CBSE Test Paper 03

Chapter 4 Determinants

1. If A, B and C be the three square matrices such that A=B + C, then Det A is equal to
   a. det C
   b. None of these
   c. det B + detC
   d. det B
2. The system of equations x + 2y = 5, 4x + 8y =20 has
   a. None of these
   b. no solution
   c. a unique solution
   d. infinitely many solutions
3. If A is a square matrix of order 2, then det (adj A) =.
   a. $A^2 = O$
   b. I
   c. $2A^2$
   d. $|A|$
4. Let A = $\begin{bmatrix} 1 & a(b+c) & bc \\ 1 & b(c+a) & ca \\ 1 & c(a+b) & ab \end{bmatrix}$, then Det. A is
   a. None of these
   b. 1+ab+bc+ca
   c. ab +bc+ca
   d. 0
5. If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} =$ k, then $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} =.$
   a. 6k
   b. $\frac{6}{k}$
   c. 2k
   d. 3k

6. If all the elements of a determinant above or below the main diagonal consists of zeros, then the value of the determinant is equal to the product of _______ elements.
7. If A is a matrix of order 3×3, then number of minors in determinant of A are ________.
8. If any two rows or any two columns in a determinant are identical (or proportional), then the value of the determinant is ________.
9. Write minors and cofactors of the elements of $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$.
10. Examine the consistency of the system of equations

    x + 2y = 2; 2x + 3y = 3
11. Evaluate $\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 3 & 2 & 3 \\ 3 & 1 & 3 \end{vmatrix}$.
12. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & 9 \end{bmatrix}$ find |A|.
13. Without expanding, evaluate $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$.
14. Solve the system of linear equations, using matrix method

    2x - y = - 2; 3x + 4y = 3
15. Prove that : $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3\left(a+b+c\right)\left(ab+bc+ca\right)$.
16. Using properties of determinants, prove the following

    $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$ = $a^3 + b^3 + c^3 - 3abc$
17. Find the area of Δ whose vertices are (3, 8) (-4, 2) and (5, 1).
18. Solve by matrix method

    x – y + z = 4

    2x + y – 3 z = 0

    x + y + z = 2

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Solution

1. b. None of these

   Explanation: Because, if A=B+C then |A| = |B+C|

2. a. infinitely many solutions

   Explanation: x + 2y = 5,

   4x + 8y = 20

   $\Rightarrow$ A = $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 5 \\ 20 \end{bmatrix}$

   |A| = 8 -8 = 0

   adjA = $\begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix}$

   now (adj A)B= $\begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 20 \end{bmatrix} = \begin{bmatrix} 40-40 \\ -20+20 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Rightarrow$ (adj A)B = 0

   Since, |A|=0 and (adjA)B=0

   So, the pair of equation have infinitely many solutions

3. d. |A|

   Explanation: Let A be a square matrix of order 2 then |adj A| = |A|, because

   |adj A| =$|A|^{n-1}$ where n is the order of square matrix.

4. d. 0

   Explanation: $\begin{bmatrix} 1 & a(b+c) & bc \\ 1 & b(c+a) & ca \\ 1 & c(a+b) & ab \end{bmatrix}$ $\Rightarrow$ det.A = $\begin{vmatrix} 1 & a(b+c) & bc \\ 1 & b(c+a) & ca \\ 1 & c(a+b) & ab \end{vmatrix}$

   Apply $C_2 \rightarrow C_2 + C_3$

   $\Rightarrow$ det.A = $\begin{vmatrix} 1 & ab+ac+bc & bc \\ 1 & bc+ca+ab & ca \\ 1 & ca+bc+ab & ab \end{vmatrix}$

   $\Rightarrow (ab+bc+ca)\begin{vmatrix} 1 & 1 & bc \\ 1 & 1 & ca \\ 1 & 1 & ab \end{vmatrix}$ = 0

5. a. 6k

Explanation: $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} = 3\begin{vmatrix} 2a & 2b & 2c \\ m & n & p \\ x & y & z \end{vmatrix} = 6\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = 6k$

6. diagonal
7. 9
8. zero
9. Let $\Delta = \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

   M_{11} = Minor of $a_{11} = |3| = 3$ and $A_{11} = (-1)^{1+1}M_{11} = (-1)^2(3) = 3$

   M_{12} = Minor of $a_{12} = |0| = 0$ and $A_{12} = (-1)^{1+2}M_{12} = (-1)^3(0) = 0$

   M_{21} = Minor of $a_{21} = |-4| = -4$ and $A_{21} = (-1)^{1+2}M_{21} = (-1)^3(-4) = 4$

   M_{22} = Minor of $a_{22} = |2| = 2$ and $A_{22} = (-1)^{2+2}M_{22} = (-1)^4(2) = 2$

10. Matrix form of given equations is AX = B

    $\Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

    $\therefore A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

    $\therefore |A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$

    Therefore, Unique solution and hence equations are consistent.

11. $\Delta = 0$ [R_1 and R_2 are identical]

12. Given: $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & 9 \end{bmatrix}$

    $\Rightarrow |A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & 9 \end{vmatrix}$

    Expanding along first row, $1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

    $= \{-9 - (-12)\} - \{-18 - (-15)\} - 2(8 - 5)$

    $= -9 + 12 - (-18 + 15) - 2(3)$

    $= 3(-3) - 6$

    $= 3 + 3 - 6 = 0$

13. $= \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6 \times 17 & 6 \times 3 & 6 \times 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

$= 6 \begin{vmatrix} 17 & 3 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \times 0 = 0$

[R_1 and R_3 are identical]

14. Matrix form of given equations is AX = B

$\Rightarrow \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Here $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$ = 8 - (-3) = 8 + 3 $= 11 \neq 0$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(adj.\,A)\,B$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \frac{1}{11} \begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix}$

$= \begin{bmatrix} \frac{-5}{11} \\ \frac{12}{11} \end{bmatrix}$

Therefore, $x = \frac{-5}{11}$ and $y = \frac{12}{11}$

15. L.H.S. $= \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$[C_1 \to C_1 + C_2 + C_3]$

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$

Taking (a+b+c) common from first column

$= (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$

$[R_2 \to R_2 - R_1 \; and \; R_3 \to R_3 - R_1]$

$$= (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & -b+a \\ 0 & -c+a & 2c+a \end{vmatrix}$$

Expanding along Ist column

$$= (a+b+c).1\begin{vmatrix} 2b+a & a-b \\ a-c & 2c+a \end{vmatrix}$$

$$= (a+b+c)\left[(2b+a)(2c+a)-(a-b)(a-c)\right]$$

$$= (a+b+c)\left[4bc+2ab+2ac+a^2-a^2+ac+ab-bc\right]$$

$$= (a+b+c)(3ab+3bc+3ac)$$

$= 3(a+b+c)(ab+bc+ac)$= R.H.S.

16. According to the question,we have to prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$ = $a^3 + b^3 + c^3$ - 3abc

We shall make use of the properties of determinants in order to prove the required result.

Let LHS = $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

Therefore, on applying $R_1 \rightarrow R_1 + R_3$, We get,

$$\text{LHS} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

On taking common (a + b + c) from R_1, we get

LHS = (a+b+c) $\begin{vmatrix} 1 & 1 & 1 \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

Therefore,on applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get,

LHS = (a + b + c) $\begin{vmatrix} 1 & 0 & 0 \\ a-b & 2b-a-c & b+c-2a \\ b+c & a-b & a-c \end{vmatrix}$

On expanding along R_1, we get,

LHS = (a + b + c) · 1 · { (2b - a - c) (a - c) - (a - b) (b + c - 2a)}

=(a+ b + c)

{2ab - a^2 - ac - 2bc + ac + c^2 - ab - ac + $2a^2$ + b^2 + bc - 2ab}

= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)

= a^3 + b^3 + c^3 - 3abc

17. $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$= \frac{1}{2}[3(2-1) - 8(-4-5) + 1(-4-10)]$

$= \frac{1}{2}[3 + 72 - 14] = \frac{61}{2}$

18. $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$

$= 10 \neq 0$

Also, adj $A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(adJA)$

$= \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

System of equation can be written is

$X = A^{-1}B$

$= \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

x=2, y = -1, z = 1