HOTS (Higher Order Thinking Skills)

Que 1. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

Sol. Let x be the total number of camels.

Then, number of camels in the forest = $\frac{x}{4}$

Number of camels on mountains = $2\sqrt{x}$

And number of camels on the bank or river = 15

Thus, total number of camels = $\frac{x}{4} + 2\sqrt{x} + 15$

Now, by hypothesis, we have

$\frac{x}{4} + 2\sqrt{x} + 15 = x \quad \Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0$

Let $\sqrt{x} = y,$ then x = y^2

$\Rightarrow \quad 3y^2 - 8y - 60 = 0 \quad \Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow \quad 3y(y-6) + 10(y-6) = 0 \quad \Rightarrow \quad (3y+10)(y-6) = 0$

$\Rightarrow$ y = 6 or $y = -\frac{10}{3}$

Now, $y = -\frac{10}{3} \quad \Rightarrow \quad x = \left(-\frac{10}{3}\right)^2 = \frac{100}{3} \quad (\because\ x = y^2)$

But the number of camels cannot be a fraction.

$\therefore$ y = 6 $\Rightarrow$ $x = 6^2 = 36$

Hence, the number of camels = 36

Que 2. Solve the following quadratic equation:

$\boldsymbol{9x^2 - 9\,(a+b)\,x + [2a^2 + 5ab + 2b^2] = 0}$.

Sol. Consider the equation $9x^2 - 9\,(a+b)\,x + [2a^2 + 5ab + 2b^2] = 0$.

Now comparing with $Ax^2 + Bx + C = 0$, we get

$A = 9, B = -9\,(a+b)\ and\ C = [2a^2 + 5ab + 2b^2]$

Now discriminant

$D = B^2 - 4AC$

$= \{-9(a+b)\}^2 - 4 \times 9(2a^2 + 5ab + 2b^2) = 9^2(a+b)^2 - 4 \times 9\,(2a^2 + 5ab + 2b^2)$

$= 9\{9(a+b)^2 - 4(2a^2 + 5ab + 2b^2)\} = 9\{9a^2 + 9b^2 + 18ab - 8a^2 - 20ab - 8b^2\}$

$= 9\{a^2 + b^2 - 2ab\} = 9(a-b)^2$

Now using the quadratic formula,

$x = \frac{-B \pm \sqrt{D}}{2A}$, we get $\quad x = \frac{9(a+b) \pm \sqrt{9(a-b)^2}}{2 \times 9}$

$\Rightarrow \quad x = \frac{9(a+b) \pm 3\,(a-b)}{2 \times 9} \qquad \Rightarrow \quad x = \frac{3(a+b) \pm (a-b)}{6}$

$\Rightarrow \quad x = \frac{(3a+3b)+(a-b)}{6}$ and $x = \frac{(3a+3b)-(a-b)}{6}$

$\Rightarrow \quad x = \frac{(4a+2b)}{6}$ and $x = \frac{(2a+4b)}{6}$

$\Rightarrow \quad x = \frac{2a+b}{3}$ and $x = \frac{a+2b}{3}$ are required solutions.

Que 3. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the order to fil it, find the time in which each pipe would fill the cistern.

Sol. Let, time taken by faster pipe to fill cistern be x minutes
Therefore, time taken by slower pipe to fill the cistern = (x + 3) minutes
Since the faster pipe takes x minutes to fill the cistern.

∴ Portion of the cistern filled by the faster pipe in one minute = $\frac{1}{x}$

Portion of the cistern filled by the slower pipe in one minute = $\frac{1}{x+3}$

Portion of the cistern filled by the two pipes together in one minute = $\frac{1}{\frac{40}{13}} = \frac{13}{40}$

According to question,

$\Rightarrow \quad \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40} \qquad \Rightarrow \quad \frac{x+3+x}{x(x+3)} = \frac{13}{40}$

$\Rightarrow \quad 40(2x+3) = 13x(x+3) \qquad \Rightarrow \quad 80x + 120 = 13x^2 + 39x$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0 \qquad \Rightarrow \quad 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x-5) + 24(x-5) = 0 \qquad \Rightarrow \quad (x-5)(13x+24) = 0$

Either x – 5 = 0 or 13x + 24 = 0

$\Rightarrow \quad x = 5 \quad or \quad x = \frac{-24}{13}$

$\Rightarrow \quad x = 5 \qquad [\because\ x\ cannot\ be\ negative]$

Hence, time taken by faster pipe to fill the cistern = x = 5 minutes
and time taken by slower pipe = x + 3 = 5 + 3 = 8 minutes.

Que 4. In the centre of a rectangular lawn of dimension 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.

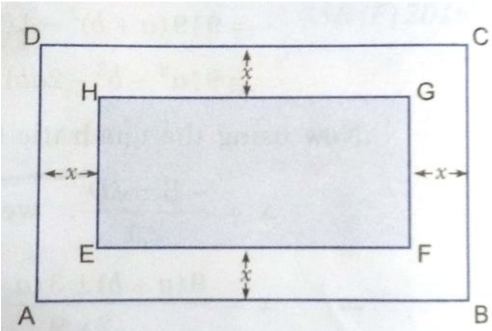

Fig. 4.1

Sol. Let ABCD be rectangular lawn and EFGH be rectangular pound. Let x m be the width of grass area, which is same around the pound.

Given, Length of lawn = 50 m

Width of lawn = 40 m

$\Rightarrow$ Length of pond = $(50 - 2x)$ m

Breadth of pond = $(40 - 2x)$ m

Also given,

Area of grass surrounding the pond = 1184 m_2

$\Rightarrow$ Area of rectangular lawn – Area of pond = 1184 m^2

$\Rightarrow$ $50 \times 40 - \{(50 - 2x) \times (40 - 2x) = 1184$

$\Rightarrow$ $2000 - (2000 - 80x - 100x + 4x^2) = 1184$

$\Rightarrow$ $4x^2 - 180x + 1184 = 0$ $\Rightarrow$ $x^2 - 45x + 296 = 0$

$\Rightarrow$ $x^2 - 37x - 8x + 296 = 0$ $\Rightarrow$ $x(x - 37) - 8(x - 37) = 0$

$\Rightarrow$ $(x - 37)(x - 8) = 0$ $\Rightarrow$ $x - 37 = 0 \; or \; x - 8 = 0$

$\Rightarrow$ $x = 37 \; or \; x = 8$

x = 37 is not possible as in this case length of pond becomes $50 - 2 \times 37 = -24$ (not possible)

Hence, x = 8 is acceptable

$\therefore$ Length of pond = $50 - 2 \times 8 = 34$ m

Breadth of pond = $40 - 2 \times 8 = 24$ m