Chapter 7. Coordinate Geometry

Question-1

Find the centroid of the triangle whose vertices are : (3, -5), (-7, 4), (10, -2).

Solution:

The centroid of the triangle whose vertices are (3, -5), (-7, 4), (10, -2) is

∵ The centroid of the triangle ABC = $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$

∴ The centroid of the triangle whose vertices A(3, -5), B(-7, 4), C(10, -2) = $\left(\frac{3-7+10}{3}, \frac{-5+4-2}{3}\right)$

= (2, -1).

Question-2

Find the centroid of the triangle whose vertices are : (2, 1), (5, 2), (3, 4).

Solution:

∵ The centroid of the triangle ABC = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

∴ The centroid of the triangle whose vertices (2, 1), (5, 2), (3, 4) = $\left(\frac{2+5+3}{3}, \frac{1+2+4}{3}\right)$ = (10/3, 7/3).

Question-3

Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin.

Solution:

Given, the two vertices of the triangle are (-3, 1), (0, -2).

Let the third vertex be (x, y)

Also given the centroid of the triangle = (0, 0)

∵ The centroid of the triangle = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

∴ $\left(\frac{-3+0+x}{3}, \frac{1-2+y}{3}\right) = (0, 0)$

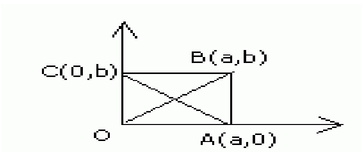
⇒ -3 + x = 0, -1 + y = 0

∴ x = 3, y = 1

Question-4

Prove that the diagonals of a rectangle bisect each other and are equal. (Hint: With O as origin, let the vertices of the rectangle be (0, 0), (a, 0), (a, b) and (0, b)).

Solution:



AC and OB are diagonals

In the figure let the intersecting point of OB and AC be P
To show that diagonals bisect each other we have to prove that OP = PB and PA = PC
The co-ordinates of P is obtained by

$$\left(\frac{O+a}{2}, \frac{O+b}{2}\right)$$

$\therefore$ P is the point $\left(\frac{a}{2}, \frac{b}{2}\right)$

$$OP^2 = \left(\frac{a}{2} - O\right)^2 + \left(\frac{b}{2} - O\right)^2$$

$$= \frac{a^2}{4} + \frac{b^2}{4} \qquad \therefore OP = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$PB^2 = \left(a - \frac{a}{2}\right)^2 + \left(b - \frac{b}{2}\right)^2$$

$$= \frac{a^2}{4} + \frac{b^2}{4} \qquad PB = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

OP = PB
Similarly we can prove that PC = PA
Thus diagonals bisect each other in a rectangle.

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0 - a)^2 + (b - 0)^2}$$
$$= \sqrt{b^2 + a^2}$$
$$OB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(a - 0)^2 + (0 - b)^2}$$
$$= \sqrt{a^2 + b^2}$$
$$AC = BO$$

$\therefore$ The diagonals of a rectangle bisects each other and equal.

Question-5

Show that the points A(1, 0), B(5, 3), C(2, 7) and D(-2, 4) are the vertices of a parallelogram. (Hint: Diagonals of a parallelogram bisect each other).

Solution:

The midpoint of diagonal AC is $\left(\frac{1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

The midpoint of diagonal BD is $\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

The diagonals AC and BD bisect each other ⇒ ABCD is a parallelogram.

Question-6

In what ratio does the y -axis divide the line segment joining the points P(-4, 5) and Q(3, 7)?

Solution:

Let the required ratio be k : 1. Then, the coordinates of the point of division are, $\left(\frac{3k-4}{k+1}, \frac{7k+5}{k+1}\right)$

But, it is a point on y-axis on which x-coordinate of every point is zero.

Therefore $\frac{3k-4}{k+1} = 0$

3k - 4 = 0

k = 4/3

Thus, the required ratio is k = 4/3 or 4 : 3.

Question-7

Find the cirumcentre of the triangle whose vertices are (-2, -3), (-1, 0), (7, -6).

Solution:

Let the centre of the circle be O(x, y). The points are A(-2, -3), B(-1, 0) and C(7, -6).

OA = $\sqrt{(-2-x)^2 + (-3-y)^2}$

OB = $\sqrt{(-1-x)^2 + (0-y)^2}$

OC = $\sqrt{(7-x)^2 + (-6-y)^2}$

OA = OB = OC = radius

$\sqrt{(-2-x)^2 + (-3-y)^2} = \sqrt{(-1-x)^2 + (0-y)^2}$

$(2 + x)^2 + (3 + y)^2 = (1 + x)^2 + y^2$

$4 + 4x + x^2 + 9 + 6y + y^2 = 1 + 2x + x^2 + y^2$

4 + 4x + 9 + 6y = 1 + 2x

2x + 6y = -12

x + 3y = -6 ....(i)

$\sqrt{(-1-x)^2+(0-y)^2} = \sqrt{(7-x)^2+(-6-y)^2}$

$(1 + x)^2 + y^2 = (7 - x)^2 + (6 + y)^2$

$1 + 2x + x^2 + y^2 = 49 - 14x + x^2 + 36 + 12y + y^2$

$1 + 2x = 49 - 14x + 36 + 12y$

$16x - 12y = 84$

$4x - 3y = 21$ ....(ii)

Solving (i) and (ii)

$x + 3y = -6$

$\underline{4x - 3y = 21}$

$5x \quad = 15$

$x = 3$

Substituting x = 3 in (i)

$4(3) - 3y = 21$

$12 - 3y = 21$

$-3y = 9$

$y = -3$

Therefore the centre of the circle is (3, -3).

Question-8

The three vertices of a parallelogram are (1, 1), (4, 4) and (4, 8). Find the fourth vertex.

Solution:

Let A(1, 1), B(4, 4), C(4, 8) and D(x, y) be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

Therefore coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$\left(\frac{1+4}{2}, \frac{1+4}{2}\right) = \left(\frac{4+x}{2}, \frac{8+y}{2}\right)$

$\left(\frac{5}{2}, \frac{5}{2}\right) = \left(\frac{4+x}{2}, \frac{8+y}{2}\right)$

4 + x = 5, 8 + y = 5

x = 1, y = -3

Therefore (1, -3) is the fourth vertex.

Question-9

Show that the points (2, 1), (5, 2), (6, 4) and (3, 3) are the angular points of a parallelogram. Is the figure a rectangle?

Solution:

Let A(2, 1), B(5, 2), C(6, 4) and D(3, 3) be the vertices of a parallelogram ABCD. Since, the diagonals of a parallelogram bisect each other.

$AC^2 = (6 - 2)^2 + (4 - 1)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$

$BC^2 = (6 - 5)^2 + (4 - 2)^2 = (1)^2 + (2)^2 = 1 + 4 = 5$

$AB^2 = (5 - 2)^2 + (2 - 1)^2 = (3)^2 + (1)^2 = 9 + 1 = 10$

$DC^2 = (6 - 3)^2 + (4 - 3)^2 = (3)^2 + (1)^2 = 9 + 1 = 10$

$AD^2 = (3 - 2)^2 + (3 - 1)^2 = (1)^2 + (2)^2 = 1 + 4 = 5$

Since BC = AD and DC = AB, ABCD is a parallelogram.

$AB^2 + BC^2 = 10 + 5 = 15$

$AB^2 + BC^2 \neq AC^2$

∴ ΔABC is not right angled. Therefore parallelogram ABCD is not a rectangle.

Question-10

Find the third vertex of a triangle, if two of its vertices are (-3, 1) and (0, -2) and the centroid is at the origin.

Solution:

Let the third vertex of the triangle be C(x, y) and the other vertices A(-3, 1) and B(0, -2).

Coordiantes of the centroid of the triangle = (0, 0)

$\therefore \left(\frac{-3+0+x}{3}, \frac{1-2+y}{3}\right) = (0, 0)$

$-3 + x = 0$

$x = 3$

And $-1 + y = 0$

$y = 1$

∴ The third vertex of the triangle is (3, 1).

Question-11

If the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3, 4), find its vertices.

Solution:

Let the vertices of the triangle be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Let the midpoints of the sides of a triangle are

D(1, 1), E(2, -3) and F(3, 4).

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (1, 1)$

$x_1 + x_2 = 2$ ....(i)
$y_1 + y_2 = 2$ ....(ii)
$\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) = (2, -3)$
$x_2 + x_3 = 4$ ....(iii)
$y_2 + y_3 = -6$ ....(iv)
$\left(\frac{x_3 + x_1}{2}, \frac{y_3 + y_1}{2}\right) = (3, 4)$
$x_3 + x_1 = 6$ ....(v)
$y_3 + y_1 = 8$ ....(vi)
Add the equations (i), (iii), (v) we get,
$2(x_1 + x_2 + x_3) = 12$
$\Rightarrow x_1 + x_2 + x_3 = 6$..............(vii)
Substitute eqn.(i) in (vii) then $x_3 = 4$.
Substitute eqn.(iii) in (vii) then $x_1 = 2$.
Substitute eqn.(v) in (vii) then $x_2 = 0$.

Add the equations (ii), (iv), (vi) we get,

$2(y_1 + y_2 + y_3) = 4$

$\Rightarrow y_1 + y_2 + y_3 = 2$..............(viii)

Substitute eqn.(ii) in (viii) then $y_3 = 0$.

Substitute eqn.(iv) in (viii) then $y_1 = 8$.

Substitute eqn.(vi) in (viii) then $y_2 = -6$.

∴ The vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ = A(2, 8), B(0, -6) and C(4, 0)

Question-12

Find the ratio in which the line points (6, 4) and (1, - 7) is divided internally by the axis of x.

Solution:

Let the required ratio be k : 1. Then, the coordinates of the point of division are, $\left(\frac{k+6}{k+1}, \frac{-7k+4}{k+1}\right)$

But, it is a point on y-axis on which x-coordinate of every point is zero.

Therefore $\frac{-7k+4}{k+1} = 0$

7k - 4 = 0

k = 4/7

Thus, the required ratio is k = 4/7 or 4 : 7.

Question-13

If the points (-2, -1), (1, 0), (x, 3) and (1, y) form a parallelogram, find the values of x and y.

Solution:

Let the vertices of the parallelogram be A(-2, -1), B(1, 0), C(x, 3) and D(1, y). Since the diagonals of a parallelogram bisect each other the coordinates of the mid-point of AC = coordinates of the mid-point of BD.

$$\left(\frac{-2+x}{2}, \frac{-1+3}{2}\right) = \left(\frac{1+1}{2}, \frac{0+y}{2}\right)$$

$$\left(\frac{-2+x}{2}, 1\right) = \left(1, \frac{y}{2}\right)$$

∴ x = 4, y = 2.

Question-14

If the mid-points of the sides of a triangle PQR are A(-1, -3), B(2, 1) and C(4, 5), find the coordinates of P, Q and R.

Solution:

Coordinate of mid point P is $\left(\frac{-1+2}{2}, \frac{-3+1}{2}\right) = \left(\frac{1}{2}, -1\right)$

Coordinate of mid point Q is $\left(\frac{2+4}{2}, \frac{1+5}{2}\right) = (3, 3)$

Coordinate of mid point R is $\left(\frac{-1+4}{2}, \frac{-3+5}{2}\right) = \left(\frac{3}{2}, 1\right)$

∴ The coordinates of the P $\left(\frac{1}{2}, -1\right)$, Q(3, 3) and R $\left(\frac{3}{2}, 1\right)$.

Question-15

Three consecutive vertices of a parallelogram are (-2, -1), (1, 0) and (4, 3). Find the fourth vertex.

Solution:

Let A(-2, -1), B(1, 0), C(4, 3) and D(x, y) be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

Therefore coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$$\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = \left(\frac{1+x}{2}, \frac{0+y}{2}\right)$$

$$(1, 1) = \left(\frac{1+x}{2}, \frac{y}{2}\right)$$

x + 1 = 2, y = 2

Therefore (1, 2) is the fourth vertex.

Question-16

Determine the ratio in which 2x + 3y - 30 = 0 divides the line segment joining A (3, 4) and B (7, 8) and the point at which it divides.

Solution:

Let the P(a, b) be the point which divides the line segment joining A (3, 4) and B (7, 8) in the ratio k : 1.

Then coordinates of the point P is $\left(\frac{7k+3}{k+1}, \frac{8k+4}{k+1}\right)$.

This point lies on the line 2x + 3y - 30 = 0.

$\therefore 2\left(\frac{7k+3}{k+1}\right) + 3\left(\frac{8k+4}{k+1}\right) - 30 = 0.$

$\therefore 2(7k + 3) + 3(8k + 4) - 30(k + 1) = 0.$

$\therefore 14k + 6 + 24k + 12 - 30k - 30 = 0.$

$\therefore 8k - 12 = 0.$

$\therefore k = 3/2$

$\therefore$ The required ratio is 3 : 2.

The coordinates of the point P is $\left(\frac{21+6}{5}, \frac{24+8}{5}\right) = \left(\frac{27}{5}, \frac{32}{5}\right)$.

Question-17

Prove that the points (2a, 4a), (2a, 6a), (2a + $\sqrt{3}$ a, 5a) are the vertices of an equilateral triangle.

Solution:

Let A(2a, 4a), B(2a, 6a) and C(2a + $\sqrt{3}$ a, 5a) be the vertices of an equilateral triangle.

AB = $\sqrt{(2a-2a)^2+(6a-4a)^2} = \sqrt{0^2+(2a)^2} = 2a$

BC = $\sqrt{(2a+\sqrt{3}a-2a)^2+(5a-6a)^2} = \sqrt{3a^2+a^2} = 2a$

CA = $\sqrt{(2a+\sqrt{3}a-2a)^2+(5a-4a)^2} = \sqrt{(\sqrt{3}a)^2+a^2} = \sqrt{3a^2+a^2} = 2a$

AB = BC = CA.

$\therefore$ the vertices are of an equilateral triangle.

Question-18

The points A(0, 3), B(-2, a) and C(-1, 4) are the vertices of a Δ ABC right – angled at A. Find the value of a.

Solution:

Given, the vertices of a Δ ABC are right −angled at A.

$\therefore AB^2 + AC^2 = BC^2$

$AB^2 = (-2-0)^2 + (a-3)^2 = 4 + (a-3)^2$

$BC^2 = (-1+2)^2 + (4-a)^2 = 1 + (4-a)^2$

$AC^2 = (-1-0)^2 + (4-3)^2 = 1 + 1 = 2$

Since, $AB^2 + AC^2 = BC^2$

$4 + (a-3)^2 + 2 = 1 + (4-a)^2$

$4 + a^2 + 9 - 6a + 2 = 1 + 16 + a^2 - 8a$

$2a = 2$

$\therefore a = 1.$

Question-19

The points A(2, 0), B(9, 1), C(11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Solution:

$AB = \sqrt{(9-2)^2+(1-0)^2} = \sqrt{7^2+1^2} = \sqrt{50}$

$BC = \sqrt{(11-9)^2+(6-1)^2} = \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$

$CD = \sqrt{(4-11)^2+(4-6)^2} = \sqrt{49+4} = \sqrt{53}$

$DA = \sqrt{(2-4)^2+(0-4)^2} = \sqrt{4+16} = \sqrt{20}$

No it is not a rhombus as all sides are not equal.

Question-20

The vertices of a triangle are A(3, 4) , B(7, 2)and C(-2, -5). Find the length of the median through the vertex A.

Solu

Let D the mid point of BC. Then the coordinate of D is $\left(\frac{7-2}{2}, \frac{2-5}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$.

Length of the median is $AD = \sqrt{\left(\frac{5}{2}-3\right)^2 + \left(\frac{-3}{2}-4\right)^2}$

$= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{-11}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{121}{4}}$

$= \frac{\sqrt{122}}{2}.$

Question-21

Two vertices of a triangle are (3, -5) and (-7, 4). If its centroid is (2, -1), find the third vertex.

Solution:

Let P(3, -5) and Q(-7, 4) be the two vertices of a triangle. Centroid of a triangle is G(2, -1).

The coordinates of the centroid of Δ ABC is $\left(\frac{x_1 + 3 - 7}{3}, \frac{y_1 - 5 + 4}{3}\right) = (2, -1)$

$\left(\frac{x_1 - 4}{3}, \frac{y_1 - 1}{3}\right) = (2, -1)$

$x_1 = 6 + 4$, $y_1 = -3 + 1$

$x_1 = 10$, $y_1 = -2$

∴ The third vertex is (10, -2).

Question-22

Are the points (-2, 2), (8, -2) and (-4, 3) are the vertices of a right angled triangle.

Solution:

Let the points A(-2, 2), B(8, -2) and C(-4, 3) be the vertices of a triangle ABC.

$AB^2 = (8 + 2)^2 + (-2 - 2)^2 = 10^2 + 4^2 = 100 + 16 = 116$

$BC^2 = (-4 - 8)^2 + (3 + 2)^2 = (-12)^2 + 5^2 = 144 + 25 = 169$

$AC^2 = (-4 + 2)^2 + (3 - 2)^2 = (-2)^2 + 1 = 4 + 1 = 5$

$AB^2 + AC^2 \neq BC^2$

The above vertices are not points of a right angled triangle.

Question-23

If (-2, 3), (4, -3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of its centroid.

Solution:

Let P(-2, 3), Q(4, -3), R(4, 5) be the mid-points of sides AB, BC and CA respectively of a triangle ABC. Let A(x_1 , y_1), B(x_2 , y_2) and C(x_3, y_3) be the vertices of triangle ABC. Then,

P is the midpoint of AB.

$\frac{x_1 + x_2}{2} = -2$, $\frac{y_1 + y_2}{2} = 3$

$x_1 + x_2 = -4$ and $y_1 + y_2 = 6$ ....(i)

Q is the midpoint of BC.

$\frac{x_2 + x_3}{2} = 4$, $\frac{y_2 + y_3}{2} = -3$

$x_2 + x_3 = 8$ and $y_2 + y_3 = -6$ ....(ii)

R is the midpoint of CA.

$\frac{x_1 + x_3}{2} = 4$, $\frac{y_1 + y_3}{2} = 5$

$x_1 + x_3 = 8$ and $y_1 + y_3 = 10$ ....(iii)

From (i), (ii) and (iii)

$2(x_1 + x_2 + x_3) = -4 + 8 + 8 = 12$

$x_1 + x_2 + x_3 = 6$

$2(y_1 + y_2 + y_3) = 6 - 6 + 10 = 10$

$y_1 + y_2 + y_3 = 5$

Therefore the coordinates of the centroid of Δ ABC are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

or, (6/3, 5/3) = (2, 5/3).

Question-24

If the points (-1, 3), (1, -1) and (5, 1) are vertices of a triangle, find the length of the median through third vertex.

Solution:

Let the points A(-1, 3), B(1, -1) and C(5, 1) be the vertices of a triangle ABC.

Let D be the mid point of AB. Then the coordinate of D is $\left(\frac{-1+1}{2}, \frac{3-1}{2}\right)$=(0, 1).

Length of the median is CD = $\sqrt{(0-5)^2 + (1-1)^2}$

$= \sqrt{25+0}$

= 5.

Question-25

Find the lengths of the sides of the triangle whose vertices are A(3, 4), B(2, -1) and C(4, -6).

Solution:

AB = $\sqrt{(2-3)^2 + (-1-4)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$

BC = $\sqrt{(4-2)^2 + (-6+1)^2} = \sqrt{(2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$

AC = $\sqrt{(4-3)^2 + (-6-4)^2} = \sqrt{(1)^2 + (-10)^2} = \sqrt{1+100} = \sqrt{101}$

Therefore the lengths of the sides of the triangle are $\sqrt{26}$, $\sqrt{29}$ and $\sqrt{101}$.

Question-26

A line is of length 10 and one end is at the point (-3, 2). If the ordinate of the other end be 10, prove that the abscissa will be 3 or −9.

Solution:

The two points are (-3, 2) and (x, 10). Their length is 10.

$\sqrt{(x+3)^2 + (10-2)^2} = 10$

$(x+3)^2 + 8^2 = 10^2$

$x^2 + 6x + 9 + 64 = 100$

$x^2 + 6x - 27 = 0$

$x^2 + 9x - 3x - 27 = 0$

$x(x + 9) - 3(x + 9) = 0$

$(x - 3)(x + 9) = 0$

$x = 3$ or -9.

Therefore the required abscissa will be 3 or −9.

Question-27

Show that the points (-2, 6), (5, 3), (-1, -11) and (-8, -8) are the vertices of a rectangle.

Solution:

Let A(-2, 6), B(5, 3), C(-1, -11) and D(-8, -8) are the vertices of a rectangle.

(i) $AB = \sqrt{(5+2)^2 + (3-6)^2} = \sqrt{(7)^2 + (-3)^2} = \sqrt{49+9} = \sqrt{58}$

$CD = \sqrt{(-8+1)^2 + (-8+11)^2} = \sqrt{(-7)^2 + 3^2} = \sqrt{49+9} = \sqrt{58}$

$BC = \sqrt{(-1-5)^2 + (-11-3)^2} = \sqrt{(-6)^2 + (-14)^2} = \sqrt{36+196} = \sqrt{232}$

$DA = \sqrt{(-8+2)^2 + (-8-6)^2} = \sqrt{(-6)^2 + (-14)^2} = \sqrt{36+196} = \sqrt{232}$

AB = CD and BC = DA.

(ii) $AC = \sqrt{(-1+2)^2 + (-11-6)^2} = \sqrt{(1)^2 + (-17)^2} = \sqrt{1+289} = \sqrt{290}$

$BD = \sqrt{(-8-5)^2 + (-8-3)^2} = \sqrt{(-13)^2 + (-11)^2} = \sqrt{169+121} = \sqrt{290}$

AC = DB

Since the opposite sides and the diagonals are equal

∴ ABCD is a rectangle.

Question-28

If a point (x, y) is equidistant from (6, -1) and (2, 3), find the relation between x and y.

Solution:

Let the points be P(x, y), A(6, -1) and B(2, 3).

$AP^2 = (x - 6)^2 + (y + 1)^2$

$BP^2 = (x - 2)^2 + (y - 3)^2$

Given, (x, y) is equidistant from (6, -1) and (2, 3)

$(x - 6)^2 + (y + 1)^2 = (x - 2)^2 + (y - 3)^2$

$x^2 - 12x + 36 + y^2 + 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$

$-12x + 36 + 2y + 1 = -4x + 4 - 6y + 9$

$-8x + 8y = -24$

$-x + y = -3$

$x - y = 3.$

Question-29

Coordinates of A and B are (-3, a) and (1, a + 4). The mid-point of AB is (-1, 1). Find the value of a.

Solution:

Mid point of AB = $\left(\frac{-3+1}{2}, \frac{a+a+4}{2}\right)$

$$\left(\frac{-3+1}{2}, \frac{a+a+4}{2}\right) = (-1, 1)$$

$$\left(\frac{-2}{2}, \frac{2a+4}{2}\right) = (-1, 1)$$

$$a + 2 = 1$$

$$a = -1.$$

Question-30

Find a point on the line through A(5, -4) and B(-3, 2), that is, twice as far from A as from B.

Solution:

Let the required point be P(x, y).

Then, AP = 2PB

AP/PB = 2/1

or AP : PB = 2 : 1

Therefore $x = \left[\frac{2\times(-3)+1\times 5}{2+1}\right] = \frac{-1}{3}$ and $y = \left[\frac{2\times 2+1\times(-4)}{2+1}\right] = 0$

So, the required point is (-1/3, 0).

Question-31

Determine the ratio in which y – x + 2 = 0 divides the line joining (3, -1) and (8, 9).

Solution:

Let the required ratio be k : 1.

Then, the point of division is $\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$.

This point must lie on y – x + 2 = 0.

Therefore $\frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$ or $k = \frac{2}{3}$.

So, the required ratio is $\frac{2}{3}$:1

i.e. 2 : 3.

Question-32

If the points (2, 1) and (1, -2) are equidistant from the point (x, y), show that x + 3y = 0.

Solution:

Let the points A(2, 1) and B(1, -2) be at equidistant from the point P(X, Y).

AP = $\sqrt{(x-2)^2 + (y-1)^2}$

AB = $\sqrt{(x-1)^2 + (y+2)^2}$

$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y+2)^2}$

$(x - 2)^2 + (y - 1)^2 = (x - 1)^2 + (y + 2)^2$

$x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 + 4y + 4$

$-4x + 4 - 2y + 1 = -2x + 1 + 4y + 4$

$-2x - 6y = 0$

$x + 3y = 0.$

Question-33

Find the ratio in which the point (2, y) divides the join of (-4, 3) and (6, 3) and hence find the value of y.

Solution:

Let the required ratio be k : 1.

Then, $2 = \frac{6k - 4 \times 1}{k + 1} \Rightarrow k = \frac{3}{2}$.

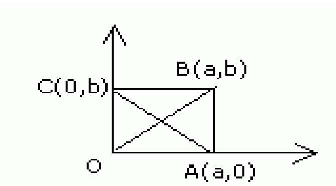
Therefore the required ratio is $\frac{3}{2}$: 1 i.e. 3 : 2.

Also, $y = \frac{3 \times 3 + 2 \times 3}{3 + 2} = 3$.

Question-34

Prove that the diagonals of a rectangle bisect each other and are equal. (Hint: With O as origin, let the vertices of the rectangle be (0, 0), (a, 0), (a, b) and (0, b).

Solution:



ABCO is a rectangle with vertices A(a, 0), B(a, b), C(0, b) and O(0, 0).

The midpoint of AC is $\left(\frac{a+0}{2}, \frac{0+b}{2}\right)$ = (a/2, b/2)

The midpoint of OB is $\left(\frac{a+0}{2}, \frac{b+0}{2}\right)$ = (a/2, b/2)

Hence the diagonals bisect each other.

The length of the diagonal AC = $\sqrt{(a-0)^2+(0-b)^2} = \sqrt{a^2+b^2}$ units

The length of the diagonal OB = $\sqrt{(a-0)^2+(b-0)^2} = \sqrt{a^2+b^2}$ units

Hence the diagonals are equal.

Question-35

Show that the points A(1, 0), B(5, 3), C(2, 7) and D(-2, 4) are the vertices of a parallelogram. (Hint: Diagonals of a parallelogram bisect each other).

Solution:

The midpoint of diagonal AC is $\left(\frac{1+2}{2}, \frac{0+7}{2}\right)$ = (3/2,7/2)

The midpoint of diagonal BD is $\left(\frac{5-2}{2}, \frac{3+4}{2}\right)$ = (3/2,7/2)

The diagonals AC and BD bisect each other ⇒ ABCD is a parallelogram.

Question-36

If the distances of A(x, y) from P(a + b, b - a) and Q(a - b, a + b) are equal, prove that bx = ay.

Solution:

$AP^2 = (a + b - x)^2 + (b - a - y)^2$

$AQ^2 = (a - b - x)^2 + (a + b - y)^2$

AP = AQ (Given)

\ $AP^2 = AQ^2$

$(a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$

$a^2 + b^2 + x^2 + 2ab - 2ax - 2bx + b^2 + a^2 + y^2 - 2ba - 2by + 2ay = a^2 + b^2 + x^2$

$- 2ab + 2bx - 2ax + a^2 + b^2 + y^2 + 2ab - 2ay - 2by - 2bx + 2ay = 2bx - 2ay$

4ay = 4bx

ay = bx

Hence proved.

Question-37

Prove that the points A (0, 1), B(1, 4), C(4, 3) and D(3, 0) are the vertices of a square.

Solution:

Let A (0, 1), B(1, 4), C(4, 3) and D(3, 0) be the four points.

AB = $\sqrt{(1-0)^2+(4-1)^2} = \sqrt{1+9} = \sqrt{10}$ units

BC = $\sqrt{(4-1)^2+(3-4)^2} = \sqrt{9+1} = \sqrt{10}$ units

CD = $\sqrt{(3-4)^2+(0-3)^2} = \sqrt{1+9} = \sqrt{10}$ units

DA = $\sqrt{(0-3)^2+(1-0)^2} = \sqrt{9+1} = \sqrt{10}$ units

Therefore the points A (0, 1), B(1, 4), C(4, 3) and D(3, 0) are the vertices of a square.

Question-38

Show that the points (1, 1), (-2, 7) and (3, -3) are collinear.

Solution:

Let A(1, 1), B(-2, 7) and C(3, -3) be the three points.

AB = $\sqrt{(-2-1)^2+(7-1)^2}$ = $\sqrt{9+36}$ = $\sqrt{45}$ = $3\sqrt{5}$ units

BC = $\sqrt{(3+2)^2+(-3-7)^2}$ = $\sqrt{25+100}$ = $\sqrt{125}$ = $5\sqrt{5}$ units

AC = $\sqrt{(3-1)^2+(-3-1)^2}$ = $\sqrt{4+16}$ = $\sqrt{20}$ = $2\sqrt{5}$ units

AB + AC = BC. Hence A, B, C are collinear.

Question-39

Find the ratio in which the points (2, 5) divides the line-segment joining the points (-1, 2) and (4, 7).

Solution:

Let A(-1, 2) and B(4, 7) be points. Let the point at which it is divided be C(2, 5).

$AC^2 = (2 + 1)^2 + (5 - 2)^2 = (3)^2 + (3)^2 = 9 + 9 = 18$

$BC^2 = (2 - 4)^2 + (5 - 7)^2 = (-2)^2 + (-2)^2 = 4 + 4 = 8$

AC/ BC = = 18/8 = 9/4

The ratio in which the line segment is divided is 9 : 4.