

Type 9: Properties of Root-Locus

For Concept, refer to Control Systems K-Notes, Root Locus Technique

Common Mistake:

Try to remember clearly how to distinguish between Breakaway and Break-in points.

Sample Problem 9:

The transfer function of this system is $G(s) = \frac{K(s^2 - 2s + 2)}{(s + 2)(s + 3)}$. The break point is

- (A) Break-away at $s = -1.29$
- (B) Break-in at $s = -2.43$
- (C) Break-away at $s = -2.43$
- (D) Break-in at $s = -1.29$

Solution: (C) is correct option

The point, at which multiple roots are present, are known as break point. These are obtained from:

$$\frac{dK}{ds} = 0$$

Here, characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s^2 - 2s + 2)}{(s + 2)(s + 3)} = 0 \quad \Rightarrow \quad K = \frac{-(s + 2)(s + 3)}{(s^2 - 2s + 2)} = \frac{-(s^2 + 5s + 6)}{(s^2 - 2s + 2)}$$

Now, differentiating eq(1) w.r.t s and equating zero we have

$$\frac{dK}{ds} = \frac{-(s^2 - 2s + 2)(2s + 5) + (s^2 + 5s + 6)(2s - 2)}{(s^2 - 2s + 2)^2} = 0$$

$$7s^2 + 8s - 22 = 0$$

which gives $s = +1.29$ and $s = -2.43$ out of which $s = -2.43$ is break-away point

Unsolved Problems:

Q.1 The open loop transfer function of a system is $G(s) = \frac{K}{s(s + 4)(s^2 + 4s + 8)}$ the value of k at

($s = -2$) in the root locus is

- (A) 4
- (B) 8
- (C) 16
- (D) 32

Q.2 The OLTF of a unity feedback control system is $G(s) = \frac{K}{(s + 1)(s^2 + 4s + 5)}$. The angle of

departure at the pole ($-2 - j1$) is

- (A) $+60^\circ$
- (B) -60°
- (C) $+45^\circ$
- (D) -45°

Q.3 A unity feedback system has open loop poles at $s = -2 \pm j2$, $s = -1$, and $s = 0$; and a zero at $s = -3$. The angle made by the root locus asymptotes with the real and the point of Intersection of the asymptotes are

- (A) $(60^\circ, -60^\circ, 180^\circ)$ and $-3/2$ (B) $(60^\circ, -60^\circ, 180^\circ)$ and $-2/3$
 (C) $(45^\circ, -45^\circ, 180^\circ)$ and $-2/3$ (D) $(45^\circ, -45^\circ, 180^\circ)$ and $-4/3$

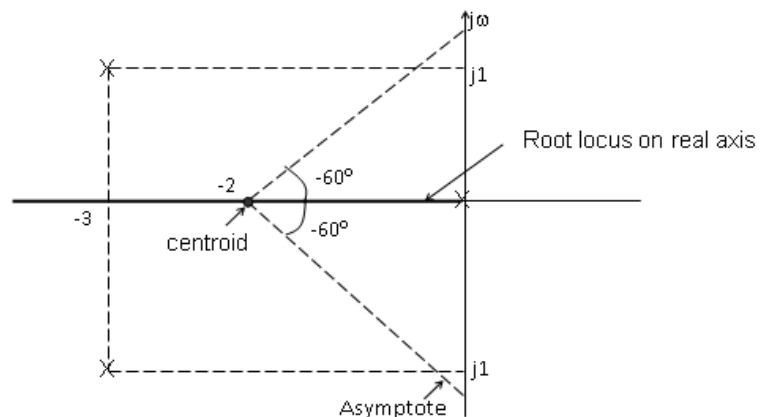
Q.4 Consider the points $S_1 = -3 + j4$ and $S_2 = -3 - j2$ in the S – plane, then for a system with the open loop transfer function $G(s)H(s) = \frac{K}{(s+1)^4}$

- (A) S_1 is on the root locus, but not S_2 (B) S_2 is on the root locus, but not S_1
 (C) Both S_1 and S_2 are on the root locus (D) Neither S_1 nor S_2 is on the root locus

Q.5 Figure shows the asymptote, root locus on real axis and location of poles and centroid.

Break-in point of the root locus is

- (A) -3
 (B) -2
 (C) -1.18
 (D) -2.82



Q.6 OLTF of an unity feedback system is $\frac{K(s+1)}{s(s-1)}$. For complex roots, the RLD is circle of centre, whose coordinates are

- (A) $-0.5, 0$ (B) $1, 0$ (C) $-1.5, 0$ (D) $-1, 0$