To Study the Relation Between the Length of a Given Wire and Tension for Constant Frequency Using Sonometer

Aim

To study the relation between the length of a given wire and tension for constant frequency using sonometer.

Apparatus

A sonometer, a set of eight tuning forks, 1\2 kg hanger, seven 1\2 kg slotted weights, rubber pad, paper rider, metre scale, screw gauge.

Theory

If stretched wire (string) vibrates in resonance with a tuning fork of frequency v, then the string also has same frequency v.
If the string has a length l, diameter D, material of density p and tension T, then

$$\nu = \frac{1}{lD}\sqrt{\frac{T}{\pi\rho}} \qquad \text{...(1)}$$

Relation between frequency (ν) and length (l). From Eq. (1) above, $\nu \propto \frac{1}{l}$

or $$\nu l = \text{Constant}.$$

A graph between ν and $\frac{1}{l}$ will be a straight line, while a graph between ν and l will be a hyperbola.

Relation between length (l) and tension (T). From Eq. (1) above,

$$\frac{\sqrt{T}}{l} = \text{Constant}$$

or $$\sqrt{T} \propto l$$

or $$T \propto l^2$$

A graph between T and l^2 will be a straight line.

Diagram

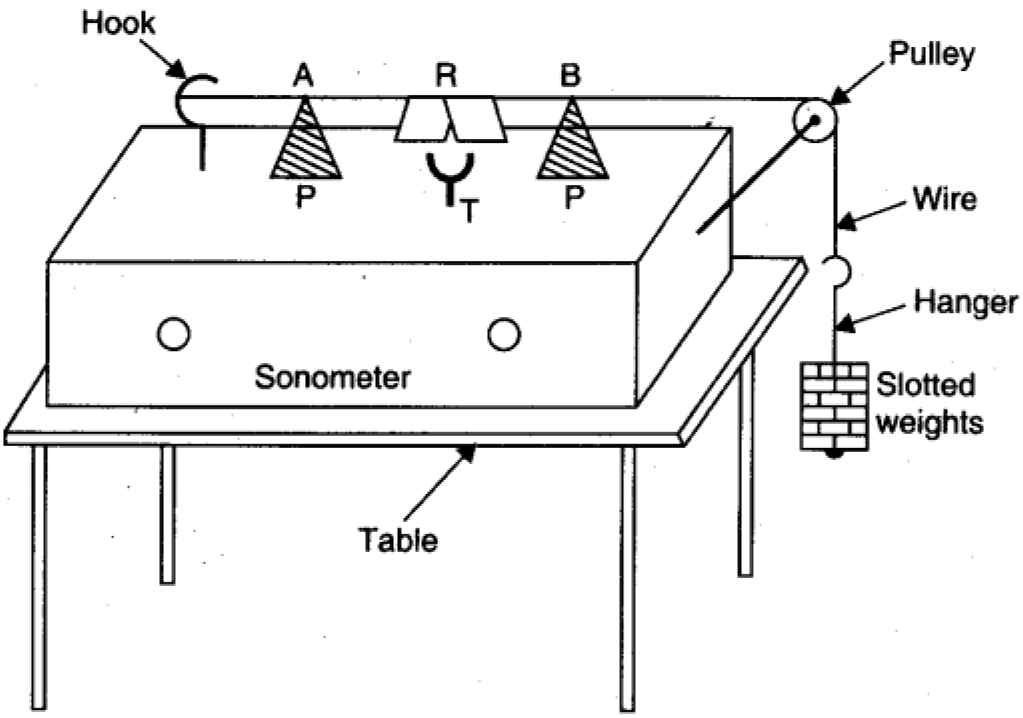


Fig. Sonometer in experimental set up.

Procedure

1. Repeat steps 1 to 13 of previous part with a tuning fork of frequency 256 Hz and load 4 kg.
2. Remove slotted weights one by one to reduce load to 3.5 kg, 3 kg,……, 1 kg and repeat above steps with same tuning fork.
3. Record your observations as given below.

Observations

Constant frequency of tuning fork,
v = 256 Hz.

Table for load (tension) and length

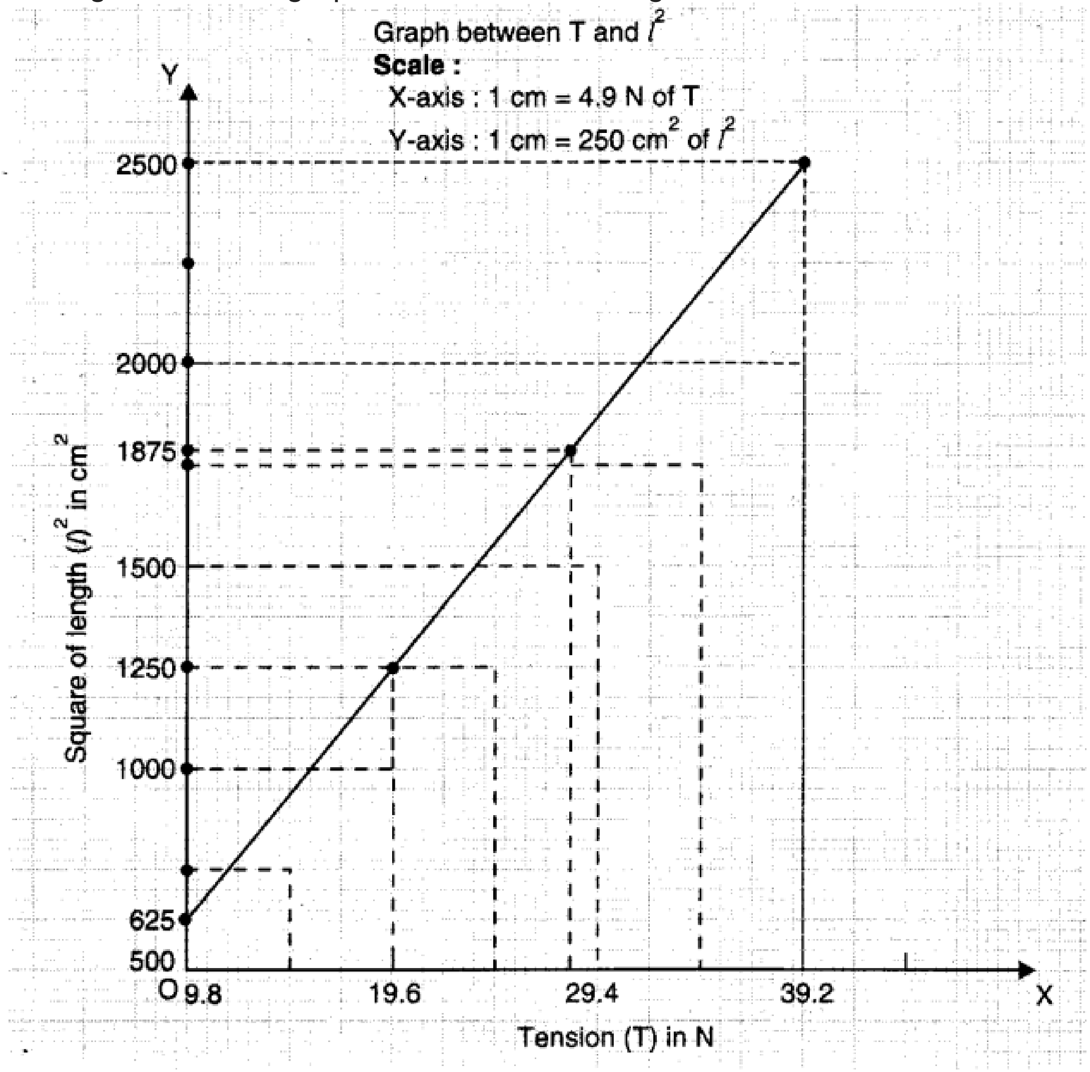
Serial No. of Obs.	Load M (kg)	Tension Mg T(N)	*Resonant length of wire*			l^2 (cm^2)	$\frac{l^2}{T}$ ($cm^2 N^{-1}$)
			Length increasing l_1 (cm)	Length decreasing l_2 (cm)	Mean $= \frac{l_1 + l_2}{2}$ l(cm)		
1.	4.0	39.2	50.1	49.9	$= \frac{50.1 + 49.9}{2}$ = 50	2500	$\frac{2500}{39.2}$ = 63.78
2.	3.5	34.3					
3.	3.0	29.4					
4.	2.5	24.5					
5.	2.0	19.6					
6.	1.5	14.7					
7.	1.0	9.8	25.1	24.9	$\frac{25.1 + 24.9}{2}$ = 25		$\frac{625}{9.8}$ = 63.78

(**Note.** *Ideal observations are as sample*)

Calculations

1. Find mean length l and write it in column 4c.
2. Find l^2 and write it in column 5.
3. Find l^2/T and write it in column 6.
4. Plot a graph between T(column 3) and l^2 (column 5), taking T along X-axis and

l^2 along Y-axis. The graph comes to be a straight line as shown in below.



Graph between T and l^2. It is a straight line.

Result

1. From column 6, we find that $\frac{l^2}{T}$ = constant
2. From the graph in Fig. 18.12, we conclude that $T \propto l^2$ for a constant frequency ν. It is in agreement with the formula,

$$\nu = \frac{1}{lD}\sqrt{\frac{T}{\pi\rho}}$$

This indirectly verifies the law of tension of transverse vibrations of strings.

Precautions

1. Pulley should be friction less.
2. Wires used should be kink less and of uniform cross-section.
3. Loading of wires should not be beyond elastic limit keep max. load = one-third of breaking load.
4. Tuning fork should be vibrated by striking its prongs against soft rubber pad.
5. Readings for length decreasing and increasing should be noted and their mean used in calculations.
6. To transfer vibrations of the tuning fork to the wire through sonometer board, lower end of handle of tuning fork should be touched gently with sonometer board.
7. Weight of hanger should be included in the load.
8. Load should be removed after the experiment.

Sources of error

1. Wire may not be rigid and of uniform cross-sectional area.
2. Pulley may not be friction less.
3. Weights used may not be correct.
4. Bridges may not be sharp.

Viva Voce

Question. 1. Define superposition of waves.
Answer. The phenomena of intermixing of two or more waves to produce a new wave, is called superposition of waves.

Question. 2. State superposition principle.
Answer. It states "the resultant displacement of a particle is equal to the vector sum of the individual displacements given to it by the superposing waves.

Question. 3. Define stationary waves.
Answer. Superposition of two waves of same frequency and same amplitude and travelling with same velocity in opposite direction produces stationary waves.

Question. 4. Under what situation are stationary waves produced ?
Answer. Stationary waves are produced when a reflected wave gets superposed with its incident wave.

Question. 5. Define nodes.
Answer. In stationary wave, the regions of zero displacement and maximum strain, are called nodes. They are denoted by N.

Question. 6. What is the distance between two consecutive nodes ?
Answer. The distance between the two consecutive nodes is equal to half the wavelength of the stationary waves. $NM=\lambda/2$

Question. 7. Define antinode.
Answer. In a stationary wave, the regions of maximum displacement and zero strain, are called antinodes. They are denoted by A.

Question. 8. What is the distance between two consecutive antinodes ?
Answer. The distance between two consecutive antinode is equal to half the wavelength of the stationary waves. AA=$\lambda/2$

Question. 9. What is the distance between a node and nearest antinode ?
Answer. The distance between a node and nearest antinodes is equal to quarter wavelength of the stationary waves. NA=$\lambda/4$

Question. 10. How can transverse stationary waves be produced in a stretched string ?
Answer. Transverse stationary waves can be produced in a string by stretching it with a tension and fixing it at its two ends. When plucked in the middle, transverse stationary waves are produced in it.

Question. 11. What is fundamental frequency of a vibrating body ?
Answer. The least frequency of a vibrating body, is called its fundamental frequency. It is also called first harmonic.

Question. 12. What does the number of a harmonic represent ?
Answer. The number of a harmonic gives the ratio of its frequency with fundamental frequency.

Question. 13. What is an overtone ?
Answer. All frequencies of a vibrating body, other than the fundamental, are called overtones.

Question. 14. How are the overtones numbered ?
Answer. They are numbered in order of their increasing frequency after the fundamental.

Question. 15. Give formula for frequency of transverse vibrations of a stretched string.
Answer.

The formula is, $\nu = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$.

Question. 16. What is law of length ?
Answer.
Its mathematical form is, $\nu \propto \frac{1}{l}$. (T, m constant)

Question. 17. What is law of tension ?
Answer.
Its mathematical form is, $\nu \propto \sqrt{T}$. (l, m constant)

Question. 18. What is law of mass ?
Answer.
Its mathematical form is, $\nu \propto \frac{1}{\sqrt{m}}$. ($l$, T constant)

Question. 19. What is law of diameter ?
Answer.
Its mathematical form is, $\nu \propto \frac{1}{D}$.

Question. 20. What is law of density ?
Answer.
Its mathematical form is, $\nu \propto \frac{1}{\sqrt{\rho}}$.

Question. 21. How can longitudinal stationary waves be produced in a rod ?
Answer. Longitudinal stationary waves can be produced in a rod by clamping it and then rubbing it along its length by a resined cloth.

Question. 22. Describe different cases of vibration of a clamped rod.
Answer. Different cases are :
1. Rod clamped at one end, —> l = λ/4.
2. Rod clamped in the middle, —> l=λ/2.
3. Rod clamped both ends, —> l = λ/2.
4. Rod clamped at a point at quarter length distance from one end—> l = λ.

Question. 23. Define pitch of a sound.
Answer. It is a sensation which makes us feel whether the sound is shrill or grave (hoarse).

Question. 24. On what factor pitch depends ?
Answer. Pitch depends upon frequency of sound.

Question. 25. Which sound is shrill ?
Answer. A sound of more frequency has a high pitch. The sound is shrill.

Question. 26. Give some example of shrill sound.
Answer. Sound produced by the humming of a mosquito is shrill.

Question. 27. Which sound is grave (or hoarse) ?
Answer. A sound of less frequency has a low pitch. The sound is grave (or hoarse).

Question. 28. Give some example of grave (hoarse) sound. .
Answer. The sound produced by the roaring of a lion is grave (or hoarse).

Question. 29. What is a tuning fork?
Answer. It is a source of standard frequency useful for sound experiment with sonometer or resonance tube.

Question. 30. How do its different parts vibrate ?
Answer. Its prongs vibrates in transverse mode, while its handle vibrates in longitudinal mode.

Question. 31. How is it that its prongs stop vibrating simply on touching with a finger, while its handle continues vibrating even when held in hand tightly ?
Answer. It is because, vibrations of prongs are transverse and those of handle are longitudinal.

Question. 32. What is a sonometer ?
Answer. It is a hollow and rectangular wooden box.

Question. 33. Why are there two holes in the side of the sonometer ?
Answer. To make air vibrating inside the box to come in contact of outside air to enhance vibrations.

Question. 34. How sonometer board helps in experiment ?
Answer. It increases loudness of sound produced by the vibrating wire.

Question. 35. What is the nature of vibrations of the sonometer board ?
Answer. The vibrations of sonometer board are forced vibrations.

Question. 36. What is the nature of vibrations of the tuning fork ?
Answer. The vibrations of the tuning fork are free vibrations.

Question. 37. What is the nature of vibrations of the sonometer wire when the rider falls ?
Answer. The vibrations are resonant vibrations.

Question. 38. What is meant by resonant vibrations of the wire ?
Answer. It means that frequency of the wire equals tuning fork frequency.

Production of longitudinal stationary waves in air column

Let a tube, open at both ends, have its lower end dipped in water taken in a vessel (Fig.). The part of the tube outside water encloses an air column BD.
When a vibrating tuning fork is held, with its prongs vibrating in a vertical plane, just above the end B of the tube, longitudinal waves are sent down the air column. These waves reflect from water surface at D and travel upwards. In the air column, incident and reflected waves get superposed. This superposition produces longitudinal stationary waves.

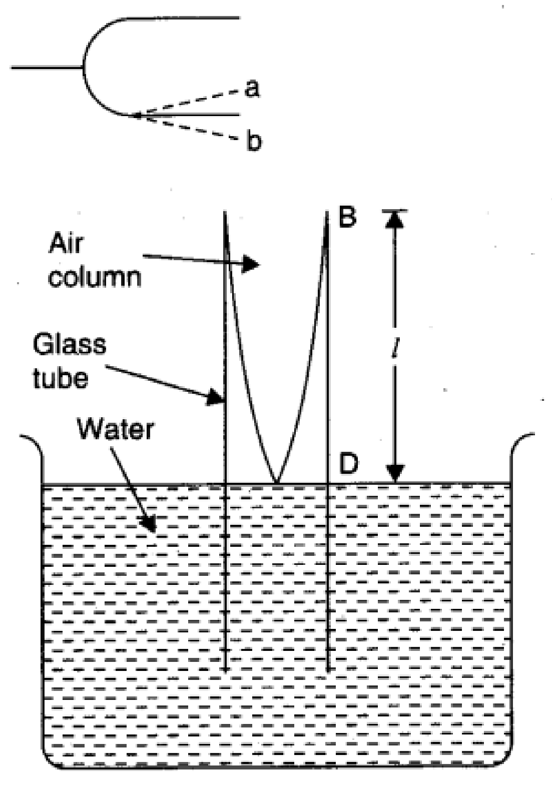


Fig. Resonance of air column.

Node is produced at the water surface where displacement is minimum due to denser water medium. Antinode is produced at the open end where displacement is maximum due to rarer medium of air outside the column.
Resonance. Let the tuning fork have frequency v and time period T. Let the air column

have length l.
When prong of tuning fork moves downward from position a, it sends down a compression. This compression is reflected as such from denser water surface. When this compression reaches open end B, it becomes rarefaction due to reflection from a rarer medium (outside air). The rarefaction moves downwards.
If by this time, prong has reached from a to b and starts moving up, it sends a new rarefaction. The two rarefactions combine to increase amplitude of vibration of particles. This combination is repeated and vibrations become so violent that a loud sound is produced in the air column. It is said that the air column is in resonance with the tuning fork.
Calculations. For resonance to take place, sound waves must travel from B to D and D to B, a total distance 21, in same time in which prong of T.F. comes from a to b in time T/2.
Distance travelled by sound in T/2 equals $\lambda/2$, where λ is the wavelength of sound waves having same frequency (v) as that of T.F.
Hence, for resonance, $2l = \lambda/2$
or $l=\lambda/4$
Thus, air column will be in resonance with T.F., only when the length of the air column is equal to one-fourth of wavelength of sound waves having frequency equal to frequency of T.F.

Determination of velocity of sound in air by resonance tube

Principle. The resonance tube works on the phenomena of resonance of air column .

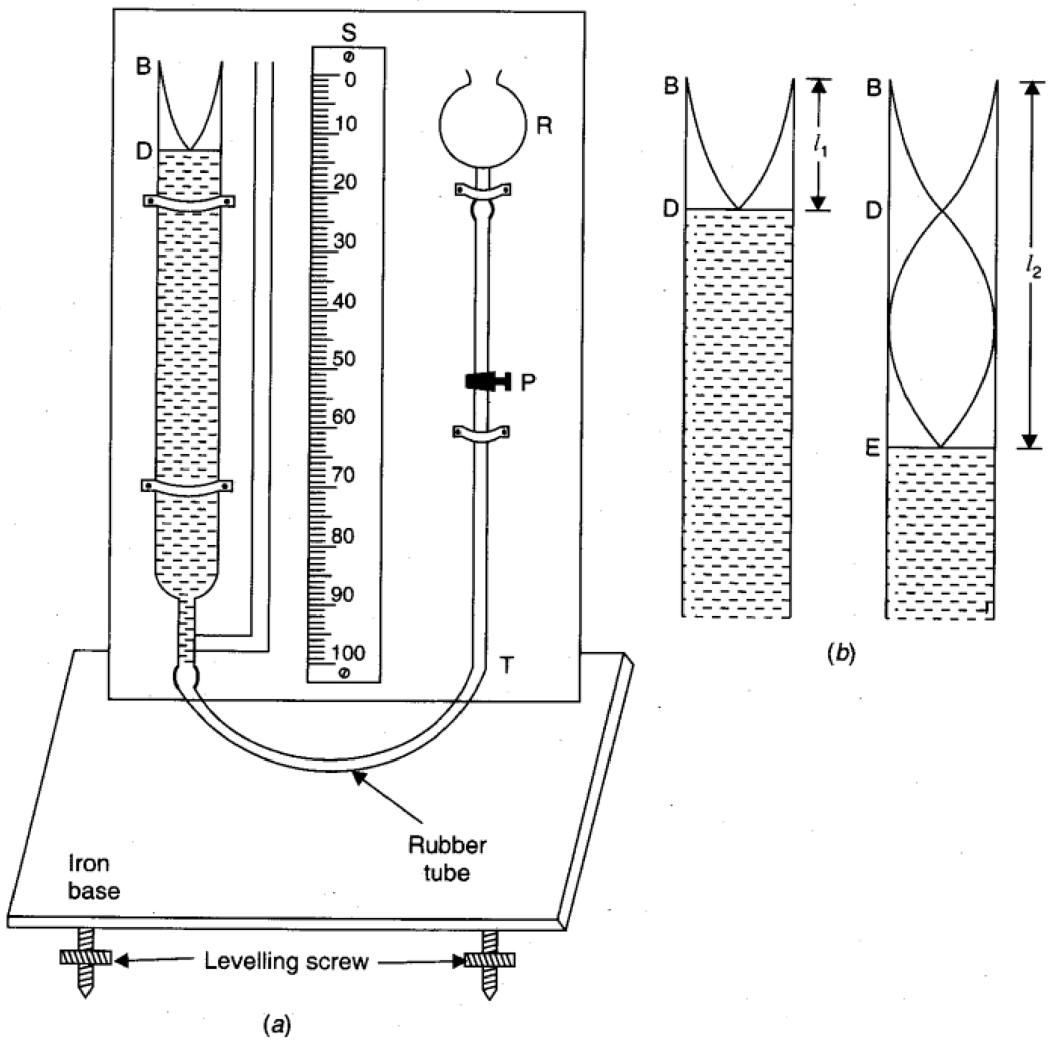


Fig. Resonance tube.

(b) Construction. It consists of a vertical wooden board fixed on a heavy and broad horizontal iron base provided with levelling screws [Fig]. On the wooden board a metallic brass tube of length 1 metre and diameter 5 cm is fixed. A metre scale S is also fixed on the board parallel to the tube. A glass tube of small bore is fixed along the metre scale and is connected to metallic tube near its lower end.
The lower end of metallic tube is connected by a rubber tube T to a reservoir R which can slide up and down along the scale and clamped in any position. To keep the level of water column fixed in the tube, a pinch cock P is clamped to the rubber tube. The tube and part of reservoir are filled with water. The metallic tube forms an air column BD over the water surface.
The length of the air column can be changed by changing water level in the tube. It is done by raising or lowering the reservoir and loosening the pinch cock.

(c) Working. The tube is made vertical by making the base horizontal. The water level is raised to a position near B. Reservoir is fixed near the base.
A tuning fork of known frequency is taken. It is vibrated and held just over the end B of

the tube, such that its prongs vibrate in a vertical plane. Pinch cock is made loose a little so that water level falls and length of the air column increases slowly. For some position of falling water level, a loud sound is heard from the air column. The position of water level is noted. The length of air column is measured with the help of the metre scale.

(*d*) **Observations.** Let,

Frequency of tuning fork $= \nu$

Length of air column in resonance $= l$.

(*e*) **Calculation.** Then,

Frequency of sound waves produced $= \nu$

Wavelength of sound waves produced $= 4l$

Hence, velocity of sound, $v = \nu\lambda = \nu 4l$

or $$\boxed{v = 4\nu l}$$

which can be calculated.

This gives velocity of sound in air in the air column at the temperature of the air column.

If t be the temperature of air column (as measured by a thermometer suspended in it), then velocity of sound at 0°C can be calculated by formula,

$$\frac{v}{v_0} = \sqrt{\frac{273 + t}{273}}$$

or $$v_0 = v\sqrt{\frac{273}{273 + t}}$$

It must come out to be 332 m s^{-1}.

End correction

Rayleigh found that reflection of sound at open end of the tube does not take place from the position of the edge of the tube but from a slightly higher region. It is because the tube medium (air) continues even beyond the edge.

According to Rayleigh, the distance of the region of reflection from the edge of the tube is

0.3 D, where D is the internal diameter of the tube. This distance is called end correction and is represented by the symbol x, i.e., x = 0.3 D.

Applying end correction, we get

$l + x = \lambda/4$

$\lambda = 4(l + x)$

Hence, formula for velocity of sound becomes,

$\upsilon = 4\, v(l + x)$.

Determination of velocity of sound eliminationg end correction

The resonance found earlier with air column length l = λ/4, is called first resonance. If this length l be increased to about three times [BE in Fig.] resonance will again be produced. This is called second resonance.

For second resonance, sound travels a total distance 2l (2 BE) in time 3T/2 (three times of that for first resonance). As sound reaches back to open end B, it finds the prong of T.F. at b (which is the condition for producing resonance).

For second resonance, $$2l = \frac{3}{2}\lambda$$

$$l = \frac{3}{4}\lambda.$$

If l_1 and l_2 be the length of air column for first and for second resonance and x be the end correction, then

$$l_1 + x = \frac{\lambda}{4}$$

$$l_2 + x = \frac{3}{4}\lambda.$$

Subtracting, we get

$$l_2 - l_1 = \frac{\lambda}{2}$$

or $$\boxed{\lambda = 2(l_2 - l_1)}$$

Thus, formula for velocity of sound becomes

$$v = \nu\lambda = 2\nu(l_2 - l_1)$$

$$\boxed{v = 2\nu(l_2 - l_1)}$$

Expression for end correction :

We have, $$l_1 + x = \frac{\lambda}{4}$$

i.e., $$3l_1 + 3x = \frac{3}{4}\lambda$$

Also, $$l_2 + x = \frac{3}{4}\lambda$$

Equating, $$3l_1 + 3x = l_2 + x$$

Transposing, $$3x - x = l_2 - 3l_1$$

$$2x = l_2 - 3l_1$$

or $$\boxed{x = \frac{l_2 - 3l_1}{2}}$$

which can be calculated.