Chapter 6 Polynomials and Polynomial Functions

Ex 6.3

Answer 1e.

When a function becomes the domain of another function, the resultant function is said to be the composition of functions. The composition of a function g with a function f is denoted as $h(x) = g(f(x))$, where $h(x)$ is the resulting function.

The given sentence can be completed as, "The function $h(x) = g(f(x))$ is called the **composition** of the function g with the function f."

Answer 1gp.

Substitute $-2x^{2/3}$ for $f(x)$, and $7x^{2/3}$ for $g(x)$ in $f(x) + g(x)$.

$f(x) + g(x) = -2x^{2/3} + 7x^{2/3}$

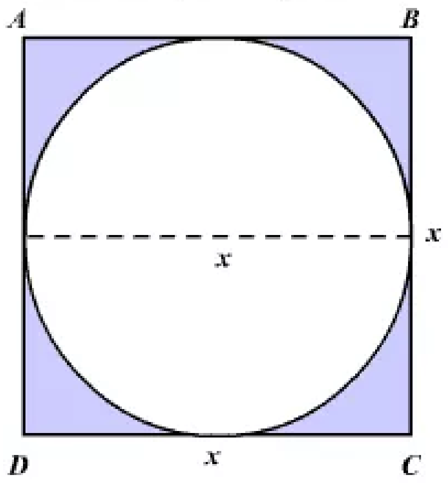
Combine the like terms using the distributive property.

$$\begin{aligned} -2x^{2/3} + 7x^{2/3} &= (-2+7)x^{2/3} \\ &= 5x^{2/3} \end{aligned}$$

Therefore, $f(x) + g(x)$ evaluates to $5x^{2/3}$.

Answer 1mr.

Redraw the given figure.



(a) The area of the square is given by the formula $A = a^2$, where a is the side of the square.

Substitute $s(x)$ for A, and x for a in the formula.
$s(x) = x^2$

The function for the area of square is $s(x) = x^2$.

(b) The area of the circle is given by the formula $A = \pi r^2$, where r is the radius of the circle.

We have the radius of circle as $\frac{x}{2}$.

Substitute $s(x)$ for A, and $\frac{x}{2}$ for r in the formula.

$$\begin{aligned} A &= \pi\left(\frac{x}{2}\right)^2 \\ &= \pi\left(\frac{x^2}{4}\right) \\ &= \frac{\pi x^2}{4} \end{aligned}$$

The function for the area of the circle is $c(x) = \frac{\pi x^2}{4}$.

(c) We know that the area of shaded region is the difference between the area of the square and the area of the circle.

$$\begin{aligned} r(x) &= s(x) - c(x) \\ &= x^2 - \frac{\pi x^2}{4} \\ &= x^2\left(1 - \frac{\pi}{4}\right) \end{aligned}$$

The area of shaded region is $r(x) = x^2\left(1 - \frac{\pi}{4}\right)$.

Answer 2e.

We know that , a function $f : R^{+} \to R$ defined by $f(x) = x^{n}$ where $n \in R$ is a called power function.

Eg:- $f(x) = \sqrt{x}$ is the square root function defined from $[0, \infty)$ to R. The power of $f(x)$ is $\frac{1}{2}$, because $\sqrt{x} = x^{\frac{1}{2}}$

The sum of two power functions need not be a power function.

For example, Suppose $f(x) = x^{\frac{1}{2}}$ and $g(x) = x^{\frac{3}{4}}$

Then

$$f(x) + g(x) = x^{\frac{1}{2}} + x^{\frac{3}{4}}$$

$$= x^{\frac{1}{2}}(1 + x^{\frac{1}{4}})$$

This is not in the form of x^{n}

$\therefore$ The sum of two power functions need not be a power function.

Answer 2gp.

Let us consider the function $f(x) = -2x^{2/3}$

And $g(x) = 7x^{2/3}$

Claim: To find $f(x) - g(x)$

$\therefore \quad f(x) - g(x) = -2x^{-2/3} - 7x^{2/3}$

$f(x) - g(x) = (-2 - 7)x^{2/3}$ [Use the distributive rule]

$f(x) - g(x) = -9x^{2/3}$

$\therefore \quad \boxed{f(x) - g(x) = -9x^{2/3}}$

Answer 2mr.

(a) Let us consider the formula for the volume 'V' of a sphere in terms of its surface area 'S' is given by $V = 3^{-1} \cdot (4\pi)^{-1/2} \cdot (S^3)^{1/2}$

Consider, $V = 3^{-1}(4\pi)^{-1/2} \cdot (S^3)^{1/2}$ [write the orginal formula]

$$\Rightarrow \quad V = \frac{1}{3} \cdot \frac{1}{(4\pi)^{1/2}} \cdot (S^3)^{1/2} \qquad \left[\text{Write the rule } a^{-n} = \frac{1}{a^n}\right]$$

$$\Rightarrow \quad V = \frac{1}{3} \cdot \frac{1}{4^{\frac{1}{2}} \cdot \pi^{\frac{1}{2}}} \cdot (S^3)^{\frac{1}{2}} \qquad \left[\text{Use the rule } (a \cdot b)^m = a^m \cdot b^m\right]$$

$$\Rightarrow \quad V = \frac{1}{3} \cdot \frac{1}{(2 \cdot 2)^{1/2} \cdot \pi^{1/2}} \cdot (S^3)^{1/2} \qquad [\text{Write 4 as 2.2}]$$

$$\Rightarrow \quad V = \frac{1}{3} \cdot \frac{1}{(2^{1+1})^{1/2} \cdot \pi^{1/2}} \cdot (S^3)^{1/2} \qquad \left[\text{Use the rule } a^m \cdot a^n = a^{m+n}\right]$$

$$\Rightarrow \quad V = \frac{1}{3} \cdot \frac{1}{(2^2)^{1/2} \cdot \pi^{1/2}} \cdot (S^3)^{1/2}$$

$$\Rightarrow \quad V = \frac{1}{3} \cdot \frac{1}{2^{2 \cdot 1/2} \cdot \pi^{1/2}} \cdot (S^3)^{1/2} \qquad \left[\text{Use the rule } (a^m)^n = a^{m \cdot n}\right]$$

$$\Rightarrow \quad V = \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{\pi^{1/2}} (S^3)^{1/2}$$

$$\Rightarrow \quad V = \frac{1}{6} \cdot \frac{(S^3)^{\frac{1}{2}}}{\pi^{\frac{1}{2}}} \cdot$$

$$\Rightarrow \quad V = \frac{1}{6} \left(\frac{S^3}{\pi}\right)^{\frac{1}{2}}$$

$$\Rightarrow \quad V = \frac{1}{6} \cdot \sqrt{\frac{S^3}{\pi}} \qquad \left[\text{Use the rule, } a^{\frac{1}{n}} = \sqrt[n]{a} \text{ for any integer n} > 1\right]$$

$$\therefore \quad \boxed{V = \frac{1}{6} \sqrt{\frac{S^3}{\pi}}}$$

(b) Let a candle pin bowling ball has a surface area S is 79 square inches.

Claim To find Volume "V"

$$V = \frac{1}{6}\sqrt{\frac{S^3}{\pi}} \qquad \left[\text{Write the formula for Volume } V\right]$$

$$\Rightarrow \quad V = \frac{1}{6}\sqrt{\frac{(79)^3}{\pi}} \qquad \left[\text{Substitute 79 for } S\right]$$

$$\Rightarrow \quad V = \frac{1}{6}\sqrt{\frac{493,039}{\pi}}$$

$$\Rightarrow \quad V = \frac{1}{6}\sqrt{\frac{493,039}{3.142}} \qquad \left[\text{Substitute 3.142 for } \pi\right]$$

$$\Rightarrow \quad V = \frac{1}{6}\sqrt{156,918.8415}$$

$$\Rightarrow \quad V = \frac{1}{6}(396.1298291) \qquad \left[\text{Use the calculator to find } \sqrt{156,918.84}\right]$$

$$\Rightarrow \quad V = 66.02163818$$

The volume $\boxed{V = 66.02163818}$

(c) Let us consider 10-pin bowling ball has a surface of about "S" is 232 square inches.

To find the 10 pin bowling ball volume

$$\therefore \quad V = \frac{1}{6}\sqrt{\frac{S^3}{\pi}} \qquad \left[\begin{array}{l}\text{Write the formula for} \\ \text{volume "}V\text{"}\end{array}\right]$$

$$\Rightarrow \quad V = \frac{1}{6}\sqrt{\frac{(232)^3}{\pi}} \qquad \left[\text{Substitute 232 for } S\right]$$

$$\Rightarrow \quad V = \frac{1}{6}\sqrt{\frac{12,487,167}{\pi}}$$

$$\Rightarrow \quad V = \frac{1}{6}\sqrt{\frac{12,487,168}{3.142}} \qquad \left[\text{ Substitute 3.142 for } \pi\right]$$

$$\Rightarrow \quad V = \frac{1}{6}\sqrt{3,974,273.711}$$

$$\Rightarrow \quad V = \frac{1}{6}[1,993.558053]$$

$$\Rightarrow \quad V = \boxed{332.2596755}$$

The volume V is 332.259655

(d) A candle pin bowling ball has a surface are is 79 square inches.
A 10-pin bowling ball has a surface area is 232 square inches.
The volume of a candle P in bowling ball has a volume is 66.02163818
The volume of a candle P in bowling hall has a volume is 332.2596755

The surface area of a candle in bowling ball is three times of the candle in bowling ball is three times of the 10- pin bowling ball approximately. The volume of the candle pin bowling ball is five times of the 10-pin bowling ball approximately.

Answer 3e.

Substitute $-3x^{1/3}+4x^{1/2}$ for $f(x)$, and $5x^{1/3}+4x^{1/2}$ for $g(x)$ in $f(x)+g(x)$.

$$f(x)+g(x) = -3x^{1/3}+4x^{1/2}+5x^{1/3}+4x^{1/2}$$

Group the like terms.

$$-3x^{1/3}+4x^{1/2}+5x^{1/3}+4x^{1/2} = \left(-3x^{1/3}+5x^{1/3}\right)+\left(4x^{1/2}+4x^{1/2}\right)$$

Combine the like terms using the distributive property.

$$\begin{aligned}\left(-3x^{1/3}+5x^{1/3}\right)+\left(4x^{1/2}+4x^{1/2}\right) &= (-3+5)\,x^{1/3}+(4+4)\,x^{1/2}\\ &= 2x^{1/3}+8x^{1/2}\end{aligned}$$

The domain of both $f(x)$ and $g(x)$ is the set of non-negative real numbers, since the value of x cannot be a negative real number. Thus, $f(x)+g(x)$ also has the same domain.

Therefore, $f(x)+g(x)$ evaluates to $2x^{1/3}+8x^{1/2}$ with the domain as the set of non-negative real numbers.

Answer 3gp.

We have $f+g$ as $5x^{2/3}$, and $f-g$ as $-9x^{2/3}$.

We know that domain is the set of x-values. The domain of both $f(x)$ and $g(x)$ is the set of non-negative real numbers, since the value of x cannot be a negative real number.

Therefore, $f+g$ and $f-g$ will also have the same domain, the set of non-negative real numbers.

Answer 3mr.

Let x represents your sales.
We know that if sales are greater than $100,000, then the bonus will be 3% of your sales.

This means bonus = $\frac{3}{100}(x-100{,}000)$ or $0.03(x-100{,}000)$.

Let us find $f(g(x))$. For this, substitute $0.03x$ for $g(x)$.
$f(0.03x) = 0.03x - 100{,}000$

Next, find $g(f(x))$. For this, substitute $x - 100{,}000$ for $f(x)$.
$g(x - 100{,}000) = 0.03(x - 100{,}000)$

We can see that $g(f(x))$ represents the condition for bonus.

Answer 4e.

Let us consider the function $f(x) = -3x^{1/3} + 4x^{1/2}$

And $g(x) = 5x^{1/3} + 4x^{1/2}$

Claim: To find $f(x) + g(x)$

Consider $f(x) + g(x) = -3x^{1/3} + 4x^{1/2} + 5x^{1/3} + 4x^{1/2}$

$\therefore f(x) = -3x^{1/3} + 4x^{1/2}$

$g(x) = 5x^{1/3} + 4x^{1/2}$

$\Rightarrow \quad f(x) + g(x) = -3x^{1/3} + 5x^{1/3} + 4x^{1/2} + 4x^{1/2}$

$\Rightarrow \quad f(x) + g(x) = (-3+5)x^{1/3} + (4+4)x^{1/2}$ [Use the distributive property]

$\Rightarrow \quad f(x) + g(x) = 2x^{1/3} + 8x^{1/2}$ [Simplify]

$\therefore \quad \boxed{f(x) + g(x) = 2x^{1/3} + 8x^{1/2}}$

The function f and g each have the same domain all non negative real numbers. So, the domain of $f(x) + g(x)$ also consists of all non negative real numbers.

Answer 4gp.

Let us consider the functions $f(x) = 3x$

And $g(x) = x^{1/5}$

Claim: To find $f(x).g(x)$

$\therefore \quad f(x) \cdot g(x) = 3x \cdot x^{1/5}$

$f(x) \cdot g(x) = 3x^{1+\frac{1}{5}}$ [Use the rule $a^m \cdot a^n = a^{m+n}$]

$f(x) \cdot g(x) = 3x^{\frac{5+1}{5}}$ [The L.C.M. of 1 and 5 is 5]

$\therefore \quad \boxed{f(x) \cdot g(x) = 3x^{6/5}}$

Answer 5e.

Substitute $-3x^{1/3}+4x^{1/2}$ for $f(x)$ in $f(x)+f(x)$.

$$f(x)+f(x) = -3x^{1/3}+4x^{1/2}+\left(-3x^{1/3}\right)+4x^{1/2}$$

Group the like terms.

$$-3x^{1/3}+4x^{1/2}+\left(-3x^{1/3}\right)+4x^{1/2} = \left[-3x^{1/3}+\left(-3x^{1/3}\right)\right]+\left(4x^{1/2}+4x^{1/2}\right)$$

Combine the like terms using the distributive property.

$$\begin{aligned}\left[-3x^{1/3}+\left(-3x^{1/3}\right)\right]+\left(4x^{1/2}+4x^{1/2}\right) &= \left[-3+(-3)\right]x^{1/3}+(4+4)x^{1/2} \\ &= -6x^{1/3}+8x^{1/2}\end{aligned}$$

The domain of $f(x)$ is the set of non-negative real numbers, since the value of x cannot be a negative real number. Thus, $f(x)+f(x)$ also has the same domain.

Therefore, $f(x)+f(x)$ evaluates to $-6x^{1/3}+8x^{1/2}$ with the domain as the set of non-negative real numbers.

Answer 5gp.

Substitute $3x$ for $f(x)$, and $x^{1/5}$ for $g(x)$ in $\dfrac{f(x)}{g(x)}$.

$$\frac{f(x)}{g(x)} = \frac{3x}{x^{1/5}}$$

Use the quotient of powers property and simplify.

$$\begin{aligned}\frac{3x}{x^{1/5}} &= 3x^{1-1/5} \\ &= 3x^{4/5}\end{aligned}$$

Thus, $\dfrac{f(x)}{g(x)} = 3x^{4/5}$.

Answer 5mr.

Let the two different functions be $f(x) = \frac{1}{x}$ and $f(x) = \frac{x}{x^2}$.

In order to find $f(f(x))$, we can substitute either $\frac{1}{x}$ or $\frac{x}{x^2}$ for $f(x)$ in $f(f(x))$. Similarly, we can find the resulting $f(x)$ by replacing x with either $\frac{1}{x}$ or $\frac{x}{x^2}$.

Let us substitute $\frac{1}{x}$ for $f(x)$ in $f(f(x))$.

$$f(f(x)) = f\left(\frac{1}{x}\right)$$

Find $f\left(\frac{1}{x}\right)$ by replacing x in $f(x) = \frac{1}{x}$ with $\frac{x}{x^2}$.

$$\begin{aligned} f\left(\frac{1}{x}\right) &= \frac{1}{\frac{x}{x^2}} \\ &= \frac{x^2}{x} \\ &= x \end{aligned}$$

We get $f(f(x))$ as x.

Therefore, the two different functions may be $f(x) = \frac{1}{x}$ and $f(x) = \frac{x}{x^2}$.

Answer 6e.

Let us consider the function $g(x) = 5x^{1/3} + 4x^{1/2}$

To find the value of $f(x) + g(x)$

Consider $g(x) + g(x) = 5x^{1/3} + 4x^{1/2} + 5x^{1/3} + 4x^{1/2}$ $\quad \left[g(x) = 5x^{1/3} + 4x^{1/2}\right]$

$\Rightarrow \quad g(x) + g(x) = 5x^{1/3} + 5x^{1/3} + 4x^{1/2} + 4x^{1/2}$

$\Rightarrow \quad g(x) + g(x) = (5+5)x^{1/3} + (4+4)x^{1/2}$

$\Rightarrow \quad g(x) + g(x) = 10x^{1/3} + 8x^{1/2}$ $\quad$ [Use the distributive property]

$\Rightarrow \quad g(x) + g(x) = 10x^{1/3} + 8x^{1/2}$ $\quad$ [Add]

The function "f" and "g" each have the same domain all non negative real numbers. So, the domain of $g(x) + g(x)$ also consists of all non negative real numbers.

Answer 6gp.

Let us consider the functions $f(x)=3x$

And $g(x)=x^{1/5}$

The functions f and g each have domain all non zero numbers. So, the domain of $f.g$ and $\frac{f}{g}$ also consist of all non zero real numbers.

Answer 6mr.

Let use consider the expression $\left(\frac{16^{1/2}}{4^{1/2}}\right)^5$

$$\therefore \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=\left[\left(\frac{16}{4}\right)^{1/2}\right]^5 \qquad \left[\text{Use the rule } \frac{a^m}{b^m}=\left(\frac{a}{b}\right)^m\right]$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=\left(\frac{16}{4}\right)^{\frac{1}{2}\cdot 5} \qquad \left[\text{Use the rule } \left(a^m\right)^n=a^{m\cdot n}\right]$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=\left(\frac{16}{4}\right)^{\frac{5}{2}}$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=\left(4^{2-1}\right)^{\frac{5}{2}} \qquad \left[\text{Use the rule } \frac{a^m}{a^n}=a^{m-n}\right]$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=\left(4^1\right)^{\frac{5}{2}}$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=(2\cdot 2)^{\frac{5}{2}} \qquad \left[\text{Write 4 as } 2\cdot 2\right]$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=\left(2^{1+1}\right)^{\frac{5}{2}} \qquad \left[\text{Use the rule } a^m\cdot a^n=a^{m+n}\right]$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=\left(2^2\right)^{5/2}$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=2^{2\cdot\frac{5}{2}} \qquad \left[\text{Use the rule } \left(a^m\right)^n=a^{m\cdot n}\right]$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=2^5$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=2\cdot 2\cdot 2\cdot 2\cdot 2$$

$$\Rightarrow \quad \left(\frac{16^{1/2}}{4^{1/2}}\right)^5=32$$

$$\therefore \quad \boxed{\left(\frac{16^{1/2}}{4^{1/2}}\right)^5=32}$$

Answer 7e.

Substitute $-3x^{1/3}+4x^{1/2}$ for $f(x)$, and $5x^{1/3}+4x^{1/2}$ for $g(x)$ in $f(x)-g(x)$.

$$f(x)-g(x) = -3x^{1/3}+4x^{1/2}-\left(5x^{1/3}+4x^{1/2}\right)$$

Open the parentheses using the distributive property.

$$-3x^{1/3}+4x^{1/2}-\left(5x^{1/3}+4x^{1/2}\right) = -3x^{1/3}+4x^{1/2}+\left(-5x^{1/3}\right)+\left(-4x^{1/2}\right)$$

Group the like terms.

$$-3x^{1/3}+4x^{1/2}+\left(-5x^{1/3}\right)+\left(-4x^{1/2}\right) = \left[-3x^{1/3}+\left(-5x^{1/3}\right)\right]+\left[4x^{1/2}+\left(-4x^{1/2}\right)\right]$$

Combine the like terms using the distributive property.

$$\begin{aligned}\left[-3x^{1/3}+\left(-5x^{1/3}\right)\right]+\left[4x^{1/2}+\left(-4x^{1/2}\right)\right] &= \left[-3+(-5)\right]x^{1/3}+\left[4+(-4)\right]x^{1/2}\\ &= -8x^{1/3}+0x^{1/2}\\ &= -8x^{1/3}\end{aligned}$$

The domain of $f(x)$ and $g(x)$ is the set of non-negative real numbers, since the value of x cannot be a negative real number. Thus, $f(x)-g(x)$ also has the same domain.

Therefore, $f(x)-g(x)$ evaluates to $-8x^{1/3}$ with the domain as the set of non-negative real numbers.

Answer 7gp.

The heart rate of rhino can be determined using the relation $r(m)\cdot s(m) = \left(1.446\times 10^9\right)m^{-0.05}$.

Substitute 1.7×10^5 for m in the equation.

$$r(m)\cdot s(m) = \left(1.446\times 10^9\right)\left(1.7\times 10^5\right)^{-0.05}$$

Use the power of a product property and simplify.

$$\begin{aligned}r(m)\cdot s(m) &= \left(1.446\times 10^9\right)\left(1.7^{-0.05}\times 10^{-0.25}\right)\\ &\approx \left(1.446\times 10^9\right)\left(0.974\times 10^{-0.25}\right)\end{aligned}$$

Use multiplication properties to rewrite and simplify.

$$\begin{aligned}r(m)\cdot s(m) &= (1.446\times 0.974)\left(10^9\times 10^{-0.25}\right)\\ &\approx 1.41\left(10^9\times 10^{-0.25}\right)\end{aligned}$$

Use the product of powers property.

$$r(m)\cdot s(m) \approx 1.41\times 10^{8.75}$$

For the given body mass, the rhino's number of heart beats is approximately $1.41\times 10^{8.75}$.

Answer 7mr.

Solve the given formula for r.

Divide each side by $\frac{4}{3}\pi$.

$$\frac{V}{\frac{4}{3}\pi} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi}$$

$$\frac{3V}{4\pi} = r^3$$

$$r^3 = \frac{3V}{4\pi}$$

Take the cube root on both the sides.

$$\sqrt[3]{r^3} = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Substitute 900 for V, and 3.14 for π.

$$r = \sqrt[3]{\frac{3(900)}{4(3.14)}}$$

Simplify.

$$\sqrt[3]{\frac{3(900)}{4(3.14)}} = \sqrt[3]{\frac{2700}{12.56}}$$

$$\approx \sqrt[3]{214.97}$$

$$\approx 5.99$$

Therefore, the radius of the sphere will be about 5.99 inches.

Answer 8e.

Let us consider the functions $f(x) = -3x^{1/3} + 4x^{1/2}$

And $g(x) = 5x^{1/3} + 4x^{1/2}$

To find the value of $g(x) - f(x)$

Consider $g(x) - f(x) = 5x^{1/3} + 4x^{1/2} - \left[-3x^{1/3} + 4x^{1/2}\right]$ $\begin{bmatrix} f(x) = -3x^{1/3} + 4x^{1/2} \\ g(x) = 5x^{1/3} + 4x^{1/2} \end{bmatrix}$

$\Rightarrow \quad g(x) - f(x) = 5x^{1/3} + 4x^{1/2} + 3x^{1/3} - 4x^{1/2}$

$\Rightarrow \quad g(x) - f(x) = (5+3)x^{1/2} + (4-4)x^{1/3}$ [Use the distributive property]

$\Rightarrow \quad g(x) - f(x) = 8x^{1/3} + 0 \cdot x^{1/2}$ [Simplify]

$\Rightarrow \quad g(x) - f(x) = 8x^{1/3}$

The function "f" and "g" each have the same domain all non negative real numbers. So, the domain of $g(x) - f(x)$ also consists of all non negative real numbers.

Answer 8gp.

Let us consider the function $f(x) = 3x - 8$

And $g(x) = 2x^2$

Claim : To find the value of $g(f(5))$

First find the value of $f(5)$

Since $f(x) = 3x - 8$, so

$$f(5) = 3 \cdot 5 - 8$$
$$= 15 - 8$$
$$= 7$$

$\therefore \quad g[f(5)] = g(7)$

$\Rightarrow \quad g[f(5)] = g(7)$

$\Rightarrow \quad g[f(5)] = 2[7]^2$

$\Rightarrow \quad g[f(5)] = 2(49)$

$\Rightarrow \quad g[f(5)] = 98$

$\therefore \quad \boxed{g[f(5)] = 98}$

Answer 9e.

Substitute $-3x^{1/3} + 4x^{1/2}$ for $f(x)$ in $f(x) - f(x)$.

$$f(x) - f(x) = -3x^{1/3} + 4x^{1/2} - \left(-3x^{1/3} + 4x^{1/2}\right)$$

Open the parentheses using the distributive property.

$$-3x^{1/3} + 4x^{1/2} - \left(-3x^{1/3} + 4x^{1/2}\right) = -3x^{1/3} + 4x^{1/2} + 3x^{1/3} + \left(-4x^{1/2}\right)$$

Group the like terms.

$$--3x^{1/3}+4x^{1/2}+3x^{1/3}+\left(-4x^{1/2}\right)=\left[-3x^{1/3}+3x^{1/3}\right]+\left[4x^{1/2}+\left(-4x^{1/2}\right)\right]$$

Combine the like terms using the distributive property.

$$\begin{aligned}\left[-3x^{1/3}+3x^{1/3}\right]+\left[4x^{1/2}+\left(-4x^{1/2}\right)\right] &= [-3+3]x^{1/3}+[4+(-4)]x^{1/2}\\ &= 0x^{1/3}+0x^{1/2}\\ &= 0\end{aligned}$$

The domain of $f(x)$ is the set of non-negative real numbers, since the value of x cannot be a negative real number. Thus, $f(x)-f(x)$ also has the same domain.

Therefore, $f(x)-f(x)$ evaluates to 0 with the domain as the set of non-negative real numbers.

Answer 9gp.

First, we have to evaluate $g(5)$.

Substitute 5 for x in $g(x)$.
$g(5)=2(5)^2$

Simplify.

$$\begin{aligned}2(5)^2 &= 2(25)\\ &= 50\end{aligned}$$

Thus, $g(5)$ is 50.

Replace $g(5)$ in $f(g(5))$ with 50.
$f(g(5))=f(50)$

Substitute 50 for x in $f(x)$ to find $f(50)$.
$f(50)=3(50)-8$

Evaluate.

$$\begin{aligned}3(50)-8 &= 150-8\\ &= 142\end{aligned}$$

Therefore, the function $f(g(5)$ evaluates to 142.

Answer 10e.

Let us consider the function $g(x) = 5x^{1/3} + 4x^{1/2}$

To find the value of $g(x) - g(x)$

Consider $g(x) - g(x) = 5x^{1/3} + 4x^{1/2} - \left[5x^{1/3} + 4x^{1/2}\right]$ $\left[\because g(x) = 5x^{1/3} + 4x^{1/2}\right]$

$\Rightarrow \quad g(x) - g(x) = 5x^{1/3} + 4x^{1/2} - 5x^{1/3} - 4x^{1/2}$

$\Rightarrow \quad g(x) - g(x) = 5x^{1/3} - 5x^{1/3} + 4x^{1/2} - 4x^{1/2}$

$\Rightarrow \quad g(x) - g(x) = (5-5)x^{1/3} + (4-4)x^{1/2}$

$\Rightarrow \quad g(x) - g(x) = 0 \cdot x^{1/3} + 0 \cdot x^{1/2}$

$\Rightarrow \quad \boxed{g(x) - g(x) = 0}$

The function "g" and "g" each have the same domain all non negative real numbers. So, the domain of $g(x) - g(x)$ also consists of all non negative real numbers.

Answer 10gp.

Let us consider the function $f(x) = 3x - 8$

Claim: To find the value of $f\left[f(5)\right]$

First find the value of $f(5)$

Since $f(x) = 3x - 8$, so

$$\begin{aligned} f(5) &= 3 \cdot 5 - 8 \\ &= 15 - 8 \\ &= 7 \end{aligned}$$

$\therefore \quad f\left[f(5)\right] = f(8) f(7)$

$\Rightarrow \quad g\left[f(5)\right] = 3(7) - 8$

$\Rightarrow \quad g\left[f(5)\right] = 21 - 8$

$\Rightarrow \quad g\left[f(5)\right] = 13$

$\therefore \quad \boxed{g\left[f(5)\right] = 13}$

Answer 11e.

Substitute $-7x^{-2/3} - 1$ for $f(x)$, and $2x^{2/3} + 6$ for $g(x)$ in $f(x) + g(x)$.
$f(x) + g(x) = -7x^{-2/3} - 1 + 2x^{2/3} + 6$

Use the distributive property to add the like radicals.

$$\begin{aligned} -7x^{2/3} - 1 + 2x^{2/3} + 6 &= (-7 + 2)x^{2/3} + (-1 + 6) \\ &= -5x^{2/3} + 5 \end{aligned}$$

Thus, $f(x) + g(x)$ evaluates to $-5x^{2/3} + 5$, which matches with **choice B.**

Answer 11gp.

First, we have to evaluate $g(5)$.

Substitute 5 for x in $g(x)$.
$g(5) = 2(5)^2$

Simplify.

$$2(5)^2 = 2(25) = 50$$

Thus, $g(5)$ is 50.

Replace $g(5)$ in $g(g(5))$ with 50.
$g(g(5)) = g(50)$

Substitute 50 for x in $g(x)$ to find $g(50)$.
$g(50) = 2(50)^2$

Evaluate.

$$2(50)^2 = 2(2500) = 5000$$

Therefore, the function $g(g(5)$ evaluates to 5000.

Answer 12e.

Let us consider the function $f(x) = 4x^{2/3}$

And $g(x) = 5x^{1/2}$

To find the value of $f(x)\cdot g(x)$

Consider $f(x)\cdot g(x) = 4x^{2/3}\cdot 5x^{1/2}$ $\quad \left[\begin{matrix} f(x) = 4x^{2/3} \\ g(x) = 5x^{1/2} \end{matrix}\right]$

$\Rightarrow \quad f(x)\cdot g(x) = 4\cdot 5\cdot x^{2/3}\cdot x^{1/2}$

$\Rightarrow \quad f(x)\cdot g(x) = 20x^{2/3}\cdot x^{1/2}$

$\Rightarrow \quad f(x)\cdot g(x) = 20x^{\frac{2}{3}+\frac{1}{2}}$ $\quad$ [use the rule $a^m\cdot a^n = a^{m+n}$]

$\Rightarrow \quad f(x)\cdot g(x) = 20x^{\frac{2\cdot 2+1\cdot 3}{6}}$ $\quad$ [the L.C.M. of 2 and 3 is 6]

$\Rightarrow \quad f(x)\cdot g(x) = 20x^{\frac{4+3}{6}}$

$\Rightarrow \quad f(x)\cdot g(x) = 20x^{\frac{7}{6}}$

$\therefore \quad \boxed{f(x)\cdot g(x) = 20x^{7/6}}$

The function "f" and "g" each have the same domain all non negative real numbers. So, the domain of $f(x)\cdot g(x)$ also consists of all non negative real numbers.

Answer 12gp.

Let use consider the function $f(x)=2x^7$

And $g(x)=2x+7$

To find the value of $f(g(x))$

Consider $f(g(x))=f(2x+7)$ $\quad [g(x)=2x+7]$

$$=2(2x+7)^7$$

$$=2\cdot\frac{1}{2x+7} \qquad \left[\text{Use the rule } a^{-n}=\frac{1}{a^n}\right]$$

$$=\frac{2}{2x+7}$$

$$f(g(x))=\frac{2}{2x+7}$$

The domain of $f(g(x))$ consist of all real number expect $x=\frac{-7}{2}$ because $g\left(\frac{-7}{2}\right)=0$ is not in the domain of f.

To find the value of $g(f(x))$

Consider $g(f(x))=g(x^{-1})$ $\quad [f(x)=x^{-1}]$

$$=g\left(\frac{1}{x}\right) \qquad \left[\text{Use the rule } a^{-n}=\frac{1}{a^n}\right]$$

$$=2\cdot\frac{1}{x}+7 \qquad [\because g(x)=2x+7]$$

$$\therefore \quad g(f(x))=2\cdot\frac{1}{x}+7$$

The domain of $g[f(x)]$ consist of all real number expect $x=0$, because $f(0)=\frac{1}{0}$ is not in the domain of f.

To find the value of $f(f(x))$

Consider $f(f(x))=f(x^{-1})$ $\quad [f(x)=x^7]$

$$=(x^7)^7$$

$$=(x^{-1})^{-1} \qquad [\because f(x)=x^{-1}]$$

$$=x^1$$

$$\therefore \quad f(f(x))=x^1$$

The domain of $f(f(x))$ consist of all real number expect $x=0$ because $f(0)=\frac{1}{0}$ is not in the domain of "f".

Answer 13e.

Substitute $4x^{2/3}$ for $f(x)$ and $5x^{1/2}$ for $g(x)$ in $g(x)\cdot f(x)$.

$$g(x)\cdot f(x) = 5x^{1/2}\cdot 4x^{2/3}$$

Group the terms such that the exponential expressions with the same base appear together.

$$5x^{1/2}\cdot 4x^{2/3} = (5\cdot 4)\left(x^{1/2}\cdot x^{2/3}\right)$$
$$= 20\left(x^{1/2}\cdot x^{2/3}\right)$$

Use the product of powers property and simplify.

$$20\left(x^{1/2}\cdot x^{2/3}\right) = 20\left(x^{1/2+2/3}\right)$$
$$= 20\,x^{7/6}$$

Thus, $g(x)\cdot f(x)$ evaluates to $20x^{7/6}$.

The domain of $g(x)\cdot f(x)$ consists of the x-values that are in the domains of both $f(x)$ and $g(x)$.
Since the domains of $f(x)$ and $g(x)$ consist of all nonnegative real numbers, the domain of $g(x)\cdot f(x)$ will also consists of all non-negative real numbers.

Answer 13gp.

STEP1 **Find** the total amount of your purchase. It is given that the cost of a can of paint is \$30, and for painting supplies is \$25.
Thus, the total amount will be \$30 + \$25, or \$55.

STEP2 **Write** functions for the discount. Let the regular price be x. Take $f(x)$ as the price after \$15 gift certificate is applied. Thus, $f(x)$ will be the difference between regular price and \$15.
Similarly, take $g(x)$ as the price after the store discount 20% is applied.
Thus, $g(x)$ will be the difference between regular price and $\frac{20}{100}x$ or $0.2x$.

Function for \$15 gift certificate: $f(x) = x - 15$.

Function for 20% discount: $g(x) = x - 0.2x$ or $0.8x$.

STEP3 **Compose** the function.

We know that $g(f(x))$ represents the sales price when \$15 gift certificate is applied before the 20% discount.

Let us find $g(f(x))$. For this, first substitute $x - 15$ for $f(x)$.
$g(f(x)) = g(x - 15)$

Next step is to find $g(x - 15)$. For this, substitute $x - 15$ for x in the expression $g(x) = 0.8x$.

$g(x - 15) = 0.8(x - 15)$

We know that $f(g(x))$ represents the sales price when 20% discount is applied before the \$15 gift certificate.

Let us find $f(g(x))$. For this, first substitute $0.8x$ for $g(x)$.
$f(g(x)) = f(0.8x)$

Next step is to find $f(0.8x)$. For this, substitute $0.8x$ for x in the expression $f(x) = x - 15$.

$f(0.8x) = 0.8x - 15$

STEP4 **Evaluate** $g(f(x))$, and $f(g(x))$ when the value of x is 55.

$$\begin{aligned} g\left(f(55)\right) &= 0.8(55 - 15) \\ &= 0.8(40) \\ &= \$32 \end{aligned} \qquad \begin{aligned} f\left(g(55)\right) &= 0.8(55) - 15 \\ &= 44 - 15 \\ &= \$29 \end{aligned}$$

Thus, the sales price is \$32 when the \$15 gift certificate is applied before the 20% discount.

The sales price is \$29 when the 20% discount is applied before the \$15 gift certificate.

Answer 14e.

Let us consider the function $f(x) = 4x^{2/3}$

To find the value of $f(x) \cdot f(x)$

Consider $f(x) \cdot f(x) = 4x^{2/3} \cdot 4x^{2/3}$ $\quad [\because f(x) = 4x^{2/3}]$

$\Rightarrow \quad f(x) \cdot f(x) = 4 \cdot 4 \cdot x^{2/3} \cdot x^{2/3}$

$\Rightarrow \quad f(x) \cdot f(x) = 16 \cdot x^{\frac{2}{3}+\frac{2}{3}}$ $\quad$ [Use the rule $a^m \cdot a^n = a^{m+n}$]

$\Rightarrow \quad f(x) \cdot f(x) = 16x^{\frac{2+2}{3}}$

$\Rightarrow \quad f(x) \cdot f(x) = 16x^{\frac{4}{3}}$

$\therefore \quad \boxed{f(x) \cdot f(x) = 16x^{4/3}}$

The function $f(x)$ and $f(x)$ each have the same domain: all real number. So, the domain of $f(x) \cdot f(x)$ also consists of all real numbers.

Answer 15e.

Substitute $5x^{1/2}$ for $g(x)$ in $g(x) \cdot g(x)$.

$g(x) \cdot g(x) = 5x^{1/2} \cdot 5x^{1/2}$

Group the terms such that the exponential expressions with the same base appear together.

$$5x^{1/2} \cdot 5x^{1/2} = (5 \cdot 5)\left(x^{1/2} \cdot x^{1/2}\right)$$
$$= 25\left(x^{1/2} \cdot x^{2/3}\right)$$

Use the product of powers property and simplify.

$$25\left(x^{1/2} \cdot x^{2/3}\right) = 25\left(x^{1/2+1/2}\right)$$
$$= 25x$$

Thus, $g(x) \cdot g(x)$ evaluates to $25x$.

The domain of $g(x) \cdot g(x)$ consists of the x-values in the domain of $g(x)$.
Since the domain of $g(x)$ consists of all nonnegative real numbers, the domain of $g(x) \cdot g(x)$ will also consists of all non-negative real numbers.

Answer 16e.

Let us consider the functions $f(x) = 4x^{2/3}$

And $g(x) = 5x^{1/2}$

To find the value of $\frac{f(x)}{g(x)}$

Consider $\frac{f(x)}{g(x)} = \frac{4x^{2/3}}{5x^{1/2}}$ $\left[\because f(x) = 4x^{2/3} \text{ and } g(x) = 5x^{1/2}\right]$

$$\Rightarrow \frac{f(x)}{g(x)} = \frac{4}{5} \cdot \frac{x^{2/3}}{x^{1/2}}$$

$$\Rightarrow \frac{f(x)}{g(x)} = \frac{4}{5} \cdot x^{2/3} \cdot x^{-1/2} \qquad \left[\text{Use the rule } \frac{1}{a^n} = a^{-n}\right]$$

$$\Rightarrow \frac{f(x)}{g(x)} = \frac{4}{5} \cdot x^{\frac{2}{3} - \frac{1}{2}}$$

$$\Rightarrow \frac{f(x)}{g(x)} = \frac{4}{5} x^{\frac{4-3}{6}}$$

$$\Rightarrow \frac{f(x)}{g(x)} = \frac{4}{5} x^{\frac{1}{6}}$$

$$\therefore \boxed{\frac{f(x)}{g(x)} = \frac{4}{5} x^{\frac{1}{6}}}$$

The function $f(x)$ and $g(x)$ each have the same domain: all non negative real numbers.

So, the domain of $\frac{f(x)}{g(x)}$ also consists of all non negative real numbers.

Answer 17e.

Substitute $4x^{2/3}$ for $f(x)$ and $5x^{1/2}$ for $g(x)$ in $\frac{g(x)}{f(x)}$.

$$\frac{g(x)}{f(x)} = \frac{5x^{1/2}}{4x^{2/3}}$$

$$= \frac{5}{4}\left(\frac{x^{1/2}}{x^{2/3}}\right)$$

Use the quotient of powers property and simplify.

$$\frac{5}{4}\left(\frac{x^{1/2}}{x^{2/3}}\right) = \frac{5}{4}\left(x^{1/2 - 2/3}\right)$$

$$= \frac{5}{4} x^{-1/6}$$

According to the negative exponent property, $a^{-m} = \dfrac{1}{a^m}$.

$$\frac{5}{4}x^{-1/6} = \frac{5}{4x^{1/6}}$$

Thus, $\dfrac{g(x)}{f(x)}$ evaluates to $\dfrac{5}{4x^{1/6}}$.

The domain of $\dfrac{g(x)}{f(x)}$ consists of the x-values that are in the domains of both $f(x)$ and $g(x)$ except those values for which $f(x) = 0$.

The domains of $f(x)$ and $g(x)$ consists of all nonnegative real numbers. Since $f(0)$ is 0, the domain of $\dfrac{g(x)}{f(x)}$ is restricted to all positive real numbers.

Answer 18e.

Let us consider the function $f(x) = 4x^{2/3}$

To find the value of $\dfrac{f(x)}{f(x)}$

Consider $\dfrac{f(x)}{f(x)} = \dfrac{4}{4}\cdot\dfrac{x^{2/3}}{x^{2/3}}$ $\quad \left[\because \begin{matrix} f(x) = 4x^{2/3} \\ g(x) = 5x^{1/2} \end{matrix}\right]$

$\Rightarrow \quad \dfrac{f(x)}{f(x)} = \dfrac{4}{4}\cdot\dfrac{x^{2/3}}{x^{2/3}}$

$\Rightarrow \quad \dfrac{f(x)}{f(x)} = 1\cdot\dfrac{x^{2/3}}{x^{2/3}}$ $\quad \left[\text{Use the rule } \dfrac{1}{a^n} = a^{-n}\right]$

$\Rightarrow \quad \dfrac{f(x)}{f(x)} = x^{\frac{2}{3}-\frac{2}{3}}$ $\quad \left[\text{Use the rule } a^m \cdot a^{-n} = a^{m-n}\right]$

$\Rightarrow \quad \dfrac{f(x)}{f(x)} = x^{\frac{2-2}{3}}$

$\Rightarrow \quad \dfrac{f(x)}{f(x)} = x^{\frac{0}{3}}$ $\quad$ [Use the L.C.M of 3 and 3 is 3]

$\Rightarrow \quad \dfrac{f(x)}{f(x)} = x^0$ $\quad \left[\because a^0 = 1\right]$

$\therefore \quad \boxed{\dfrac{f(x)}{f(x)} = 1}$

The function $f(x)$ and $f(x)$ each have the same domain: all real numbers. So, the domain of $\dfrac{f(x)}{g(x)}$ also consists of all real numbers.

Answer 19e.

Substitute $5x^{1/2}$ for $g(x)$ in $\frac{g(x)}{g(x)}$.

$$\frac{g(x)}{g(x)} = \frac{5x^{1/2}}{5x^{1/2}}$$

$$= 1$$

Thus, $\frac{g(x)}{g(x)}$ evaluates to 1.

The domain of $\frac{g(x)}{g(x)}$ consists of the x-values in the domain of $g(x)$ except those values for which $g(x) = 0$.

The domain of $g(x)$ consists of all nonnegative real numbers. Since $g(0)$ is 0, the domain of $\frac{g(x)}{g(x)}$ is restricted to all positive real numbers.

Answer 20e.

Let us consider the function $f(x) = 3x + 2$

And $g(x) = -x^2$

Claim: To find the value of $f(g(-3))$

First find the value of $g(-3)$

Since $g(x) = -x^2$, so

$$g(-3) = -(-3)^2$$
$$= -9$$

$\Rightarrow \quad f(g(-3)) = f(-9)$

$\Rightarrow \quad f(g(-3)) = 3(-9) + 2 \qquad [\because f(x) = 3x + 2]$

$\Rightarrow \quad f(g(-3)) = -27 + 2$

$\Rightarrow \quad f(g(-3)) = -25$

$\therefore \quad \boxed{f(g(-3)) = -25}$

Answer 21e.

First, we have to evaluate $f(2)$.

Substitute 2 for x in $f(x)$.
$f(2) = 3(2) + 2$

Simplify.
$$3(2) + 2 = 6 + 2$$
$$= 8$$

Thus, $f(2)$ is 8.

Replace $f(2)$ in $g(f(2))$ with 8.
$g(f(2) = g(8)$

Substitute 8 for x in $g(x)$ to find $g(8)$.
$g(8) = -8^2$

Evaluate.
$-8^2 = -64$

Therefore, the function $g(f(2)$ evaluates to -64.

Answer 22e.

Let us consider the functions $f(x) = 3x + 2$

And $h(x) = \frac{x-2}{5}$

Claim: To find the value of $h[f(-9)]$

First find the value of $f(-9)$

Since $f(x) = 3x + 2$, so $f(-9) = 3(-9) + 2$

Consider $h[f(-9)] = h[3 \cdot (-9) + 2]$

$\Rightarrow \quad h[f(-9)] = h[-27 + 2]$

$\Rightarrow \quad h[f(-9)] = h[-25]$

$\Rightarrow \quad h[f(-9)] = \frac{-25-2}{5} \qquad \left[h(x) = \frac{x-2}{5}\right]$

$\Rightarrow \quad h[f(-9)] = \frac{-27}{5}$

$\therefore \quad \boxed{h[f(-9)] = \frac{-27}{5}}$

Answer 23e.

First, we have to evaluate $h(8)$.

Substitute 8 for x in $h(x)$, and simplify.

$$h(8) = \frac{8-2}{5}$$
$$= \frac{6}{5}$$

Thus, $h(8)$ is $\frac{6}{5}$.

Replace $h(8)$ in $g(h(8))$ with $\frac{6}{5}$.

$$g(h(8)) = g\left(\frac{6}{5}\right)$$

Substitute $\frac{6}{5}$ for x in $g(x)$ to find $g\left(\frac{6}{5}\right)$, and evaluate.

$$g\left(\frac{6}{5}\right) = -\left(\frac{6}{5}\right)^2$$
$$= -\frac{36}{25}$$

Therefore, the function $g(h(8)$ evaluates to $-\frac{36}{25}$.

Answer 24e.

Let us consider the functions $g(x) = -x^2$

And $h(x) = \frac{x-2}{5}$

To find the value of $h[g(5)]$

Consider $h[g(5)] = h\left[-(5)^2\right]$ $\quad \left[\because g(x) = -x^2\right]$

$\Rightarrow \quad h[g(5)] = h[-25]$

$\Rightarrow \quad h[g(5)] = \left[\frac{-25-2}{5}\right]$ $\quad \left[h(x) = \frac{x-2}{5}\right]$

$\Rightarrow \quad h[g(5)] = \frac{-27}{5}$

$\therefore \quad \boxed{h[g(5)] = \frac{-27}{5}}$

Answer 25e.

First, we have to evaluate $f(7)$.

Substitute 7 for x in $f(x)$.
$f(7) = 3(7) + 2$

Simplify.
$$3(7) + 2 = 21 + 2$$
$$= 23$$

Thus, $f(7)$ is 23.

Replace $f(7)$ in $f(f(7))$ with 23.
$f(f(7)) = f(23)$

Again, substitute 23 for x in $f(x)$ to find $f(23)$.
$f(23) = 3(23) + 2$

Evaluate.
$$3(23) + 2 = 69 + 2$$
$$= 71$$

Therefore, the function $f(f(7))$ evaluates to 71.

Answer 26e.

Let us consider the function $h(x) = \frac{x-2}{5}$

To find the value of $h(h(-4))$

Consider $h(h(-4)) = h\left(\frac{-4-2}{5}\right)$ $\quad\left[h(x) = \frac{x-2}{5}\right]$

$$\Rightarrow \quad h(h(-4)) = h\left(\frac{-6}{5}\right)$$

$$\Rightarrow \quad h(h(-4)) = \left[\frac{\frac{-6}{5} - \frac{2}{1}}{5}\right] \quad \left[\because h(x) = \frac{x-2}{5}\right]$$

$$\Rightarrow \quad h(h(-4)) = \left[\frac{\frac{-6-10}{5}}{5}\right] \quad [\text{The L.C.M of 5 and 1 is 5}]$$

$$\Rightarrow \quad h(h(-4)) = \frac{-16}{5 \cdot 5}$$

$$\Rightarrow \quad h(h(-4)) = \frac{-16}{25}$$

$$\therefore \quad \boxed{h(h(-4)) = \frac{-16}{25}}$$

Answer 27e.

First, we have to evaluate $g(-5)$.

Substitute -5 for x in $g(x)$, and simplify.

$$\begin{aligned} g(-5) &= -(-5)^2 \\ &= -(25) \\ &= -25 \end{aligned}$$

Thus, $g(-5)$ is -25.

Replace $g(-5)$ in $g(g(-5))$ with -25.
$g(g(-5)) = g(-25)$

Substitute -25 for x in $g(x)$ to find $g(-25)$, and evaluate.

$$\begin{aligned} g(-25) &= -(-25)^2 \\ &= -(625) \\ &= -625 \end{aligned}$$

Therefore, the function $g(g(-5))$ evaluates to -625.

Answer 28e.

Let us consider the function $f(x) = 3x^{-1}$

And $g(x) = 2x - 7$

Claim : To find the value of $f(g(x))$

Consider $f(g(x)) = f(2x-7)$ $\quad [\because g(x) = 2x-7]$

$\Rightarrow \quad f(g(x)) = 3\cdot(2x-7)^{-1}$ $\quad [\because f(x) = 3x^{-1}]$

$\Rightarrow \quad f(g(x)) = 3\cdot\dfrac{1}{2x-7}$ $\quad \left[\text{Use the rule } a^{-n} = \dfrac{1}{a^n}\right]$

$\therefore \quad \boxed{f(g(x)) = \dfrac{3}{2x-7}}$

The domain of $f(g(x))$ consist of all real numbers except $x = \dfrac{7}{2}$ because $g\left(\dfrac{7}{2}\right) = 0$ is not in the domain of "f". not that $f(0) = \dfrac{3}{0}$ which is undefined.

Answer 29e.

In order to find $g(f(x))$, first substitute $3x^{-1}$ for $f(x)$.
$g(f(x)) = g(3x^{-1})$

Next step is to find $g(3x^{-1})$. For this, substitute $3x^{-1}$ for x in the expression $g(x) = 2x - 7$.
$g(3x^{-1}) = 2(3x^{-1}) - 7$

Open the parentheses using the distributive property.
$2(3x^{-1}) - 7 = 6x^{-1} - 7$

Use the negative exponent property.
$$6x^{-1} - 7 = \frac{6}{x} - 7$$

Thus, $g(f(x))$ evaluates to $\frac{6}{x} - 7$.

Since $f(0) = \frac{3}{0}$ is undefined, 0 is not in the domain of f. Thus, the domain of $g(f(x))$ consists of all real numbers except $x = 0$.

Answer 30e.

Let us consider the functions $f(x) = 3x^{-1}$

And $h(x) = \frac{x+4}{3}$

Claim: To find the value of $h(f(x))$

Consider $h(f(x)) = h[3x^{-1}]$ $\quad [\because f(x) = 3x^{-1}]$

$$h[f(x)] = h\left[3 \cdot \frac{1}{x}\right] \quad \left[\text{Use the rule } a^{-n} = \frac{1}{a^n}\right]$$

$$h[f(x)] = h\left[\frac{3}{x}\right]$$

$$h[f(x)] = \frac{\frac{3}{x} + 4}{3} \quad \left[\because h(x) = \frac{x+4}{3}\right]$$

$$h[f(x)] = \frac{\frac{3+4x}{x}}{3}$$

$$h[f(x)] = \frac{3+4x}{3x}$$

$$\therefore \boxed{h[f(x)] = \frac{3+4x}{3x}}$$

The domain of $h[f(x)]$ consist of all real numbers except $x = 0$ because $f(0) = \frac{3}{0}$ which is undefined

Answer 31e.

In order to find $g(h(x))$, first substitute $\frac{x+4}{3}$ in $h(x)$.

$$g(h(x)) = g\left(\frac{x+4}{3}\right)$$

Next step is to find $g\left(\frac{x+4}{3}\right)$. For this, substitute $\frac{x+4}{3}$ for x in the expression for $g(x)$.

$$g\left(\frac{x+4}{3}\right) = 2\left(\frac{x+4}{3}\right) - 7$$

Multiply the expression using the least common denominator, 3.

$$2\left(\frac{x+4}{3}\right) - 7 = \frac{2(x+4) - 3(7)}{3}$$

Open the parentheses using the distributive property.

$$\frac{2(x+4) - 3(7)}{3} = \frac{2x + 8 - 21}{3}$$

$$= \frac{2x - 13}{3}$$

Thus, $g(h(x))$ evaluates to $\frac{2x-13}{3}$.

The domain of $g(h(x))$ consists of the x-values that are in the domains of both $g(x)$ and $h(x)$.
Since the domains of $g(x)$ and $h(x)$ consist of all real numbers, the domain of $g(h(x))$ also consists of all real numbers.

Answer 32e.

Let us consider the functions $g(x) = 2x - 7$

And $h(x) = \frac{x+4}{3}$

Claim : To find the value of $h[g(x)]$

Consider $h[g(x)] = \frac{2x-7+4}{3}$ $\quad \left[\because \quad h(x) = \frac{x+4}{3} \text{ and } g(x) = 2x-7\right]$

$$h[g(x)] = \frac{2x-3}{3}$$

$$\therefore \quad \boxed{h[g(x)] = \frac{2x-3}{3}}$$

The domain of $h[g(x)]$ consist of all real numbers, because the denominator of $h[g(x)]$ is constant

Answer 33e.

In order to find $f(f(x))$, first substitute $3x^{-1}$ for $f(x)$.
$f(f(x)) = f(3x^{-1})$

Next step is to find $f(3x^{-1})$. For this, substitute $3x^{-1}$ for x in the expression for $f(x)$.

$$f\left(3x^{-1}\right) = 3\left(3x^{-1}\right)^{-1}$$

Apply the power of a product property.

$$3\left(3x^{-1}\right)^{-1} = 3 \cdot 3^{-1} \cdot x^{(-1)(-1)}$$
$$= 3^1 \cdot 3^{-1} \cdot x$$

Simplify using the product of powers property.

$$3^1 \cdot 3^{-1} \cdot x = 3^{1+(-1)} \cdot x$$
$$= 3^0 \cdot x$$
$$= 1 \cdot x$$
$$= x$$

Thus, $f(f(x))$ evaluates to x.

The domain of $f(f(x))$ consists of the x-values in the domains of $f(x)$.
Since $f(0) = \frac{3}{0}$ is undefined, 0 is not in the domain of f. The domain of $f(f(x))$ consists of all real numbers except $x = 0$.

Answer 34e.

Let us consider the function $h(x) = \frac{x+4}{3}$

To find the value of $h[h(x)]$

Consider $h[h(x)] = h\left[\frac{x+4}{3}\right]$ $\left[h(x) = \frac{x+4}{3}\right]$

$$h[h(x)] = h\left[\frac{x+4}{3}\right] \qquad \left[\because h(x) = \frac{x+4}{3}\right]$$

$$h\ [h(x)] = \frac{\frac{x+4+4\cdot 3}{3}}{3} \qquad \text{[The L.C.M. of 3 and 2 is 3]}$$

$$h[h(x)] = \frac{x+16}{9}$$

$$\therefore \boxed{h[h(x)] = \frac{x+16}{9}}$$

The domain of $h[h(x)]$ consist of all real numbers.

Answer 35e.

In order to find $g(g(x))$, first substitute $2x - 7$ in $g(x)$.
$g(g(x)) = g(2x - 7)$

Next step is to find $g(2x - 7)$. For this, substitute $2x - 7$ for x in the expression for $g(x)$.
$g(2x - 7) = 2(2x - 7) - 7$

Open the parentheses using the distributive property.

$$\begin{aligned} 2(2x - 7) - 7 &= 4x - 14 - 7 \\ &= 4x - 21 \end{aligned}$$

The domain of $g(g(x))$ consists of the x-values in the domains of $g(x)$.
Since the domain of $g(x)$ consists of all real numbers, the domain of $g(g(x))$ also consists of all real numbers.

Answer 36e.

Let us consider the function $f(x) = x^2 - 3$

And $g(x) = 4x$

To find the value of $f[g(x)]$

Consider $f[g(x)] = f(4x) \qquad [\because g(x) = 4x]$

$f[g(x)] = f(4x) \qquad [\because f(x) = x^2 - 3]$

$f(g(x)) = 16x^2 - 3$

$\therefore \boxed{f(g(x)) = 16x^2 - 3}$

Answer 37e.

In order to find $g(f(x))$, first substitute $x^2 - 3$ for $f(x)$.
$g(f(x)) = g(x^2 - 3)$

Next step is to find $g(x^2 - 3)$. For this, substitute $x^2 - 3$ for x in the expression for $g(x)$.
$g(x^2 - 3) = 4(x^2 - 3)$

Open the parentheses using the distributive property.
$4(x^2 - 3) = 4x^2 - 12$

Thus, $g(f(x))$ evaluates to $4x^2 - 12$.

Answer 38e.

Let us consider the function $f(x)=7x^2$

And $g(x)=3x^{-2}$

To find the value of $g[f(x)]$

Consider $g[f(x)]=g[7x^2]$ $\quad [\because f(x)=7x^2]$

$$g[f(x)]=3\cdot[7x^2]^{-2} \quad [\because g(x)=3x^{-2}]$$

$$g[f(x)]=3\cdot\frac{1}{(7x^2)^2} \quad \left[\text{Use the rule } a^{-n}=\frac{1}{a^n}\right]$$

$$g[f(x)]=3\frac{1}{7^2\cdot(x^2)^2} \quad \left[\text{Use the rule } (a\cdot b)^m=a^m\cdot b^m\right]$$

$$g[f(x)]=3\cdot\frac{1}{49\cdot x^{2\cdot 2}} \quad \left[\text{Use the rule } (a^m)^n=a^{m\cdot n}\right]$$

$$g[f(x)]=\frac{3}{49\cdot x^4}$$

$$g[f(x)]=\frac{3}{49x^4}$$

$$\therefore \boxed{g[f(x)]=\frac{3}{49x^4}}$$

The Answer is A

Answer 39e.

Let $f(x)$ be $x+1$, and $g(x)$ be $x+2$.

Find $f(g(x))$.

Substitute $x+2$ for $g(x)$ in $f(g(x))$.

$f(g(x))=f(x+2)$

In order to find $f(x+2)$, replace x in $f(x)$ with $x+2$.

$$\begin{aligned} f(x+2) &= x+2+1 \\ &= x+3 \end{aligned}$$

We get $f(g(x))$ as $x+3$.

Next, find $g(f(x))$.

Substitute $x + 1$ for $f(x)$ in $g(f(x))$.
$g(f(x)) = g(x + 1)$

Replace x in $g(x)$ with $x + 1$ to find $g(x + 1)$.

$$g(x+1) = x+1+2$$
$$= x+3$$

Both the functions evaluate to the same value.

Therefore, the function f may be $x + 1$, and g may be $x + 2$.

Answer 40e.

Let us consider the function $h(x)=\sqrt[3]{x+2}$

Claim: To find the function $f(x)$ and $g(x)$ such that $f[g(x)]=h(x)$

Consider $h(x)=\sqrt[3]{x+2}$

$$h(x)=\sqrt[3]{x+2}$$
$$h(x)=\sqrt[3]{g(x)} \qquad [\text{Assume } g(x)=x+2]$$
$$h(x)=f[g(x)] \qquad [\text{Assume } f(x)=\sqrt[3]{x}]$$
$$\therefore \boxed{f(x)=\sqrt[3]{x}} \text{ and } \boxed{g(x)=x+2}$$

Answer 41e.

Let $f(x)$ be $4x$, and $g(x)$ be $\dfrac{1}{3x^2+7}$.

Find $f(g(x))$.

Substitute $\dfrac{1}{3x^2+7}$ for $g(x)$ in $f(g(x))$.

$$f(g(x)) = f\left(\frac{1}{3x^2+7}\right)$$

In order to find $f\left(\dfrac{1}{3x^2+7}\right)$, replace x in $f(x)$ with $\dfrac{1}{3x^2+7}$.

$$f\left(\frac{1}{3x^2+7}\right) = 4\left(\frac{1}{3x^2+7}\right)$$
$$= \frac{4}{3x^2+7}$$

The value of $f(g(x))$ is $\dfrac{4}{3x^2+7}$, which is same as that of $h(x)$.

Therefore, the function f can be $4x$, and g can be $\dfrac{1}{3x^2+7}$.

Answer 42e.

Let us consider the function $h(x)=|2x+9|$

To find the function $f(x)$ and $g(x)$ such that

$$f[g(x)]=h(x)$$

Consider $h(x)=|2x+9|$

$$h(x)=|2x+8+1| \quad [\because \text{Write "9" as } 8+1]$$

$$h(x)=|2(x+4)+1| \quad [\because \text{ use the distributive rule}]$$

$$h(x)=|2g(x)+1| \quad [\because \text{Let } g(x)=x+4]$$

$$h(x)=f[g(x)] \quad [\because \text{ Let} f(x)=2x+1]$$

$$\therefore \boxed{h(x)=f[g(x)]}$$

$$\therefore \boxed{\begin{array}{r} f(x)=2x+1 \\ \text{and } g(x)=x+4 \end{array}}$$

Answer 43e.

Substitute the values of $b(w)$ and $d(w)$ in the given model.

$$r(w) = \frac{1.1w^{0.734}}{0.007w - 0.002w}$$

Factor out w in the denominator.

$$\frac{1.1w^{0.734}}{0.007w - 0.002w} = \frac{1.1w^{0.734}}{w(0.007 - 0.002)}$$

Subtract the exponents on w, and simplify.

$$\frac{1.1w^{0.734}}{w(0.007 - 0.002)} = \frac{1.1w^{0.734-1}}{0.007 - 0.002}$$

$$= \frac{1.1w^{-0.266}}{0.005}$$

$$= 220w^{-0.266}$$

The function $r(w)$ simplifies to $220w^{-0.266}$.

Next, replace w with 6.5, and simplify.

$$\begin{aligned} 220w^{-0.266} &= 220(6.5)^{-0.266} \\ &\approx 220(0.61) \\ &\approx 134 \end{aligned}$$

Thus, the breathing rate is about 134 for body weights of 6.5 grams.

Replace w with 300, and simplify.

$$\begin{aligned} 220w^{-0.266} &= 220(300)^{-0.266} \\ &\approx 220(0.22) \\ &\approx 48.3 \end{aligned}$$

Thus, the breathing rate is about 48.3 for body weights of 300 grams.

Replace w with 70,000 and simplify.

$$\begin{aligned} 220w^{-0.266} &= 220(70,00)^{-0.266} \\ &\approx 220(0.05) \\ &\approx 11.3 \end{aligned}$$

Therefore, the breathing rate is about 11.3 for body weights of 70,000 grams.

Answer 44e.

Let us consider the cost of producing "x" sneakers in a factory is given by $c(x)=60x+750$

Let us consider the number of sneakers produced in 't' hours is given by $x(t)=50t$

To find the value of $c\left[x(t)\right]$

Consider $c\left[x(t)\right]=c(50t)$ $\qquad \left[\because x(t)=50t\right]$

$$c\left[x(t)\right]=60(50t)+750 \qquad \left[\begin{array}{l}\because c(x)=60x+750 \\ c[50t]=60(50t+750)\end{array}\right]$$

$$c\left[x(t)\right]=60\cdot 50\cdot t+750$$

$$c\left[x(t)\right]=300t+750$$

$$\therefore \boxed{c\left[x(t)\right]=300t+750}$$

The composition $c\left[x(t)\right]$ represents the cost of producing x sneakers in a factory when the number of sneakers produced in 't' hours

To find the value of $c[x(5)]$

Consider $c[x(5)] = c[50(5)]$ $\quad \left[\begin{array}{l} \because x(t) = 50t \\ x(5) = 50 \cdot 5 \end{array}\right]$

$c[x(5)] = c[250]$ [Multiply]

$c[x(5)] = 60(250) + 750$ $[\because c(t) = 60x + 750]$

$c[x(5)] = 15{,}000 + 750$

$c[x(5)] = 15{,}750$ [Add]

$\boxed{c[x(5)] = 15{,}750}$

The composition $c[x(5)]$ represents the cost of producing "x" sneakers in a factory when the number of sneakers produced in 5 hours.

Answer 45e.

(a) **Find** the total amount of your purchase.
Since the sales price is \$85, the total amount of your purchase will be \$85.

Let the regular price be x, and $f(x)$ be the price after \$15 discount is applied. Thus $f(x)$ will be difference between the regular price and \$15.
Similarly, take $g(x)$ as the price after the store discount 10% is applied.

Function for \$15 discount: $f(x) = x - 15$.
Function for 10% discount: $g(x) = x - 0.1x$, or $g(x) = 0.9x$.

Compose the function.

We know that $g(f(x))$ represents the sales price when \$15 discount is applied before the 10% discount.

Let us find $g(f(x))$. For this, first substitute $x - 15$ for $f(x)$.
$g(f(x)) = g(x - 15)$

Next step is to find $g(x - 15)$. For this, substitute $x - 15$ for x in the equation for $g(x)$.

$g(0.9x) = 0.9(x - 15)$

Evaluate $g(f(x))$ when the value of x is 85.

$g(f(55)) = 0.9(85 - 15)$

Simplify.

$$\begin{aligned} 0.9(85 - 15) &= 0.9(70) \\ &= \$63 \end{aligned}$$

The sales price is \$63 when the \$15 discount is applied before the 10% discount.

(b) We know that $f(g(x))$ represents the sales price when 10%discount is applied before the $15 discount.

Let us find $f(g(x))$. For this, first substitute $0.9x$ for $g(x)$.
$f(g(x)) = f(0.9x)$

Next step is to find $f(0.9x)$. For this, substitute $0.9x$ for x in the equation for $f(x)$.

$f(0.9x) = 0.9x - 15.$

Evaluate $f(g(x))$, when the value of x is 85.

$f(g(85)) = 0.9(85) - 15$

Simplify.

$$\begin{aligned} 0.9(85) - 15 &= 76.5 - 15 \\ &= \$61.50 \end{aligned}$$

The sales price is $61.50 when the 10% discount is applied before the $15 discount.

(c) If you apply the 10% discount before the $15 discount you have to pay $61.5, while if you apply the $15 discount before the 10% discount you have to pay $63. Thus, it is clear that the 10% discount before the $15 discount is better deal.

Answer 47e.

(a) In order to find $f(x)$, substitute 1 for x, and 2 for n in $f(x) = \dfrac{x + \frac{n}{x}}{2}$.

$$f(1) = \frac{1 + \frac{2}{1}}{2}$$

Simplify.

$$\begin{aligned} \frac{1 + \frac{2}{1}}{2} &= \frac{1+2}{2} \\ &= \frac{3}{2} \end{aligned}$$

In order to find $f(f(x))$, substitute $\frac{3}{2}$ for x in $f(x)$.

$$f\left(\frac{3}{2}\right) = \frac{1 + \frac{2}{\left(\frac{3}{2}\right)}}{2}$$

Simplify.

$$\frac{\frac{3}{2} + \frac{2}{\left(\frac{3}{2}\right)}}{2} = \frac{\frac{3}{2} + \frac{4}{3}}{2}$$
$$= \frac{\left(\frac{17}{6}\right)}{2}$$
$$= \frac{17}{12} \quad \text{or} \quad 1.416666$$

Similarly, substitute $\frac{17}{12}$ for x in $f(x) = \frac{x + \frac{n}{x}}{2}$, to find $f(f(f(x)))$.

We will get $f(f(f(x)))$ as $\frac{577}{408}$ or 1.41425686 .

Find $f(f(f(f(x))))$.
$f(f(f(f(x)))) = 1.414213562$

(b) We have $\sqrt{2} = 1.414213562$. Approximate the value of $\sqrt{2}$ to three decimal places.
$1.414213562 \approx 1.414$

If we approximate the value of $f(f(f(f(x))))$ to three decimal places, we will get 1.414.

Thus, three times we have to compose the function in order for the result to approximate $\sqrt{2}$ to three decimal places.

Similarly, approximate the value of $\sqrt{2}$ to six decimal places.
$1.414213562 \approx 1.414213$

If we approximate the value of $f(f(f(f(x))))$ to six decimal places, we will get 1.414213.

Thus, four times we have to compose the function in order for the result to approximate $\sqrt{2}$ to six decimal places.

Answer 48e.

Let us consider the function $y-2x=12$

Claim To solve the equation for "y"

$\therefore \quad y-2x=12 \quad$ [Write the original equation]

$y=2x+12 \quad$ [add $2x$ to each other]

$\therefore \quad \boxed{y=2x+12}$

Answer 49e.

In order to solve for y, we have to isolate y on one side of the equation.

Subtract $3x$ from both the sides.

$$3x-2y-3x = 10-3x$$

$$-2y = 10-3x$$

Divide each term by -2.

$$\frac{-2y}{-2} = \frac{10}{-2} - \frac{3x}{-2}$$

$$y = -5+\frac{3}{2}x$$

$$= \frac{3}{2}x-5$$

Therefore, the solution is $y = \frac{3}{2}x-5$.

Answer 50e.

Let us consider the equation $x=-3y+9$

Claim To solve the equation for "y".

$\therefore \quad x=-3y+9 \quad$ [Write the original equation]

$\Rightarrow \quad x-9=-3y \quad$ [Substract "9" to each side]

$\Rightarrow \quad \frac{x-9}{-3}=y \quad$ [Divide "-3" to each side]

$\Rightarrow \quad \frac{-1}{3}x-\frac{9}{(-3)}=y \quad \left[\text{Use the rule } \frac{a-b}{c}=\frac{a}{b}-\frac{b}{c}\right]$

$\Rightarrow \quad \frac{-x}{3}+\frac{9}{3}=y$

$\Rightarrow \quad \frac{-x}{3}+3=y \quad$ [Simplify]

$\therefore \quad \boxed{y=\frac{-1}{3}x+3}$

Answer 51e.

In order to solve for y, we have to isolate y to one side of the equation.

Subtract $3x$ from both the sides.

$$3x - 4y - 3x = 7 - 3x$$
$$-4y = 7 - 3x$$

Divide each term by -4.

$$\frac{-4y}{-4} = \frac{7}{-4} - \frac{3x}{-4}$$
$$y = -\frac{7}{4} + \frac{3}{4}x$$
$$= \frac{3}{4}x - \frac{7}{4}$$

Therefore, the solution is $y = \frac{3}{4}x - \frac{7}{4}$.

Answer 52e.

Let us consider the equation $x - y = 12$

<u>Claim</u> To solve the equation for "y".

$\therefore \quad x - y = 12$ [Write the original equation]

$\Rightarrow \quad -y = 12 - x$ [Substract "x" to each side]

$\Rightarrow \quad y = -(12 - x)$ [Multiply "-1" to each side]

$\Rightarrow \quad y = -12 - (-x)$ [Use the distributive property]

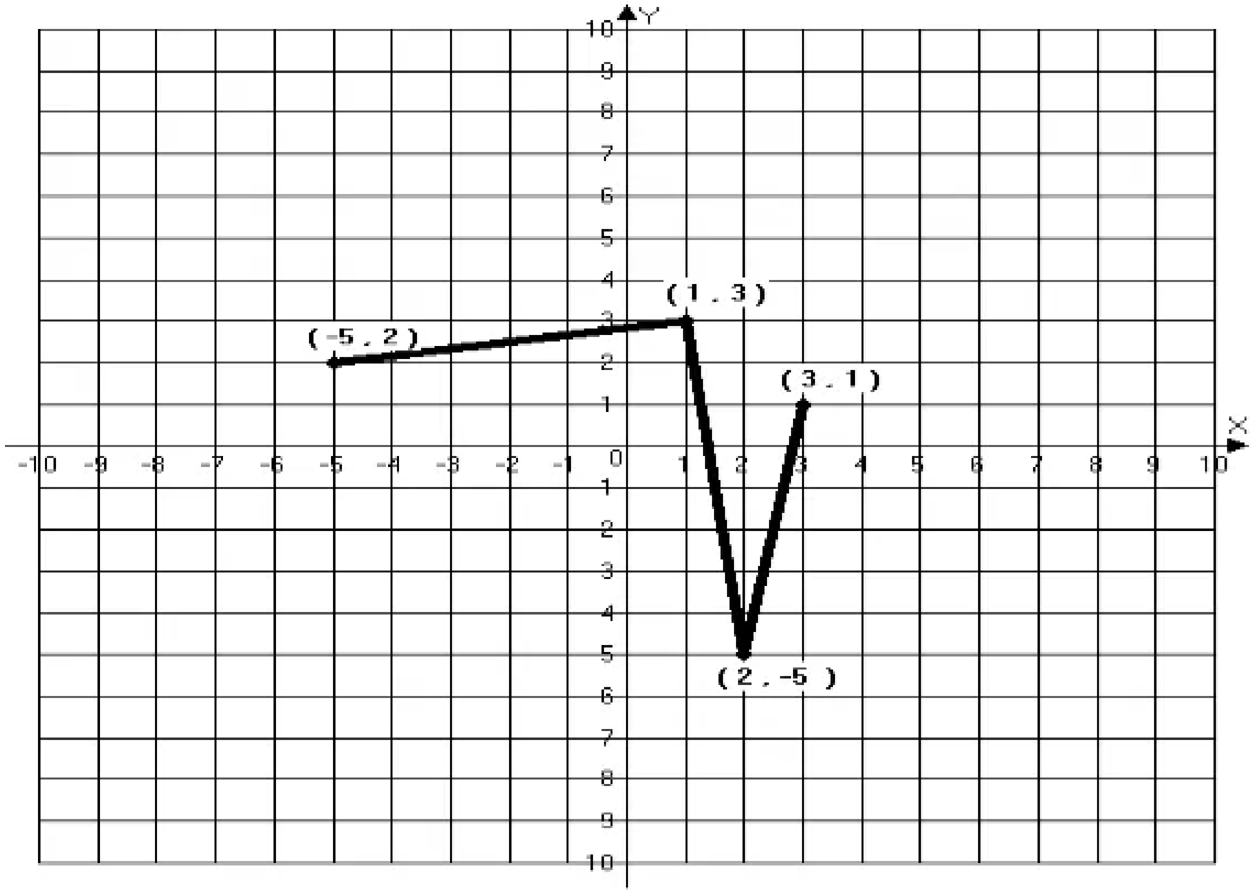
$\Rightarrow \quad y = -12 + x$

$\therefore \quad \boxed{y = x - 12}$

Answer 54e.

Let us consider the points $(-5,2)$, $(1,3)$, $(3,1)$ and $(2,-5)$

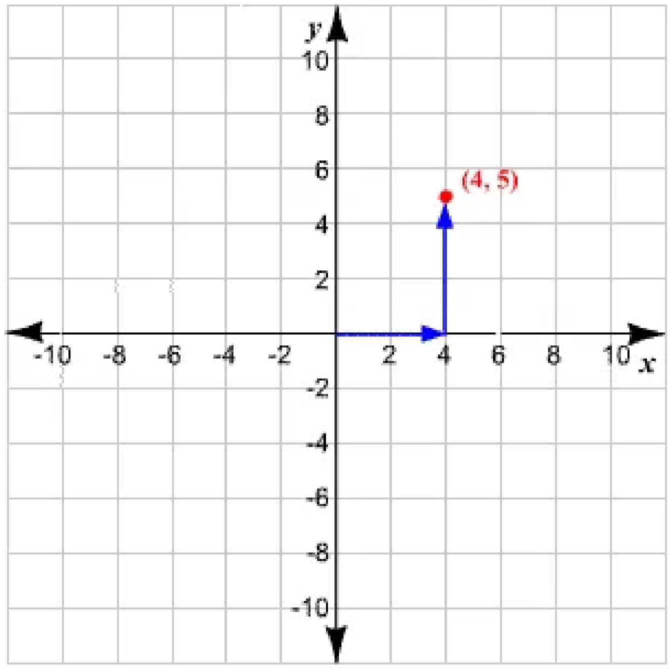
The graph of the above points are shown below



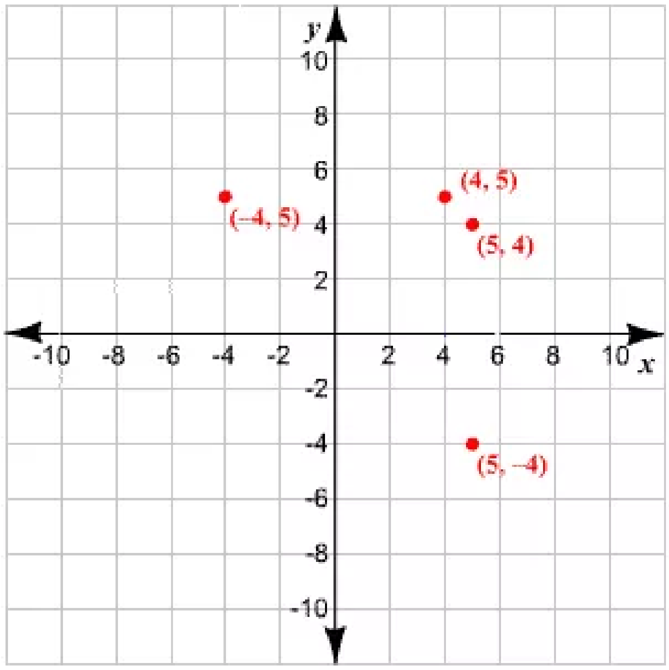
Answer 55e.

Consider the first point, (4, 5).

From the origin, move 4 units to the right since the x-coordinate is 4, and then move 5 units up since the y-coordinate is 5.



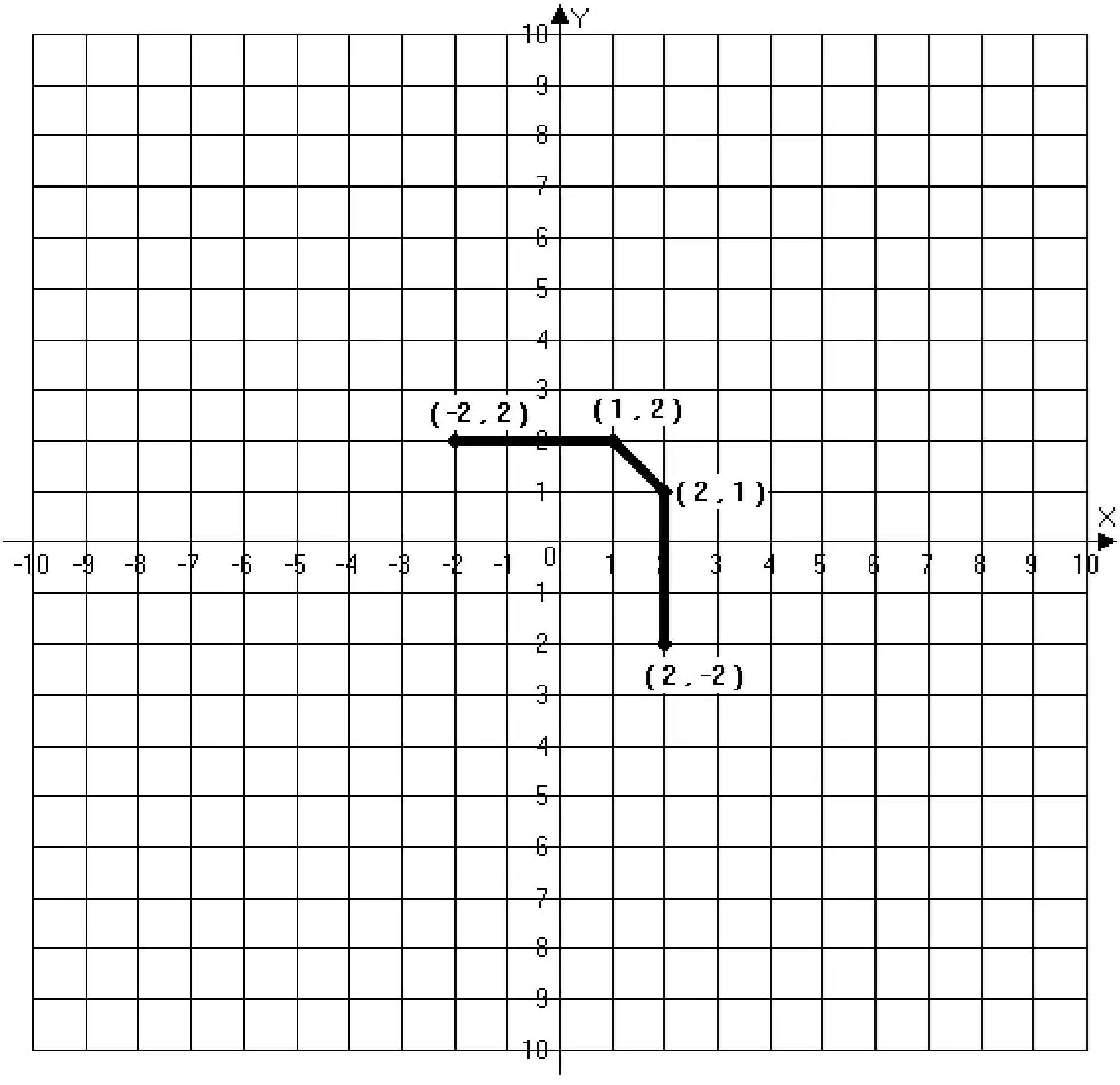
Similarly, plot the other points.



Answer 56e.

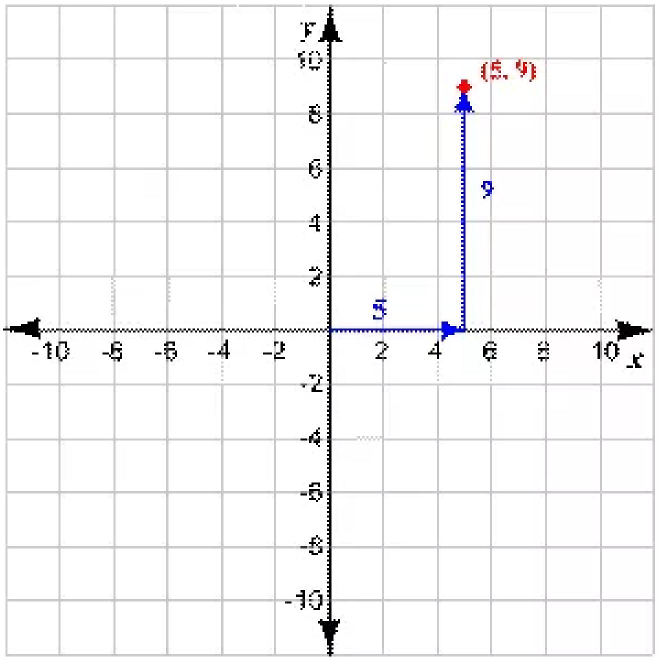
Let us consider the points $(-2,2)$, $(1,2)$, $(2,1)$ and $(2,-2)$

The graph of the above points are shown below

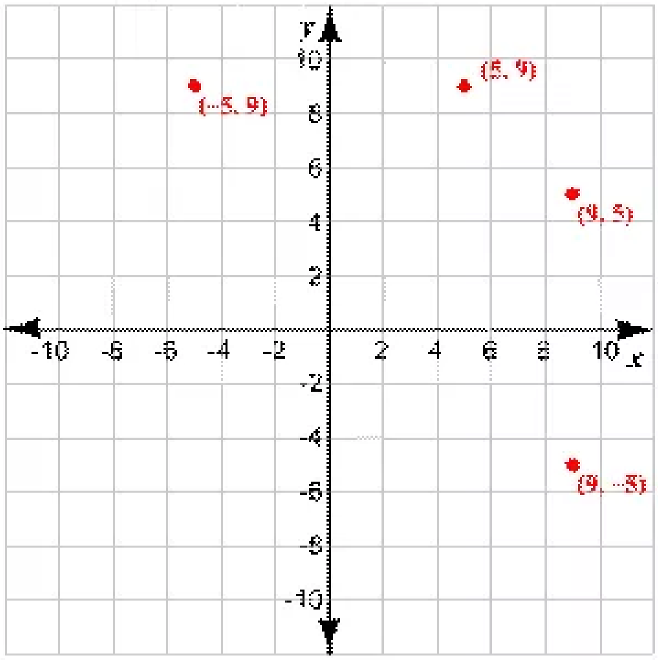


Answer 57e.

Consider the first point, (5, 9).
From the origin, move 5 units to the right as the x-coordinate is 5, and then move 9 units up as the y-coordinate is 9.



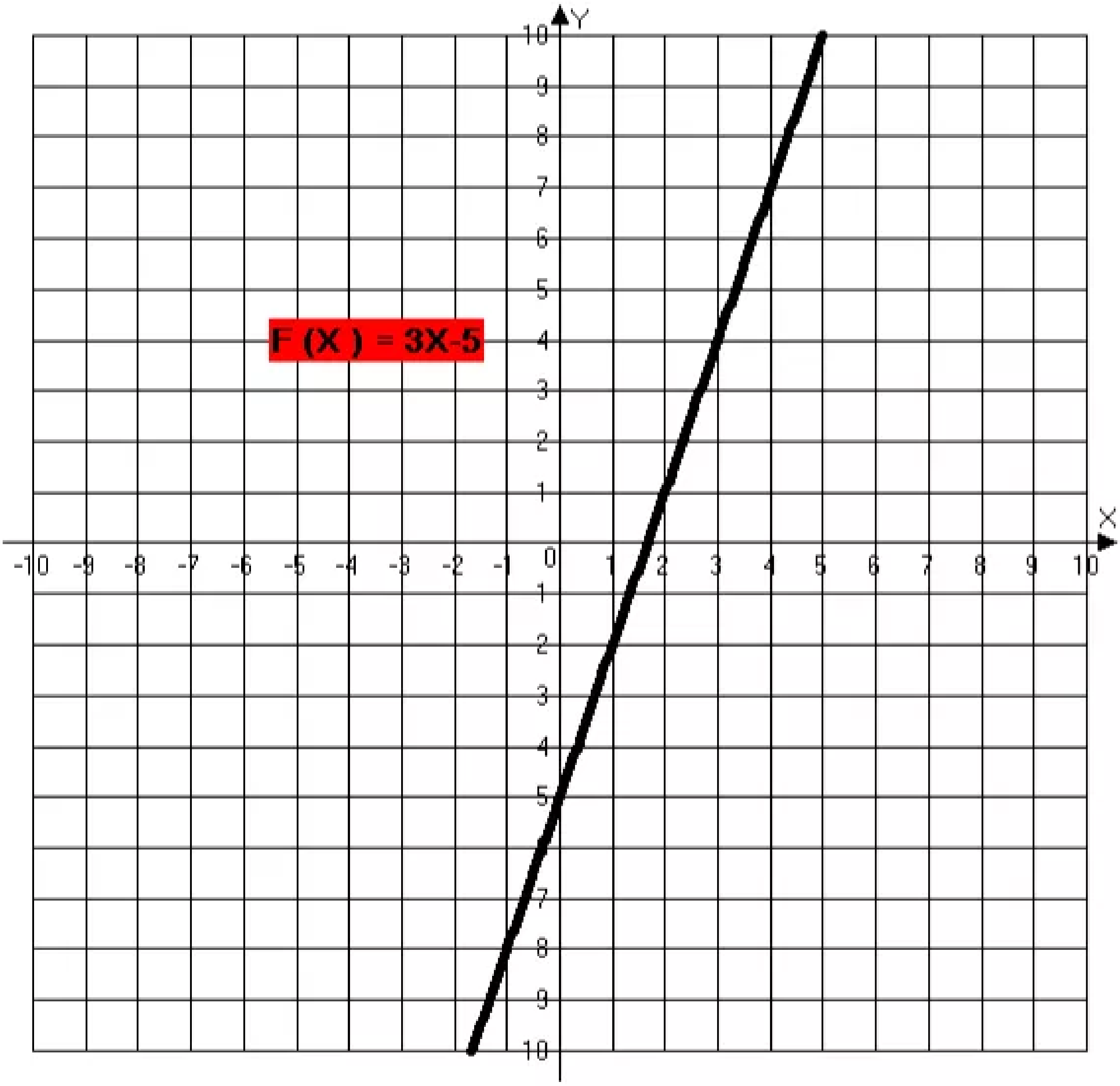
Similarly, plot the other points.



Answer 58e.

Let us consider the function $f(x)=3x-5$

The graph of the above function is shown below



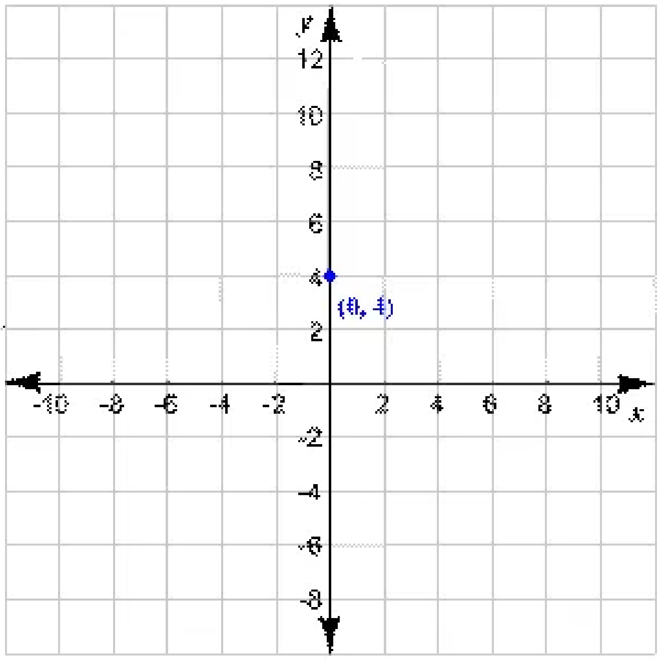
Answer 59e.

The given equation is in slope-intercept form. So, let us graph the function using the y-intercept and the slope.

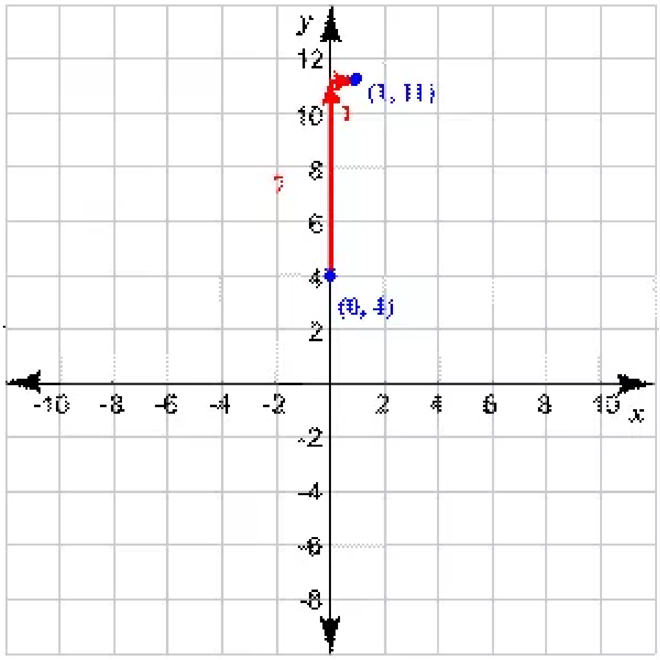
STEP 1 The slope-intercept form of a linear equation is $y = mx + b$, where m denotes the slope of the line, and b denotes the y-intercept.

On comparing, we find that m is 7, and b is 4 for the given function.

STEP 2 The y-intercept is 4. Plot the point (0, 4) on a coordinate plane.

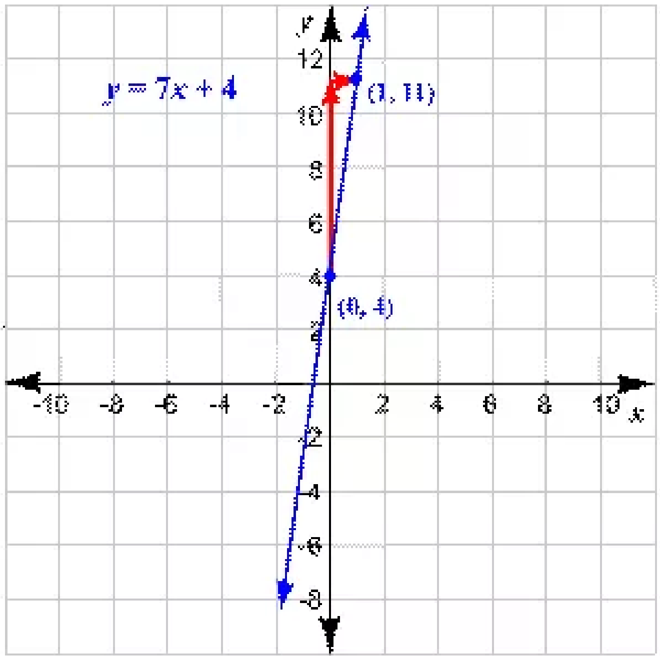


STEP 3 Use the slope to plot a second point on the line. Since the slope is 7 or $\frac{7}{1}$, start at (0, 4) and move 7 units up. Then, move 1 unit to the right.



The second point is (1, 11).

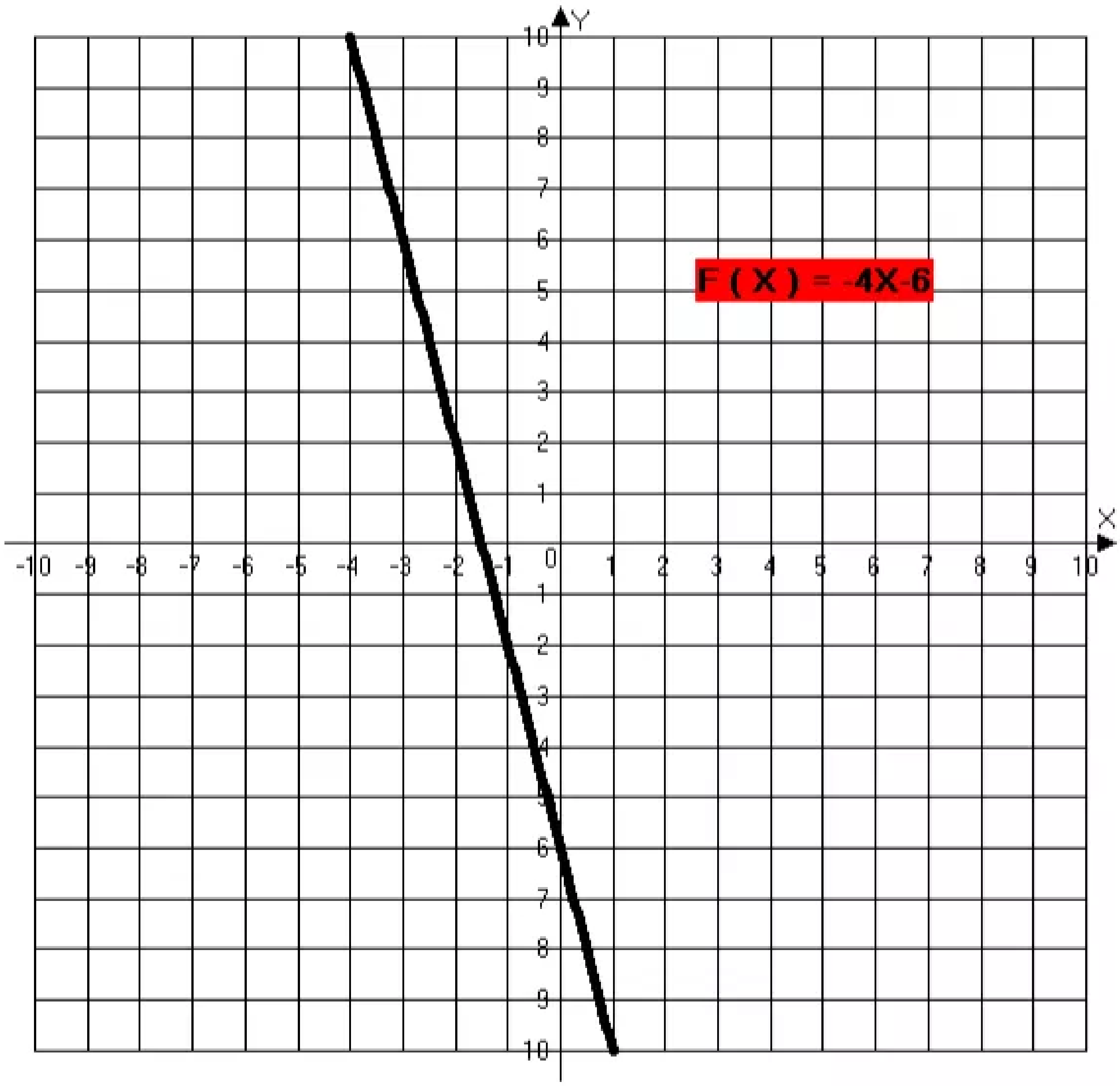
STEP 4 Finally, draw a line through the two points.



Answer 60e.

Let us consider the function $f(x) = -4x - 6$

The graph of the above function is shown below



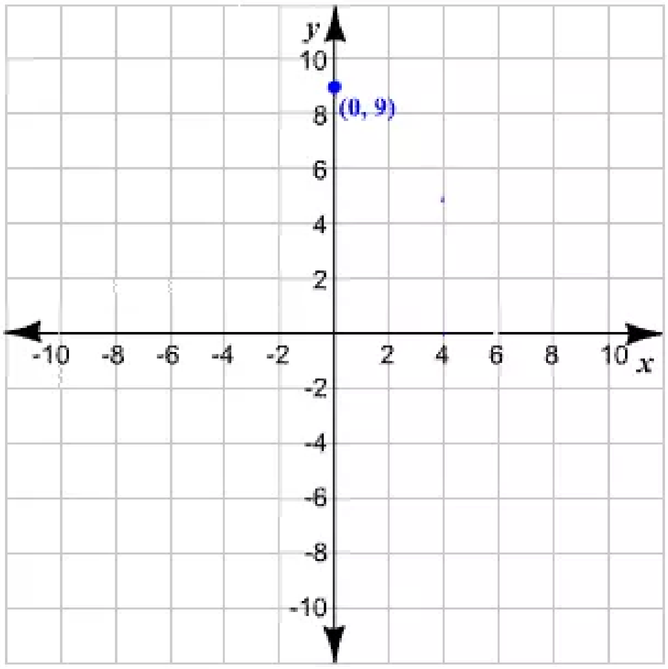
Answer 61e.

The given equation is in the slope-intercept form. Let us graph the function using the y-intercept and the slope.

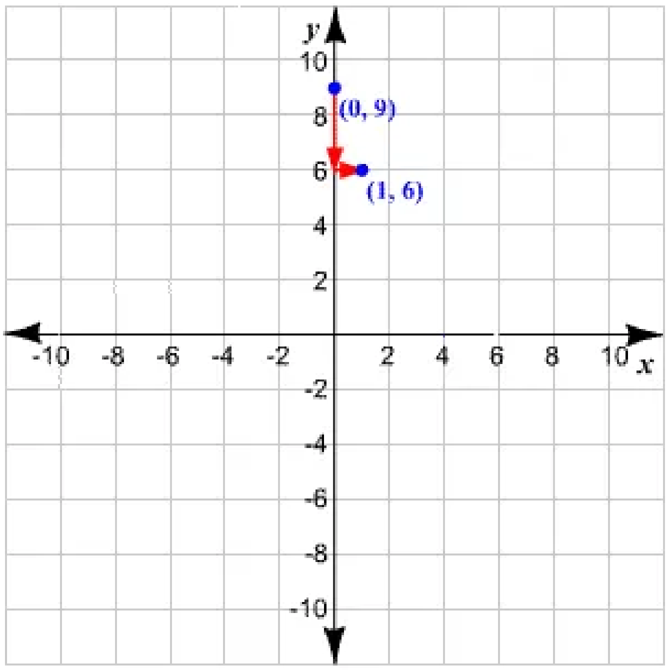
STEP 1 The slope-intercept form of a linear equation is $y = mx + b$, where m denotes the slope of the line, and b denotes the y-intercept.

On comparing, we find that m is -3, and b is 9 for the given function.

STEP 2 The y-intercept is 9. Plot the point (0, 9) on a coordinate plane where the line crosses the y-axis.

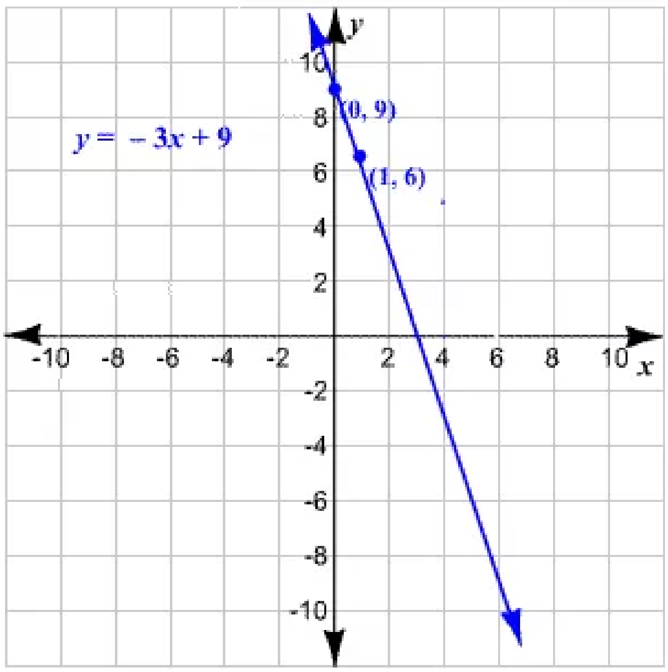


STEP 3 Use the slope to plot a second point on the line. Since the slope is -3 or $\frac{-3}{1}$, start at (0, 9) and then move 3 units down. Now, move 1 unit to the right.



The second point is (1, 6).

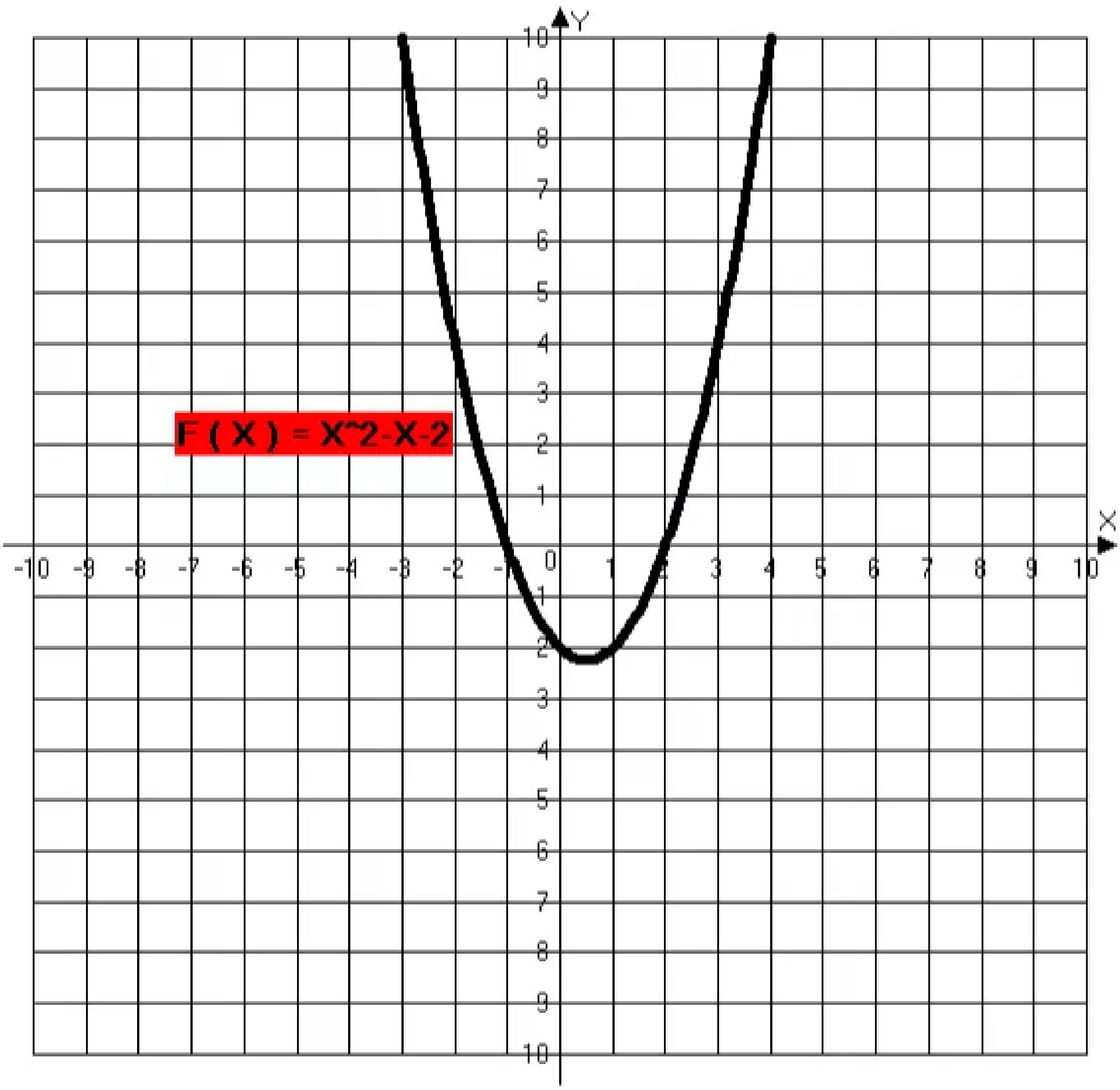
STEP 4 Finally, draw a line through the two points.



Answer 62e.

Let us consider the function $f(x) = x^2 - x - 2$

The graph of the above function is shown below



Answer 63e.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y = ax^2 + bx + c$. On comparing, we have a as 3, b as 20, and c as –7. Since $a = 3 > 0$, the graph opens upwards.

STEP 2 Find the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute –4 for a, and 8 for b in $-\frac{b}{2a}$ and evaluate.

$$-\frac{b}{2a} = -\frac{20}{2(3)}$$
$$= -\frac{20}{6}$$
$$= -\frac{10}{3} \text{ or } -3.33$$

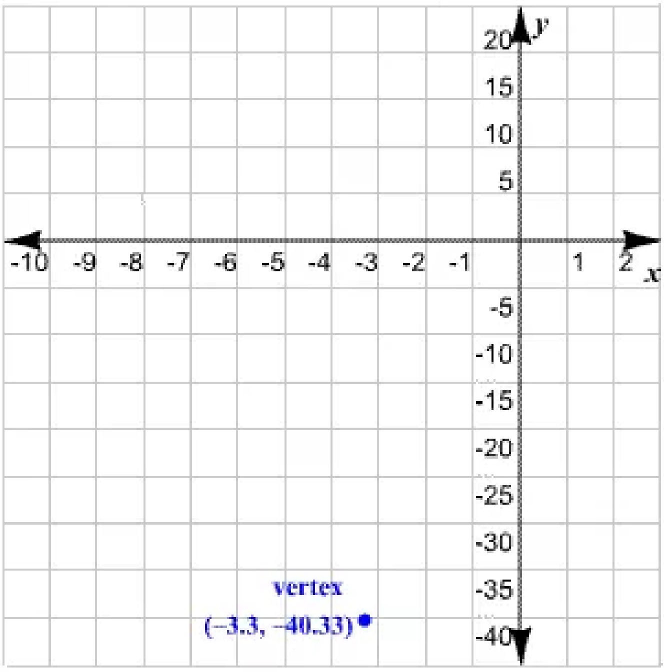
The x-coordinate of the vertex is $-\frac{10}{3}$ or -3.33.

Substitute –3.33 for x in the given function to find the y-coordinate.

$$y = 3(-3.33)^2 + 20(-3.33) - 7$$
$$\approx 3(11.09) - 66.6 - 7$$
$$\approx -40.33$$

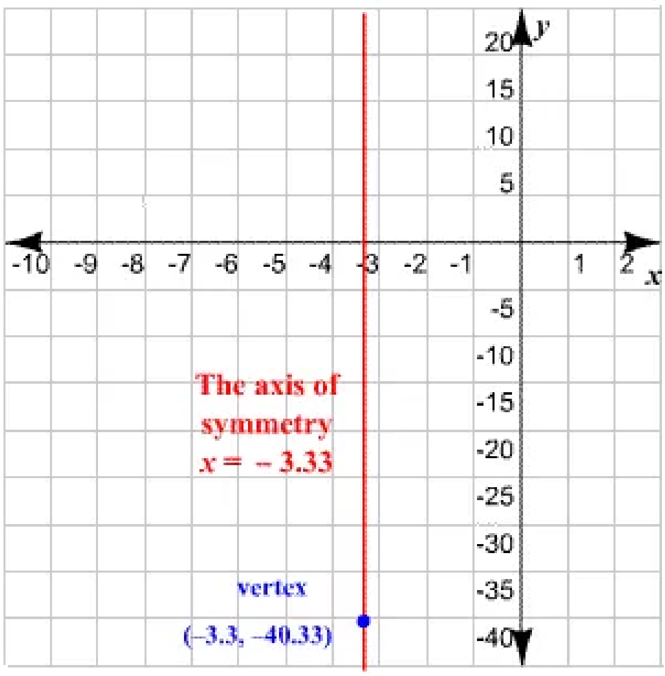
Thus, the vertex of the graph of the given function is (–3.33, –40.33).

Plot the vertex on a coordinate plane.



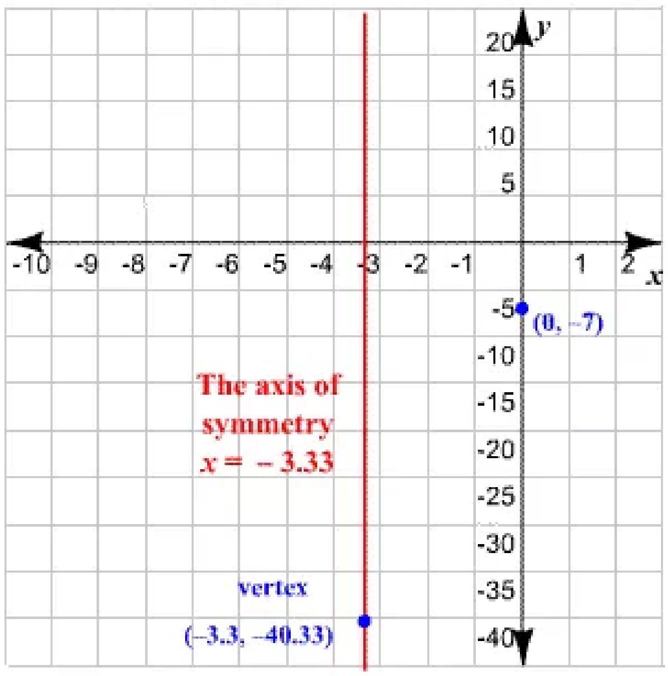
STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$.

The axis of symmetry of the given function is the line $x = -3.33$. Now, draw the axis of symmetry $x = -3.33$.

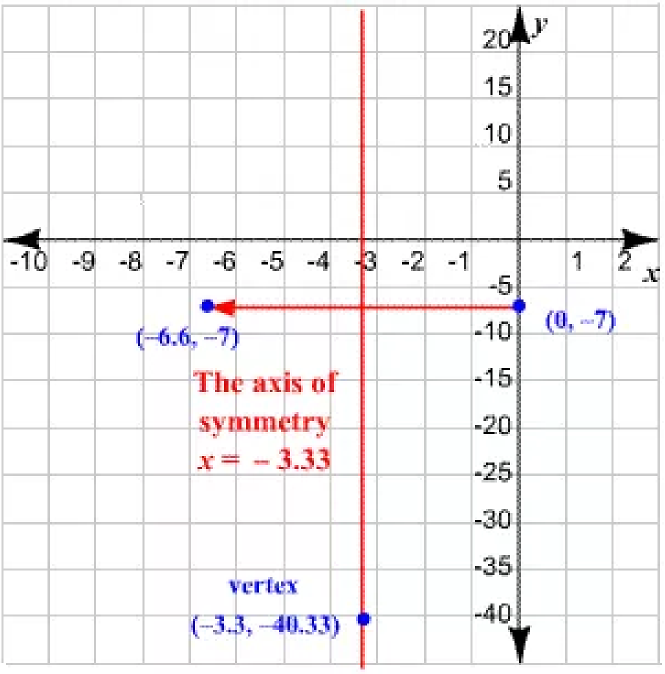


STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola. Thus, the y-intercept of the given function is -7 and $(0, -7)$ is on the parabola.

Plot the point $(0, -7)$ on the same coordinate plane.



Now, reflect the point (0, –7) in the axis of symmetry to get another point.



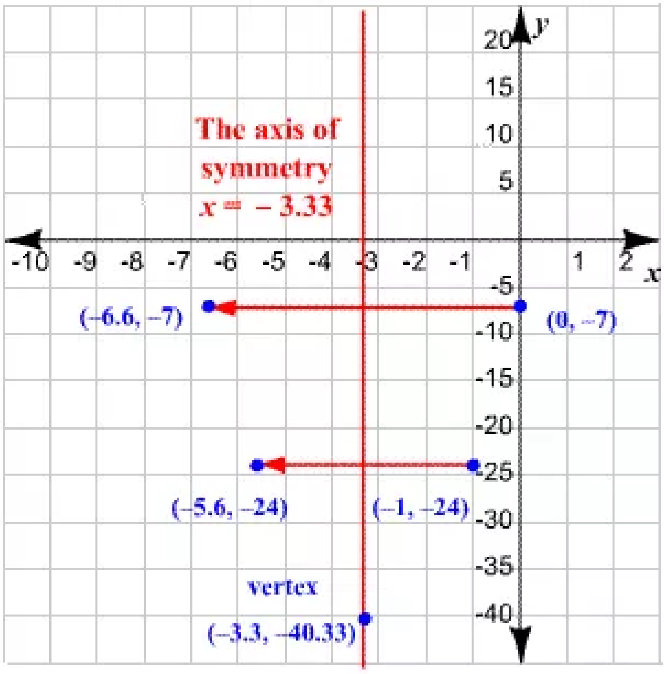
STEP 5 Evaluate the given function for another value of x, say, –1.

Substitute –1 for x in the function and simplify.

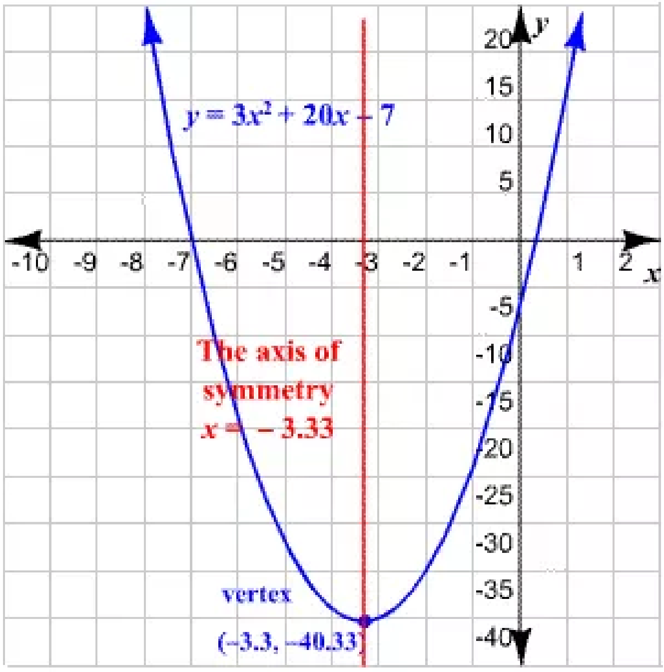
$$\begin{aligned} y &= 3(-1)^2 + 20(-1) - 7 \\ &= 3 - 20 - 7 \\ &= -24 \end{aligned}$$

Thus, the point (–1, –24) lies on the graph.

Plot the point (–1, –24) and its reflection point (–5.6, –24).



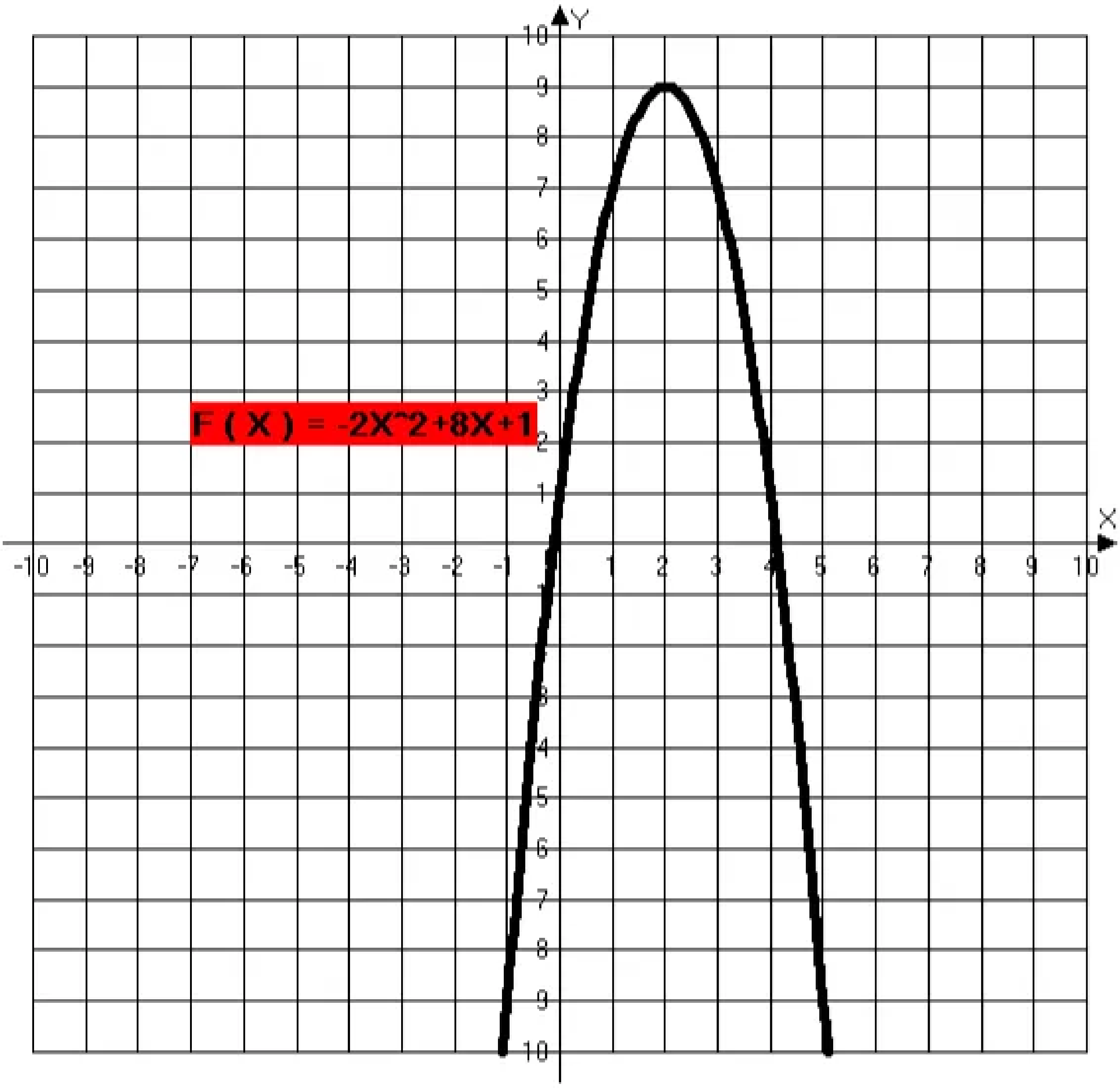
STEP 6 Draw a smooth curve through the plotted points.



Answer 64e.

Let us consider the function $f(x) = -2x^2 + 8x + 1$

The graph of the above function is shown below



Answer 65e.

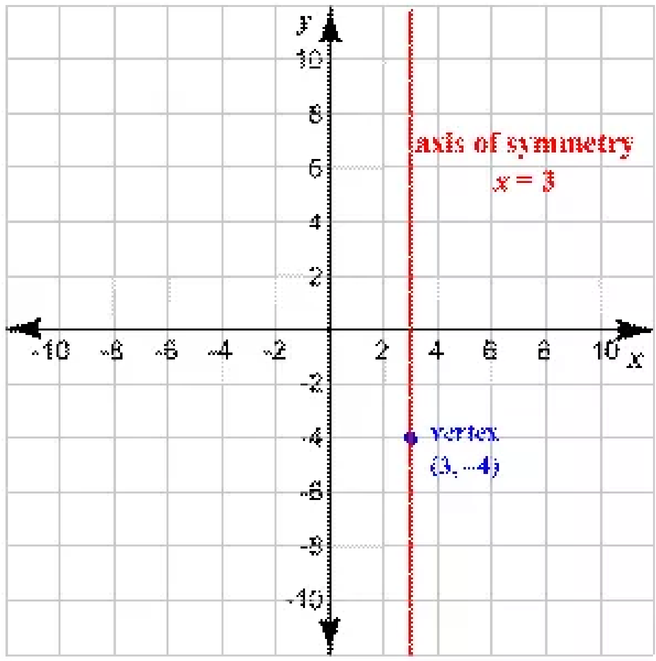
The given function is in vertex form. So, let us graph the function using the vertex.

STEP 1 The graph of a quadratic function in the vertex form $y = a(x - h)^2 + k$ has its vertex at (h, k) and $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.
On comparing the given equation with the vertex form, we find that a is 1, h is 3, and k is –4. Thus, the vertex (h, k) is (3, –4), and the axis of symmetry is $x = 3$.

Since $a < 0$, the parabola opens down.

STEP 2 Plot the vertex (3, –4) on a coordinate plane and draw the axis of symmetry, $x = 3$.



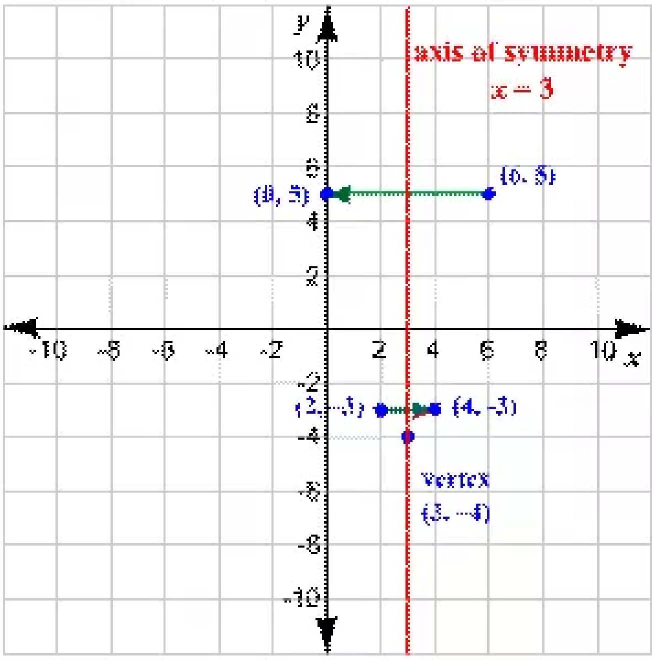
STEP 3 Evaluate the function for any two values of x.

$x = 2: \quad y = (2 - 3)^2 - 4 = -3$

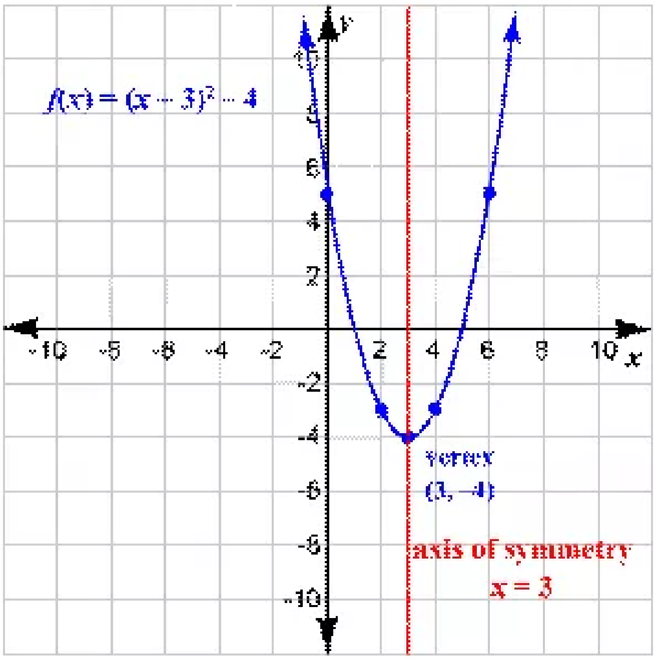
$x = 6: \quad y = (6 - 3)^2 - 4 = 5$

Thus, (2, –3) and (6, 5) are two points on the graph.

Now, plot the points (2, −3) and (6, 5) and their reflections in the axis of symmetry.



STEP 4 Draw a parabola through the points plotted.



Answer 66e.

Let us consider the function $f(x)=(x+4)^2-6$

The graph of the above function is shown below