CBSE Class 12 - Mathematics
Sample Paper 06 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Give an example of a function which is onto but not one-one.

OR

Show that the relation R on the set A = {1 ,2 ,3} given by R = {(1,1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

2. A Relation R : A $\rightarrow$ A is said to be Reflexive if ______ for every a $\in$ A where A is non empty set.

OR

Let f : R $\rightarrow$ R be defined by

$$f(x) = \begin{cases} 2x+3, & \text{when } x < -2 \\ x^2 - 2, & \text{when } -2 \leq x \leq 3 \\ 3x - 1, & \text{when } x > 3 \end{cases}$$

find f(-1)

3. Let A be the set of all students of a boys school. Show that the relation R in A given by R = {(a, b) : a is a sister of b} is the empty relation and R' = {(a, b) : the difference between heights of a and b is less than 3 meters} is the universal relation.

4. From the following matrix equation, find the value of x.

$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$

5. Construct a 3×4 matrix A = [a_{ij}] whose elements are given by:

a_{ij} = i + j

OR

If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, write the value of x.

6. Find adj (A), if $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$.

7. Evaluate: $\int \frac{2e^x}{\sqrt{4-e^{2x}}} dx$

OR

Evaluate: $\int_{\pi}^{3\pi/2} \sqrt{1-\cos 2x}\, dx$

8. The area of the bounded by the lines y = 2, x = 1, x = a and the curve y = f(x), which cuts the last two lines above the first line for all $a \geq 1$, is equal to

$\frac{2}{3}\left[(2a)^{3/2} - 3a + 3 - 2\sqrt{2}\right]$. Find f(x)

9. How many arbitrary constants are there in the general solution of the differential equation of order 3.

OR

Write the degree of the differential equation $a^2\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{4}}$.

10. Find the intervals of function $f(x) = x^3 - 12x^2 + 36x + 17$ is
    a. increasing
    b. decreasing.
11. Prove that f(x) = ax + b, where a, b are constants and a > 0 is strictly increasing function on R.
12. Classify Acceleration as scalars and vector quantity.
13. Write the vector equation of a line passing through a point having position vector $\vec{\alpha}$ and parallel to vector $\vec{\beta}$.
14. If the equations of a line AB is $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$ find the directions ratio of line parallel to AB.
15. For what value of x, the given matrix A = $\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix?
16. Let A and B be the events such that P(A) = $\frac{5}{11}$, P(B) = $\frac{6}{11}$ and P(A ∪ B) = $\frac{7}{11}$, find P(A/B).

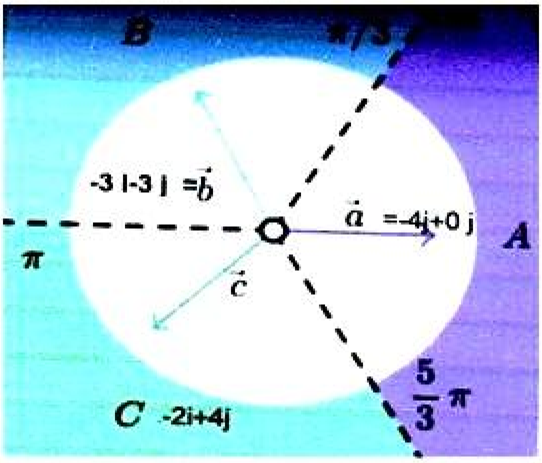
Section - II

17. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness, and Leadership. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with total award money of Rs. 2200.

    School Q wants to spend Rs 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs1200, using matrices, find the following:

i. What is award money for Tolerance?
   a. 350
   b. 300
   c. 500
   d. 400
ii. What is the award money for Leadership?
   a. 300
   b. 280
   c. 450
   d. 500
iii. What is the award money for Kindness?
   a. 500
   b. 400
   c. 300
   d. 550
iv. If a matrix A is both symmetric and skew-symmetric, then
   a. A is a diagonal matrix
   b. A is a scalar matrix
   c. A is a zero matrix
   d. A is a square matrix
v. If A and B are two matrices such that AB = B and BA = A, then B^2 is equal to
   a. B
   b. A
   c. 1
   d. 0

18.



Three friends Ganesh, Dinesh and Ramesh went for playing a Tug of war game. Team A, B, and C belong to Ganesh, Dinesh and Ramesh respectively.

Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below).

Team A pulls with $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team B $\rightarrow F_2 = -2\hat{i} + 4\hat{j}$ KN

Team C $\rightarrow F_3 = -3\hat{i} - 3\hat{j}$ KN

i. Which team will win the game?
   a. Team B
   b. Team A
   c. Team C
   d. No one

ii. What is the magnitude of the teams combine Force?
   a. 7 KN
   b. 1.4 KN
   c. 1.5 KN
   d. 2 KN

iii. In with direction approx the ring getting pulls:
   a. 2.0 radian
   b. 2.5 radian
   c. 2.4 radian
   d. 3 radian

iv. What is the magnitude of the force of Team B?
   a. $2\sqrt{5}$ KN

b. 6 KN
c. 2 KN
d. $\sqrt{6}$ KN

v. How many KN Force is applied by Team A?

a. 5 KN
b. 4 KN
c. 2 KN
d. 16 KN

Part - B Section - III

19. Find the principal values of $\sec^{-1}\left(2\sin\frac{3\pi}{4}\right)$.

20. Let $\begin{vmatrix} 4 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 4 & 1 \end{vmatrix}$ find all the possible values of x and y if x and y are natural numbers.

OR

Write the cofactor of a_{12} in the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

21. Find $\frac{dy}{dx}$, if $y = x^{\tan x} + \sqrt{\frac{x^2+1}{2}}$

22. Find the points of local maxima or local minima and corresponding local maximum and local minimum values of the function. Also, find the points of inflection, if any: $f(x) = x^3 - 2ax^2 + a^2x$, $a > 0$, $x \in R$.

23. Integrate: $\int \frac{1}{1+\cos 3x} dx$

OR

Evaluate: $\int_{-1}^{1} (x+3) dx$

24. Using integration, find the area of region bounded by the triangle whose vertices are (-1, 0), (1, 3) and (3, 2).

25. Solve $\left(1+x^2\right)\frac{dy}{dx} + y = \tan^{-1} x$.

26. Write the set of values of **a** for which $f(x) = \log_a x$ is decreasing in its domain.

27. Find the equations of the planes parallel to the plane x + 2y - 2z + 8 = 0 which are at distance of 2 units from the point (2, 1, 1).

28. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs not more than one bulb will fuse after 150 days of use.

OR

A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize exactly once?

Section - IV

29. Classify the function f : R $\rightarrow$ R, defined by f (x) = x^3 - x as injection, surjection or bijection.
30. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at x = 1 .
31. Differentiate w.r.t. x: $(x + 1)^2 + (x + 2)^3.(x + 3)^4$.

OR

If y = e^{-x} cos x, show that $\frac{d^2y}{dx^2} = 2e^{-x}$ sin x

32. Find the equation of the tangent to the curve y = $(x^3 - 1)(x - 2)$ at the points where the curve cuts the x-axis.
33. Evaluate: $\int \frac{1}{1+\cot x} dx$
34. Find the area of the region bounded by y = $|x - 1|$ and y = 1.

OR

Find the area of the region included between the parabola $y = \frac{3}{4}x^2$ and the line 3x - 2y + 12 = 0

35. Solve the differential equation $\frac{dy}{dx} + 2xy = y$.

Section - V

36. There are two bags. The first bag contains 5 white and 3 black balls and the second bag contains 3 white and 5 black balls. Two balls are drawn at random from the first bag and are put into the second bag without noticing their colours. Then two balls are drawn from the second bag. Find the probability that the balls are white and black.

OR

20% of the bulbs produced by a machine are defective. Find the probability distribution of the number of defective bulbs in a sample of 4 bulbs chosen at random.

37. Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

OR

Find the vector equation of the plane that contains the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$. Also, find the length of perpendicular drawn from the point (2, 1, 4) to the plane thus obtained.

38. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has at most Rs 120 to spend on petrol and one hour time. He wishes to find the maximum distance that he can travel. Determine the maximum distance that the man can travel.

OR

A manufacturer produces nuts and bolts. It takes 1 hours of work on machine A and 3 hours on machine B to product a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7 per package of bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day? From the above as a linear programming problem and solve it graphically.

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Solution

Part - A Section - I

1. Consider a function f: Z→N such that f(x) = |x|
   Since the Z maps to every single element in N twice, this function is onto but not one - one.

   OR

   Since 1, 2, 3 ∈ A and (1,1), (2, 2), (3, 3) ∈ R i.e. for each a ∈ A, (a, a) ∈ R. So, R is reflexive.
   We observe that (1, 2) ∈ R but (2,1) ∈ R. So, R is not symmetric.
   Also, (1,2) ∈ R and (2, 3) ∈ R but (1, 3) ∈ R. So, R is not transitive.
2. (a, a) ∈ R

   OR

   The function is $f(x)=\begin{cases}2x+3, & \text{when } x<-2\\ x^2-2, & \text{when } -2\leq x\leq 3\\ 3x-1, & \text{when } x>3\end{cases}$
   Since f(x) = x^2 - 2 , when x = - 1
   ∴ f(- 1) = (- 1)^2 - 2 = 1 - 2 = - 1
   ∴ f(- 1) = - 1.
3. Since the school is a boys school, no student of the school can be the sister of any student of the school. Hence, R = ϕ, showing that R is the empty relation.
   It is also obvious that the difference between the heights of any two students of the school has to be less than 3 meters. This shows that R' = A × A is the universal relation.
4. According to the question,
   $\begin{bmatrix}x+y & 4\\ -5 & 3y\end{bmatrix}=\begin{bmatrix}3 & 4\\ -5 & 6\end{bmatrix}$
   Equating the corresponding elements,
   $x+y=3$ ...(i)
   and $3y=6$ ...(ii)

From Eq. (ii), we get

$\Rightarrow y = 2$

On substituting y = 2 in Eq. (i), we get

x + 2 = 3

$\Rightarrow$ x = 1

5. $a_{ij} = i + j$

$a_{11} = 1 + 1 = 2$, $a_{12} = 1 + 2 = 3$, $a_{13} = 1 + 3 = 4$, $a_{14} = 1 + 4 = 5$

$a_{21} = 2 + 1 = 3$, $a_{22} = 2 + 2 = 4$, $a_{23} = 2 + 3 = 5$, $a_{24} = 2 + 4 = 6$

$a_{31} = 3 + 1 = 4$, $a_{32} = 3 + 2 = 5$, $a_{33} = 3 + 3 = 6$, $a_{34} = 3 + 4 = 7$

Therefore, A = $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$

OR

According to the question,

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

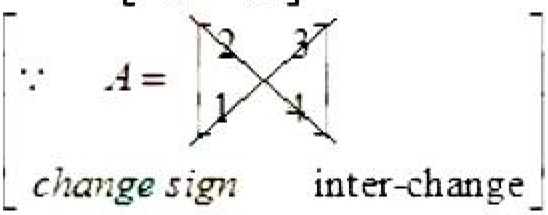
$$\Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

Equating the corresponding elements,

x = 13.

6. $\text{adj}A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$



7. Let, $I = \int \frac{2e^x dx}{\sqrt{4-e^{2x}}}$

$= 2\int \frac{d(e^x)}{\sqrt{2^2-(e^x)^2}}$

$= 2\sin^{-1}\left(\frac{e^x}{2}\right) + C$ $\left(\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C\right)$

OR

I = $\int_{\pi}^{3\pi/2} \sqrt{1-\cos 2x dx}$

= $\int_{\pi}^{\frac{3\pi}{2}} \sqrt{1-(1-2\sin^2 x)}dx$

= $\int_{\pi}^{\frac{3\pi}{2}} \sqrt{2\sin^2 xdx}$

= $\sqrt{2}\int_{\pi}^{\frac{3\pi}{2}} \sin xdx$

= $\sqrt{2}[-\cos x]_{\pi}^{\frac{3\pi}{2}}$

= $\sqrt{2}[-\cos\frac{3\pi}{2} + cos\pi]$

= $\sqrt{2}[0+1]$

= $\sqrt{2}$

8. we are given,

$\int_{a}^{1}[f(x)-2]dx = \frac{2}{3}\left[(2a)^{3/2}-3a+3-2\sqrt{2}\right]$

Differentiating w.r.t a, we get

f(a) - 2 $= \frac{2}{3}\left[\frac{3}{2}\sqrt{2a}.2-3\right]$

f(a) = 2$\sqrt{2a}, a \geq 1$

$\therefore\ f(x) = 2\sqrt{2x}, x \geq 1$

9. There are as many arbitrary constants present in the general solution of D.E as the order of D.E, i.e; 3.

OR

we are given that

$a^2\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{4}}$

$\left(a^2\frac{d^2y}{dx^2}\right)^4 = 1+\left(\frac{dy}{dx}\right)^2$

$\left(a^8\left(\frac{d^2y}{dx^2}\right)^4\right) - \left(\frac{dy}{dx}\right)^2 - 1 = 0$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 4, so

Degree of equation = 4

10. f (x) = $x^3 - 12x^2 + 36x + 17$

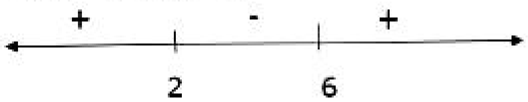
f'(x) = $3x^2$ - 24x + 36

f'(x) = 3(x^2 - 8x + 12)

= 3(x - 6)(x - 2)

$\Rightarrow$ f'(x) =3(x - 6)(x - 2)= 0

$\Rightarrow$ x = 2 or x = 6



Function f(x) is decreasing for x $\in$ [2, 6] and increasing in x $\in (-\infty, 2) \cup (6, \infty)$

11. We are given that,

$f(x) = ax + b, a > 0$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow ax_1 > ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b > ax_2 + b$ for some b

$\Rightarrow f(x_1) > f(x_2)$

$\therefore f(x)$ is strictly increasing function of R

12. The acceleration is a vector quantity as it involves both magnitudes as well as direction.

13. The vector equation of the line passing through the point having position vector $\vec{\alpha}$ and parallel to vector $\vec{\beta}$ is, $\vec{r} = \vec{a} + \lambda\vec{\beta}$

14. The given line is,

$\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$. Therefore, direction ratios of a line parallel to AB are (1, -2, 4)

15. For A to be singular matrix its determinant should be equal to 0.

0 = (3 - 2x) × 4 - (x + 1) × 2

0 = 12 - 8x - 2x - 2

0 = 10 - 10 x

Thus for x = 1, the given matrix is singular

16. A and B be the events such that P(A) $= \frac{5}{11}, P(B) = \frac{6}{11}$ and

$\mathrm{P(A \cap B)}$

$= \mathrm{P(A) + P(B) - P(A \cup B)}$

$= \frac{5}{11} + \frac{6}{11} - \frac{7}{11} = \frac{4}{11}$

$\mathrm{P(A/B)} = \frac{P(A \cap B)}{P(B)}$

$= \frac{4}{11} \div \frac{6}{11} = \frac{4}{6} = \frac{2}{3}$

Section - II

17. Three equations are formed from the given statements:

$3x + 2y + z = 2200$

$4x + y + 3z = 3100$ and

$x + y + z = 1200$

Converting the system of equations in matrix form we get,

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

i.e. $AX = B$

where $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

$\Rightarrow X = A^{-1} B$

$A^{-1} = \frac{1}{|A|} (\text{Adj } A)$

$|A| = 3(1 - 3) - 2(4 - 3) + 1(4 - 1) = -6 - 2 + 3 = -5$

$$\text{Adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{5} \begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 4400 + 3100 - 6000 \\ 2200 - 6200 + 6000 \\ -6600 + 3100 + 6000 \end{bmatrix}$$

$$= \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

$\Rightarrow x = 300, y = 400$ and $z = 500$

i.e. The award money for each value are Rs.300 for Tolerance, Rs.400 for Kindness and

Rs.500 for Leadership.

i. (b) 300

ii. (d) 500

iii. (b) 400

iv. (c) A is a zero matrix

v. (a) B

18. i. (a) Team B

ii. (b) 1.4 KN

iii. (c) 2.4 radian

iv. (a) $2\sqrt{5}$ KN

v. (b) 4 KN

Part - B Section - III

19. $\sec^{-1}\left(2\sin\frac{3\pi}{4}\right) = \sec^{-1}\left(2\times\left(\frac{1}{\sqrt{2}}\right)\right) = \sec^{-1}(\sqrt{2})$

We know that, for any x ∈ R - (-1, 1), $\sec^{-1}$x represents an angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ whose tangent is x.

$\therefore \sec^{-1}(\sqrt{2}) = \frac{\pi}{4}$

$\therefore$ Principal value of $\sec^{-1}\left(2\sin\frac{3x}{4}\right)$ is $\frac{\pi}{4}$.

20. 4 – xy = 4 - 8

xy = 8

If x = 1 , x = 4 , x = 2 , x = 8,

then, y = 8 , y =2 , y = 4 or y = 1 respectively

OR

Given the matrix is, $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

We need to find the cofactor of a_{12} in the matrix

$$\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$$

Firstly we know what the element at position a_{12} in the matrix is.

a_{12} = -3

And as discussed above, the sign at a_{12} is (-).

For cofactor of -3, eliminate first row and second column in the matrix.

Cofactor of -3 = $\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix}$

⇒ Cofactor of -3 = (6 × -7) – (4 × 1)

⇒ Cofactor of -3 = -42 – 4

⇒ Cofactor of -3 = -46

Since, the sign of cofactor of -3 is (-), then

Cofactor of -3 = -(-46)

⇒ Cofactor of -3 = 46

Thus, the cofactor of -3 is 46.

21. We have, $\frac{dy}{dx}$, if $y = x^{\sin x} + \sqrt{\frac{x^2+1}{2}}$

Taking, u = xtan x and $= \sqrt{\frac{x^2+1}{2}}$

log u = tan x log x .... (i)

and $v^2 = \frac{x^2+1}{2}$ .... (ii)

On, differentiating eq. (i) w.r.t. x, we get

$\frac{1}{u} \cdot \frac{du}{dx} = \tan x \cdot \frac{1}{x} + \log x \sec^2 x$

$\Rightarrow \frac{du}{dx} = u\left[\frac{\tan x}{x} + \log x \cdot \sec^2 x\right]$

$= x^{\sin x}\left[\frac{\tan x}{x} + \log x \cdot \sec^2 x\right]$ .... (iii)

also, differentiating Eq. (ii) w.r.t. x, we get

$2v \cdot \frac{dv}{dx} = \frac{1}{2}(2x) \Rightarrow \frac{dv}{dx} = \frac{1}{4v} \cdot (2x)$

$\Rightarrow \frac{dv}{dx} = \frac{1}{4 \cdot \sqrt{\frac{x^2+1}{2}}} \cdot 2x = \frac{x \cdot \sqrt{2}}{2\sqrt{x^2+1}}$

$\Rightarrow \frac{dv}{dx} = \frac{x}{\sqrt{2(x^2+1)}}$ .... (iv)

Now, y = u + v

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= x^{\tan x}\left[\frac{\tan x}{x} + \log x \cdot \sec^2 x\right] + \frac{x}{\sqrt{2(x^2+1)}}$

22. Given: $f(x) = x^3 - 2ax^2 + a^2x$

$\because f'(x) = 3x^2 - 4ax + a^2$

$f''(x) = 6x - 4a$,

For maxima and minima, we must have

f'(x) = 0

$\Rightarrow 3x^2 - 4ax + a^2 = 0$

$\because x = \frac{4a \pm \sqrt{16a^2 - 12a^2}}{6}$

$= \frac{4a \pm 2a}{6} = a, \frac{a}{3}$

Now,

f''(a) = 2a > 0 as a > 0

$\because$ x = a is point of local minima

$f''\left(\frac{a}{3}\right) = -2a < 0$ as a > 0

$\because x = \frac{a}{3}$ is point of local maxima

Hence,

Local max value = $f\left(\frac{a}{3}\right) = \frac{4a^3}{27}$

Local min value = f(a) = 0.

23. Let I = $\int \frac{1}{1+\cos 3x} dx$. Then,

$I = \int \frac{1}{1+\cos 3x} \times \frac{1-\cos 3x}{1-\cos 3x} \times dx$

$= \int \frac{1-\cos 3x}{1-\cos^2 3x} \times dx$

$= \int \frac{1-\cos 3x}{\sin^2 3x} \times dx$

$= \int \left(\frac{1}{\sin^2 3x} - \frac{\cos 3x}{\sin^2 3x}\right) dx$

$= \int (\text{cosec}^2 3x - \text{cosec} 3x \cot 3x)\, dx$

$= \frac{-\cot 3x}{3} + \frac{\cos ec 3x}{3} + c$

$= \frac{-1}{3} \times \frac{\cos 3x}{\sin 3x} + \frac{1}{3} \times \frac{1}{\sin 3x} + c$

$= \frac{1-\cos 3x}{3\sin 3x} + c \therefore I = \frac{1-\cos 3x}{3\sin 3x} + c$

OR

Let $I = \int_{-1}^{1} (x+3) dx$

We know that,

$\int_a^b f(x) dx = \lim_{h \to 0} h$ [f(a) + f(a + h) + f(a + 2h) + ... + f(a + (n - 1)h)]

Where $h = \frac{b-a}{n}$

Here a = -1, b = 1 and f(x) = x + 3

Therefore, $h = \frac{2}{n} \Rightarrow nh = 2$

Thus, we have

$I = \int_{-1}^{1}(x+3)dx$

$I = \lim\limits_{h \to 0}$ h[f(-1) + f(-1 + h) + f(-1 + 2h) + ... + f(-1 + (n - 1)h)]

$= \lim\limits_{h \to 0}$ h[2 + (2 + h) + (2 + 2h) + ... + ({(n - 1)h + 2}]

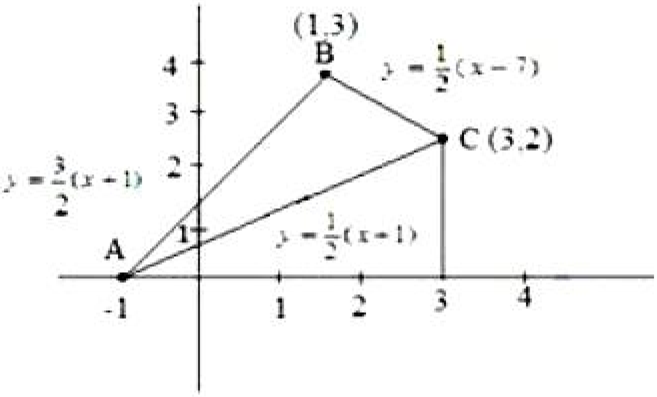
$= \lim\limits_{h \to 0}$ h[2n + h(1 + 2 + 3 + ...-1]]

$= \lim_{h \to 0} h\left[2n + h\frac{n(n-1)}{2}\right] \left[\because h = \frac{2}{n} \& \text{ if } h \to 0 \Rightarrow n \to \infty\right]$

$= \lim_{n \to \infty} \frac{2}{n}\left[2n + \frac{2}{n}\frac{n(n-1)}{2}\right]$

$= \lim_{n \to \infty} 4 + \frac{2n^2}{n^2}\left(1 - \frac{1}{n}\right)$

= 4 + 2 = 6

24. 

A (-1, 0) B (1, 3) C (3, 2)

Equation of AB

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{3-0}{1+1}(x+1)$

$y = \frac{3}{2}(x+1)$

Similarly,

Equation of BC $y = \frac{-1}{2}(x - 7)$

Equation of AC $= \frac{1}{2}(x+1)$

Area $\Delta ABC = \int_{-1}^{1} \frac{3}{2}(x+1)\,dx + \int_{1}^{3} \frac{1}{2}(x-7)\,dx - \int_{-1}^{3} \frac{1}{2}(x+1)\,dx$

$= \frac{3}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{1} + \frac{1}{2}\left[7x - \frac{x^2}{2}\right]_{1}^{3} - \left[\frac{x^2}{2} + x\right]_{-1}^{3}$

$= \frac{3}{2}\left[\left(\frac{1}{2} + 1\right) - \left(\frac{1}{2} - 1\right)\right] + \frac{1}{2}\left[\left(21 - \frac{9}{2}\right) - \left(7 - \frac{1}{2}\right]\right.$

$-\frac{1}{2}\left[\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} - 1\right)\right]$

$= \frac{3}{2}(2) + \frac{1}{2}(10) - \frac{1}{2}(8) = 3 + 5 - 4$

= 4 sq. units

25. The given differential equation may be written as

$\frac{dy}{dx} + \frac{1}{(1+x^2)} \cdot y = \frac{\tan^{-1} x}{(1+x^2)}$.

This is of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{(1+x^2)}$ and $Q = \frac{\tan^{-1} x}{(1+x^2)}$.

Thus, the given equation is linear.

$\text{IF} = e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1} x}$.

$\therefore$ the required solution is

$y \times \text{IF} = \int [Q \times \text{IF}] dx + C$,

i.e., $y \times e^{\tan^{-1} x} = \int \left\{ \frac{\tan^{-1} x}{(1+x^2)} \cdot e^{\tan^{-1} x} \right\} dx + C$

$= \int (te^t)\, dt + C$ where $\tan^{-1}x = t$

$= te^t - \int 1 \cdot e^t dt + C$ [integrating by parts]

$= te^t - e^t + C = e^t(t - 1) + C$

$= e\tan^{-1}x\,(\tan^{-1}x - 1) + C$

$\Rightarrow y = \left(\tan^{-1} x - 1\right) + Ce^{-\tan^{-1} x}$

Hence, $y = \left(\tan^{-1} x - 1\right) + Ce^{-\tan^{-1} x}$ is the required solution.

26. Given: $f(x) = \log_a x$

Case 1: Let a > 1

Here $x_1 < x_2$

$\Rightarrow \log_a x_1 < \log_a x_2$

$\Rightarrow f(x_1) < f(x_2)$

$\therefore x_1 < x_2 \Rightarrow f(x_1) < f(x_2), \forall x_1, x_2 \in (0, \infty)$

so , f(x) is increasing for a>1 on $(0, \infty)$

Case 2: Let 0 < a < 1

Here

$x_1 < x_2$

$\Rightarrow \log_a x_1 > \log_a x_2$

$\Rightarrow f(x_1) > f(x_2)$

$\therefore x_1 < x_2 \Rightarrow f(x_1) > f(x_2), \forall x_1, x_2 \in (0, \infty)$

So f(x) is decreasing on $(0, \infty)$

Thus, for 0 < a < 1 f(x) is decreasing in its domain

27. Since the planes are parallel to $x + 2y - 2z + 8 = 0$, they must be of the form:

$x + 2y - 2z + \theta = 0$

We know, the distance of point (x_1, y_1, z_1) from the plane

π: ax + by + cz + d = 0 is given by:

$$p = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

According to the question, the distance of the planes from (2, 1, 1) is 2 units.

$$\Rightarrow \left|\frac{(1)(2)+(2)(1)+(-2)(1)+\theta}{\sqrt{1^2+2^2+(-2)^2}}\right| = 2$$

$$\Rightarrow \left|\frac{2+\theta}{3}\right| = 2$$

$$\Rightarrow \frac{2+\theta}{3} = 2 \quad \frac{2+\theta}{3} = -2$$

$\Rightarrow \theta = 4$ or -8

$\Rightarrow$The required planes are: x + 2y – 2z + 4 = 0 and x + 2y – 2z – 8 = 0.

28. Let x = number of bulbs that will fuse after 150 days of use in an experiment of 5 trials.

Given that trials are made with replacement, thus, the trials are Bernoulli trials.

Also given that p = 0.05

Thus, q = 1 - p = 1 - 0.05 = 0.95

Here, we can clearly observe that x has a Binomial representation with n = 5 and p = 0.05

Thus, $P(X = x) = {}^nC_x q^{n-x} p^x$, where x = 0, 1, 2, ...n

$= {}^5C_x (0.95)^{5-x} (0.05)^x$

Probability of not more than one such bulb in a random drawing of 5 bulbs = $P(X \leq 1)$

= P(X = 0) + P(X = 1)

$= {}^5C_0 (0.95)^{5-0} (0.05)^0 + {}^5C_1 (0.95)^{5-1} (0.05)^1$

$= 1 \times 0.95^5 + 5 \times (0.95)^4 \times 0.05$

$= (0.95)^4 (0.95 + 0.25)$

$= (0.95)^4 \times 1.2$

OR

Let X represents the number of prizes winning in 50 lotteries and the trials are Bernoulli trials

Here clearly, X follows Binomial Distribution, where n = 50 and $p = \frac{1}{100}$

Thus, q = 1 - p

$= 1 - \frac{1}{100}$

$= \frac{99}{100}$

$\therefore P(X = x) = {}^{n}C_x q^{n-x} p^x$

$= {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^x$

Probability of winning in lottery exactly once = P(X = 1)

$= {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^1$

$= 50 \left(\frac{1}{100}\right) \left(\frac{99}{100}\right)^{49}$

$= \frac{1}{2} \left(\frac{99}{100}\right)^{49}$

Section - IV

29. $f : R \rightarrow R$ defined by $f(x) = x^3 - x$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow x^3 - x = y^3 - y$

$\Rightarrow x^3 - x = y^3 - y$

$\Rightarrow (x - y)(x^2 + xy + y^2 - 1) = 0$

$\because x^2 + xy + y^2 \geq 0 \Rightarrow x^2 + xy + y^2 - 1 \geq -1$

$\therefore x^2 + xy + y^2 - 1 \neq 0$

$\Rightarrow x - y = 0 \Rightarrow x = y$

$\therefore$ f is not one-one.

Surjective:

Let $y \in R$, then

$f(x) = y$

$\Rightarrow x^3 - x - y = 0$

We know that a degree 3 equation has atleast one real solution.

let $x = \alpha$ be that real solution

$\therefore \quad \alpha^3 - \alpha = y$

$\Rightarrow f(\alpha) = y$

$\therefore$ For each $y \in R$, there exist $x = \alpha \in R$ such that $f(\alpha) = y$

$\therefore$ f is onto.

Thus, it is surjective but not injective.

30. Given: $f(x) = [x], 0 < x < 3$

$Rf'(1) = \lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h \to 0} \frac{[1+h]-1}{h} = \lim_{h \to 0} \frac{1-1}{h}$

$= \lim_{h \to 0} \frac{0}{h} = 0$

And $Lf'(1) = \lim_{h \to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim_{h \to 0} \frac{[1-h]-1}{-h}$

$= \lim_{h \to 0} \frac{0-1}{-h} = \infty$

Since $Rf'(1) \neq Lf'(1)$

Therefore, $f(x) = [x]$ is not differentiable at x =1.

31. Let y = $(x + 1)^2 + (x + 2)^3(x + 3)^4$

Taking log on both sides, we get

$\therefore \log y = \log\{(x+1)^2 \cdot (x+2)^3(x+3)^4\}$

$= \log(x + 1)^2 + \log(x + 2)^3 + \log(x + 3)^4$

and $\frac{d}{dy}\log y \cdot \frac{dy}{dx} = \frac{d}{dx}[2\log(x+1)] + \frac{d}{dx}[3\log(x+2)] + \frac{d}{dx}[4\log(x+3)]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2}{(x+1)} \cdot \frac{d}{dx}(x+1) + 3 \cdot \frac{1}{(x+2)} \cdot \frac{d}{dx}(x+2)$

$+4 \cdot \frac{1}{(x+3)} \cdot \frac{d}{dx}(x+3) \quad \ldots \left[\because \frac{d}{dx}(\log x) = \frac{1}{x}\right]$

$= \left[\frac{2}{x+1} + \frac{3}{x+2} + \frac{4}{x+3}\right]$

$\therefore \frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{3}{x+2} + \frac{4}{x+3}\right]$

$= (x+1)^2 \cdot (x+2)^3 \cdot (x+3)^4 \left[\frac{2}{x+1} + \frac{3}{x+2} + \frac{4}{x+3}\right]$

$= (x+1)^2 \cdot (x+2)^3 \cdot (x+3)^4$

$\left[\frac{2(x+2)(x+3)+3(x+1)(x+3)+4(x+1)(x+2}{(x+1)(x+2)(x+3)}\right]$

$= \frac{(x+1)^2(x+2)^3(x+3)^4}{(x+1)(x+2)(x+3)}$

$[2(x^2 + 5x + 6) + 3(x^2 + 4x + 3) + 4(x^2 + 3x + 2)]$

$= (x + 1)(x + 2)^2(x + 3)^3$

$[2x^2 + 10x + 12 + 3x^2 + 12x + 9 + 4x^2 + 12x + 8]$

$= (x + 1)(x + 2)^2(x + 3)^3 (9x^2 + 34x + 29)$

OR

Given, $y = e^{-x} \cos x$

To prove: $\frac{d^2y}{dx^2} = 2e^{-x}\sin x$

From the given question we need to find double derivative of the given function .

As, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So lets first find dy/dx and differentiate it again.

$\therefore \frac{dy}{dx} = \frac{d}{dx}(e^{-x}\cos x)$

Let $u = e^{-x}$ and $v = \cos x$

As, $y = u \cdot v$

$\therefore$ using product rule of differentiation:

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$\therefore \frac{dy}{dx} = e^{-x}\frac{d}{dx}(\cos x) + \cos x\frac{dy}{dx}e^{-x}$

$\frac{dy}{dx} = -e^{-x}\sin x - e^{-x}\cos x$

$\left[\because \frac{d}{dx}(\cos x) = -\sin x \& \frac{d}{dx}e^{-x} = -e^{-x}\right]$

Again differentiating w.r.t. x:

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-e^{-x}\sin x - e^{-x}\cos x)$

$= \frac{d}{dx}(-e^{-x}\sin x) - \frac{d}{dx}(e^{-x}\cos x)$

Again using the product rule:

$\frac{d^2y}{dx^2} = -e^{-x}\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}e^{-x} - e^{-x}\frac{d}{dx}(\cos x) - \cos x\frac{d}{dx}(e^{-x})$

$\frac{d^2y}{dx^2} = -e^{-x}\cos x + e^{-x}\sin x + e^{-x}\sin x + e^{-x}\cos x$

$\left[\because \frac{d}{dx}(\cos x) = -\sin x, \frac{d}{dx}e^{-x} = -e^{-x}\right]$

$\frac{d^2y}{dx^2} = 2e^{-x}\sin x$

Hence proved

32. Here,it is given that

the equation of the curve is

$y = (x^3 - 1)(x - 2)$ ... (i)

It cuts x-axis at y = 0. Thus, putting y = 0 in (i), we obtain

$(x^3 - 1)(x - 2) = 0$

$\Rightarrow (x - 1)(x - 2)(x^2 + x + 1) = 0$

$\Rightarrow x - 1 = 0, x - 2 = 0 \; [\because x^2 + x + 1 \neq 0]$

$\Rightarrow$ x = 1, 2

Therefore, the points of intersection of curve (i) with x-axis are (1, 0) and (2, 0)

Now, y = $(x^3 - 1)(x - 2)$

$\Rightarrow \quad \frac{dy}{dx} = 3x^2(x - 2) + (x^3 - 1)$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,0)} = -3$ and , $\left(\frac{dy}{dx}\right)_{(2,0)} = 7$

The equations of the tangents at (1, 0) and (2, 0) are respectively.

$y - 0 = \left(\frac{dy}{dx}\right)_{(1,0)}(x - 1)$ and $y - 0 = \left(\frac{dy}{dx}\right)_{(2,0)}(x - 2)$

$\Rightarrow$ y - 0 = 3(x - 1) and y - 0 = 7(x - 2)

$\Rightarrow$ y + 3x - 3 = 0 and 7x - y - 14 = 0

33. Let the given integral be,

I = $\int \frac{1}{1+\cot x} dx$

= $\int \frac{1}{1+\frac{\cos x}{\sin x}} dx$

= = $\int \frac{\sin x}{\sin x+\cos x} dx$

Let sin x = $\lambda \frac{d}{dx}$ - (sin x + cos x) + μ (sin x + cos x)

i.e. sin x = λ (cos x - sin x) + μ(sin x + cos x)

Comparing the coefficients of sin x and cos x on both sides, we get

0 = $\lambda + \mu$ = and 1 = $-\lambda + \mu \Rightarrow \mu = -\frac{1}{2}$ and $\lambda = -\frac{1}{2}$

$$\therefore I = \int \frac{\lambda(\cos x - \sin x) + \mu(\sin x + \cos x)}{\sin x + \cos x} dx$$

$\Rightarrow \quad I = \lambda \int \frac{\cos x - \sin x}{\sin x + \cos x} dx + \mu \int \frac{\sin x + \cos x}{\sin x + \cos x} dx$

$\Rightarrow \quad I = \lambda \int \frac{dt}{t} + \mu \int 1 \cdot dx$, where t = sin x + cos x

$\Rightarrow \quad I = \lambda \log |t| + \mu x + C$

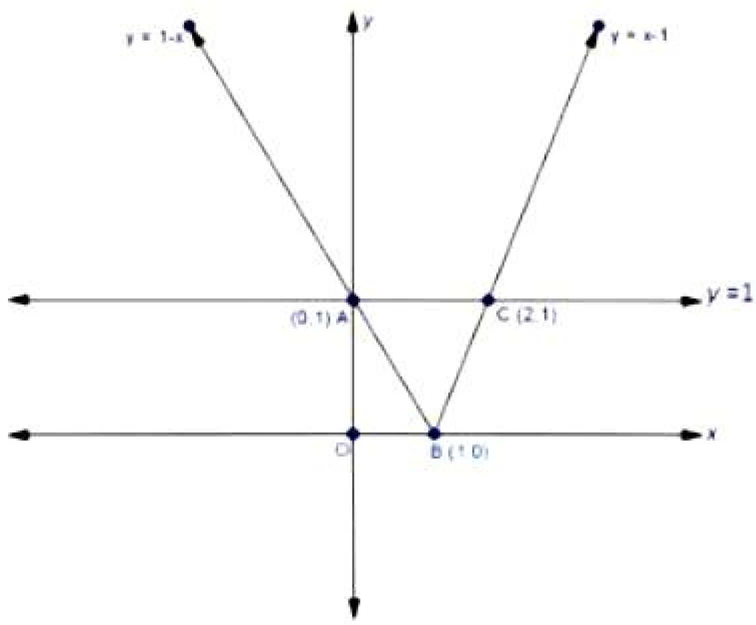
$\Rightarrow \quad I = -\frac{1}{2} \log |\sin x + \cos x| + \frac{1}{2} x + C$

34. To find area bounded by y = 1 and

$y = |x - 1|$

$$y = \begin{cases} x - 1, \text{ if } x \geq 0 & \ldots(1) \\ 1 - x, \text{ if } x < 0 & \ldots(2) \end{cases}$$

A rough sketch of the curve is as under:-



Bounded reigon is the required region. So

Required area of bounded region=Area of Region ABCA

A = Region ABDA + Region BCDB

$= \int_0^1 (y_1 - y_2)\, dx + \int_1^2 (y_1 - y_3)\, dx$

$= \int_0^1 (1 - 1 + x) dx + \int_1^2 (1 - x + 1) dx$

$= \int_0^1 x dx + \int_1^2 (2 - x) dx$

$= \left(\frac{x^2}{2}\right)_0^1 + \left(2x - \frac{x^2}{2}\right)_1^2$

$= \left(\frac{1}{2} - 0\right) + \left[(4 - 2) - \left(2 - \frac{1}{2}\right)\right]$

$= \frac{1}{2} + \left(2 - 2 + \frac{1}{2}\right)$

A = 1 sq. unit

OR

The equations of given curves are

$y = \frac{3}{4}x^2$ ....(1)}

and $3x - 2y + 12 = 0$ ....(2)

From (2), $2y = 3x + 12$

$\therefore\ y = \frac{3x+12}{2}$

putting this value of y in (1), we get

$\Rightarrow \frac{3x+12}{2} = \frac{3}{4}x^2$

$\Rightarrow 6x + 24 = 3x^2$

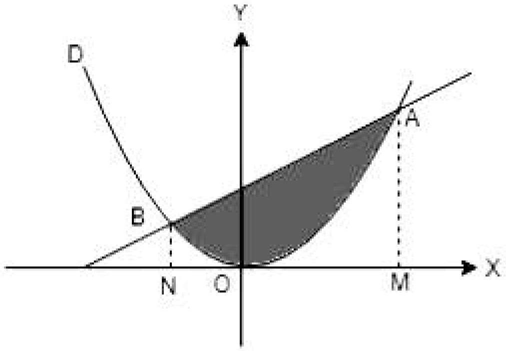
$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x+2)(x-4)=0$

$\Rightarrow x=-2,4$

$\therefore\ y=3,12$

Thus, curves (1) and (2) intersect in points A(4,12) and B(-2,3).



From A, draw AM$\perp$ x-axis and from B, draw BN$\perp$ x-axis.

Required area= the area of the region included between the parabola $y=\frac{3}{4}x^2$ and the line $3x-2y+12=0$

= area of trapezium BNMA - (area BNO + area OMA)

=$\frac{1}{2}(3+12)\times 6-\int_{-2}^{4}\frac{3}{4}x^2\,dx$ [using (1)]

=$45-\frac{3}{4}\int_{-2}^{4}x^2\,dx$

=$45-\frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$

=$45-\frac{1}{4}[x^3]_{-2}^{4}$

=$45-\frac{1}{4}[64+8]$

= 27 sq.units.

35. Given that, $\frac{dy}{dx}+2xy=y$

$\Rightarrow \frac{dy}{dx}+2xy-y=0$

$\Rightarrow \frac{dy}{dx}+(2x-1)y=0$

Which is a linear differential equation.

On comparing it with $\frac{dy}{dx}+Py=Q$, we get

P = (2x - 1), Q = 0

$I.F=e^{\int pdx}=e^{\int(2x-1)dx}$

$=e^{\left(\frac{2x^2}{2}-x\right)}=e^{x^2-x}$

The complete solution is

$y.e^{x^2-x}=\int Q.e^{x^2-x}=0+C$

$\Rightarrow y.e^{x^2-x}=0+C$

$\Rightarrow y = Ce^{x-x^2}$

Section - V

36. A white and a black ball can be drawn from the second bag in the following mutually exclusive ways:

    i. By transferring 2 black balls from first bag to the second bag and then drawing a white and a black ball from it.

    ii. By transferring 2 white balls from first bag to the second bag and then drawing a white and a black ball from it.

    iii. By transferring one white and one black ball from first bag to the second bag and then drawing a white and a black ball from it.

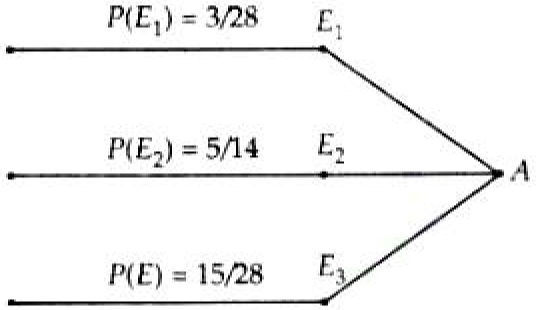
    Consider the following events:

    E_1 = Two black balls are drawn from the first bag,

    E_2 = Two white balls are drawn from the first bag,

    E_3 = One white and one black ball is drawn from the first bag,

    A = Two balls drawn from the second bag are white and black.



We have, $P(E_1) = \frac{^3C_2}{^8C_2} = \frac{3}{28}, P(E_2) = \frac{^5C_2}{^8C_2} = \frac{5}{14}$ and $P(E_3) = \frac{^3C_1 \times {^5C_1}}{^8C_2} = \frac{15}{28}$

If E_1 has already occurred, that is, if two black balls have been transferred from the first bag to the second bag, then the second bag will contain 3 white and 7 black balls.

Therefore, we have,

Probability of drawing a white and a black ball from the second bag is $\frac{^3C_1 \times {^7C_1}}{^{10}C_2}$

$\therefore \quad P(A/E_1) = \frac{^3C_1 \times {^7C_1}}{^{10}C_2} = \frac{7}{15}$

Similarly, we have

$P(A/E_2) = \frac{^5C_1 \times {^5C_1}}{^{10}C_2} = \frac{5}{9}$ and $P(A/E_3) = \frac{^4C_1 \times {^6C_1}}{^{10}C_2} = \frac{8}{15}$

By the law of total probability, we have P(A) = P (E_1) P (A/E_1) + P (E_2) P (A/E_2) + P (E_3) P (A/E_3)$= \frac{3}{28} \times \frac{7}{15} + \frac{5}{14} \times \frac{5}{9} + \frac{15}{28} \times \frac{8}{15} = \frac{673}{1260}$

OR

In order to find probability distribution of a number of defective bulbs in a sample of 4 bulbs chosen at random we use following
formulae:

Where P(x) = ${}^nC_xp^xq^{n-x}$

Xi	X1	X2	X3	X4
P(X)	(1)	(2)	(3)	(4)

Here p is the probability of getting a defective bulb. q = 1 - p (Probability that the bulb is not defective) Let the total number of bulbs produced by a machine be x
Using the given condition number of defective bulbs produced by a machine = $\frac{20x}{100} = \frac{x}{5}$
X denotes the number of defective bulbs in a sample of 4 bulbs chosen at random.
X = {0,1,2,3,4}
p = $\frac{1}{5}$
q = 1 - p = 1 - $\frac{1}{5} = \frac{4}{5}$
q = $\frac{4}{5}$
n = 4(Number of bulbs chosen at random)
Possible values of X are 0,1,2,3,4

P(x) = ${}^nC_xp^xq^{n-x}$

P(0) = ${}^4C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$
P(1) = ${}^4C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^3 = \frac{256}{625}$
P(2) = ${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = \frac{96}{625}$
P(3) = ${}^4C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^1 = \frac{16}{625}$
P(4) = ${}^4C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^0 = \frac{1}{625}$
The probability distribution of a number of defective bulbs in a sample of 4 bulbs chosen at random

0	1	2	3	4
$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

37. Suppose the required line is parallel to vector $\vec{b}$

Which is given by $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

We know that the position vector of the point (1, 2, 3) is given by

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$ ...(i)

The equation of the given planes are

$\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ ...(ii)

and $\vec{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$ ...(iii)

The line in Equation (i) and plane in Eq. (ii) are parallel.

Therefore, the normal to the plane of Eq. (ii) is perpendicular to the given line

$\therefore (\hat{i} - \hat{j} + 2\hat{k}) \cdot \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$

$\Rightarrow (b_1 - b_2 + 2b_3) = 0$ ....(iv)

Similarly, from Eqs. (i) and (iii), we get

$(3\hat{i} + \hat{j} + \hat{k}).(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$

$\Rightarrow (3b_1 + b_2 + b_3) = 0$......(v)

On solving Equations . (iv) and (v) by cross-multiplication, we get

$\frac{b_1}{(-1)\times1-1\times2} = \frac{b_2}{2\times3-1\times1} = \frac{b_3}{1\times1-3(-1)}$

$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$

Therefore, the direction ratios of $\vec{b}$ are (-3, 5, 4).

$\vec{b} = -3\hat{i} + 5\hat{j} + 4\hat{k}\left[\because \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right]$

On substituting the value of b in Equation (i), we get

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$

which is the equation of the required line.

OR

Now, given equation of lines are

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

and $\vec{r} = (\hat{i} + \hat{j}) + \mu(\hat{-i} + \hat{j} - 2\hat{k})$

On comparing these equations with standard equation of line, $\vec{r} = \vec{a} + \lambda\vec{b}$, we get

$\overrightarrow{a_1} = \hat{i} + \hat{j}, \overrightarrow{a_2} = \hat{i} + \hat{j}, \overrightarrow{b_1} = \hat{i} + 2\hat{j} - \hat{k}$and

$\overrightarrow{b_2} = -\hat{i} + \hat{j} - 2\hat{k}$

Now, the equation of plane containing both the lines is given by

$(\vec{r} - \vec{a}_1) \cdot \vec{n} = 0$

where, $\vec{n}$ is normal to both the lines.

Clearly, $\vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$

$= \hat{i}(-4+1) - \hat{j}(-2-1) + \hat{k}(1+2)$

$= -3\hat{i} + 3\hat{j} + 3\hat{k}$

The required equation of plane is

$[\overrightarrow{r} - (\hat{i} + \hat{j})] \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k}) = 0$

$\Rightarrow \vec{r} \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k}) = (\hat{i} + \hat{j}) \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k})$

$\Rightarrow \vec{r} \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k}) = -3 + 3 = 0$

$\Rightarrow \quad \vec{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 0$

Now, the length of perpendicular from the point (2, 1, 4) to the above plane

$= \frac{|(2\hat{i}+\hat{j}+4\hat{k}) \cdot (-\hat{i}+\hat{j}+\hat{k})|}{\sqrt{(-1)^2+1^2+1^2}}$

$= \frac{|-2+1+4|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}} = \sqrt{3}.units$

38. Let the man rides to his motorcycle to a distance x km at the speed of 50 km/h and to a distance y km at the speed of 80 km/h.

Therefore, cost on petrol is 2x + 3y.

Since, he has to spend Rs 120 at most on petrol.

$\therefore 2x + 3y \leq 120$ .... (i)

Also, he has at most 1 hour time.

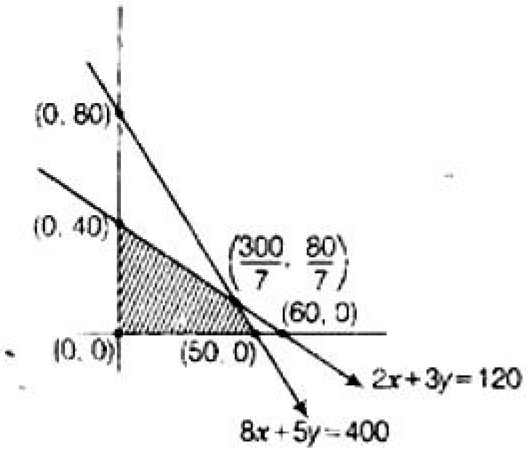
$\therefore \frac{x}{50} + \frac{y}{80} \leq 1$

$\Rightarrow 8x + 5y \leq 400$ ..... (ii)

Also, we have $x \geq 0, y \geq 0$ [non-negative constraints]

Thus, required LPP to travel maximum distance by him is

Maximise Z = x + y, subject to $2x + 3y \leq 120, 8x + 5y \leq 400, x \geq 0, y \geq 0$.



Maximise Z = x + y, subject to

2x + 3y $\leq$ 120, 8x + 5y $\leq$ 400, x $\geq$ 0, y $\geq$ 0

On solving, we get

8x + 5y = 400 and 2x + 3y = 120, we get

$x = \frac{300}{7}, y\frac{80}{7}$

From the shaded feasible region, it is clear that coordinates of corner points are (0, 0), (50, 0), $\frac{300}{7}, \frac{80}{7}$ and (0, 40).

Corner Points	Corresponding Value of Z = x + y
(0, 0)	0
(50, 0)	50
$\left(\frac{300}{7}, \frac{80}{7}\right)$	$\frac{300}{7} = 54\frac{2}{7}km$ (maximum)
(0, 40)	40

Hence, the maximum distance that the man can travel is $54\frac{2}{7}km$.

OR

Let x be the no. of nuts & y be the bolts.

We have to maximize x & y if the factory worked at capacity of 12hours per day.

Clearly, x ≥ y, y ≥ 0

Since we have only 12 hours of machine, we will use the following constraints:-

$x + 3y \leq 12$

$3x + y \leq 12$

The profit of nuts is Rs.17.5 and bolts is Rs. 7 .

We need to maximize 17.5x + 7y

We can see that the feasible region is bounded and in the first quadrant.

On solving the equations, x + 3 y = 12 and 3 x + y = 12, we get,

x = 3 and y = 3.

Therefore, feasible points are (0,0), (0,4), (3,3), (4,4)

Values of z are z = 17.5x + 7y

The maximum profit = 73.5 which is when 3 packets each of nuts & bolts are produced.