Sample Question Paper - 14
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. What is the common difference of an AP in which $a_{21} - a_7 = 84$?

OR

Which term of the AP 27, 24, 21,.....is zero?

2. A tower stands vertically on the ground. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.

3. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

4. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm^3.

5. A solid sphere of radius r melted and recast into the shape of a solid cone of height r. Find the radius of the base of a cone.

6. For the following distribution what is the modal class:

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

OR

In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. What is the lower limit of the class?

Section B

7. An isosceles triangle ABC, with $AB = AC$, circumscribes a circle, touching BC at P, AC at Q and AB at R. Prove that the contact point P bisects BC.

8. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. $\left(\text{Use } \pi = \frac{22}{7}\right)$

9. Heights of students of class X are given in the following distribution :

Heights (in cm)	150-155	155-160	160-165	165-170	170-175
Number of students	15	8	20	12	5

Find the modal height.

10. A school conducted a test (of 100 marks) in English for students of Class X. The marks obtained by students are shown in the following table :

Marks obtained	0- 10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	1	2	4	15	15	25	15	10	2	1

Find the modal marks.

OR

The mode of a distribution is 55 and the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10. Find the frequency of the modal class.

Section C

11. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

12. Draw a circle of radius 3 cm. From a point P, 7 cm away from centre draw two tangents to the circle. Measure the length of each tangent.

OR

Draw a circle of radius of 3 cm. Take two points P and Q one of its diameter extended on both sides, each at a distance of 7 cm on opposite sides of its centre. Draw tangents to the circle from these two points.

13. Fencing a Backyard : Tina and Shriya have just purchased a purebred German Shepherd, and need to fence in their backyard so the dog can run.

(i) What is the maximum rectangular area they can enclose with 200 ft of fencing, if they use fencing material along all four sides? What are the dimensions of the rectangle?

(ii) What is the maximum area if they use the house as one of the sides? What are the dimensions of this rectangle?

Figure is given on next Page.

14. Silo : A silo is a structure for storing bulk materials. Silos are used in agriculture to store grain or fermented feed known as silage. Silos are commonly used for bulk storage of grain, coal, cement, carbon black, woodchips, food products and sawdust.

A silo is in the shape of cylinder surmounted by a conical top. The height and diameter of cylindrical part are 40 feet and 42 feet respectively and the slant height of conical part is 29 feet.

(i) How much metal sheet is required to make this silo ?

(ii) What is the storage capacity of silo ?

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. What is the common difference of an AP in which $a_{21} - a_7 = 84$?

Sol :

Let the first term of an AP be a and common difference be d.

$$a_{21} - a_7 = 84$$

$$a + (21-1)d - \left[a + (7-1)d\right]$$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = 6$$

or

Which term of the AP 27, 24, 21,.....is zero?

Sol :

We have, AP : 27, 24, 21, ...

$$a = 27,\ d = 24 - 27 = -3$$

Let n^{th} term of the AP is zero.

i.e $a_n = 0$

$$a + (n-1)\,d = 0 \qquad [a_n = a + (n-1)\,d]$$

$$27 + (n-1)(-3) = 0$$

$$27 - 3n + 3 = 0$$

$$3n = 30 \Rightarrow n = 10$$

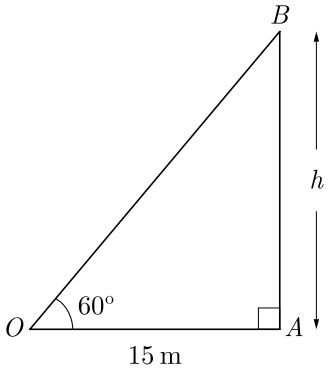
Thus 10^{th} term of the given AP is zero.

2. A tower stands vertically on the ground. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.

Sol :

Let AB be the tower standing vertically on the ground and O be the position of the observer as shown in figure.

We have$OA = 15\,\text{m}$, $\angle OAB = 90^\circ$, $\angle AOB = 60^\circ$



Now, in right $\Delta\, OAB$, we have

$$\tan 60^\circ = \frac{AB}{OA}$$

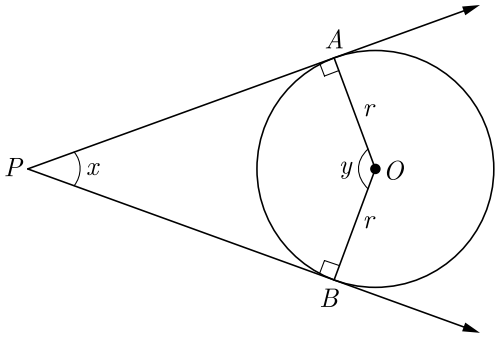
$$\sqrt{3} = \frac{h}{15}$$

$$h = 15\sqrt{3} = 25.95\,\text{m}$$

3. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Sol :

As per question we draw figure shown below.



Now $OA \perp AP$ and $OB \perp BP$ because tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle A = \angle B = 90^\circ$

Since, $AOBP$ is a quadrilateral,

$$\angle A + \angle B + x + y = 360^\circ$$
$$90^\circ + 90^\circ + x + y = 360^\circ$$
$$180 + x + y = 360^\circ$$
$$x + y = 180^\circ$$

Therefore opposite angle are supplementary.

4. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm^3.

Sol :

Let r be the radius of bowl, then circumference of bowl,

$$2\pi r = 132$$
$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Capacity i.e volume of the bowl,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$
$$= 19404 \text{ cm}^3$$

5. A solid sphere of radius r melted and recast into the shape of a solid cone of height r. Find the radius of the base of a cone.

Sol :

Let the radius of cone be R cm.

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 \times r$$
$$4r^3 = R^2 r$$
$$R^2 = 4r^2$$
$$R = 2r$$

6. For the following distribution what is the modal class:

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

Sol :

Marks	Number of students
0-10	$3 - 0 = 3$
10-20	$12 - 3 = 9$
20-30	$27 - 12 = 15$
30-40	$57 - 27 = 30$
40-50	$75 - 57 = 18$
50-60	$80 - 75 = 5$

Class 30-40 has the maximum frequency 30, therefore this is model class.

or

In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. What is the lower limit of the class?

Sol :

Let x be the upper limit and y be the lower limit.

Since the mid value of the class is 10.

Hence, $\frac{x+y}{2} = 10$

$$x + y = 20 \quad \ldots(1)$$

Since width of the class is 6,

$$x - y = 6 \quad \ldots(2)$$

Solving (1) and (2), we get $y = 7$

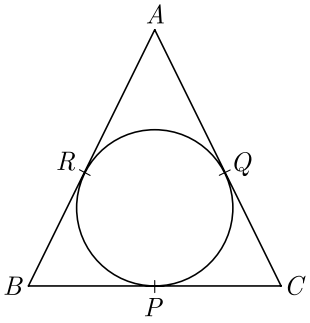
Hence, lower limit of the class is 7.

Section B

7. An isosceles triangle ABC, with $AB = AC$, circumscribes a circle, touching BC at P, AC at Q and AB at R. Prove that the contact point P bisects BC.

Sol :

As per given information we have drawn the figure below.



Since, the tangents drawn from externals points are equal,

$$AR = AQ$$
$$BR = BP$$
$$CP = CQ$$

Now we have, $AB = AC$

$$AR + BR = AQ + CQ$$
$$AR + BP = AQ + CP$$
$$AQ + BP = AQ + CP$$
$$BP = CP$$

Hence, the point of contact P bisects BC.

8. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol :

Let r be the radius of base of cylinder and h be height.

Volume of a right circular cylinder $= 25\frac{1}{7}$ cm

$$\pi r^2 h = \frac{176}{7}$$
$$\frac{22}{7} \times h^2 \times h = \frac{176}{7}$$
$$h^3 = \frac{176}{22} = 8 = 2^3.$$

Hence, height of the cylinder$= 2$ cm.

9. Heights of students of class X are given in the following distribution :

Heights (in cm)	150-155	155-160	160-165	165-170	170-175
Number of students	15	8	20	12	5

Find the modal height.

Sol :

Class 160-165 has the maximum frequency 20, therefore this is model class.

Now $l = 160,\ f_1 = 20,\ f_0 = 8,\ f_2 = 12,\ h = 5$

$$\text{Mode, } M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$
$$= 160 + \left(\frac{20 - 8}{40 - 8 - 12}\right) \times 5$$
$$= 160 + \left(\frac{12}{20}\right) \times 5$$
$$= 163$$

Thus modal height is 163 cm.

10. A school conducted a test (of 100 marks) in English for students of Class X. The marks obtained by students are shown in the following table :

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	1	2	4	15	15	25	15	10	2	1

Find the modal marks.

Sol :

Class 50-60 has the maximum frequency 25, therefore this is model class.

Here $l = 50,\ f_1 = 25,\ f_0 = 15,\ f_2 = 15,\ h = 10$

$$\text{Mode, } M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$
$$= 50 + \frac{25 - 15}{50 - 15 - 15} \times 10$$
$$= 50 + \frac{10}{20} \times 10$$
$$= 50 + 10 = 55$$

or

The mode of a distribution is 55 and the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10.

Find the frequency of the modal class.

Sol :

Mode, $M_o = 55$

Modal class $= 45 - 60$

Frequency of the class preceding,

$f_1 = 5$

Frequency of the class succeeding the modal class,

$f_2 = 10$

Let the frequency of modal class be f.

Mode $M_o = l + \dfrac{f - f_1}{2f - f_1 - f_2} \times h$

$$55 = 45 + \frac{f-5}{2f-5-10} \times 15$$

$$10 = \left(\frac{f-5}{2f-15}\right) \times 15$$

$$\frac{10}{15} = \frac{f-5}{2f-15}$$

$$\frac{2}{3} = \frac{f-15}{2f-15}$$

$$4f - 30 = 3f - 15$$

$$4f - 3f = -15 + 30 \Rightarrow f = 15$$

Section C

11. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

Sol :

Given AP is 45, 39, 33, ...

Here, $a = 45$, $d = 39 - 45 = -6$ and $S_n = 180$

Now $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$$180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)]$$

$$360 = n(90 - 6n + 6$$

$$360 = n(96 - 6n)$$

$$60 = n(16 - n)$$

$$n^2 - 16n + 60 = 0$$

$$n^2 - 6n - 10n + 60 = 0$$

$$n(n-6) - 10(n-6) = 0$$

$$(n-10)(n-6) = 0$$

$$n = 10 \text{ or } n = 6$$

Hence, 10 terms or 6 terms can be taken to get the sum of AP as 180.

Now, sum of 6 terms,

$$S_6 = \frac{6}{2}[2 \times 45 + (6-1)(-6)]$$

$$= 3(90 - 30)$$

$$= 3 \times 60 = 180$$ Hence, verified.

and sum of 10 terms,

$$S_{10} = \frac{10}{2}[2 \times 45 + (10-1)(-6)]$$

$$= 5(90 - 54)$$

$$= 5 \times 36 = 180$$ Hence, verified.

Here we have two values of n because d is negative. There will be negative terms after some positive terms. Thus first 6 term will give sum 180 and after 10 term it will be again 180 because negative term cancel positive term.

Series will be : 45, 39, 33, 27, 21, 15, 9, 3, -3, -9 ...

Here it may be easily seen that sum of initial 6 terms is 180. Sum of next 4 terms is zero. Thus sum of 10 terms is also 180.

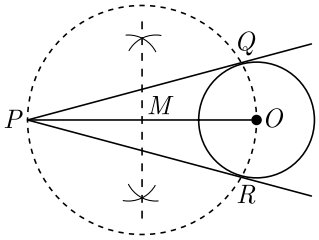
12. Draw a circle of radius 3 cm. From a point P, 7 cm away from centre draw two tangents to the circle. Measure the length of each tangent.

Sol :

Steps of Construction :

1. Draw a line segment PO of length 7 cm.
2. Draw a circle with centre O and radius 3 cm.
3. Draw a perpendicular bisector of PO. Let M be the mid-point of PO.
4. Taking M as centre and OM as radius draw a circle. Let this circle intersects the given circle at the point Q and R.
5. Join PQ and PR. On measuring we get

$$PQ = PR = 6.3 \text{ cm}.$$



or

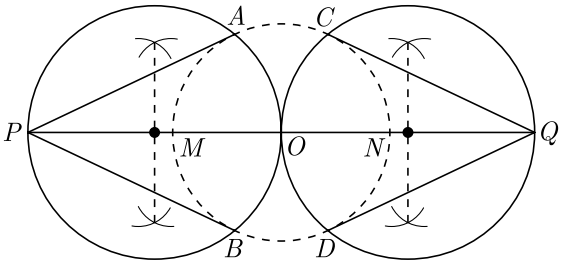
Draw a circle of radius of 3 cm. Take two points P and Q one of its diameter extended on both sides,

each at a distance of 7 cm on opposite sides of its centre. Draw tangents to the circle from these two points.

Sol :

Steps of Construction :

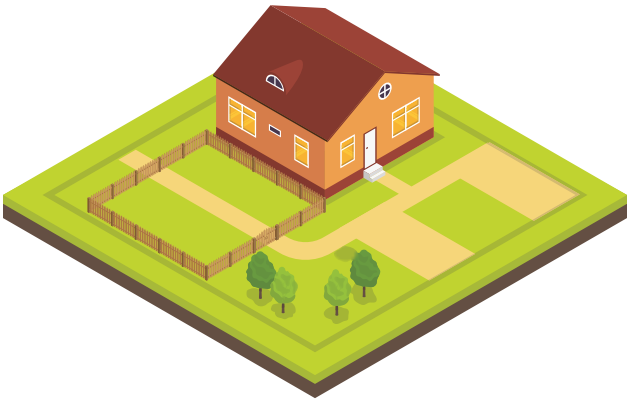
1. Draw a circle with centre O and radius 3 cm.
2. Draw its diameter MON and extend it to both the sides to P and Q. Such that $OP = OQ = 7$ cm.
3. Taking diameters as OP and OQ draw two circles each of which intersects the first circle at the points A, B and C, D respectively.
4. Join PA, PB, QC and QO to get the required tangents



13. Fencing a Backyard : Tina and Shriya have just purchased a purebred German Shepherd, and need to fence in their backyard so the dog can run.

(i) What is the maximum rectangular area they can enclose with 200 ft of fencing, if they use fencing material along all four sides? What are the dimensions of the rectangle?

(ii) What is the maximum area if they use the house as one of the sides? What are the dimensions of this rectangle?



Sol :

(i) Let x represent the width, then $100 - x$ represent the length.

Now area, $A(x) = x(100 - x)$

$$= 100x - x^2$$
$$= -x^2 + 100x - 50^2 + 50^2$$
$$= -(x^2 - 100x + 50^2) + 2500$$
$$= -(x - 50)^2 + 2500$$

From above equation it is clear that $A(x)$ is maximum at $x = 50$ and this maximum value is 2500.

Length, $= 100 - x = 100 - 50 = 50$

Thus dimensions is 50 ft $\times$ 50 ft.

(ii) Let x represent the width if they use the house as one of the sides. In this cases length will be $200 - 2x$.

Now are, $A(x) = x(200 - 2x)$

$$= 200x - 2x^2$$
$$= -2(x^2 - 100x)$$
$$= -2(x^2 - 100x + 50^2 - 50^2)$$
$$= -2(x^2 - 100x + 50^2 - 2500)$$
$$= -2(x^2 - 100x + 50^2) + 5000$$
$$= -2(x - 50) + 5000$$

From above equation it is clear that $A(x)$ is maximum at $x = 50$ and this maximum value is 5000.

14. Silo : A silo is a structure for storing bulk materials. Silos are used in agriculture to store grain or fermented feed known as silage. Silos are commonly used for bulk storage of grain, coal, cement, carbon black, woodchips, food products and sawdust.

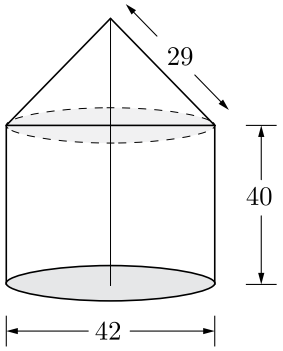
A silo is in the shape of cylinder surmounted by a conical top. The height and diameter of cylindrical part are 40 feet and 42 feet respectively and the slant height of conical part is 29 feet.

(i) How much metal sheet is required to make this silo ?

(ii) What is the storage capacity of silo ?

Sol :

(i) The structure of silo will be as given below.



Area of metal sheet required will be surface area of silo.

Height of cylinder part,

$$h_{cy} = 40 \text{ feet}$$

Diameter of cylindrical part is 42 feet i.e radius is 21 feet. Since radius of cylinder part is equal to the radius of conical part, for both part

$$r = 21 \text{ feet}$$

Slant height of conical part,

$$l = 29 \text{ feet}$$

Height of conical part,

$$h_{co} = \sqrt{29^2 - 21^2} = 20 \text{ feet}$$

Surface area of silo,

$$\begin{aligned} S_{silo} &= C.S.A \text{ of cone} + C.S.A \text{ of cylinder.} \\ &= \pi rl + 2\pi r h_{cy} \\ &= \pi r(l + 2h_{cy}) \\ &= \frac{22}{7} \times 21\big(29 + 2 \times 40 \\ &= 22 \times 3 \times 109 = 7194 \text{ feet}^2 \end{aligned}$$

(ii) Storage capacity of silo is the volume of silo.

Volume,
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h_{co} + \pi r^2 h_{cy} \\ &= \pi r^2\Big(\frac{1}{3}h_{co} + h_{cy}\Big) \\ &= \frac{22}{7} \times 21^2\Big(\frac{20}{3} + 40\Big) \\ &= \frac{22}{7} \times 21^2 \times \frac{140}{3} \\ &= 22 \times 21 \times 140 = 61600 \text{ feet}^3 \end{aligned}$$