Class XII Session 2023-24
Subject - Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If I is a unit matrix, then 3I will be **[1]**

a) None of these
b) A unit matrix
c) A triangular matrix
d) A scalar matrix

2. If A is a 2-rowed square matrix and IAI = 6 then A·adj A = ? **[1]**

a) $\begin{bmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{6} \end{bmatrix}$
b) None of these
c) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
d) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

3. Let X = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If AX = B, then X is equal to **[1]**

a) $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$
b) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$
c) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$
d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

4. $\frac{d}{dx}(\tan^{-1}(\sec x + \tan x)$ is equal to **[1]**

a) $-\frac{1}{2}$
b) $\frac{1}{2}$
c) $\frac{1}{2\sec x(\sec x+\tan x)}$
d) None of these

5. Distance between the two planes: 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12 is **[1]**

a) 2 units

b) 8 units

c) 4 units

d) $\frac{2}{\sqrt{29}}$ units

6. Integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$ is **[1]**

a) -x

b) $\sqrt{1-x^2}$

c) $\frac{x}{1+x^2}$

d) $\frac{1}{2}\log(1-x^2)$

7. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is **[1]**

a) q = 3p

b) q = 2p

c) p = q

d) p = 2q

8. If θ is the angle between two unit vectors $\hat{a}$ and $\hat{b}$ then $\frac{1}{2}|\hat{a}-\hat{b}| = ?$ **[1]**

a) $\tan\frac{\theta}{2}$

b) none of these

c) $\sin\frac{\theta}{2}$

d) $\cos\frac{\theta}{2}$

9. $\int_0^{\pi/4} \tan^2 x dx = ?$ **[1]**

a) $\left(1+\frac{\pi}{4}\right)$

b) $\left(1-\frac{\pi}{4}\right)$

c) $\left(1+\frac{\pi}{2}\right)$

d) $\left(1-\frac{\pi}{2}\right)$

10. Out of the following matrices, choose that matrix which is a scalar matrix: **[1]**

a) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

11. In an LPP, if the objective function z = ax + by has the same maximum value on two corner points of the feasible region, then the number of points at which z_{max} occurs is: **[1]**

a) finite

b) 0

c) infinite

d) 2

12. If $\vec{a}, \vec{b}$ represent the diagonals of a rhombus, then **[1]**

a) $\vec{a} \times \vec{b} = \vec{0}$

b) $\vec{a} + \vec{b} = 1$

c) $\vec{a} \times \vec{b} = \vec{a}$

d) $\vec{a} \cdot \vec{b} = 0$

13. If A is a non singular matrix of order 3 , then $|adj(A^3)|$= **[1]**

a) None of these

b) $|A|^8$

c) $|A|^6$

d) $|A|^9$

14. Let A and B be two events such that P(A) = 0.6, P(B) = 0.2, and P(A | B) = 0.5. Then P (A′|B′) equals **[1]**

a) $\frac{3}{10}$

b) $\frac{6}{7}$

c) $\frac{3}{8}$

d) $\frac{1}{10}$

15. Solution of $x\,dy - y dx = \sqrt{x^2+y^2}\,dx$ is **[1]**

a) $y+\sqrt{x^2+y^2}=Cx^2$
b) $y+\sqrt{3+y^2}=Cx^4$
c) $y+\sqrt{x^3+y^2}=Cx^2$
d) $y+\sqrt{x^2+y^3}=Cx^2$

16. Let a vector $\vec{r}$ make angles 60°, 30° with X and Y-axes, respectively. What are the direction cosines of $\vec{r}$? **[1]**

a) $< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 >$
b) $< \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 >$
c) $< -\frac{1}{2}, \frac{\sqrt{3}}{2}, 0 >$
d) $< \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 >$

17. If $x=a(\cos\theta+\theta\sin\theta)$ and $y=a(\sin\theta-\theta\cos\theta)$ $\frac{dy}{dx}=$? **[1]**

a) a cot θ
b) cot θ
c) tan θ
d) a tan θ

18. The line $\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{4}$ meets the plane 2x + 3y - z = 14 in the point. **[1]**

a) (6, 5, 3)
b) (5, 7, 3)
c) (2, 5, 7)
d) (3, 5, 7)

19. **Assertion (A):** If manufacturer can sell x items at a price of ₹$\left(5-\frac{x}{100}\right)$ each. The cost price of x items is ₹ $\left(\frac{x}{5}+500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items. **[1]**
Reason (R): The profit for selling x items is given by $\frac{24}{5}x-\frac{x^2}{100}$ - 300.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** A function f: N $\to$ N be defined by $f(n)=\begin{cases}\frac{n}{2} & \text{if } n \text{ is even}\\ \frac{(n+1)}{2} & \text{if } n \text{ is odd}\end{cases}$ for all n $\in$ N; is one-one. **[1]**
Reason (R): A function f: A $\to$ B is said to be injective if a $\neq$ b then f(a) $\neq$ f(b).

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$ **[2]**

OR

Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

22. Find the point of local maxima or local minima and the corresponding local maximum and minimum values of function: f(x) = -(x - 1) 3(x + 1)^2 **[2]**

23. A man is walking at the rate of 6.5 km/hr towards the foot of a tower 120 m high. At what rate is he approaching the top of the tower when he is 50 m away from the tower **[2]**

OR

Show that the function given by f(x) = $\frac{\log x}{x}$ has maximum at x = e.

24. Evaluate: $\int$ log (x + 1) dx **[2]**

25. Find the intervals of function f(x) = 2x^3 - 24x + 5 is **[2]**

a. increasing

b. decreasing.

Section C

26. Evaluate: $\int \frac{2x}{2+x-x^2} dx$ **[3]**

27. Ramesh appears for an interview for two posts, A and B, for which the selection is independent. The probability for his selection for Post A is (1/6) and for Post B, it is (1/7). Find the probability that Ramesh is selected for at least one post. **[3]**

28. Evaluate the integral: $\int x^3 (\log x)^2 dx$ **[3]**

OR

Evaluate: $\int \frac{(3+4x-x^2)}{(x+2)(x-1)} dx$.

29. Find the general solution for differential equation: $x\frac{dy}{dx} + y = y^2$ **[3]**

OR

Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by (x + y + 1) = A (1 - x - y - 2xy), where A is parameter.

30. Solve the Linear Programming Problem graphically: **[3]**

Maximize Z = 3x + 3y, if possible, Subject to the constraints

x - y $\leq$ 1

x + y $\geq$ 3

x, y $\geq$ 0

OR

Solve the Linear Programming Problem graphically:

Maximize Z = x + y Subject to

- 2x + y $\leq$ 1

x $\leq$ 2

x + y $\leq$ 3

x ,y $\geq$ 0

31. Find $\frac{dy}{dx}$ when y = $(\tan x)^{\cot x} + (\cot x)^{\tan x}$ **[3]**

Section D

32. Using method of integration find the area of the triangle ABC, co-ordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3). **[5]**

33. Show that the function f : R $\rightarrow$ R defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in R$, is neither one-one nor onto. **[5]**

OR

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

34. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness, and Leadership. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with total award money of Rs2200. **[5]**

School Q wants to spend Rs 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs1200, using matrices, find the award money for each value.

35. Find the vector equation of the line passing through (1, 2, 3) and parallel to each of the planes **[5]**

$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$.

OR

A line with direction ratios (2, 2, 1) intersects the lines $\frac{x-7}{3} = \frac{y-5}{2} = \frac{z-3}{1}$ and $\frac{x-1}{2} = \frac{y+1}{4} = \frac{z+1}{3}$ at the points P and Q respectively. Find the length and the equation of the intercept PQ.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) Find the probability that she gets grade A in all subjects.

(ii) Find the probability that she gets grade A in no subjects.

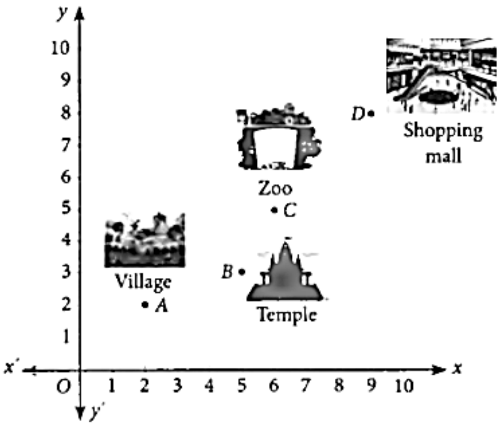
(iii) Find the probability that she gets grade A in two subjects.

OR

Find the probability that she gets grade A in at least one subject.

37. **Read the text carefully and answer the questions:** **[4]**

Girish left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Girish at different places is given in the following graph.



(i) Find position vector of B

(ii) Find position vector of D

(iii) Find the vector $\overrightarrow{BC}$ in terms of $\hat{i}, \hat{j}$.

OR

Find the length of vector $\overrightarrow{AD}$.

38. **Read the text carefully and answer the questions:** **[4]**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Find the volume of the open box formed by folding up the cutting each corner with x cm.

(ii) Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

Solution

Section A

1.

(d) A scalar matrix

Explanation: A scalar matrix

2.

(d) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

Explanation: A.(adj A) = |A|I

$= 6 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

3.

(b) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

Explanation: Given that

X = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

Also AX = B and we have to find the value of X ,

Pre-multiplying A^{-1} both sides we get,

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$ ($\because A^{-1}A = I$)

$X = A^{-1}B$ ($\because IX = X$) …(i)

Now,

$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix}$ = 1(0 - 2) + 1(2 - 3) +2(4 - 0) = -2 - 1 + 8 = 5

And adjA = $\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$

$\therefore \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

On comparing both sides we get,

$x_1 = -1$, $x_2 = 2$ and $x_3 = 3$.

4.

(b) $\frac{1}{2}$

Explanation: $\frac{d}{dx}\left(\tan^{-1}(\sec x + \tan x)\right) = \frac{\sec x \tan x + \sec^2 x}{1+(\sec x+\tan x)^2} = \frac{\sec x(\sec x+\tan x)}{2\sec x(\sec x+\tan x)} = \frac{1}{2}$.

Which is the required solution.

5.

(d) $\frac{2}{\sqrt{29}}$ units

Explanation: Distance between two parallel planes Ax + By + Cz = d_1 and Ax + By + Cz = d_2 is $\left|\frac{d_1-d_2}{\sqrt{A^2+B^2+C^2}}\right|$

Given that,
First Plane is 2x + 3y + 4z = 4
Comparing with Ax + By + Cz = d_1, we get

A = 2, B = 3, C = 4, d_1 = 4

Second Plane is 4x + 6y + 8z = 12
After Dividing by 2,
2x + 3y + 4z = 6
Comparing with Ax + By + Cz = d_1, we get

A = 2, B = 3, C = 4, d_2 = 6

So,
Distance between two planes
$= \left|\frac{4-6}{\sqrt{2^2+3^2+4^2}}\right|$
$= \left|\frac{-2}{\sqrt{4+9+16}}\right|$
$= \frac{2}{\sqrt{29}}$

6.

(b) $\sqrt{1-x^2}$

Explanation: We have, $(1 - x^2)\frac{dy}{dx}$ - xy = 1
$\Rightarrow \frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{1}{1-x^2}$
This is a linear differential equation.
On comparing it with $\frac{dy}{dx} + Py = Q$, we get.
$\therefore IF = e^{\int pdx} = e^{-\int \frac{x}{1-x^2}dx}$
Let, $1 - x^2 = t \Rightarrow 2xdx = -dt$
$= e^{\frac{1}{2}\int \frac{dt}{t}} = e^{\frac{1}{2}logt} = \sqrt{t} = \sqrt{1-x^2}$

7. **(a)** q = 3p
Explanation: Since Z occurs maximum at (15, 15) and (0, 20), therefore, 15p + 15q = 0p + 20q ⇒ q = 3p.

8.

(c) $\sin\frac{\theta}{2}$
Explanation: Since a and b are unit vectors |a| = |b| =1
Let angle be x between vectors a and b

$|a - b| = \{|a|^2 + |b|^2 + 2|a||b|\cos(180 - \theta)\}^{1/2}$
Putting values in R.H.S.

$|a - b| = \{1 + 1 + 2 \times 1 \times 1\times(-\cos\theta)\}^{1/2}$

$= (2 - 2\cos\theta)^{1/2}$

$= 2(1-\cos\theta)^{1/2}$

$=\{2 \times 2(\sin\frac{\theta}{2})^2\}^{1/2}$
$= 2\sin\frac{\theta}{2}$
$|a - b| = 2\sin\frac{\theta}{2}$
$\frac{1}{2}|a - b| = \sin\frac{\theta}{2}$

9.

(b) $\left(1-\frac{\pi}{4}\right)$

Explanation: $I = \int_0^{\frac{\pi}{4}} \left(\sec^2 x - 1\right)dx$
$= (\tan x - x)_0^{\frac{\pi}{4}}$
$= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)$
$= 1 - \frac{\pi}{4}$

10.

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Explanation: ∵ Scalar Matrix is a matrix whose all off-diagonal elements are zero and all on-diagonal elements are equal.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

11.

(c) infinite

Explanation: In a LPP, if the objective f^n Z = ax + by has the maximum value on two corner point of the feasible region then every point on the line segment joining these two points gives the same maximum value.

hence, Z_{max} occurs at infinite no of times.

12.

(d) $\vec{a} \cdot \vec{b} = 0$

Explanation: Diagonals of a rhombus are perpendicular to each other

$\Rightarrow \vec{a} \cdot \vec{b} = 0$

13.

(c) $|A|^6$

Explanation: If A is a non singular matrix of order 3, then $|adj(A^3)| = (|A^3|)^2 = (|AAA|)^2 = (|A|\,|A|\,|A|)^2 = (|A|^3)^2 = |A|^6$.

14.

(c) $\frac{3}{8}$

Explanation: P (A∩B) = P (A | B) P (B)

= 0.5 × 0.2 = 0.1

$P(A'|B') = \frac{P(A'\cap B')}{P(B')} = \frac{P[(A\cup B)']}{1-P(B)} = \frac{1-P(A\cup B)}{1-P(B)}$

$= \frac{1-\mathrm{P(A)}-\mathrm{P(B)}+\mathrm{P(A\cap B)}}{1-0.2} = \frac{3}{8}$

15. **(a)** $y + \sqrt{x^2+y^2} = Cx^2$

Explanation: $\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Put y=vx, we have ; $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \sqrt{1+v^2} + v \Rightarrow \int \frac{1}{\sqrt{1+v^2}}dv = \int \frac{1}{x}dx$

$\Rightarrow \log|\sqrt{1+v^2}+v| = \log|x| + \log C \Rightarrow |\sqrt{1+v^2}+v| = Cx$

$\Rightarrow (y+\sqrt{x^2+y^2})^2 = C^2x^4$

$\Rightarrow (y+\sqrt{x^2+y^2}) = Cx^2$

16.

(b) $< \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 >$

Explanation: Given that, A vector $\vec{r}$ make an angle 60° with X-axis, i.e. l = cos 60° = $\frac{1}{2}$ and a vector r make an angle 30° with Y-axis,

i.e., $m = \cos 30^\circ = \frac{\sqrt{3}}{2}$)

The direction cosine of the vector

$\vec{r}$ = < l, m, n>

$\Rightarrow \vec{r}$ = <cos 60^o, cos 30^o, cos 90^o>

$\Rightarrow \vec{r} = < \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 >$

17.

(c) tan θ

Explanation: $x = a(\cos\theta + \theta\sin\theta)$,we get

$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$

$y = a(\sin\theta - \theta\cos\theta)$,we get

$\therefore \frac{\mathrm{d}y}{\mathrm{d}\theta} = a(\cos\theta - (\cos\theta + \theta(-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = \mathrm{a}\cos\theta - \mathrm{a}\cos\theta + \theta\mathrm{a}\sin\theta$
$\Rightarrow \frac{\mathrm{dy}}{\mathrm{d}\theta} = \mathrm{a}\theta\sin\theta$
$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{dy}}{\mathrm{d}\theta} \times \frac{\mathrm{d}\theta}{\mathrm{dx}}$
$\Rightarrow \frac{dy}{dx} = a\theta\sin\theta \times \frac{1}{a\theta\cos\theta}$
$\Rightarrow \frac{dy}{dx} = \tan\theta$

18.

(d) (3, 5, 7)

Explanation: Let $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda(\text{ say })$

A general point on this line is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$.

For some value of λ, let the given line meet the plane 2x + 3y - z = 14 at a point $P(2\lambda+1, 3\lambda+2, 4\lambda+3)$.

Then, $2(2\lambda+1) + 3(3\lambda+2) - (4\lambda+3) = 14$

$\Rightarrow 9\lambda = 9 \Rightarrow \lambda = 1$

So, the required point is P(2 + 1, 3 + 2, 4 + 3),

i.e., P(3, 5, 7).

19.

(c) A is true but R is false.

Explanation: Let S(x) be the selling price of x items and let C(x) be the cost price of x items.

Then, we have

S(x) = (5 - $\frac{x}{100}$)x = 5x - $\frac{x^2}{100}$

and C(x) = $\frac{x}{5}$ + 500

Thus, the profit function P(x) is given by

P(x) = S(x) - C(x) = 5x - $\frac{x^2}{100} - \frac{x}{5}$ - 500

i.e. P(x) = $\frac{24}{5}x - \frac{x^2}{100}$ - 500

On differentiating both sides w.r.t. x, we get

P'(x) = $\frac{24}{5} - \frac{x}{50}$

Now, P'(x) = 0 gives x = 240.

Also, P'(x) = $\frac{-1}{50}$.

So, P'(240) = $\frac{-1}{50}$ < 0

Thus, x = 240 is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

20.

(d) A is false but R is true.

Explanation: Assertion is false because distinct elements in N has equal images.

for example f(1) = $\frac{(1+1)}{2}$ = 1

f(2) = $\frac{2}{2}$ = 1

Reason is true because for injective function if elements are not equal then their images should be unequal.

Section B

21. We have, $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

$\cos\left[\cos^{-1}\left(-cos\frac{\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left(\frac{5\pi}{6} + \frac{\pi}{6}\right)\{\because \cos^{-1}\cos x = x,\ x \in [0,\pi]\}$

$= \cos\left(\frac{6\pi}{6}\right)$

$= \cos(\pi) = -1$

OR

We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1)$.

$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$ $\left[\begin{array}{c}\because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0,\ \pi) \\ and\ \tan^{-1}(-x) = -\tan^{-1} x\end{array}\right]$

$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi+4\pi-3\pi}{12}$

$= \frac{-5\pi+4\pi}{12} = \frac{-\pi}{12}$

22. We have Local max. value is 0, at each of the points x = 1 and x = -1 and local min. value is $\frac{-3456}{3125}$ at $x = -\frac{1}{5}$

F'(x) = -(x - 1)^{3}2(x + 1) -3(x - 1)^2(x + 1)^2 = 0

x = 1, -1, $-\frac{1}{5}$

Since, f''(1) and f''(-1) < 0, 1 and -1 are the points of local maximum.

$F'\left(| -\frac{1}{5}\right) > 0, -\frac{1}{5}$ is the point of local minimum.

F(1) = f(-1)=0

Also, $f\left(-\frac{1}{5}\right) = -\frac{3456}{3125}$

23. Let at any time t, the man be at distances of x and y metres from the foot and top of the tower respectively. Then,

$y^2 = x^2 + (120)^2$ ...(i)

$\Rightarrow \quad 2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\Rightarrow \quad \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$

given: $\frac{dx}{dt} = -6 \cdot 5\text{km/hr}$ negative sign due to decreasing,

therefore;

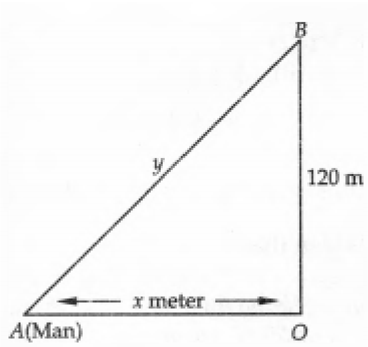
$\frac{dy}{dt} = -\frac{6 \cdot 5x}{y}$ ... (ii)

Putting x = 50 in (i) we get $y = \sqrt{50^2 + 120^2} = 130$

Putting x = 50, y = 130 in (ii), we get

$\frac{dy}{dt} = -\frac{6 \cdot 5 \times 50}{130} = -2 \cdot 5$

Thus, the man is approaching the top of the tower at the rate of 2.5 km/hr.



OR

It is given that f(x) = $\frac{\log x}{x}$

Then, f'(x) = $\frac{x\left(\frac{1}{x}\right) - \log x}{x^2} = \frac{1 - \log x}{x^2}$

Now, f'(x) = 0

$\Rightarrow$ 1 - log x = 0

$\Rightarrow$ log x = 1

$\Rightarrow$ log x = log e

$\Rightarrow$ x = e

Further, $f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)(2x)}{x^4}$

= $\frac{-x - 2x(1 - \log x)}{x^4}$

= $\frac{-3 + 2\log x}{x^3}$

Now, f''(e) = $\frac{-3 + 2\log e}{e^3} = \frac{-3+2}{e^3} = \frac{-1}{e^3} < 0$

Therefore, by second derivative test, f is the maximum at x = e.

24. Let,

I = $\int$ log (x + 1) dx,

= $\int 1 \cdot log$ (x + 1) dx

Taking log (x + 1) as the first function and 1 as the second function, we get

I = log (x + 1) $\int$ 1 dx - $\int \left[\frac{d}{dx}\{\log(x+1)\}\int 1 dx\right] dx$

= x log (x + 1) - $\int \frac{x}{x+1} dx$

= x log (x + 1) - $\int \frac{x+1}{x+1} - \frac{1}{x+1} dx$

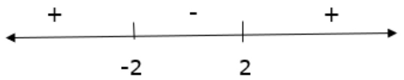
= x log (x + 1) - x + log |x + 1| + C

25. Given function is f(x) = $2x^3$ - 24x + 5

f'(x) = $6x^2$ - 24

f'(x) = $6(x^2 - 4)$

= 6(x - 2)(x + 2)



Function f(x) is decreasing for x∈[-2, 2] and increasing in x∈$(-\infty, -2) \cup (2, \infty)$.

Section C

26. Let the integral be I = $\int \frac{2x}{2+x-x^2} dx$

write the numerator in the following form

$2x = \lambda\left\{\frac{d}{dx}\left(2+x-x^2\right)\right\} + \mu$

i.e. 2x = λ{-2x + 1} + μ

Equating the coefficients will given the values of λ, μ

λ = -1, μ = 1

$\therefore\ I = \int \frac{2x}{2+x-x^2} dx = \int \frac{\lambda\{-2x+1\}+\mu}{2+x-x^2} dx$

Using the values of λ and μ gives

$I = \int \frac{-1(-2x+1)+1}{2+x-x^2} dx$

$= \int \frac{-1\{-2x+1\}}{2+x-x^2} dx + \frac{1}{2+x-x^2} dx$

= -log |2 + x - x^2| + $\int \frac{1}{2+x-x^2} dx$

= -log |2 + x - x^2| - $\int \frac{1}{(x^2-x-2)} dx$

= -log |2 + x - x^2| - $\int \frac{1}{\left(x^2-x+\frac{1}{4}-2-\frac{1}{4}\right)} dx$

= -log |2 + x - x^2| - $\int \frac{1}{\left(x^2-x+\frac{1}{4}-\frac{9}{4}\right)} dx$

= -log |2 + x - x^2| - $\int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2\right)} dx$

= -log |2 + x - x^2| - $\frac{1}{3}\log\left|\frac{\left(x-\frac{1}{2}\right)-\left(\frac{3}{2}\right)}{\left(x-\frac{1}{2}\right)+\left(\frac{3}{2}\right)}\right| + C$

= -log |2 + x - x^2| - $\frac{1}{3}\log\left|\frac{(x-2)}{(x+1)}\right| + C$

27. Let E_1 = event that Ramesh is selected for the post A, and E_2 = event that Ramesh is selected for the post B.

Therefore,we have,

P(E_1) = $\frac{1}{6}$ and P(E_2) = $\frac{1}{7}$

Clearly, E_1 and E_2 are independent events.

$\therefore$ P($E_1 \cap E_2$) = P(E_1) $\times$ P(E_2) = $\left(\frac{1}{6} \times \frac{1}{7}\right) = \frac{1}{42}$

$\therefore$ P(Ramesh is selected for at least one post)

= P($E_1 \cup E_2$)

= P(E_1) + P(E_2) - P($E_1 \cap E_2$)

$= \left(\frac{1}{6} + \frac{1}{7} - \frac{1}{42}\right) = \frac{12}{42} = \frac{2}{7}$

28. Solving integration by parts we have....$\int x_{II}^3 \cdot \left(\log_I x\right)^2 \cdot dx$

$= \left(\log x^2\right) \int x^3 dx - \int \frac{2\log x}{x} \times \frac{x^4}{4} dx$

$= (\log x)^2 \times \frac{x^4}{4} - \frac{1}{2} \int \log_I x \cdot x_{II}^3 \mathrm{d}x$

$= (\log x)^2 \times \frac{x^4}{4} - \frac{1}{2}\left[\log x \int x^3 dx - \int \left\{\frac{d}{dx}(\log x) \int x^3 dx\right\} dx\right]$

$= (\log x)^2 \times \frac{x^4}{4} - \frac{1}{2}\left[\log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \times \frac{x^4}{4} dx\right]$

$= (\log x)^2 \times \frac{x^4}{4} - \frac{1}{2}\left[\log x \cdot \frac{x^4}{4} - \frac{1}{4} \int x^3 dx\right]$

$= (\log x)^2 \times \frac{x^4}{4} - \frac{1}{2}\left[\log x \cdot \frac{x^4}{4} - \frac{x^4}{16}\right] + C$

$= (\log x)^2 \times \frac{x^4}{4} - \frac{\log x \cdot x^4}{8} + \frac{x^4}{32} + C$

OR

Let the given integral be,

$I = \int \frac{3+4x-x^2}{(x+2)(x-1)} dx$

by long division we have,

$= \int \left(-1 + \frac{5x+1}{(x+2)(x-1)}\right) dx$

$= \int -dx + \int \frac{5x+1}{(x+2)(x-1)} dx$

= -x + I_1 ,Now

$I_1 = \int \frac{5x+1}{(x+2)(x-1)} dx$

Put $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$

A(x - 1) + B(x + 2) = 5x + 1

Now put x - 1 = 0

Therefore, x = 1

A(0) + B(1 + 2) = 5 + 1 = 6

B = 2

Now put x + 2 = 0

Therefore, x = -2

A(-2 - 1) + B(0) = 5 × (-2) + 1

A = 3

Now From equation (1) we get,

$\frac{5x+1}{(x+2)(x-1)} = \frac{3}{(x+2)} + \frac{2}{(x-1)}$

$\int \frac{5x+1}{(x+2)(x-1)} dx = 3 \int \frac{1}{(x+2)} dx + 2 \int \frac{1}{(x-1)} dx$

$3 \log |x+2| + 2 \log |x-1| + c$

Therefore,

$I = -x + 3 \log |x+2| + 2 \log |x-1| + c$

29. The given differential equation is,

$x \cdot \frac{dy}{dx} + y = y^2$

$x \cdot \frac{dy}{dx} = y^2 - y$

$\frac{1}{y^2-y} dy = \frac{1}{x} dx$

$\frac{1}{y(y-1)} dy = \frac{1}{x} dx$

Integrating on both the sides,

$\int \frac{1}{y(y-1)} dy = \int \frac{1}{x} dx$

LHS.

Let $\frac{1}{y(y-1)} dy = \frac{A}{y} + \frac{B}{(y-1)}$

$\frac{1}{y(y-1)} dy = A(y-1) + By$

1 = A(y - 1) + By

1 = Ay + By - A

Comparing coefficients in both the sides,we have,

A = - 1, B = 1

$\frac{1}{y(y-1)} dy = -\frac{1}{y} + \frac{1}{(y-1)}$

$\int \frac{1}{y(y-1)} dy = \int \left[-\frac{1}{y} + \frac{1}{(y-1)}\right] dy$

= -y + log (y-1)

$= \log\left(\frac{y-1}{y}\right)$

RHS:

$\int \frac{1}{x}dx$

$\int \frac{1}{x}dx = \log x + \log C$

Therefore the solution of the given differential equation is

$\log\left(\frac{y-1}{y}\right) = \log x + \log C$

$\frac{y-1}{y} = x.c$

y - 1 = yxc

⇒y = 1 + xyc

OR

It is given that $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{y^2+y+1}{x^2+x+1}\right)$

$\Rightarrow \frac{dy}{y^2+y+1} = \frac{-dx}{x^2+x+1}$

$\Rightarrow \frac{dy}{y^2+y+1} + \frac{dx}{x^2+x+1} = 0$

On integrating both sides, we get,

$\int \frac{dy}{y^2+y+1} + \int \frac{dx}{x^2+x+1} = c$

$\Rightarrow \int \frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \int \frac{dy}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = c$

$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\mathrm{y}+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right] + \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\mathrm{x}+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right] = \mathrm{C}$

$\Rightarrow \tan^{-1}\left[\frac{2y+1}{\sqrt{3}}\right] + \tan^{-1}\left[\frac{2x+1}{\sqrt{3}}\right] = C$

$\Rightarrow \tan^{-1}\left[\frac{\frac{2y+1}{\sqrt{3}}+\frac{2x+1}{\sqrt{3}}}{1-\frac{2y+1}{\sqrt{3}}.\frac{2x+1}{\sqrt{3}}}\right] = \frac{\sqrt{3}}{2}C$

$\Rightarrow \tan^{-1}\left[\frac{\frac{2\mathrm{x}+2\mathrm{y}+2}{\sqrt{3}}}{1-\left(\frac{4\mathrm{xy}+2\mathrm{x}+2\mathrm{y}+1}{3}\right)}\right] = \frac{\sqrt{3}}{2}\mathrm{C}$

$\Rightarrow \tan^{-1}\left[\frac{2\sqrt{3}(x+y+1)}{3-4xy-2x-2y-1}\right] = \frac{\sqrt{3}}{2}C$

$\Rightarrow \tan^{-1}\left[\frac{2\sqrt{3}(x+y+1)}{2(1-x-y-2xy)}\right] = \frac{\sqrt{3}}{2}C$

$\Rightarrow \frac{\sqrt{3}(x+y+1)}{(1-x-y-2xy)} = \tan\left(\frac{\sqrt{3}}{2}c\right)$

Let $\tan\left(\frac{\sqrt{3}}{2}C\right) = B$

Then $\mathrm{x}+\mathrm{y}+1 = \frac{2\mathrm{B}}{\sqrt{3}}(1-\mathrm{x}-\mathrm{y}-2\mathrm{xy})$

Now let $A = \frac{2B}{\sqrt{3}}$ then, we have,

x + y + 1 = A(1 - x - y - 2xy)

30. Firstly, we will convert the given inequations into equations, now we will get the equitions:

x - y = 1, x + y = 3, x = 0 and y = 0

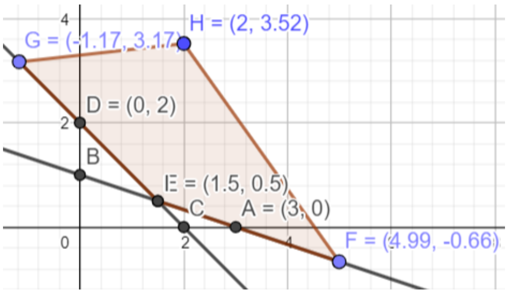
Region represented by x - y≤1: The line x - y = 1 meets the coordinate axes at A(1,0) and B(0, - 1) respectively. By joining these points we obtain the line x - y = 1 Clearly (0,0) satisfies the inequation x + y $\leq$ 8 . So, the region in x y plane which contain the origin represents the solution set of the inequation x - y $\leq$ 1.

The region represented by x + y $\geq$ 3 :

The line x + y = 3 meets the coordinate axes at C(3,0) and D(0,3) respectively. By joining these points we obtain the line x + y = 3. Clearly (0,0) satisfies the inequation x + y $\geq$ 3 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation x + y $\geq$ 3

The region represented by x $\geq$ 0 and y $\geq$ 0 since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0.

The feasible region determined by subject to the constraints are x - y $\leq$ 1, x + y $\geq$ 3, and the non-negative restrictions x $\geq$ 0 and y $\geq$ 0 are as follows.



The feasible region is unbounded. We would obtain the maximum value at infinity. Therefore, maximum value will be infinity i.e. the solution is unbounded.

OR

We need to maximize z = x + y

First, we will convert the given inequations into equations, we obtain the following equations:

-2x + y = 1, x = 2, x + y = 3, x = 0 and y = 0

The line -2x + y = 1 meets the coordinate axis at $A\left(\frac{-1}{2}, 0\right)$ and B(0, 1). Join these points to obtain the line -2x + y = 1.

Clearly, (0, 0) satisfies the inequation -2x + y $\leq$ 1. So, the region in xy-plane that contains the origin represents the solution set of the given equation.

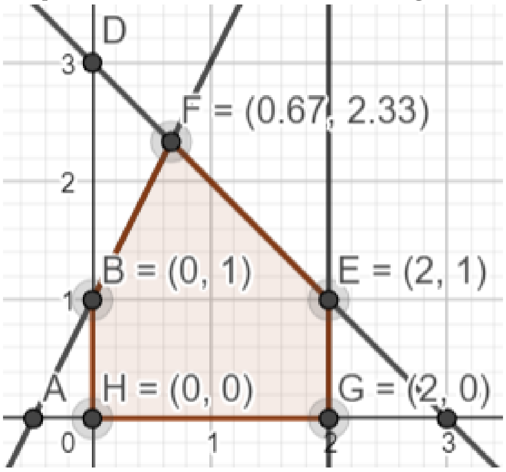
x = 2 is the line passing through (2,0) and parallel to the Y axis.

The region below the line x = 2 will satisfy the given inequation. The line x + y = 3 meets the coordinate axis at C(3, 0) and D(0, 3). Join these points to obtain the line x + y = 3.

Clearly, (0,0) satisfies the inequation x + y $\leq$ 3 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

Region represented by x $\geq$ 0 and y $\geq$ 0 (non -negative restrictions)

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the regior represented by the inequations. These lines are drawn using a suitable scale.



The corner points of the feasible region are O(0,0), G(2,0), E(2,1) and $F\left(\frac{2}{3}, \frac{7}{3}\right)$

The values of objective function at the corner points are as follows:

Corner point : Z = x + y

O(0, 0) : 0 + 0 = 0

C(2, 0) : 2 + 0=2

E(2, 1) : 2 + 1=3

$F\left(\frac{2}{3}, \frac{7}{3}\right) : \frac{2}{3} + \frac{7}{3} = \frac{9}{3} = 3$

We see that the maximum value of the objective function z is 3 which is at E(2,1) and $F\left(\frac{2}{3}, \frac{7}{3}\right)$

Thus, the optimal value of objective function z is 3.

31. We have, y = $(\tan x)^{\cot x} + (\cot x)^{\tan x}$

$y = e^{\log(\tan x)^{\cot x}} + e^{\log(\cot x)^{\tan x}}$

$\Rightarrow$ y = $e^{\cot x \log \tan x} + e^{\tan x \log(\cot x)}$

Differentiating with respect to x using chain rule and product rule,

$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cot x \log \tan x}\right) + \frac{d}{dx}\left(e^{\tan x \log \cot x}\right)$

$= e^{\cot x \log \tan x} \frac{d}{dx}\left(\cot x \log \tan x\right) + e^{\tan x \log \cot x} \frac{d}{dx}\left(\tan x \log \cot x\right)$

$= e^{\log(\tan x)^{\cot x}}\left[\cot x \frac{d}{dx}(\log \tan x) + \log \tan x \frac{d}{dx}(\cot x)\right] + e^{\log(\cot x)\tan x}\left[\tan x \frac{d}{dx}(\log \cot x) + \log \cot x \frac{d}{dx}(\tan x)\right]$

$= (\tan x)^{\cot x}\left[\cot x \times \left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log \tan x\left(-cosec^2 x\right)\right]$

$+(\cot x)^{\tan x}\left[\tan x \times \left(\frac{1}{\cot x}\right)\frac{d}{dx}(\cot x) + \log \cot x\left(\sec^2 x\right)\right]$

$= (\tan x)^{\cot x}\left[\left(\frac{cosec^2 x}{\sec^2 x}\right)\left(\sec^2 x\right) - cosec^2 x \log \tan x\right] + (\cot x)^{\tan x}\left[\left(\frac{\sec^2 x}{cosec^2 x}\right)\left(-cosec^2 x\right) + \sec^2 x \log \cot x\right]$

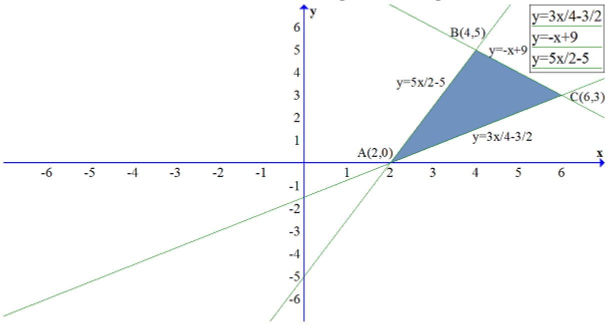
= $(\tan x)^{\cot x}[\text{cosec}^2 x - \text{cosec}^2 x \log \tan x] + (\cot x) \tan x [\sec^2 x \log \cot x - \sec^2 x]$

= $(\tan x) \cot x \,\text{cosec}^2 x[1 - \log \tan x] + (\cot x) \tan x \sec^2 x[\log \cot x^{-1}]$

The differentiation of the given function y is as above.

Section D

32. Points in the form of line in the given diagram



The equation of side AB is,

$y - 0 = \frac{5-0}{4-2}(x - 2)$

$\Rightarrow y = \frac{5}{2}(x - 2)$

The equation of side BC is,

$y - 3 = \frac{5-3}{4-6}(x - 6)$

$\Rightarrow y - 3 = \frac{2}{-2}(x - 6)$

$\Rightarrow y - 3 = -1(x - 6)$

$\Rightarrow y = -x + 9$

The equation of side AC is,

$y - 0 = \frac{3-0}{6-2}(x - 2)$

$\Rightarrow y = \frac{3}{4}(x - 2)$

Area $= \frac{5}{2}\int_2^4 (x-2)dx + \int_4^6 -(x-9)dx - \frac{3}{4}\int_2^6 (x-2)dx$

$A = \int_2^4 \frac{5}{2}(x-2)dx + \int_0^1 -(x+9)dx + \int_6^2 \frac{3}{4}(x-2)dx$

On integrating we get,

$A = \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[\frac{-x^2}{2} + 9x\right]_1^0 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6$

On applying limits we get,

$A = \frac{5}{2}[8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4}[18 - 12 - 2 + 4]$

$A = 5 - 8 - \frac{3}{4}(8)$

= 13 - 6 = 7 sq. units.

Hence the required area is 7 sq. units.

33. For $x_1, x_2 \in R$, consider

$f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$

$\Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$

$\Rightarrow x_1x_2(x_2 - x_1) = x_2 - x_1$

$\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$

We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$ for instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1)\frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$. Hence f is not one-one. Also, f is not onto for if so then for $1 \in R \ \exists \ x \in R$ such that f(x) = 1 which gives $\frac{x}{x^2+1} = 1$. But there is no such x in the domain R, since the equation $x^2 - x + 1 = 0$ does not give any real value of x.

OR

A = {1, 2, 3, 4, 5} and R = {(a, b) : |a - b| is even}, then R = {(1, 3), (1, 5), (3, 5), (2, 4)}

1. For (a, a), |a - a| = 0 which is even. ∴ R is reflexive.
   If |a - b| is even, then |b - a| is also even. ∴ R is symmetric.
   Now, if |a - b| and |b - c| is even then |a - b + b - c| is even
   ⇒ |a - c| is also even. ∴ R is transitive.
   Therefore, R is an equivalence relation.
2. Elements of {1, 3, 5} are related to each other.
   Since |1 - 3| = 2, |3 - 5| = 2, |1 - 5| = 4 all are even numbers
   ⇒ Elements of {1, 3, 5} are related to each other.
   Similarly elements of (2, 4) are related to each other.
   Since |2 - 4| = 2 an even number, then no element of the set {1, 3, 5} is related to any element of (2, 4).
   Hence no element of {1, 3, 5} is related to any element of {2, 4}.

34. Three equations are formed from the given statements:

3x + 2y + z = 2200

4x + y + 3z = 3100 and

x + y + z = 1200

Converting the system of equations in matrix form we get,

$\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}2200\\3100\\1200\end{bmatrix}$

i.e. AX = B

where A = $\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}$, X = $\begin{bmatrix}x\\y\\z\end{bmatrix}$ and B = $\begin{bmatrix}2200\\3100\\1200\end{bmatrix}$

⇒ X = A^{-1} B

$A^{-1} = \frac{1}{|A|}$ (Adj A)

|A| = 3(1 − 3) −2(4 − 3) + 1(4 − 1) = −6 −2 + 3 = −5

Adj A = $\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}$

∴ $A^{-1} = \frac{1}{-5}\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix}2 & 1 & -5\\1 & -2 & 5\\-3 & 1 & 5\end{bmatrix}$

⇒ X = $\frac{1}{5}\begin{bmatrix}2 & 1 & -5\\1 & -2 & 5\\-3 & 1 & 5\end{bmatrix}\begin{bmatrix}2200\\3100\\1200\end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix}4400 + 3100 - 6000\\2200 - 6200 + 6000\\-6600 + 3100 + 6000\end{bmatrix}$

$= \begin{bmatrix}300\\400\\500\end{bmatrix}$

⇒ x = 300, y = 400 and z = 500

i.e. The award money for each value are Rs.300 for Tolerance, Rs.400 for Kindness and Rs.500 for Leadership.

35. Here the equation of two planes are: $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Since the line is parallel to the two planes.

∴ Direction of line $\vec{b} = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$

$= -3\hat{i} + 5\hat{j} + 4\hat{k}$

∴ Equation of required line is

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$ .... (i)

Any point on line (i) is $(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$

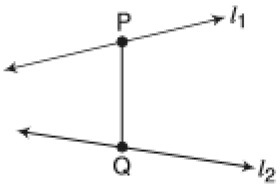
For this line to intersect the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k})$ we have

$(1 - 3\lambda)2 + (2 + 5\lambda)1 + (3 + 4\lambda)1 = 4$

$\Rightarrow \lambda = 1$

$\therefore$ Point of intersection is (4, -3, -1)

OR



Let P(3λ + 7, 2λ + 5, λ + 3) and

Q(2μ + 1, 4μ - 1, 3μ - 1)

Now, d.r′.s. of PQ = 3λ - 2μ + 6, 2λ - 4μ + 6, λ - 3μ + 4

According to question,

$\frac{3\lambda - 2\mu + 6}{2} = \frac{2\lambda - 4\mu + 6}{2} = \frac{\lambda - 3\mu + 4}{1}$

$\Rightarrow \lambda + 2\mu = 0$ and $2\mu = 2 \Rightarrow \mu = 1$

$\Rightarrow \lambda = -2\mu$

$\therefore \mu = 1, \lambda = -2$

$\therefore$ P(1, 1, 1) and Q(3, 3, 2)

PQ = $\sqrt{(3 - 1)^2 + (3 - 1)^2 + (2 - 1)^2}$

= $\sqrt{4 + 4 + 1}$

= 3

therefore length is 3 unit

Equation of PQ is $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-1}{1}$

Section E

36. **Read the text carefully and answer the questions:**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in all subjects) = P(M $\cap P \cap$ C)

= P(M) $\times$ P(P) $\times$ P(C)

= 0.2 $\times$ 0.3 $\times$ 0.5 = 0.03

(ii) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in on subjects) = $P(\overline{M} \cap \overline{P} \cap \overline{C})$

$= P(\overline{M}) \times P(\overline{P}) \times P(\overline{C})$

= 0.8 $\times$ 0.7 $\times$ 0.5 = 0.280

(iii) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 2 subjects)

$\Rightarrow$ P(grade A in M and P not in C) + P(grade A in P & C not in M) + P(grade A in M & C not in P)

$\Rightarrow$ P$\left(M \cap P \cap \overline{C}\right)$ + P$\left(P \cap C \cap \overline{M}\right)$ + P$(M \cap C \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.3 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.2 $\times 0.5 \times$ 0.7 = 0.03 + 0.12 + 0.07

P(getting grade A in 2 subjects) = 0.22

OR

P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 1 subjects)

$\Rightarrow$ P(grade A in M not in P and C) + P(grade A in P not in M and C) + P(grade A in C not in P and M)

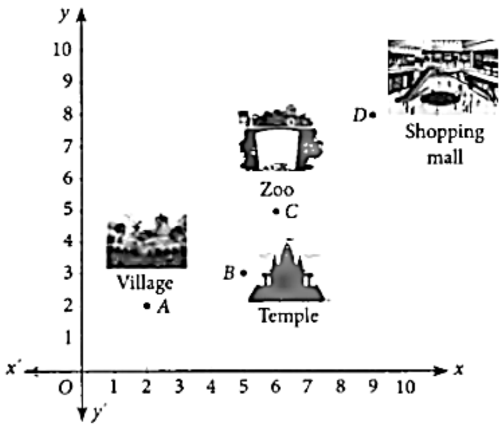
$\Rightarrow$ P$\left(M \cap \overline{P} \cap \overline{C}\right)$ + P$\left(P \cap \overline{C} \cap \overline{M}\right)$ + P$(C \cap \overline{M} \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.7 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.5 $\times 0.8 \times$ 0.7 = 0.07 + 0.12 + 0.028

P(getting grade A in 1 subjects) = 0.47

37. **Read the text carefully and answer the questions:**

Girish left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Girish at different places is given in the following graph.



(i) Here (5, 3) are the coordinates of B.

$\therefore$ Position vector of B = $5\hat{i} + 3\hat{j}$

(ii) Here (9, 8) are the coordinates of D.

$\therefore$ Position vector of D = $9\hat{i} + 8\hat{j}$

(iii) Position vector of B = $5\hat{i} + 3\hat{j}$ and Position vecto of C = $6\hat{i} + 5\hat{j}$

$\therefore \overrightarrow{BC} = (6 - 5)\hat{i} + (5 - 3)\hat{j} = \hat{i} + 2\hat{j}$

OR

Since P.V. of A = $2\hat{i} + 2\hat{j}$, P.V. of D = $9\hat{i} + 8\hat{j}$

$\therefore \overrightarrow{AD} = (9 - 2)\hat{i} + (8 - 2)\hat{j} = 7\hat{i} + 6\hat{j}$

$|\overrightarrow{AD}|^2 = 7^2 + 6^2 = 49 + 36 = 85$

$\Rightarrow |\overrightarrow{AD}| = \sqrt{85}$ units

38. **Read the text carefully and answer the questions:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Let the side of square to be cut off be 'x' cm. then, the length and the breadth of the box will be (18 - 2x) cm each and the height of the box is 'x' cm.

The volume V(x) of the box is given by $V(x) = x(18 - x)^2$

(ii) $V(x) = x(18 - 2x)^2$

$\frac{dV(x)}{dx} = (18 - 2x)^2 - 4x(18 - 2x)$

For maxima or minima = $\frac{dV(x)}{dx} = 0$

$\Rightarrow (18 - 2x)[18 - 2x - 4x] = 0$

$\Rightarrow x = 9$ or $x = 3$

$\Rightarrow x =$ not possible

$\Rightarrow x = 3$ cm

The side of the square to be cut off so that the volume of the box is maximum is x = 3 cm