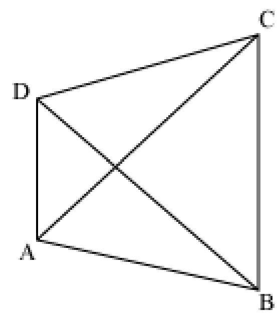
Practical Geometry

Construction of Quadrilateral when One Diagonal and Four Sides Are Given

Look at the following quadrilateral ABCD.



It has ten elements - four sides (AB, BC, CD, and DA), four angles (∠ABC, ∠BCD, ∠CDA, and ∠DAB), and two diagonals (AC and BD).

If we want to construct a unique quadrilateral, then how many elements are required?

To construct a unique quadrilateral, at least five elements are necessary.

Now, suppose we want to construct a quadrilateral whose one diagonal and four sides are given, then how will we construct it?

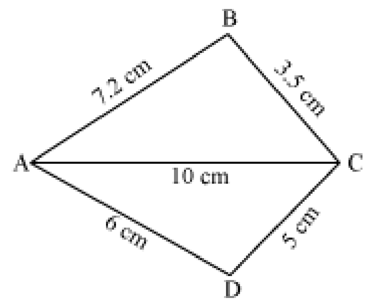
Let us now look at some more examples.

Example 1:

Construct a quadrilateral ABCD with AC = 10 cm, AD = 6 cm, DC = 5 cm, AB = 7.2 cm, and BC = 3.5 cm.

Solution:

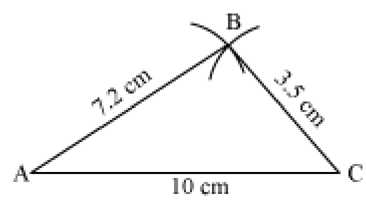
First, we draw a rough sketch of quadrilateral ABCD and indicate the given lengths as shown below.



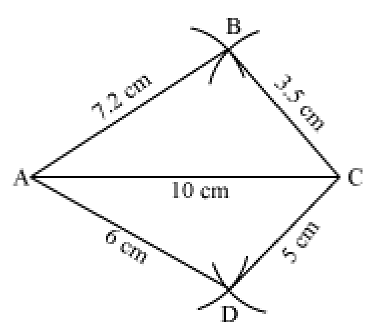
The steps of construction are as follows.

1. Draw AC = 10 cm.

2. Then, draw an arc taking A as centre and 7.2 cm as the radius. Now, draw another arc with C as the centre and 3.5 cm as the radius cutting the previous arc at B. Then, join AB and BC.



3. Draw an arc on the opposite side of B by taking A as centre and 6 cm as the radius. Then, draw another arc with C as the centre and 5 cm as the radius cutting the previous arc at D. Then, join AD and CD.



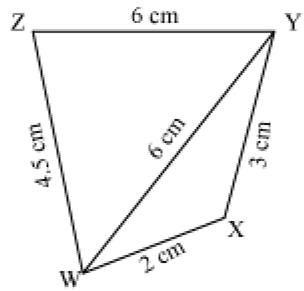
Thus, ABCD is the required quadrilateral.

Example 2:

Can you construct a quadrilateral WXYZ with WY = 6 cm, WX = 2cm, XY = 3 cm, YZ = 6 cm, and WZ = 4.5 cm? Justify your answer.

Solution:

First, we draw a rough sketch of the quadrilateral and indicate the given lengths as shown below.



Here, in ΔWXY,

WX + XY = 2 + 3 = 5 **<** WY = 6 cm

Therefore, WX + XY < WY

⇒ Sum of two sides < Third side

Therefore, ΔWXY is not possible. Thus, we cannot construct a quadrilateral with the given lengths.

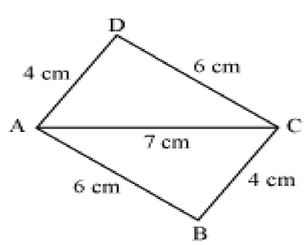
Example 3:

Construct a parallelogram whose adjacent sides are 6 cm and 4 cm and one of its diagonals is 7 cm.

Solution:

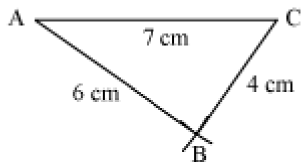
We know that opposite sides of a parallelogram are equal. If ABCD is a parallelogram where the adjacent sides AB and BC are of lengths 6 cm and 4 cm respectively and the diagonal AC is of length 7 cm, then CD = 6 cm and AD = 4 cm

Now, we have the parallelogram ABCD, where AB = CD = 6 cm, BC = AD = 4 cm, and AC = 7 cm. For this, we have to follow the below given steps.

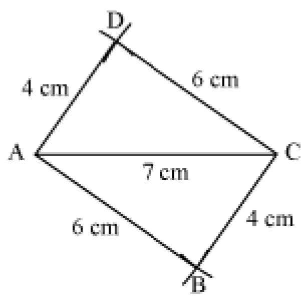


1. Draw AC = 7 cm

2. Draw an arc taking A as centre and 6 cm as the radius. Now, draw another arc with C as the centre and 4 cm as the radius cutting the previous arc at C. Then, join AB and BC.



3. Now, draw an arc taking A as centre and 4 cm as the radius. Then, draw another arc with C as the centre and 6 cm as the radius cutting the previous arc at D. Then, join AD and CD.



Thus, ABCD is the required parallelogram.

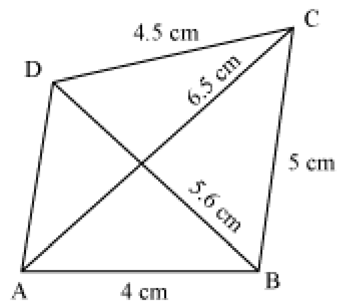
Construction of a Quadrilateral When Three Sides and Both Diagonals Are Given

We know that to construct a quadrilateral, we require five elements. Now suppose three sides and the two diagonals of a quadrilateral are given.

Then, **how will we construct the quadrilateral with these five elements?**

Let us construct a quadrilateral ABCD in which AB = 4 cm, BC = 5 cm, CD = 4.5 cm, AC = 6.5 cm, and BD = 5.6 cm.

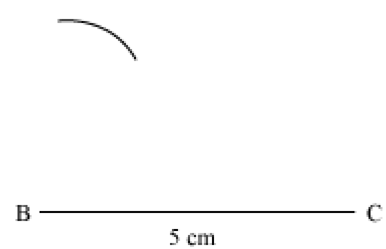
First we draw a rough sketch of the quadrilateral ABCD and indicate the given lengths in the figure as shown below.



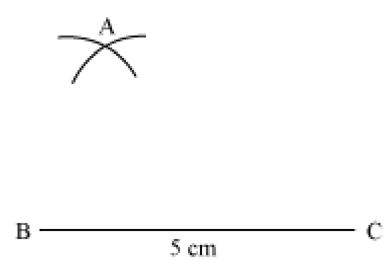
To construct the quadrilateral, we follow the below given steps.

1. Draw BC = 5 cm (the side opposite to the unknown side).

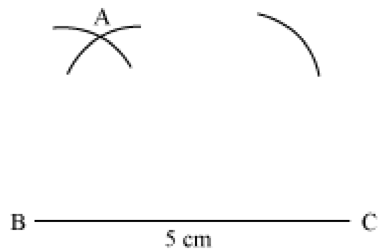
2. Then, draw an arc taking B as centre and AB (4 cm) as radius.



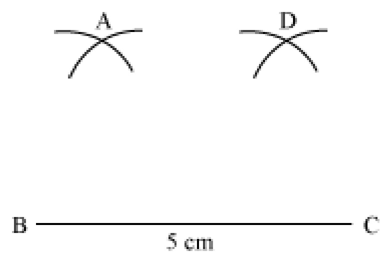
3. Then, draw another arc taking C as centre and AC (6.5 cm) as radius to intersect the previously drawn arc at a point A.



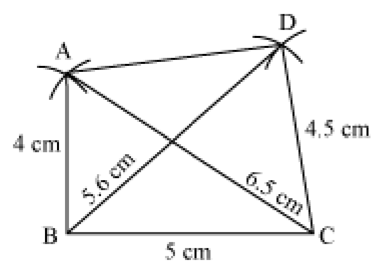
4. Then, draw an arc taking B as centre and BD (5.6 cm) as radius on the same side of BC in which point A lies.



5. Then, draw another arc taking C as centre and CD (4.5 cm) as radius to intersect the arc drawn in step IV at point D.



6. Then, join AB, AD, CD, AC, and BD.



This is the required quadrilateral ABCD.

In this way, we can construct a quadrilateral if we are given three of its sides and the two diagonals.

Let us now solve another example.

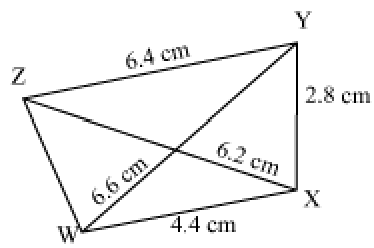
Example 1:

Construct a quadrilateral WXYZ in which WX = 4.4 cm, XY = 2.8 cm,

YZ = 6.4 cm, XZ = 6.2 cm, and WY = 6.6 cm.

Solution:

First, draw a rough sketch of quadrilateral WXYZ and indicate the given lengths as shown below.



The steps of construction are as follows.

(i) Draw XY = 2.8 cm.

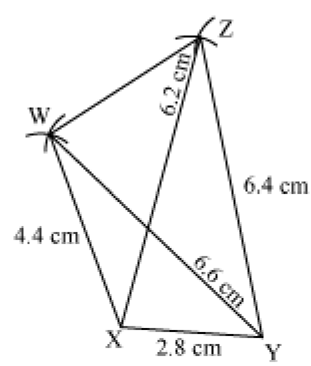
(ii) Draw an arc taking X as centre and WX (4.4 cm) as radius.

(iii) Draw another arc taking Y as centre and WY (6.6 cm) as radius to intersect the previous arc at W.

(iv) Again, draw an arc taking X as centre and XZ (6.2 cm) as radius on the same side of XY in which point W lies.

(v) Draw another arc taking Y as centre and YZ (6.4 cm) as radius to intersect the arc drawn in previous step at Z.

(vi) Join XW, WZ, YZ, XZ, and WY.



Thus, WXYZ is the required quadrilateral.

Construction of Quadrilateral when Two Adjacent Sides and Three Angles Are Given

Suppose we have to construct a quadrilateral ABCD and the following elements of the quadrilateral are given.

AB = 5.5 cm, BC = 4.9 cm, ∠B = 120°, ∠C = 90°, ∠A = 75°.

How will we construct the quadrilateral with this information?

In this way, we can construct a quadrilateral if two adjacent sides and three angles are given.

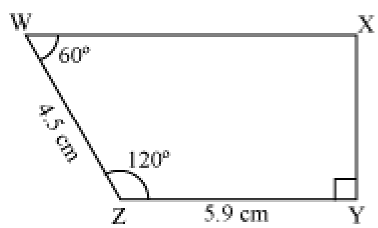
Let us now look at some more examples to understand this concept better.

Example 1:

Construct a quadrilateral WXYZ with ZY = 5.9 cm, WZ = 4.5 cm, ∠Y = 90°, ∠Z = 120°, and ∠W = 60°.

Solution:

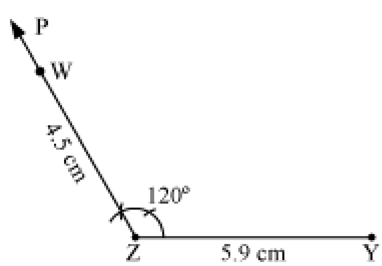
First, we draw a rough sketch of the quadrilateral WXYZ and indicate the given lengths as shown in the following figure.



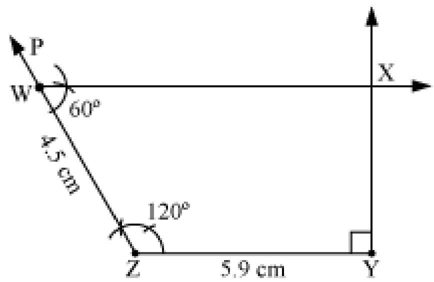
The steps of construction are as follows.

1. First, we draw a line segment ZY = 5.9 cm.

2. Then we construct an angle of 120° at Z. Mark the point W, such that WZ = 4.5 cm.



3. Now, we construct an angle of 60° at W and an angle of 90° at Y to meet the two line segments at X.



Thus, WXYZ is the required quadrilateral.

Example 2:

Can you construct a quadrilateral ABCD with AB = 5.5 cm, BC = 4.9 cm, ∠A = 130°, ∠B = 145°, and ∠C = 95°?

Solution:

We know that the sum of all the angles of a quadrilateral is 360°.

Here, we are given that ∠A = 130°, ∠B = 145°, and ∠C = 95°.

∴ ∠A + ∠B + ∠C = 130° + 145° + 95°

= 370°

This is not possible.

Thus, we cannot construct a quadrilateral with the given measurements.

Construction Of Quadrilaterals When Three Sides And Two Included Angles Are Given

We know that to construct a quadrilateral, we require five elements. Suppose three sides and two angles included between them are given. Then how will we construct the quadrilateral?

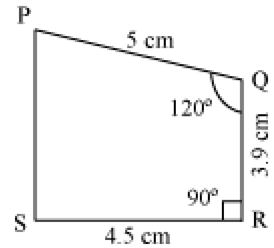
Let us now look at one more example to understand the construction of a quadrilateral better.

Example:

Construct a quadrilateral PQRS with SR = 4.5 cm, QR = 3.9 cm, PQ = 5 cm, ∠R = 90°, and ∠Q = 120°.

Solution:

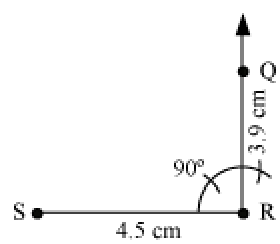
First, we draw a rough sketch of the quadrilateral PQRS as shown in the following figure.



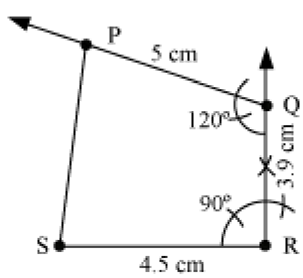
The steps of construction are as follows.

1. We draw a line segment SR of length 4.5 cm.

2. Then we construct an angle of 90° at R and locate the point Q such that RQ = 3.9 cm.



3. Now, we construct an angle of 120° at Q and locate the point P such that QP = 5 cm. Then we join PS.



Thus, PQRS is the required quadrilateral of given measures.

Construction of Special Quadrilaterals

Suppose the diagonals of the rhombus PQRS are PR = 5.8 cm and SQ = 6 cm.

Can we construct the rhombus PQRS?

Now, let us consider the case of a square.

Suppose a side of the square ABCD is 6.2 cm. **Can we construct the square?**

Let us now consider the case of a rectangle.

Suppose we have to construct a **rectangle** STUV with ST = 5 cm and TU = 4 cm. How will we construct it?

We can construct square and rectangle if the measurements of their sides are not given. Let us learn the same.

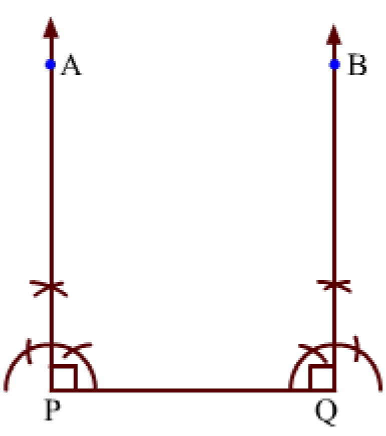
Constructing a square when the length of the side is not known:

We know the properties of square that all of its sides are of equal length and each angle measures 90°. So, if the length of the side of square is not given then we can construct a square having side of any length.

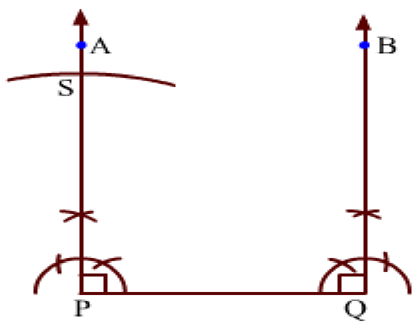
The steps of construction are as follows:

(1) Draw a line segment PQ of any length.

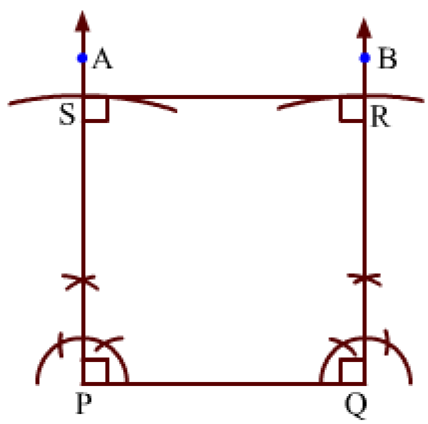
(2) Draw a ray PA perpendicular to PQ at point P. Also, draw another ray QB perpendicular to PQ at point Q.



(3) Place the point of compass at point P and take the radius equal to the length of PQ. Taking this radius and placing the point of compass at point P, draw an arc intersecting the ray PA at S.



(4) Without changing the radius and placing the point of compass at point Q, draw another arc intersecting the ray QB at R. Join R to S.



Thus, we obtain a square PQRS in which the length of side is not known.

Similarly, we can construct a rectangle of unknown length and breadth.

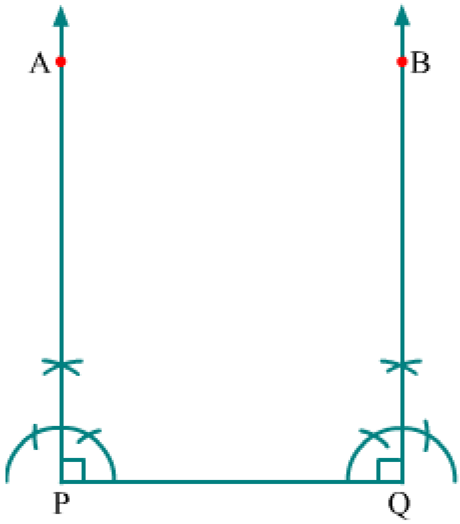
Constructing a rectangle when length and breadth are not known:

In a rectangle, opposite sides are of equal length and each angle measures 90°. These properties can be used to construct a rectangle whose length and breadth are not given.

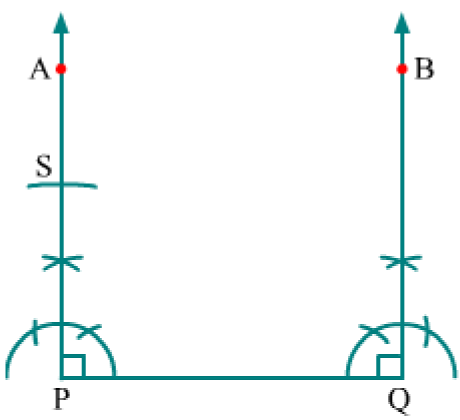
The steps of construction are as follows:

(1) Draw a line segment PQ of any length.

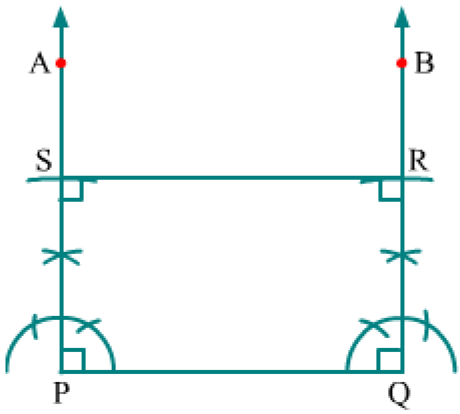
(2) Draw a ray PA perpendicular to PQ at point P. Also, draw another ray QB perpendicular to PQ at point Q.



(3) Taking any radius which is not equal to the length of PQ and placing the point of compass at point P, draw an arc intersecting the ray PA at S.



(4) Without changing the radius and placing the point of compass at point Q, draw another arc intersecting the ray QB at R. Join R to S.



Thus, we obtain a rectangle PQRS in which the length and breadth are not known.