 0.6\,\text{m}\left(\because 1\,\text{dm} = \frac{1}{10}\text{m}\right)$$

$$\text{and } d_2 = 80\,\text{cm} = \frac{80}{100} = 0.8\,\text{m}\left(\because 1\,\text{cm} = \frac{1}{100}\text{m}\right)$$

Hence, area of each tile $= \frac{1}{2} d_1 \times d_2$

$$= \left(\frac{1}{2} \times 0.6 \times 0.8 \right) \text{ m}^2$$

Area of 5000 tiles = $\left(5000 \times \frac{1}{2} \times 0.6 \times 0.8 \right) \text{ m}^2$

$$= (2500 \times 0.6 \times 0.8) \text{ m}^2$$
$$= 1200 \text{ m}^2$$

The rate of polishing the floor is Rs 70 per 3 m^2.

Hence, cost of polishing the floor $= \text{Rs } 1200 \times \frac{70}{3} = \text{Rs } 28000$

Example 2:

Area of a rhombus is $1600\sqrt{3} \text{ dm}^2$. If one of its diagonals is $40\sqrt{3} \text{ dm}$, then find the length of the other diagonal in terms of metre.

Solution:

Length of one diagonal $d_1 = 40\sqrt{3} \text{ dm}$

Length of other diagonal = d_2

Given area of the rhombus $= 1600\sqrt{3} \text{ dm}^2$

Therefore,

$$\frac{1}{2} d_1 \times d_2 = 1600\sqrt{3}$$
$$\frac{1}{2} \times 40\sqrt{3} \times d_2 = 1600\sqrt{3}$$
$$d_2 = \frac{1600\sqrt{3}}{20\sqrt{3}}$$
$$d_2 = 80 \text{ dm}$$
$$d_2 = \frac{80}{10} \text{ m} = 8 \text{ m} \left(\because 1 \text{ dm} = \frac{1}{10} \text{ m} \right)$$

Hence, the length of the other diagonal is 8 m.

Example 3:

If the ratio of the diagonals of a rhombus is 4:7 and the area of rhombus is 1400 m^2, then find the dimensions of the diagonals.

Solution:

Let the diagonals be $d_1 = 4x$ and $d_2 = 7x$.

Area of the rhombus = 1400 m^2.

Therefore,

$$\frac{1}{2} d_1 \times d_2 = 1400$$
$$\frac{1}{2} \times 4x \times 7x = 1400$$

$$14x^2 = 1400$$
$$x^2 = 100$$
$$x = \sqrt{100} = 10$$
$$\text{Thus, } d_1 = 4x = 4 \times 10 = 40\,\text{m}$$
$$\text{and } d_2 = 7x = 7 \times 10 = 70\,\text{m}$$

Hence, the dimensions of the diagonals of the rhombus are 40 m and 70 m.

Area Of A Polygon

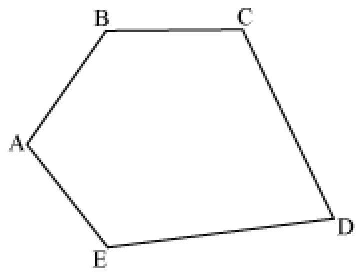
A polygon consists of different shapes of plane figures such as rectangle, square, triangle etc. The area of a polygon is the measurement of the two-dimensional region enclosed by the polygon.

To find the area of a polygon, we have to split it into different shapes. The sum of the areas of these different shapes gives the value of the area of the polygon.

But how can it be done?

It can be understood easily by an example.

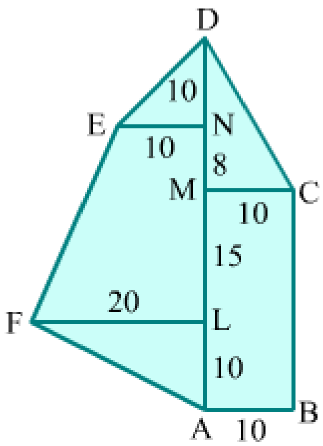
Let us consider a pentagon ABCDE as shown below.



Scale Drawing

Measurement of the area of a land:

You may observe land, field or garden of the following shape.



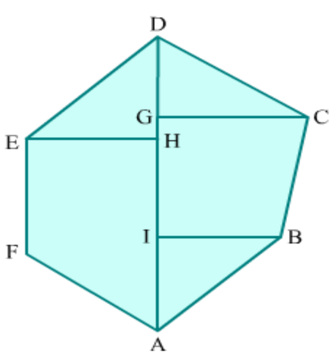
The area of such irregular shape can found under the following manner.

Step 1: Firstly divide the shape into known geometrical shaped fragments.

Step 2: Record the measurements and draw a sketch.

Step 3: Measurements are recorded in the observer's field book.

Step 4: Total area of the land is the sum of the areas of all the fragments.

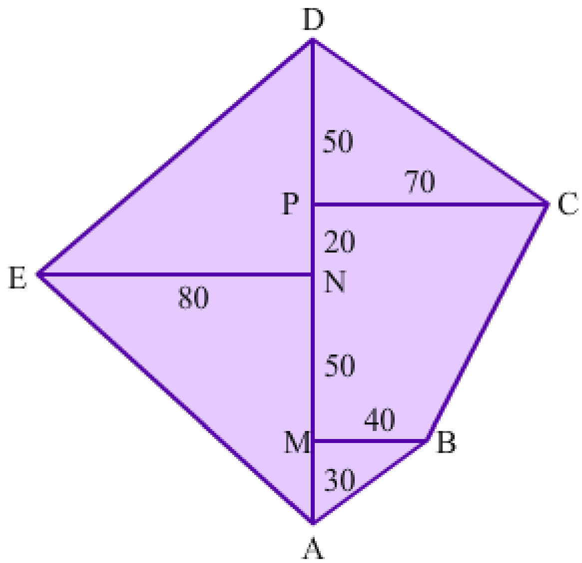


Here, the area of the land ABCDEF = Area of ΔABI + Area of trapezium IBCG + Area of ΔGCD + Area of ΔHDE+ Area of trapezium EFAH

For example, to plan out and to find the area of the field from the following notes.

	Metre to D	
To E 80	150 100 80 30	70 to C 40 to B
	From A	

Here, let us choose the scale as 20 m = 1 cm.



It can be observe that,

AM = 30 m, MN = 80 m

MP = (100-30) = 70 m

ND = (150-80) = 70 m

PD = (150-100) = 50 m

Area of ΔABM $= \frac{1}{2} \times 30 \times 40 = 600 \text{ m}^2$

Area of trapezium MBCP $= \frac{1}{2} \times 70 \times (70 + 40) = 3850 \text{ m}^2$

Area of ΔDPC $= \frac{1}{2} \times 50 \times 70 = 1750 \text{ m}^2$

Area of ΔDEN $= \frac{1}{2} \times 80 \times 70 = 2800 \text{ m}^2$

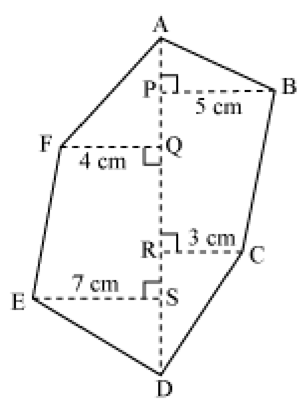
Area of ΔNEA $= \frac{1}{2} \times 80 \times 80 = 3200 \text{ m}^2$

Area of the field ABCDE = 600 + 3850 + 1750 + 2800 + 3200 = 12200 m^2

Let us discuss some more examples based on the areas of polygons.

Example 1:

Find the area of the following polygon ABCDEF with given dimensions. Also it is given that AD = 17 cm, AS = 13 cm, AR = 10 cm, AQ = 5 cm, and AP = 2 cm.



Solution:

Given AD = 17 cm, AS = 13 cm, AR = 10 cm, AQ = 5 cm, and AP = 2 cm

Therefore, PR = AR − AP = 10 − 2 = 8 cm

RD = AD − AR = 17 − 10 = 7 cm

SD = AD − AS = 17 − 13 = 4 cm

QS = AS − AQ = 13 − 5 = 8 cm

Area of right-angled triangle APB = $\frac{1}{2} \times AP \times PB$

$= \left(\frac{1}{2} \times 2 \times 5\right)\ cm^2$

$= 5\ cm^2$

Area of trapezium PBCR = $\frac{1}{2} \times PR \times (PB + RC)$

$= \left(\frac{1}{2} \times 8 \times (5+3)\right) cm^2$

$= \left(\frac{1}{2} \times 8 \times 8\right)\ cm^2$

= 32 cm^2

Area of right-angled ΔRCD = $\frac{1}{2} \times RD \times RC$

$= \left(\frac{1}{2} \times 7 \times 3\right)\ cm^2$

= 10.5 cm^2

Area of right-angled ΔESD = $\frac{1}{2} \times SD \times ES$

$= \left(\frac{1}{2} \times 4 \times 7\right)\ cm^2$

= 14 cm^2

Area of trapezium ESQF = $\frac{1}{2} \times QS \times (ES + FQ)$

$= \left(\frac{1}{2} \times 8 \times (7 + 4)\right)\ cm^2$

= 44 cm^2

Area of right-angled ΔAFQ = $\frac{1}{2} \times AQ \times FQ$

$= \left(\frac{1}{2} \times 5 \times 4\right)\ cm^2$

$= 10\ cm^2$

Hence, area of the polygon = area of right-angled ΔAPB + area of trapezium PBCR + area of right-angled ΔRCD + area of right-angled ΔESD + area of trapezium ESQF + area of right-angled ΔAFQ

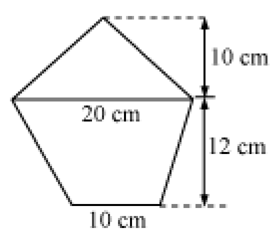
= (5 + 32 + 10.5 + 14 + 44 + 10) cm^2

= 115.5 cm^2

Hence, the area of the given polygon is 115.5 cm^2.

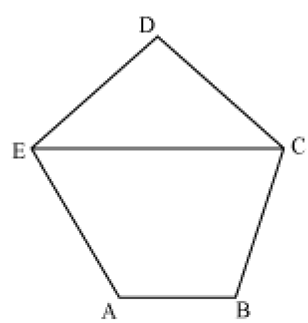
Example 2:

Sagar divided a pentagon into two parts as shown in the figure. Find the area of the pentagon?



Sagar's method

Solution:



Area of pentagon = Area of triangle EDC + Area of trapezium ABCE

$$= \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times \text{ sum of parallel sides} \times \text{Distance between parallel sides}$$

$$= \frac{1}{2} \times 20 \times 10 + \frac{1}{2}(20 + 10) \times 12$$

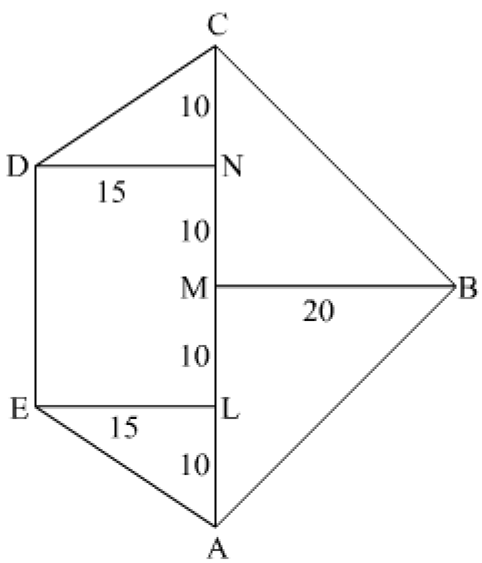
= 100 + 180

= 280 cm^2

Example 3.
Draw a plan and find the area of the field from the data taken out from the surveyor's field book.

	Km to C	
	10	
To D 15	**10**	
	10	**20 to B**
To E 15	**10**	
	From A	

Solution:



It is seen that, AL = 10 km, LM = 10 km, MN = 10 km, NC = 10 km

$$\text{Area of } \Delta ALE = \frac{1}{2}bh = \frac{1}{2} \times 10 \times 15 = 75 \text{ km}^2$$

$$\text{Area of rectangle DELN} = lb = 15 \times 20 = 300 \text{ km}^2$$

$$\text{Area of } \Delta CDN = \frac{1}{2}bh = \frac{1}{2} \times 15 \times 10 = 75 \text{ km}^2$$

$$\text{Area of } \Delta BCM = \frac{1}{2}bh = \frac{1}{2} \times 20 \times 20 = 200 \text{ km}^2$$

$$\text{Area of } \Delta ABM = \frac{1}{2}bh = \frac{1}{2} \times 20 \times 20 = 200 \text{ km}^2$$

∴ Area of the field ABCD

= 75 + 300 + 75 + 200 + 200

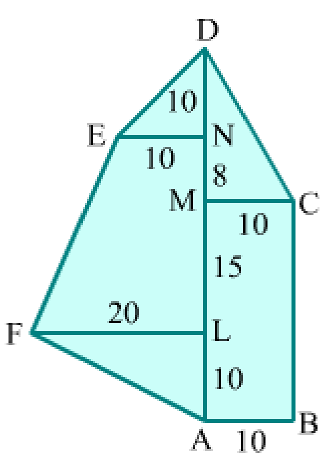
= 850 km^2

Example 4.

Sketch a plan and calculate the area of the park from the following data.

	Metre to D	
	10	
To E 10	**8**	
	15	**10 to C**
To F 20	**10**	
	From A	**10 to B**

Solution:



It is seen that, $AL = 10 \text{ m}, LM = 15 \text{ m}, MN = 8 \text{ m}, ND = 10 \text{ m}$

Area of $\Delta ALF = \frac{1}{2}bh = \frac{1}{2} \times 10 \times 20 = 100 \text{ m}^2$

Area of trapezium EFLN $= \frac{1}{2}h(a+b) = \frac{1}{2} \times 23 \times (20+10) = 23 \times 15 = 345 \text{ m}^2$

Area of $\Delta DEN = \frac{1}{2}bh = \frac{1}{2} \times 10 \times 10 = 50 \text{ m}^2$

Area of $\Delta CDM = \frac{1}{2}bh = \frac{1}{2} \times 10 \times 18 = 90 \text{ m}^2$

Area of rectangle ABCM $= lb = 25 \times 10 = 250 \text{ m}^2$

Thus, area of the park ABCDEF

$= 100 + 345 + 50 + 90 + 250$

$= 835 \text{ m}^2$

Surface Areas of Cubes and Cuboids

We give gifts to our friends and relatives at one time or another. We usually wrap our gifts in nice and colourful wrapping papers. Look, for example, at the nicely wrapped and tied gift shown below.

Clearly, the gift is packed in box that is cubical or shaped like a **cube**. Suppose you have a gift packed in a similar box. How would you determine the amount of wrapping paper needed to wrap the gift? You could do so by making an estimate of the surface area of the box. In this case, the total area of all the faces of the box will tell us the area of the wrapping paper needed to cover the box.

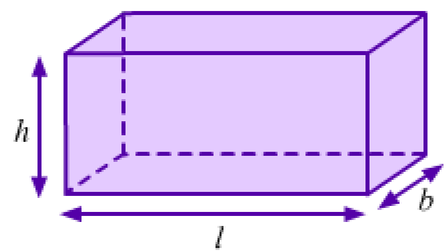
Knowledge of surface areas of the different solid figures proves useful in many real-life situations where we have to deal with them. In this lesson, we will learn the formulae for the surface areas of a cube and a **cuboid**. We will also solve some examples using these formulae.

Did You Know?

- The word 'cuboid' is made up of 'cube' and '-oid' (which means 'similar to'). So, a cuboid indicates something that is similar to a cube.
- A cuboid is also called a 'rectangular prism' or a 'rectangular parallelepiped'.

Formulae for the Surface Area of a Cuboid

Consider a cuboid of length *l*, breadth *b* and height *h*.



The formulae for the surface area of this cuboid are given as follows:

Lateral surface area of the cuboid = 2*h* (*l* + *b*)

Total surface area of the cuboid = 2 (*lb* + *bh* + *hl*)

Here, lateral surface area refers to the area of the solid excluding the areas of its top and bottom surfaces, i.e., the areas of only its four standing faces are included. Total surface area refers to the sum of the areas of all the faces.

Did You Know?

Two mathematicians named Henri Lebesgue and Hermann Minkowski sought the definition of surface area at around the twentieth century.

Know Your Scientist

Henri Lebesgue (1875–1941) was a French mathematician who is famous for his theory of integration. His contribution is one of the major achievements of modern analysis which greatly expands the scope of Fourier analysis. He also made important contributions to topology, the potential theory, the *Dirichlet* problem, the calculus of variations, the set theory, the theory of surface area and the dimension theory.

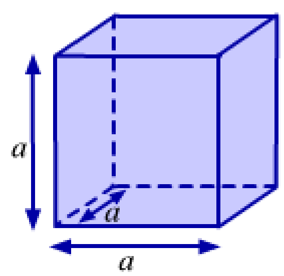
Hermann Minkowski (1864–1909) was a Polish mathematician who developed the geometry of numbers and made important contributions to the number theory, mathematical physics and the theory of relativity. His idea of combining time with the three dimensions of space, laid the mathematical foundations for Albert Einstein's theory of relativity.

Did You Know?

The concept of surface area is widely used in chemical kinetics, regulation of digestion, regulation of body temperature, etc.

Formulae for the Surface Area of a Cube

Consider a cube with edge a.



The formulae for the surface area of this cube are given as follows:

Lateral surface area of the cube = $4a^2$

Total surface area of the cube = $6a^2$

Here, lateral surface area refers to the area of the solid excluding the areas of its top and bottom surfaces, i.e., the areas of only its four standing faces are included. Total surface area refers to the sum of the areas of all the faces.

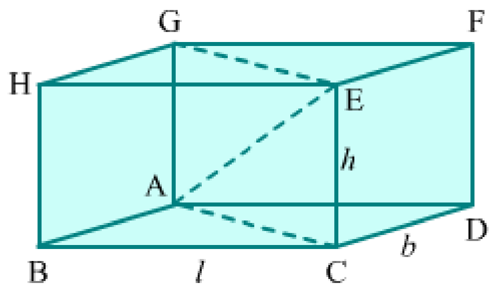
Did You Know?

- A cube can have 11 different nets.
- Cubes and cuboids are **convex polygons** that satisfy **Euler's formula**, i.e., **F + V − E = 2**.

Know More

Length of the diagonal in a cube and in a cuboid

A cuboid has four diagonals (say AE, BF, CG and DH). The four diagonals are equal in length.



Let us consider the diagonal AE.

In rectangle ABCD, length of diagonal AC = $\sqrt{l^2 + b^2}$

Now, ACEG is a rectangle with length AC and breadth CE or *h*.

So, length of diagonal AE = $\sqrt{\mathrm{AC}^2 + \mathrm{CE}^2}$

$$= \sqrt{\left(\sqrt{l^2 + b^2}\right)^2 + h^2}$$
$$= \sqrt{l^2 + b^2 + h^2}$$

∴ **Length of the diagonal of a cuboid =** $\boldsymbol{\sqrt{l^2 + b^2 + h^2}}$

A cube is a particular case of cuboid in which the length, breadth and height are equal to *a*.

∴ **Length of the diagonal of a cube =** $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \boldsymbol{\sqrt{3}a}$

Solved Examples

Easy

Example 1:

There are twenty-five cuboid-shaped pillars in a building, each of dimensions 1 m ×1 m × 10 m. Find the cost of plastering the surface of all the pillars at the rate of Rs 16 per m².

Solution:

Length (l) of one pillar = 1 m

Breadth (b) of one pillar = 1 m

Height (h) of one pillar = 10 m

∴ Lateral surface area of one pillar= $2h\ (l + b)$

= $2 \times 10 \times (1 + 1)\ m^2$

= $40\ m^2$

⇒ Lateral surface area of twenty-five pillars = $(25 \times 40)\ m^2 = 1000\ m^2$

Cost of plastering $1\ m^2$ of surface = Rs 16

⇒ Cost of plastering $1000\ m^2$ of surface = Rs (16×1000) = Rs 16000

Thus, the cost of plastering the twenty-five pillars of the building is Rs 16000.

Example 2:

Find the length of the diagonal of a cube whose surface area is 294 m².

Solution:

Let the edge of the given cube be a.

∴ Surface area of the cube = $6a^2$

It is given that the surface area of the cube is $294\ m^2$.

So, $6a^2 = 294$

$$\Rightarrow a^2 = 49\ \text{m}^2$$
$$\Rightarrow \therefore a = \sqrt{49}\ \text{m} = 7\ \text{m}$$

Now, length of the diagonal of the cube = $\sqrt{3}a = 7\sqrt{3}$ m

Medium

Example 1:

A metallic container (open at the top) is a cuboid of dimensions 7 cm × 5 cm × 8 cm. What amount of metal sheet went into making the container? Also, find the cost required for painting the outside of the container, excluding the base, at the rate of Rs 17 per 3 cm^2.

Solution:

Length (l) of the container = 7 cm

Breadth (b) of the container = 5 cm

Height (h) of the container = 8 cm

The container is open at the top. Therefore, while calculating the amount of metal sheet used, we will exclude the top part.

∴ Amount of metal sheet used = Total surface area − Area of the top part

$= 2(lb + bh + lh) - lb$

$= [2 \times (7 \times 5 + 5 \times 8 + 7 \times 8) - 7 \times 5]$ cm^2

$= [2 \times (35 + 40 + 56) - 35]$ cm^2

$= (2 \times 131 - 35)$ cm^2

$= 227$ cm^2

Thus, 227 cm^2 of metal went into making the given container.

Now, area to be painted = Lateral surface area of the cuboid

$= 2h(l + b)$

$= [2 \times 8 \times (7 + 5)]$ cm^2

$= (16 \times 12)$ cm^2

$= 192\ cm^2$

Cost of painting $3\ cm^2$ of surface = Rs 17

$\Rightarrow$ Cost of painting $1\ cm^2$ of surface = Rs $\frac{17}{3}$

$\Rightarrow$ Cost of painting $192\ cm^2$ of surface = $\text{Rs}\left(192 \times \frac{17}{3}\right)$ = Rs 1088

Therefore, the cost of painting the outside of the container is Rs 1088.

Example 2:

If the total surface area of a cube is $24x^2$, then find the surface area of the cuboid formed by joining

i)two such cubes.

ii)three such cubes.

Solution:

Total surface area of cube = $6a^2$

It is given that the total surface area of the cube is $24x^2$.

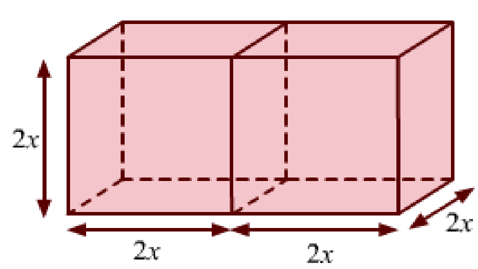
So, $6a^2 = 24x^2$

$\Rightarrow a^2 = 4x^2$

$\Rightarrow \therefore a = 2x$

So, the edge of the cube is $2x$.

i)When two cubes with edge $2x$ are joined, we obtain the following cuboid.



Length (l) of the cuboid = $2x + 2x = 4x$

Breadth (b) of the cuboid = $2x$

Height (h) of the cuboid = $2x$

∴ Surface area of the cuboid = $2 (lb + bh + lh)$

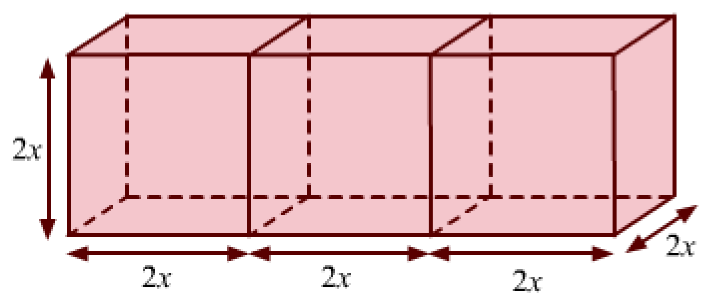
$= 2 \times (4x \times 2x + 2x \times 2x + 4x \times 2x)$

$= 2 \times (8x^2 + 4x^2 + 8x^2)$

$= 40x^2$

Thus, the surface area of the cuboid formed according to the given specifications is $40x^2$.

ii)When three cubes with edge $2x$ are joined, we obtain the following cuboid.



Length (l) of the cuboid = $2x + 2x + 2x = 6x$

Breadth (b) of the cuboid = $2x$

Height (h) of the cuboid = $2x$

∴ Surface area of the cuboid = $2 (lb + bh + lh)$

$= 2 \times (6x \times 2x + 2x \times 2x + 6x \times 2x)$

$= 2 \times (12x^2 + 4x^2 + 12x^2)$

$= 56x^2$

Thus, the surface area of the cuboid formed according to the given specifications is $56x^2$.

Hard

Example 1:

The cost of flooring a twenty-metre-long room at Rs 5 per square metre is Rs 1000. If the cost of painting the four walls of the room at Rs 15 per square metre is Rs 1800, then find the height of the room.

Solution:

The length (l) of the room is given as 20 m. Let b and h be its breadth and height respectively.

Area of the floor = $l \times b$

Cost of flooring at Rs 5 per m^2 = Rs 1000

So, $5 \times l \times b = 1000$

$$\Rightarrow 5 \times 20 \times b = 1000$$
$$\Rightarrow \therefore b = \frac{1000}{100} = 10$$

Area of the four walls = $2\,(bh + lh)$

Cost of painting the four walls at Rs 15 per m^2 = Rs 1800

So, $15 \times [2\,(bh + lh)] = 1800$

$\Rightarrow 15 \times [2 \times (10 \times h + 20 \times h)] = 1800$

$$\Rightarrow 30h = \frac{1800}{15 \times 2}$$
$$\Rightarrow \therefore h = \frac{1800}{15 \times 2 \times 30} = 2$$

Thus, the height of the room is 2 m.

Example 2:

The internal measures of a cuboidal room are 20 m × 15 m × 12 m. Dinesh wants to paint the four walls of the room with orange colour and the roof of the room with white colour. 100 m^2 of surface can be painted using each can of orange paint and 125 m^2 of surface can be painted using each can of white paint. How many cans of each colour will be required? If the orange and white paints are available at Rs 250

per can and Rs 300 per can respectively, then how much money will be spent by Dinesh to paint the room?

Solution:

Length (l) of the room = 20 m

Breadth (b) of the room = 15 m

Height (h) of the room = 12 m

Area of the room to be painted using orange colour= Area of the four walls of the room

= Lateral surface area of the room

= $2h\ (l + b)$

= $[2 \times 12\ (20 + 15)]\ \text{m}^2$

= $(24 \times 35)\ \text{m}^2$

= $840\ \text{m}^2$

It is given that $100\ \text{m}^2$ of surface can be painted using each can of orange paint.

∴ Number of cans of orange paint required $= \dfrac{\text{Area of the room painted using orange colour}}{\text{Area that can be painted using each can}}$

$= \dfrac{840}{100}$

= 8.4

= 9 (∵ 8 cans will be insufficient for the job)

Thus, 9 cans of orange paint will be required for painting the four walls of the room.

Area of the room to be painted using white colour= Area of the roof

= $l \times b$

= $(20 \times 15)\ \text{m}^2$

= $300\ \text{m}^2$

It is given that 125 m^2 of surface can be painted using each can of white paint.

∴ Number of cans of white paint required $= \dfrac{\text{Area of the room painted using white colour}}{\text{Area that can be painted using each can}}$

$= \dfrac{300}{125}$

= 2.4

= 3 (∵ 2 cans will be insufficient for the job)

Thus, 3 cans of white paint will be required for painting the roof of the room.

Cost of each can of orange paint = Rs 250

⇒ Cost of 9 cans of orange paint = 9 × Rs 250 = Rs 2250

Cost of each can of white paint = Rs 300

⇒ Cost of 3 cans of white paint = 3 × Rs 300 = Rs 900

Thus, total money that will be spent in painting the room = Rs 2250 + Rs 900 = Rs 3150

Surface Area of Right Circular Cylinders

Surface Area of a Right Circular Cylinder

We come across many objects in our surroundings which are cylindrical, i.e., shaped like a **cylinder**, for example, pillars, rollers, water pipes, tube lights, cold-drink cans and LPG cylinders. This three-dimensional figure is found almost everywhere.

We can easily make cylindrical containers using metal sheets of any length and breadth. Say we have to make an open metallic cylinder (as shown below) of radius 14 cm and height 40 cm. How will we calculate the dimensions of the metal required for making this specific cylinder?

We will do so by calculating the surface area of the required cylinder. This surface area will be equal to the area of metal sheet required to make the cylinder.

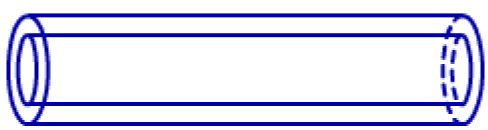
Knowledge of surface areas of three-dimensional figures is important in finding solutions to several real-life problems involving them. In this lesson, we will learn the formulae for the surface area of a right circular cylinder. We will also solve examples using these formulae.

Features of a right circular cylinder

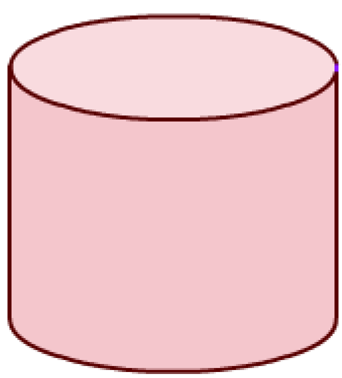
1. A right circular cylinder has two plane surfaces circular in shape.
2. The curved surface joining the plane surfaces is the lateral surface of the cylinder.
3. The two circular planes are parallel to each other and also congruent.
4. The line joining the centers of the circular planes is the axis of the cylinder.
5. All the points on the lateral surface of the right circular cylinder are equidistant from the axis.
6. Radius of circular plane is the radius of the cylinder.

Two types of cylinders are given below.

1. Hollow cylinder: It is formed by the lateral surface only. Example: A pipe



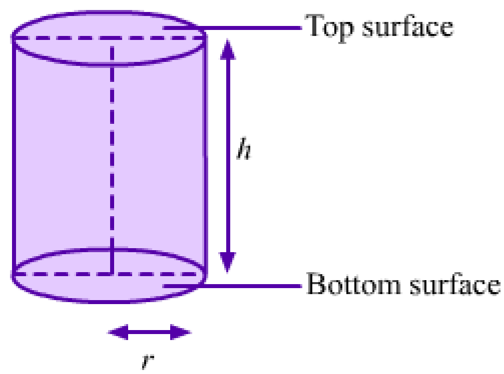
2. Solid cylinder: It is the region bounded by two circular plane surfaces with the lateral surface. Example: A garden roller



Solid cylinder

Formulae for the Surface Area of a Right Circular Cylinder

Consider a cylinder with base radius *r* and height *h*.



The formulae for the surface area of this cylinder are given as follows:

Curved surface area of the cylinder = $2\pi rh$

Area of two circular faces of cylinder = $2\pi r^2$

Total surface area of the cylinder = $2\pi r\,(r + h)$

Note: We take π as a constant and its value as $\frac{22}{7}$ or 3.14.

Here, curved (or lateral) surface area refers to the area of the curved surface excluding the top and bottom surfaces. Total surface area refers to the sum of the areas of the top and bottom surfaces and the area of the curved surface.

Did You Know?

Pi

- Pi is a mathematical constant which is equal to the ratio of the circumference of a circle to its diameter.
- It is an irrational number represented by the Greek letter 'π' and its value is approximately equal to 3.14159.
- William Jones (1706) was the first to use the Greek letter to represent this number.
- Pi is also called 'Archimedes' constant' or 'Ludolph's constant'.
- Pi is a 'transcendental number', which means that it is not the solution of any finite polynomial with whole numbers as coefficients.

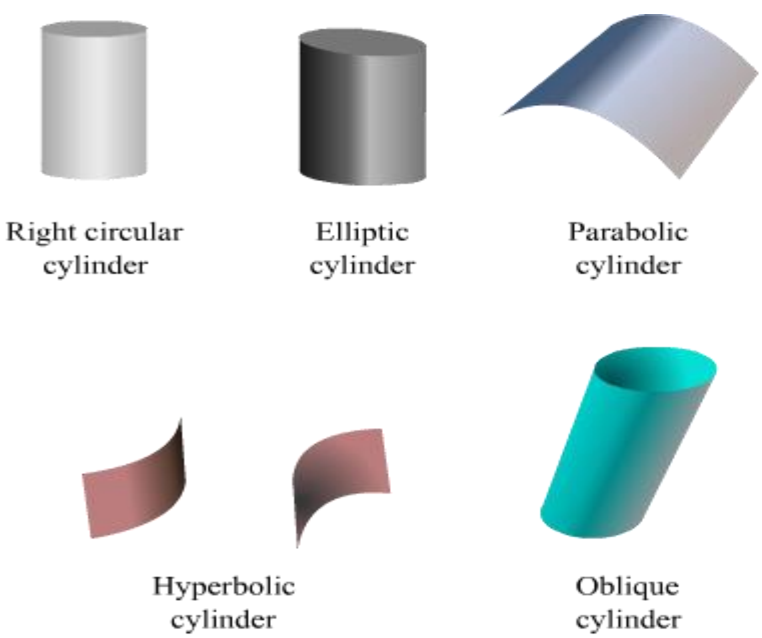
- Suppose a circle fits exactly inside a square; then, pi = $4 \times \frac{\text{Area of the circle}}{\text{Area of the square}}$

Know Your Scientist

William Jones (1675–1749) was a Welsh mathematician who is primarily known for his proposal to use the Greek letter ‘π’ for representing the ratio of the circumference of a circle to its diameter. His book ***Synopsis Palmariorum Matheseos*** includes theorems on differential calculus and infinite series. In this book, π is used as an abbreviation for perimeter.

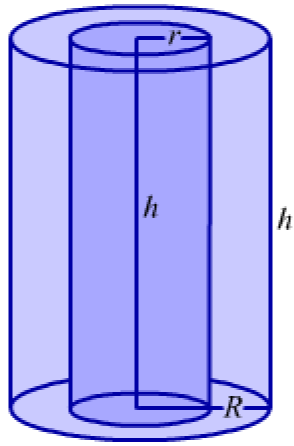
Whiz Kid

There are many types of cylinders—right circular cylinder (whose base is circular), elliptic cylinder (whose base is an ellipsis or oval), parabolic cylinder, hyperbolic cylinder, imaginary elliptic cylinder, oblique cylinder (whose top and bottom surfaces are displaced from each other), etc.



Formulae for the Surface Area of a Right Circular Hollow Cylinder

Consider a hollow cylinder of height h with external and internal radii R and r respectively,



Here, curved surface area, CSA = External surface area + Internal surface area

$$= 2\pi Rh + 2\pi rh$$
$$= 2\pi h\left(R + r\right)$$

Total surface area, TSA = Curved surface area + 2 × Base area

$$= 2\pi h\left(R + r\right) + 2 \times \left[\pi R^2 - \pi r^2\right]$$
$$= 2\pi h\left(R + r\right) + 2\pi\left(R + r\right)\left(R - r\right)$$
$$= 2\pi\left(R + r\right)\left(R - r + h\right)$$

Here, thickness of the hollow cylinder = $R - r$.

Example Based on the Surface Area of a Right Circular Cylinder

Solved Examples

Easy

Example 1:

The curved surface area of a right circular cylinder of height 7 cm is 44 cm^2. Find the diameter of the base of the cylinder.

Solution:

Let r be the radius and h be the height of the cylinder.

It is given that:

$h = 7$ cm

Curved surface area of the cylinder = 44 cm^2

So, $2\pi rh = 44\ cm^2$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 7 \text{ cm} = 44 \text{ cm}^2$$
$$\Rightarrow r = \frac{44 \times 7}{2 \times 22 \times 7} \text{ cm}$$
$$\Rightarrow \therefore r = 1 \text{ cm}$$

Thus, diameter of the base of the cylinder = $2r$ = 2 cm

Example 2:

The radii of two right circular cylinders are in the ratio 4 : 5 and their heights are in the ratio 3 : 1. What is the ratio of their curved surface areas?

Solution:

Let the radii of the cylinders be $4r$ and $5r$ and their heights be $3h$ and h.

Let S_1 be the curved surface area of the cylinder of radius $4r$ and height $3h$.

$\therefore S_1 = 2\pi \times 4r \times 3h = 24\pi rh$

Let S_2 be the curved surface area of the cylinder of radius $5r$ and height h.

$\therefore S_2 = 2\pi \times 5r \times h = 10\pi rh$

Now,

$$\frac{S_1}{S_2} = \frac{24\pi rh}{10\pi rh} = \frac{12}{5}$$
$$\Rightarrow S_1 : S_2 = 12 : 5$$

Thus, the curved surface areas of the two cylinders are in the ratio 12 : 5.

Medium

Example 1:

Find the height and curved surface area of a cylinder whose radius is 14 dm and total surface area is 1760 dm^2.

Solution:

Radius (r) of the cylinder = 14 dm

Let the height of the cylinder be h.

Total surface area of the cylinder = 1760 dm^2

So, $2\pi r(r + h) = 1760$ dm^2

$$\Rightarrow 2\times\frac{22}{7}\times 14(14+h)\text{dm} = 1760\ \text{dm}^2$$
$$\Rightarrow 14+h = \frac{1760\times 7}{2\times 22\times 14}\text{dm}$$
$$\Rightarrow 14+h = 20\ \text{dm}$$
$$\Rightarrow \therefore h = (20-14)\text{dm} = 6\ \text{dm}$$

Thus, the height of the cylinder is 6 dm.

Now, curved surface area of the cylinder = $2\pi rh$

$$= (2\times\frac{22}{7}\times 14\times 6)\ \text{dm}^2$$
$$= 528\ \text{dm}^2$$

Example 2:

There are ten identical cylindrical pillars in a building. If the radius of each pillar is 35 cm and the height is 12 m, then find the cost of plastering the surface of all the pillars at the rate of Rs 15 per m^2.

Solution:

Radius (r) of one pillar $= 35\ \text{cm} = \frac{35}{100}\text{m} = 0.35\ \text{m}$

Height (h) of one pillar = 12 m

∴ Curved surface area of one pillar = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 0.35 \times 12\right) \text{ m}^2$$
$$= 26.4 \text{ m}^2$$

⇒ Curved surface area of ten pillars = 10 × 26.4 m^2 = 264 m^2

Cost of plastering 1 m^2 of surface = Rs 15

⇒ Cost of plastering 264 m^2 of surface = Rs (15 × 264) = Rs 3960

Therefore, the cost of plastering the ten pillars of the building is Rs 3960.

Hard

Example 1:

A cylindrical road roller is of diameter 175 cm and length 1.5 m. It has to cover an area of 0.33 hectare on the ground. How many complete revolutions must the roller take to cover the ground? (1 hectare = 10000 m^2)

Solution:

Diameter of the cylindrical roller $= 175 \text{ cm} = \frac{175}{100} \text{ m} = \frac{7}{4} \text{ m}$

∴ Radius (r) of the cylindrical roller = $\frac{7}{8}$ m

Length (h) of the cylindrical roller = 1.5 m

Area covered by the roller in one complete revolution = Curved surface area of the roller

= $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times \frac{7}{8} \times 1.5\right) \text{ m}^2$$
$$= 8.25 \text{ m}^2$$

Area of the ground to be covered = 0.33 hectare = 0.33 × 10000 m^2 = 3300 m^2

$\therefore$ Number of complete revolutions $= \dfrac{\text{Area of the ground covered by the roller}}{\text{Area covered by the roller in one revolution}}$

$$= \frac{3300\text{m}^2}{8.25\text{m}^2}$$
$$= 400$$

Thus, the roller must take 400 complete revolutions to cover the ground.

Example 2:

The internal diameter, thickness and height of a hollow cylinder are 20 cm, 1 cm and 25 cm respectively. What is the total surface area of the cylinder?

Solution:

Internal diameter of the cylinder = 20 cm

$\therefore$ Internal radius (r) of the cylinder $= \dfrac{20}{2}\text{ cm} = 10\text{ cm}$

Thickness of the cylinder = 1 cm

$\therefore$ External radius (R) of the cylinder = (10 + 1) cm = 11 cm

Height (h) of the cylinder = 25 cm

Internal curved surface area of the cylinder = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 10 \times 25\right)\text{ cm}^2$$
$$= \frac{11000}{7}\text{cm}^2$$

External curved surface area of the cylinder = $2\pi Rh$

$$= \left(2 \times \frac{22}{7} \times 11 \times 25\right)\text{ cm}^2$$
$$= \frac{12100}{7}\text{cm}^2$$

The two bases of the cylinder are ring-shaped. Therefore, their area is given as follows:

Area of base = π (R^2 − r^2)

$$= \left[\frac{22}{7}\left(11^2 - 10^2\right)\right] \text{ cm}^2$$
$$= \left(\frac{22}{7} \times 21\right) \text{ cm}^2$$
$$= 66 \text{ cm}^2$$

So, total surface area of the cylinder = Internal CSA + External CSA + 2 × Area of base

$$= \left(\frac{11000}{7} + \frac{12100}{7} + 2 \times 66\right) \text{ cm}^2$$
$$= \left(\frac{23100}{7} + 132\right) \text{ cm}^2$$
$$= (3300 + 132) \text{ cm}^2$$
$$= 3432 \text{ cm}^2$$

Volumes of Cubes and Cuboids

Abhinav's mother gives him a container, asking him to go to the neighbouring milk booth and buy 2.5 L of milk. What does '2.5 L'represent? It represents the amount of milk that Abhinav needs to buy. In other words, it is the volume of milk that is to be bought.

After buying the milk, Abhinav notices that the container is full up to its brim. He says to himself, 'This container has no capacity to hold any more milk.' What does the word 'capacity' indicate? **The space occupied by a substance is called its volume. The capacity of a container is the volume of a substance that can fill the container completely.** In this case, the volume and the capacity of the container are the same. The standard units which are used to measure the volume are **cm^3 (cubic centimetre)** and **m^3 (cubic metre).**

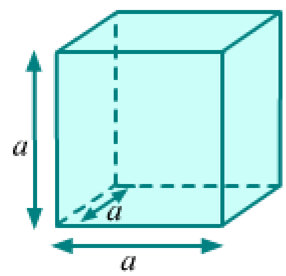
In this lesson, we will learn the formulae for the volumes or capacities of cubic and cuboidal objects. We will also solve examples using these formulae.

Did You Know?

A cube is one among the five platonic solids. This means that it is a regular and convex polyhedron with the same number of faces meeting at each vertex.

Formulae for the Volumes of a Cube and a Cuboid

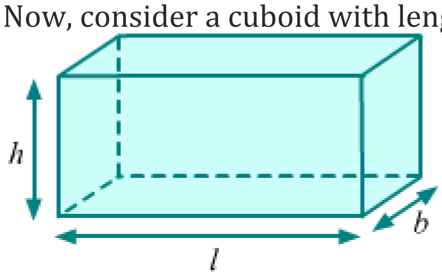
Consider a cube with an edge *a*.



The formula for the volume of this cube is given as follows:

Volume of the cube = a^3

Now, consider a cuboid with length *l*, breadth *b* and height *h*.



The formula for the volume of this cuboid is given as follows:

Volume of the cuboid = $l \times b \times h$

Concept Builder

The units of capacity and volume are interrelated as follows:

- 1 cm^3 = 1 mL
- 1000 cm^3 = 1 L
- 1 m^3 = 1 kL = 1000 L

Did You Know?

- A cube has the maximum volume among all cuboids with equal surface area.
- A cube has the minimum surface area among all cuboids with equal volume.

Solved Examples

Easy

Example 1:

Find the volumes of cubes of given sides.

(a) 2 cm (b) 5 m (c) 12 cm (d) 15 m

Solution:

(a)

Measure of side of cube = 2 cm

Volume of cube = $(\text{Side})^3 = 2^3\ \text{cm}^3 = 8\ \text{cm}^3$

(b)

Measure of side of cube = 5 m

Volume of cube = $(\text{Side})^3 = 5^3\ \text{m}^3 = 125\ \text{m}^3$

(c)

Measure of side of cube = 12 cm

Volume of cube = $(\text{Side})^3 = 12^3\ \text{cm}^3 = 1728\ \text{cm}^3$

(d)

Measure of side of cube = 15 m

Volume of cube = $(\text{Side})^3 = 15^3\ \text{m}^3 = 3375\ \text{m}^3$

Example 2:

Find the volumes of cuboids of given dimensions.

(a) length = 5 cm, breadth = 2 cm, height = 6 cm

(b) length = 15 cm, breadth = 10 cm, height = 30 cm

(c) length = 1 m, breadth = 0.5 m, height = 1.5 m

Solution:

(a)

We have

length = 5 cm, breadth = 2 cm, height = 6 cm

∴ Volume of cuboid = length × breadth × height

$= (5 \times 2 \times 6)\ cm^3$

$= 60\ cm^3$

(b)

We have

length = 15 cm, breadth = 10 cm, height = 30 cm

∴ Volume of cuboid = length × breadth × height

$= (15 \times 10 \times 30)\ cm^3$

$= 4500\ cm^3$

(c)

We have

length = 1 m, breadth = 0.5 m, height = 1.5 m

∴ Volume of cuboid = length × breadth × height

$= (1 \times 0.5 \times 1.5)\ m^3$

$= 0.75\ m^3$

Example 3:

If a cubical tank can contain 1331000 L of water, then find the edge of the tank.

Solution:

Capacity of the cubical tank = 1331000 L

= 1331 m^3 ($\because$ 1000 L = 1 m^3)

Now, capacity of the tank = Volume of water that can be contained in the tank

We know that volume of water in the tank = $(\text{Edge})^3$

$\Rightarrow (\text{Edge})^3 = 1331\ m^3$

$\Rightarrow \therefore$ Edge = 11 m

Thus, the edge of the cubical tank is 11 m.

Example 4:

Find the height of the cuboid whose volume is 840 cm^3 and the area of whose base is 120 cm^2.

Solution:

Let the length, breadth and height of the cuboid be *l*, *b* and *h* respectively.

Area of the base of the cuboid = 120 cm^2

$\therefore l \times b = 120\ cm^2$

Volume of the cuboid = 840 cm^3

$\therefore l \times b \times h = 840\ cm^3$

$\Rightarrow 120\ cm^2 \times h = 840\ cm^3$ ($\because l \times b = 120\ cm^2$)

$$\Rightarrow h = \frac{840}{120}\ \text{cm}$$

$$\Rightarrow \therefore h = 7\ \text{cm}$$

Thus, the height of the cuboid is 7 cm.

Example 5:

If the ratio of the edges of two cubes is 2 : 5, then find the ratio of their volumes.

Solution:

Let the edges of the cubes be $a = 2x$ and $b = 5x$.

Ratio of the volumes of the cubes $= \dfrac{\text{Volume of the first cube}}{\text{Volume of the second cube}}$

$$= \frac{a^3}{b^3}$$
$$= \frac{(2x)^3}{(5x)^3}$$
$$= \frac{8x^3}{125x^3}$$
$$= \frac{8}{125}$$

Thus, the volumes of the cubes are in the ratio 8 : 125.

Medium

Example 1:

A solid cube of edge 18 cm is cut into eight cubes of equal volume. Find the dimension of each new cube. Also find the ratio of the total surface area of the bigger cube to that of the new cubes formed.

Solution:

Let the edge of each new cube be x.

According to the question, we have:

Volumes of 8 cubes each of edge x = Volume of cube of edge 18 cm

$$\Rightarrow 8 \times x^3 = (18 \text{ cm})^3$$

$$\Rightarrow x^3 = \frac{18 \text{ cm} \times 18 \text{ cm} \times 18 \text{ cm}}{8} = 729 \text{ cm}^3$$

$$\Rightarrow x^3 = (9 \text{ cm})^3$$

$$\Rightarrow \therefore x = 9 \text{ cm}$$

Thus, the edge of each new cube is 9 cm.

Total surface area (S_1) of the bigger cube = 6 × (18 cm)^2

Total surface area of 8 cubes (S_2) each of edge 9 cm = 8 × [6 × (9 cm)^2]

$$\therefore \frac{S_1}{S_2} = \frac{6 \times 18^2}{8 \times 6 \times 9^2} = \frac{1}{2}$$

Hence, the required ratio is 1 : 2.

Example 2: A hostel having strength of 300 students requires on an average 36000 L of water per day. It has a tank measuring 10 m × 8 m × 9 m. For how many days will the water in the tank filled to capacity last?

Solution:

Let the cuboidal tank have length l, breadth b and height h.

It is given that l = 10 m, b = 8 m and h = 9 m.

Capacity of the tank = $l \times b \times h$ = 10 m × 8 m × 9 m = 720 m^3

∴ Amount of water in the tank filled to capacity = 720 m^3 = 720000 L (∵ 1000 L = 1 m^3)

Amount of water used by 300 students in 1 day = 36000 L

Number of days for which the water in the full tank will last $= \frac{\text{Amount of water in the full tank}}{\text{Amount of water used in a day}}$

$$= \frac{720000}{36000}$$

$$= 20$$

Thus, the water in the tank filled to capacity will last for 20 days.

Example 3: The dimensions of a wall in a godown are 25 m × 0.3 m × 10 m. How many bricks of dimensions 25 cm × 10 cm × 5 cm were used to construct the wall?

Solution:

Length (L) of the wall = 25 m = (25 × 100) cm = 2500 cm

Breadth (B) of the wall = 0.3 m = (0.3 × 100) cm = 30 cm

Height (H) of the wall = 10 m = (10 × 100) cm = 1000 cm

∴ Volume of the wall = $L \times B \times H$ = $(2500 \times 30 \times 1000)\ \text{cm}^3$

Length (l) of one brick = 25 cm

Breadth (b) of one brick = 10 cm

Height (h) of one brick = 5 cm

∴ Volume of one brick = $l \times b \times h$ = $(25 \times 10 \times 5)\ \text{cm}^3$

Number of bricks used to construct the wall $= \dfrac{\text{Volume of the wall}}{\text{Volume of one brick}}$

$$= \frac{2500 \times 30 \times 1000}{25 \times 10 \times 5}$$
$$= 60000$$

Thus, 60000 bricks of dimensions 25 cm × 10 cm × 5 cm were used to construct the wall.

Example 4:

A storeroom is in the form of a cuboid with dimensions 90 m × 150 m × 120 m. How many cubical boxes of edge 60 dm can be stored in the room?

Solution:

Length (l) of the storeroom = 90 m

Breadth (b) of the storeroom = 150 m

Height (h) of the storeroom = 120 m

∴ Volume of the storeroom = $l \times b \times h$ = $(90 \times 150 \times 120)\ \text{m}^3$

Edge (a) of one cubical box $= 60 \text{ dm} = \left(\frac{60}{10}\right) \text{ m} = 6 \text{ m}$

∴ Volume of one box = $a^3 = (6)^3 \text{ m}^3$

Number of boxes that can be stored in the room $= \frac{\text{Volume of the storeroom}}{\text{Volume of one box}}$

$$= \frac{90 \times 150 \times 120}{6 \times 6 \times 6}$$
$$= 7500$$

Thus, 7500 cubical boxes of edge 60 dm can be stored in the room.

Hard

Example 1:

A man-made canal is 5 m deep and 60 m wide. The water in the canal flows at the rate of
3 km/h. The canal empties its water into a reservoir. How much water will fall into the reservoir in 10 minutes?

Solution:

Depth (h) of the canal = 5 m

Width (b) of the canal = 60 m

Length (l) of the canal is the rate of water flowing per hour = 3 km = 3000 m

Amount of water flowing per hour = $l \times b \times h$ = $(3000 \times 60 \times 5) \text{ m}^3 = 900000 \text{ m}^3$ = 900000 kL (∵ $1 \text{ m}^3 = 1$ kL)

∴ Amount of water flowing in 60 min = 900000 kL

⇒ Amount of water flowing in 1 minute $= \left(\frac{900000}{60}\right) \text{kL}$

⇒ Amount of water flowing in 10 minutes $= \left(\frac{900000}{60} \times 10\right) \text{kL}$

= 150000 kL

Thus, 150000 kL of water will fall into the reservoir in 10 minutes.

Example 2: **The external length, breadth and height of a closed rectangular wooden box are 9 cm, 5 cm and 3 cm respectively. The thickness of the wood used is 0.25 cm. The box weighs 7.5 kg when empty and 50 kg when it is filled with sand. Find the weights of one cubic cm of wood and one cubic cm of sand.**

Solution:

External length (L) of the wooden box = 9 cm

External breadth (B) of the wooden box = 5 cm

External height (H) of the wooden box = 3 cm

∴ External volume of the wooden box = $L \times B \times H = (9 \times 5 \times 3)\ \text{cm}^3 = 135\ \text{cm}^3$

Thickness of the wood used = 0.25 cm

Internal length (l) of the wooden box = 9 cm − (0.25 cm + 0.25 cm) = 8.5 cm

Internal breadth (b) of the wooden box = 5 cm − (0.25 cm + 0.25 cm) = 4.5 cm

Internal height (h) of the wooden box = 3 cm − (0.25 cm + 0.25 cm) = 2.5 cm

∴ Internal volume of the wooden box = $l \times b \times h = (8.5 \times 4.5 \times 2.5)\ \text{cm}^3 = 95.625\ \text{cm}^3$

Now, volume of the wood = External volume of the box − Internal volume of the box

= $(135 - 95.625)\ \text{cm}^3$

= $39.375\ \text{cm}^3$

Weight of the empty box = 7.5 kg

⇒ Weight $39.375\ \text{cm}^3$ of wood = 7.5 kg

∴ Weight of $1\ \text{cm}^3$ of wood $= \left(\dfrac{7.5}{39.375}\right)\text{kg} = 0.19\ \text{kg}$

Now, volume of sand = Internal volume of the box = $95.625\ \text{cm}^3$

Weight of sand = Weight of the box filled with sand − Weight of the empty box

= (50 − 7.5) Kg

= 42.5 Kg

⇒ Weight of 95.625 cm^3 of sand = 42.5 kg

∴ Weight of 1 cm^3 of sand $=\left(\frac{42.5}{95.625}\right)$ kg $= 0.44$ kg

Volume of Right Circular Cylinders

Water tanks like the ones shown below are a common enough sight.

Clearly, these tanks are cylindrical or shaped like a cylinder. The choice of this shape for a water tank (and many other storage containers) is because a cylinder provides a large volume for a little surface area. Also, this shape can withstand much more pressure than a cube or a cuboid, which makes it easy to transport. Another example of a cylindrical storage container is the LPG cylinder.

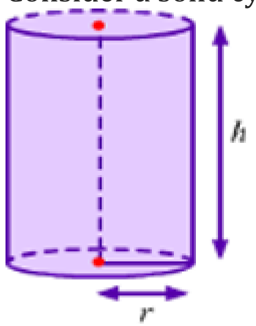
The amount of space occupied by a water tank is the same as the volume of the tank. So, to find the capacity or the amount of space occupied by a tank, we need to find the volume of the tank. In this lesson, we will learn the formula to calculate the volume of a right circular cylinder and solve some examples using the same.

Did You Know?

LPG tanks are cylinder-shaped so that they can withstand the pressure inside them. If these tanks were square or rectangular in shape, then an increase in pressure inside them would cause the tanks to reform themselves so as to gain a rounded shape. This, in turn, could result in leakage at the corners. Actual LPG tanks are designed to have no corners.

Formula for the Volume of a Right Circular Cylinder

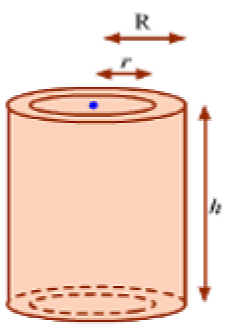
Consider a solid cylinder with *r* as the radius of the circular base and *h* as the height.



The formula for the volume of this right circular solid cylinder is given as follows:

Volume of the solid cylinder = Area of base × Height

Volume of the solid cylinder = $\pi r^2 h$



Consider a hollow cylinder with internal and external radii as *r* and *R* respectively, and height as *h*.

The formula for the volume of this right circular hollow cylinder is given as follows:

Volume of the hollow cylinder = $\pi (R^2 - r^2) h$

In right prisms, top and base surfaces are congruent and parallel while lateral faces are perpendicular to the base. Thus, their volumes can also be calculated in the same manner as that of right cylinders.

Volume of the right prism = Area of base × Height

Did You Know?

The volume of a pizza (which is always cylindrical in shape) is hidden in its name itself. If we take the radius of a pizza to be 'z' and its thickness to be 'a', then its volume is $'\pi z^2 a'$ or '**pi.z.z.a**'.

Solved Examples

Easy

Example 1:

A cylindrical tank can hold 11000 L of water. What is the radius of the base of the tank if its height is 3.5 m?

Solution:

Let *r* be the radius of the base of the cylindrical tank.

Height (*h*) of the tank = 3.5 m

Volume of the tank = 11000 L = 11 m^3 (∵ 1000 L = 1 m^3)

Volume of a cylinder = $\pi r^2 h$

In this case, we have

$$\pi r^2 h = 11 \text{ m}^3$$
$$\Rightarrow \left(\frac{22}{7} \times r^2 \times 3.5 \text{ m}\right) = 11 \text{ m}^3$$
$$\Rightarrow 11\, r^2 = 11 \text{ m}^2$$

$\Rightarrow r = 1$ m

Thus, the radius of the base of the cylindrical tank is 1 m.

Example 2:

What is the height of a cylinder whose volume is 6.16 m^3 and the diameter of whose base is 28 dm?

Solution:

Diameter of the base of the cylinder = 28 dm

$\therefore$ Radius (r) of the base $= \left(\frac{28}{2}\right)$ dm

$$= 14 \text{ dm}$$
$$= \left(\frac{14}{10}\right) \text{ m} \qquad \left(\because 1 \text{ dm} = \frac{1}{10} \text{ m}\right)$$
$$= 1.4 \text{ m}$$

Volume of the cylinder = 6.16 m^3

$$\Rightarrow \pi r^2 h = 6.16 \text{ m}^3$$
$$\Rightarrow \frac{22}{7} \times (1.4 \text{ m})^2 \times h = 6.16 \text{ m}^3$$
$$\Rightarrow h = \left[\frac{6.16 \times 7}{22 \times (1.4)^2}\right] \text{m}$$

$\Rightarrow h$ = 1 m

Thus, the height of the cylinder is 1 m.

Example 3:

The external diameter, thickness and length of a cylindrical water pipe are 22 cm, 1 cm, and 8 m respectively. What amount of material went into making this pipe?

Solution:

External diameter of the hollow cylindrical pipe = 22 cm

$\therefore$ External radius, R = $\left(\frac{22}{2}\right) \text{cm} = 11 \text{ cm}$

Thickness of the pipe = 1 cm

$\therefore$ Internal radius, r = (11 − 1) cm = 10 cm

Length (h) of the pipe = 8 m = (8 × 100) cm = 800 cm ($\because$ 1 m = 100 cm)

$\therefore$ Volume of the material used to make the pipe $= \pi\left(R^2 - r^2\right)h$

$$= \left[\frac{22}{7} \times \left(11^2 - 10^2\right) \times 800\right] \text{cm}^3$$
$$= \left[\frac{22}{7} \times 21 \times 800\right] \text{cm}^3$$
$$= 52800 \text{ cm}^3$$

Thus, 52800 cm^3 of material was used to make the water pipe.

Medium

Example 1:

The diameter and height of a solid metallic cylinder are 21 cm and 25 cm respectively. If the mass of the metal is 8 g per cm^3, then find the mass of the cylinder.

Solution:

Diameter of the cylinder = 21 cm

∴ Radius (r) of cylinder = $\left(\frac{21}{2}\right) \text{cm}$

Height (h) of the cylinder = 25 cm

To find the mass of the metallic cylinder, we have to first find the volume of the cylinder.

Volume of the cylinder $= \pi r^2 h$

$$= \left[\frac{22}{7} \times \left(\frac{21}{2}\right) \times \left(\frac{21}{2}\right) \times 25\right] \text{cm}^3$$
$$= 8662.5 \text{ cm}^3$$

Mass of 1 cm^3 of the metal = 8 g

∴ Mass of 8662.5 cm^3 of the metal= (8662.5 × 8) g

$$= 69300 \text{ g}$$
$$= \left(\frac{69300}{1000}\right) \text{kg} \qquad \left(\because 1 \text{ g} = \frac{1}{1000} \text{kg}\right)$$
$$= 69.3 \text{ kg}$$

Thus, the mass of the cylinder is 69.3 kg.

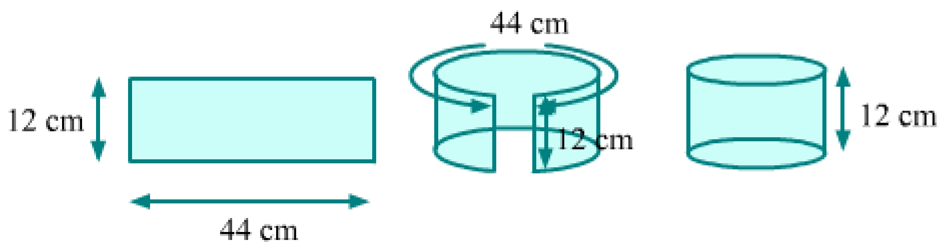
Example 2:

A rectangular sheet of paper is folded to form a cylinder of height 12 cm. If the length and breadth of the sheet are 44 cm and 12 cm respectively, then find the volume of the cylinder.

Solution:

Height (*h*) of the cylinder = 12 cm

Let *r* be the radius of the cylinder. We can find this value from the circumference of the base of the cylinder. As shown in the figure, this circumference is nothing but the length of the sheet.



So, circumference of the base of the cylinder = 44 cm

$$\Rightarrow 2\pi r = 44$$
$$\Rightarrow r = \frac{44}{2\pi}$$
$$\Rightarrow r = \frac{44 \times 7}{2 \times 22}$$

$$\Rightarrow r = 7cm$$

Now, volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 12 \text{ cm}^3$$
$$= 1848 \text{ cm}^3$$

Hard

Example 1:

The inner and outer diameters of a cylindrical iron pipe are 54 cm and 58 cm respectively and its length is 5 m. What is the mass of the pipe if 1 cm^3 of iron has a mass of 8 g?

Solution:

Inner diameter of the hollow cylindrical iron pipe = 54 cm

∴ Inner radius, r = $\left(\frac{54}{2}\right)\text{cm} = 27\text{ cm}$

Outer diameter of the pipe = 58 cm

∴ Outer radius, R = $\left(\frac{58}{2}\right)\text{cm} = 29\text{ cm}$

Length (h) of the pipe = 5 m = (5 × 100) cm = 500 cm

∴ Volume of the pipe $= \pi\left(R^2 - r^2\right)h$

$$= \left[\frac{22}{7} \times \left(29^2 - 27^2\right) \times 500\right]\text{cm}^3$$
$$= \left[\frac{22}{7} \times 112 \times 500\right]\text{cm}^3$$
$$= 176000\text{ cm}^3$$

Mass of 1 cm^3 of iron = 8 g

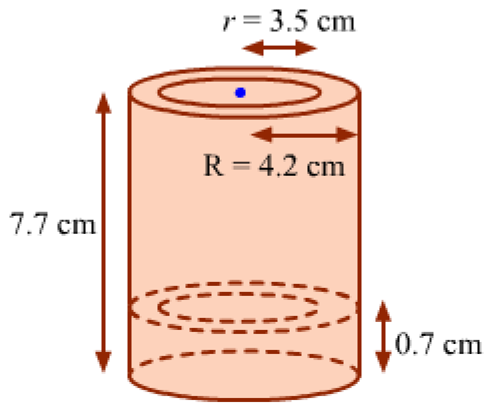
∴ Mass of 176000 cm^3 of iron = (8 × 176000) g

$$= \left(\frac{8 \times 176000}{1000}\right)\text{kg} \qquad \left(\because 1\text{ g} = \frac{1}{1000}\text{kg}\right)$$
$$= 1408\text{ kg}$$

Thus, the mass of the hollow cylindrical iron pipe is 1408 kg.

Example 2:

The internal and external radii of a cylindrical juice can (as shown in the figure) are 3.5 cm and 4.2 cm respectively. The total height of the can is 7.7 cm. The thickness of the base (i.e., a solid cylinder) is 0.7 cm. If the mass of the material used in the can is 3 g per cm^3, then find the mass of the can.



Solution:

To find the mass of the juice can, we need to first find its volume.

The juice can shown in the figure contains two cylinders. One is a solid cylinder (i.e., the base of the can) and the other is a hollow cylinder (i.e., the cylindrical part that stands on the base).

External radius (R) of the hollow cylinder = 4.2 cm

Internal radius (r) of the hollow cylinder = 3.5 cm

Thickness (h) of the base = 0.7 cm (i.e., the height of the solid cylinder)

Total height (H) of the juice can = 7.7 cm

∴ Height (h') of the hollow cylinder = (7.7 − 0.7) cm = 7 cm

Volume of the juice can = Volume of the solid base + Volume of the hollow cylinder on the base

$$= \pi R^2 h + \pi \left(R^2 - r^2\right) h'$$
$$= \pi \left[R^2 h + \left(R^2 - r^2\right) h'\right]$$
$$= \frac{22}{7}\left[(4.2)^2 \times 0.7 + \left\{(4.2)^2 - (3.5)^2\right\} \times 7\right] \text{cm}^3$$
$$= \frac{22}{7}(12.348 + 5.39 \times 7) \text{cm}^3$$
$$= \frac{22}{7}(12.348 + 37.73) \text{cm}^3$$
$$= \left(\frac{22}{7} \times 50.078\right) \text{cm}^3$$
$$= 157.388 \text{ cm}^3$$

Mass of the material per cm^3 = 3 g

∴ Mass of the material used in the container = (3 × 157.388) g

= 472.164 g

Thus, the mass of the juice can is 472.164 g.

Example 3:

A well 3.5 m in diameter and 20 m deep is dug in a rectangular field of dimensions 20 m × 14 m. The earth taken out is spread evenly across the field. Find the level of earth raised in the field.

Solution:

Length (l) of the field = 20 m

Breadth (b) of the field = 14 m

Diameter (d) of the well = 3.5 m

∴ Radius (r) of the well= $\frac{3.5}{2}$ m

Depth (h) of the well = 20 m

Volume of the dug out earth = $\pi r^2 h$

Now, the area of the field on which the dug out earth is spread is given by the difference between the area of the entire field and the area of the field covered by the cross-section of the well.

⇒ $l \times b - \pi r^2$

Let H be the level of earth raised in the field.

Volume of earth spread in the field = Volume of the dug out earth

$$\Rightarrow \left(l \times b - \pi r^2\right) H = \pi r^2 h$$

$$\Rightarrow \left(20 \times 14 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) H = \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 20$$

$$\Rightarrow \left(280 - \frac{269.5}{28}\right) H = \frac{5390}{28}$$

$$\Rightarrow \left(280 - \frac{77}{8}\right) H = \frac{385}{2}$$

$$\Rightarrow \frac{2163}{8} H = \frac{385}{2}$$

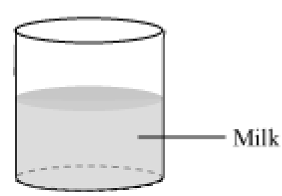
$$\Rightarrow H = \frac{385}{2} \times \frac{8}{2163}$$

$$\Rightarrow H = 0.71197 \text{ m} \approx 0.712 \text{ m}$$

Therefore, the level of earth in the field is raised by about 0.712 m.

Difference Between Volume And Capacity Of An Object

Consider a container, which is cylindrical in shape. Let us consider that 10 litres of milk can be stored in this container.



If the container is half-filled with milk, can we find the quantity of milk in the container?

Yes, we can find it. When the container is half-filled with milk, then the quantity of milk in the container is 5 litres.

Here, the quantity of milk in the container is the volume of the milk which is 5 litres.

And the container can store a maximum of 10 litres of milk, which is the capacity of the container.

If the container is completely filled with milk, then

Capacity of container = volume of milk = 10 litres

Thus we can say that,

“Volume is the amount of space occupied by an object, while capacity refers to the quantity that a container holds".

The units of volume of solid material are cm^3, m^3, dm^3 etc and the unit of volume of liquid and capacity is litre.

Let us discuss some examples based on volume and capacity.

Example 1:

A cubical container has each side measuring 20 cm. The container is half-filled with water. Metal stones are dropped in the container till the water comes up to the brim. Each stone is of volume 10 cm^3. Calculate the number of stones and the capacity of the container.

Solution:

We know that volume of cube = $(\text{side})^3$

$\therefore$ Volume of cubical container = $(20)^3$ cm^3

= 8000 cm^3

$\therefore$ Capacity of container = 8000 cm^3

= 8 litres (∵∵1 litre = 1000 cm^3)

The container is half-filled with water.

$\therefore$ Volume of water in the container = 4 litres

and, volume of metal stones = 4 litres

= 4 × 1000 cm^3

= 4000 cm^3

Volume each metal stone = 10 cm^3

$\therefore$ Number of stones = $\frac{4000}{10}$

= 400 stones

Thus, the capacity of the container is 8 litres and the number of stones is 400.

Example 2:

An oil tank is in the form of a cuboid whose dimensions are 60 cm, 30 cm, and 30 cm respectively. Find the quantity of oil that can be stored in the tank.

Solution:

It is given that

Length (l) = 60 cm

Breadth (b) = 30 cm

Height (h) = 30 cm

∴ Quantity of oil = Capacity of tank

$= l \times b \times h$

$= 60 \times 30 \times 30$

$= 54{,}000\ cm^3$

We know that,

1 litre = 1000 cm^3

∴ Quantity of oil that can be stored in the tank = 54 litres

Example 3:

Water is pouring in a cubical reservoir at a rate of 50 litres per minute. If the side of the reservoir is 1 metre, then how much time will it take to fill the reservoir?

Solution:

Side of reservoir = 1 m

∴ Capacity of reservoir = 1 m × 1 m × 1 m

$= 1\ m^3$

We know that,

1 m^3 = 1000 litres

∴ Capacity of reservoir = 1000 litres

50 litres of water is filled in 1 minute.

1 litre of water is filled in $\frac{1}{50}$ minute.

⇒⇒1000 litres of water will be filled in = $\frac{1000}{50}$ min

= 20 min

∴ Thus, the reservoir is filled in 20 minutes.

Example 4:

Orange juice is available in two packs – a tin cylinder of radius 2.1 cm and height 10 cm and a tin can with rectangular base of length 4 cm, width 3 cm, and height 12 cm. Which of the two packs has a greater capacity?

Solution:

For tin cylinder,

Radius (r) = 2.1 cm

And, height (h) = 10 cm

Capacity of cylinder = $\pi r^2 h$

$$= \left(\frac{22}{7} \times 2.1 \times 2.1 \times 10\right) \text{ cm}^3$$

= 138.60 cm^3

For tin can with rectangular base,

Length (l) = 4 cm

Width (b) = 3 cm

And height (h) = 12 cm

Capacity of tin can = $l \times b \times h$

= 144 cm^3

Therefore, the tin can with a rectangular base has greater capacity than the tin cylinder.