

4. Cubes and Cube Roots

Exercise 4A

1. Question

Evaluate:

(i) $(8)^3$

(ii) $(15)^3$

(iii) $(21)^3$

(iv) $(60)^3$

Answer

(i) To calculate the cube of $(8)^3$

We have to multiply the given number three times;

$$= (8 \times 8 \times 8) = 512$$

So, 512 is the cube of 8.

(ii) $(15)^3$

First multiply the given number three times;

$$= (15 \times 15 \times 15) = 3375$$

So, 3375 is the cube of 15

(iii) $(21)^3$

First multiply the given number three times;

$$= (21 \times 21 \times 21) = 9261$$

9261 is the cube of 21.

(iv) $(60)^3$

First multiply the given number three times;

$$= (60 \times 60 \times 60) = 216000$$

216000 is the cube of 60.

2. Question

Evaluate:

(i) $(1.2)^3$

(ii) $(3.5)^3$

(iii) $(0.8)^3$

(iv) $(0.05)^3$

Answer

(i) To calculate the cube of $(1.2)^3$

We have to multiply the given number three times;

$$= (1.2 \times 1.2 \times 1.2) = 1.728$$

Now by converting it into fraction we get,

$$= \frac{1728}{1000}$$

$$= \frac{216}{125}$$

(ii) To calculate the cube of $(3.5)^3$

We have to multiply the given three times;

$$= (3.5 \times 3.5 \times 3.5) = 42.875$$

Now by converting it into fraction we get,

$$= \frac{42875}{1000}$$

$$= \frac{343}{8}$$

(iii) To calculate the cube of $(0.8)^3$

We have to multiply the given number by its power;

$$= (0.8 \times 0.8 \times 0.8) = 0.512$$

Now by converting it into fraction we get,

$$= \frac{512}{1000}$$

$$= \frac{64}{125}$$

(iv) To calculate the cube of $(0.05)^3$

We have to multiply the given number by its power;

$$= (0.05 \times 0.05 \times 0.05) = 0.000125$$

Now by converting it into fraction we get,

$$= \frac{125}{1000000}$$

$$= \frac{1}{8000}$$

3. Question

Evaluate:

$$(i) \left(\frac{4}{7}\right)^3 \quad (ii) \left(\frac{10}{11}\right)^3$$

$$(iii) \left(\frac{1}{15}\right)^3 \quad (iv) \left(1\frac{3}{10}\right)^3$$

Answer

$$(i) \left(\frac{4}{7}\right)^3$$

By multiplying we get,

$$= \left(\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7}\right)$$

$$= \left(\frac{64}{343}\right)$$

So, cube of $\frac{4}{7}$ is $\left(\frac{64}{343}\right)$

$$(ii) \left(\frac{10}{11}\right)^3$$

Multiplying the given number three times we get,

$$= \left(\frac{10}{11} \times \frac{10}{11} \times \frac{10}{11}\right)$$

$$= \left(\frac{1000}{1331}\right)$$

$$(iii) \left(\frac{1}{15}\right)^3$$

Multiplying the given number three times we get,

$$= \left(\frac{1}{15} \times \frac{1}{15} \times \frac{1}{15}\right)$$

$$= \left(\frac{1}{3375}\right)$$

$$(iv) \left(1\frac{3}{10}\right)^3$$

Multiplying the given number three times we get,

$$= \left(\frac{13}{10}\right)^3$$

$$= \left(\frac{13}{10} \times \frac{13}{10} \times \frac{13}{10}\right)$$

$$= \left(\frac{2197}{1000}\right)$$

4. Question

Which of the following numbers are perfect cubes? In case of perfect cube, find the number whose cube is the given number.

(i) 125 (ii) 243

(iii) 343 (iv) 256

(v) 8000 (vi) 9261

(vii) 324 (viii) 3375

Answer

(i) 125

First find out the prime factors of 125,

5	125
5	25
5	5
1	

$$125 = 5 \times 5 \times 5$$

As we see a group of three 5 is made, which we can also be write as 5^3 ;

So, 125 is the product of triplets of 5.

Therefore, it is the perfect cube.

(ii) 243

The prime factorization of 256 is shown below:

3	243
3	81
3	27
3	9
3	3
1	

$$243 = 3 \times 3 \times 3 \times 3 \times 3$$

To be a perfect cube the prime factors of number should make a group of 3 but as we can see here more than 3 numbers are available in prime factors.

So, 243 is not the perfect cube.

(iii) 343

The prime factorization of 343 is shown below:

$$\begin{array}{r|l} 7 & 343 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$343 = 7 \times 7 \times 7$$

As we see a group of three 7 is formed, which we can also be write as 7^3 ;

So, 343 is the product of triplets of 7.

Therefore, it is the perfect cube.

(iv) 256

The prime factorization of 256 is shown below:

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

If the prime factors are not making the pairs of three so the number is not perfect cube.

(v) 8000

The prime factorization of 8000 is shown below:

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$$

As we can see three pairs can be made of the above prime factors, which are 2^3 , 2^3 , and 5^3 .

So, 8000 can be expressed as the product of the triplets of 2, 2 and 5, i.e.

$$2^3 \times 2^3 \times 5^3 = 20^3$$

Therefore, 8000 is a perfect cube.

(vi) 9261

The prime factorization of 9261 is shown below:

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$$9261 = 3 \times 3 \times 3 \times 7 \times 7 \times 7$$

As we can see two pairs can be made of the above prime factors, which are 3^3 , and 7^3 .

So, 9261 can be expressed as the product of the triplets of 3 and 7, i.e.

$$3^3 \times 7^3 = 21^3$$

Therefore, 9261 is a perfect cube.

(vii) 5324

The prime factorization of 5324 is shown below:

2	5324
2	2662
11	1331
11	121
11	11
	1

$$5324 = 2 \times 2 \times 11 \times 11 \times 11$$

Therefore, 5324 is not a perfect cube.

(viii) 3375

The prime factorization of 3375 is shown below:

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

As we can see two pairs can be made of the above prime factors, which are 3^3 , and 5^3 .

So, 3375 can be expressed as the product of the triplets of 3 and 5, i.e.

$$3^3 \times 5^3 = 15^3$$

Therefore, 3375 is a perfect cube.

5. Question

Which of the following are the cubes of even numbers?

(i) 216 (ii) 729

(iii) 512 (iv) 3375

(v) 1000

Answer

By the rule for even numbers, the cubes of even numbers are always even.

So, first we have to look for which given numbers are even.

216, 512 and 1000 are the even numbers.

Now, the prime factorization are as follows:

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$\therefore 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = 6^3$$

2	1000
2	500
2	250
5	125
5	25
5	5
1	

$$\therefore 1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3 = 10^3$$

Therefore, we can say that 216, 512 and 1000 are the cube of even numbers.

6. Question

Which of the following are the cubes of odd numbers?

(i) 125 (ii) 343

(iii) 1728 (iv) 4096

(v) 9261

Answer

By the rule for odd numbers, the cubes of odd numbers are always odd.

So, first we have to look for which of the given numbers are odd.

125, 343 and 9261 are the odd numbers.

5	125
5	25
5	5
1	

$$\therefore 125 = 5 \times 5 \times 5 = 5^3$$

7	343
7	49
7	7
1	

$$\therefore 343 = 7 \times 7 \times 7 = 7^3$$

3	9261
3	3087
3	1029
7	343
7	49
7	7
1	

$$\therefore 9261 = 3 \times 3 \times 3 \times 7 \times 7 \times 7 = 3^3 \times 7^3 = 21^3$$

As we can see, odd numbers are the cubes of odd number.

Therefore, 125, 343 and 9261 are the cubes of odd numbers.

7. Question

Find the smallest number by which 1323 must be multiplied so that the product is a perfect cube.

Answer

Let's find out the prime factors of the given number,

$$\begin{array}{r|l}
 3 & 1323 \\
 \hline
 3 & 441 \\
 \hline
 3 & 147 \\
 \hline
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

$$1323 = 3 \times 3 \times 3 \times 7 \times 7$$

As we can see, one 7 is required to make the pair of two triplets. So, 7 will be the smallest number to multiply 1323 to make it the perfect cube.

8. Question

Find the smallest number by which 2560 must be multiples so that the product is a perfect cube.

Answer

Let's find out the prime factors of the given number,

2	2560
2	1280
2	640
2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

$$\therefore 2560 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$$

As we can see, to make the pair of 4 triplets two 5 are required, which is 5×5 .

So, 25 will be the number multiplied to 2560, to get the perfect cube.

9. Question

What is the smallest number by which 1600 must be divided so that the quotient is a perfect cube?

Answer

Let's find out the prime factors of the given number,

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

$$1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$$

So, in these two pairs of 2 triplets, two 5 are extra. Therefore to get the perfect cube we have to divide the given number by 5×5 , which is 25.

10. Question

Find the smallest number by which 8788 must be divided so that the quotient is a perfect cube.

Answer

Let's find out the prime factors of the given number,

2	8788
2	4394
13	2197
13	169
13	13
	1

$$8788 = 2 \times 2 \times 13 \times 13 \times 13$$

So, in this pair of triplets, two 2 are extra. Therefore to get the perfect cube we have to divide the given number by 2×2 , which is 4.

Exercise 4B

1. Question

Find the value of each of the following using the short-cut method:

$$(25)^3$$

Answer

Let's take, $a = 2$ and $b = 5$

So, by using the formula,

$$\text{We have } a^3 + 3a^2b + 3ab^2 + b^3$$

4	4	25	25
$\times 2$	$\times 15$	$\times 6$	$\times 5$
8	60	150	125
+7	+16	+12	
15	76	162	

Keep the digits

By taking the highlighted digits only,

We get;

$$15625 = {}^3(25) \square$$

2. Question

Find the value of each of the following using the short-cut method:

$$(47)^3$$

Answer

Let's take, $a = 4$ and $b = 7$

So, by using the formula,

$$\text{We have, } a^3 + 3a^2b + 3ab^2 + b^3$$

16	16	49	49
4	$\times 21$	$\times 12$	$\times 7$
64	336	588	343
+39	+62	+34	
103	398	622	

By taking the highlighted digits only,

We get;

$$103823 = {}^3(47)\square$$

3. Question

Find the value of each of the following using the short-cut method:

$$(68)^3$$

Answer

Let's take,

$$a = 6 \text{ and}$$

$$b = 8$$

So, by using the formula,

$$\text{We have } a^3 + 3a^2b + 3ab^2 + b^3$$

36	36	64	64
X 6	X 24	X 18	X 8
216	864	1152	512
+98	+120	+51	
314	984	1203	

By taking the highlighted digits only,

We get;

$$314432 = {}^3(68)\square$$

4. Question

Find the value of each of the following using the short-cut method:

$$(84)^3$$

Answer

Let's take,

$$a = 8 \text{ and}$$

$$b = 4$$

So, by using the formula,

$$\text{We have } a^3 + 3a^2b + 3ab^2 + b^3$$

64	64	16	16
X 8	X 12	X 24	X 4
512	768	384	64
+80	+39	+6	
592	807	390	

By taking the highlighted digits only,

We get;

$$592704 = {}^3(84)\square$$

Exercise 4C

1. Question

Evaluate:

$$\sqrt[3]{64}$$

Answer

By prime factorization method we get;

2	64
2	32
2	16
2	8
2	4
2	2
	1

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$

$$4 = (2 \times 2) = \sqrt[3]{(2)^3 \times (2)^3}$$

2. Question

Evaluate:

$$\sqrt[3]{343}$$

Answer

By prime factorization method we get;

$$\begin{array}{r|l}
 7 & 343 \\
 \hline
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

$$343 = 7 \times 7 \times 7$$

$$= (7 \times 7 \times 7)$$

$$\therefore \sqrt[3]{343} = \sqrt[3]{7^3} = 7$$

3. Question

Evaluate:

$$\sqrt[3]{729}$$

Answer

By prime factorization method we get;

$$\begin{array}{r|l}
 3 & 729 \\
 \hline
 3 & 243 \\
 \hline
 3 & 81 \\
 \hline
 3 & 27 \\
 \hline
 3 & 9 \\
 \hline
 3 & 3 \\
 \hline
 & 1
 \end{array}$$

$$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= (3 \times 3 \times 3) \times (3 \times 3 \times 3)$$

$$\therefore \sqrt[3]{729} = (3 \times 3) = 9$$

4. Question

Evaluate:

$$\sqrt[3]{1728}$$

Answer

By prime factorization method we get;

2	1728
<hr/>	
2	864
<hr/>	
2	432
<hr/>	
2	216
<hr/>	
2	108
<hr/>	
2	54
<hr/>	
3	27
<hr/>	
3	9
<hr/>	
3	3
<hr/>	
	1

$$\begin{aligned}
 1728 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\
 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \\
 &= 2^3 \times 2^3 \times 3^3
 \end{aligned}$$

$$\therefore \sqrt[3]{1728} = (2 \times 2 \times 3) = 12$$

5. Question

Evaluate:

$$\sqrt[3]{9261}$$

Answer

By prime factorization method we get;

$$\begin{array}{r|l}
 3 & 9261 \\
 \hline
 3 & 3087 \\
 \hline
 3 & 1029 \\
 \hline
 7 & 343 \\
 \hline
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

$$9261 = 3 \times 3 \times 3 \times 7 \times 7 \times 7$$

$$= (3 \times 3 \times 3) \times (7 \times 7 \times 7)$$

$$\therefore \sqrt[3]{9261} = (3 \times 7) = 21$$

6. Question

Evaluate:

$$\sqrt[3]{4096}$$

Answer

By prime factorization method we get;

2	4096
<hr/>	
2	2048
<hr/>	
2	1024
<hr/>	
2	512
<hr/>	
2	256
<hr/>	
2	128
<hr/>	
2	64
<hr/>	
2	32
<hr/>	
2	16
<hr/>	
2	8
<hr/>	
2	4
<hr/>	
2	2
<hr/>	
	1

$$\begin{aligned}
 4096 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\
 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \\
 &= 2^3 \times 2^3 \times 2^3 \times 2^3
 \end{aligned}$$

$$\therefore \sqrt[3]{4096} = (2 \times 2 \times 2 \times 2) = 16$$

7. Question

Evaluate:

$$\sqrt[3]{8000}$$

Answer

By prime factorization method we get;

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$$

$$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (5 \times 5 \times 5)$$

$$= 2^3 \times 2^3 \times 5^3$$

$$\therefore \sqrt[3]{8000} = (2 \times 2 \times 5) = 20$$

8. Question

Evaluate:

$$\sqrt[3]{3375}$$

Answer

By prime factorization method we get;

$$\begin{array}{r|l} 5 & 3375 \\ \hline 5 & 675 \\ \hline 5 & 135 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$= (3 \times 3 \times 3) \times (5 \times 5 \times 5)$$

$$\therefore \sqrt[3]{3375} = (3 \times 5) = 15$$

9. Question

Evaluate:

$$\sqrt[3]{-216}$$

Answer

By prime factorization method we get;

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$= (2 \times 2 \times 2) \times (3 \times 3 \times 3)$$

$$\sqrt[3]{-216} = -(2 \times 3) = -6$$

10. Question

Evaluate:

$$\sqrt[3]{-512}$$

Answer

By prime factorization method we get;

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$

$$\sqrt[3]{-512} = -\sqrt[3]{(2 \times 2 \times 2)} = -8$$

11. Question

Evaluate:

$$\sqrt[3]{-1331}$$

Answer

By prime factorization method we get;

11	1331
11	121
11	11
1	

$$1331 = 11 \times 11 \times 11 = 11^3$$

$$\sqrt[3]{-1331} = -\sqrt[3]{11 \times 11 \times 11} = -(11 \times 11 \times 11)^{\frac{1}{3}} = -11$$

12. Question

Evaluate:

$$\sqrt[3]{\frac{27}{64}}$$

Answer

By prime factorization method we get;

27;

3	27
3	9
3	3
1	

$$27 = 3 \times 3 \times 3$$

64;

2	64
2	32
2	16
2	8
2	4
2	2
	1

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

Now,

$$\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}}$$

$$= \frac{\sqrt[3]{(3 \times 3 \times 3)}}{\sqrt[3]{(2 \times 2 \times 2) \times (2 \times 2 \times 2)}}$$

$$= \frac{\sqrt[3]{(3 \times 3 \times 3)}}{\sqrt[3]{(4 \times 4 \times 4)}} = \frac{3}{4}$$

$$\therefore \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

13. Question

Evaluate:

$$\sqrt[3]{\frac{125}{216}}$$

Answer

By prime factorization method we get;

$$125$$

$$\begin{array}{r|l}
 5 & 125 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

216

$$\begin{array}{r|l}
 2 & 216 \\
 \hline
 2 & 108 \\
 \hline
 2 & 54 \\
 \hline
 3 & 27 \\
 \hline
 3 & 9 \\
 \hline
 3 & 3 \\
 \hline
 & 1
 \end{array}$$

$$\therefore \sqrt[3]{\frac{125}{216}} = \frac{\sqrt[3]{5 \times 5 \times 5}}{\sqrt[3]{(2 \times 2 \times 2) \times (3 \times 3 \times 3)}}$$

$$= \frac{\sqrt[3]{5 \times 5 \times 5}}{\sqrt[3]{6 \times 6 \times 6}} = \frac{5}{6}$$

$$\sqrt[3]{\frac{125}{216}} = \frac{5}{6}$$

14. Question

Evaluate:

$$\sqrt[3]{\frac{-27}{125}}$$

Answer

By prime factorization method we get;

$$\begin{array}{c|c} 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$27 = 3 \times 3 \times 3$$

$$125$$

$$\begin{array}{c|c} 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\sqrt[3]{\frac{27}{125}} = \sqrt[3]{\frac{3 \times 3 \times 3}{5 \times 5 \times 5}}$$

$$= \sqrt[3]{\frac{-27}{125}} = \frac{-3}{5}$$

15. Question

Evaluate:

$$\sqrt[3]{\frac{-64}{343}}$$

Answer

By prime factorization method we get;

$$\begin{array}{r|l}
 2 & 64 \\
 \hline
 2 & 32 \\
 \hline
 2 & 16 \\
 \hline
 2 & 8 \\
 \hline
 2 & 4 \\
 \hline
 2 & 2 \\
 \hline
 & 1
 \end{array}$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

343;

$$\begin{array}{r|l}
 7 & 343 \\
 \hline
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

$$\begin{aligned}
 \sqrt[3]{\frac{64}{343}} &= \sqrt[3]{\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{7 \times 7 \times 7}} \\
 &= \sqrt[3]{\frac{-64}{343}} = \frac{-4}{7}
 \end{aligned}$$

16. Question

Evaluate:

$$\sqrt[3]{64 \times 729}$$

Answer

By prime factorization method we get;

2	64
<hr/>	
2	32
<hr/>	
2	16
<hr/>	
2	8
<hr/>	
2	4
<hr/>	
2	2
<hr/>	
	1

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

3	729
<hr/>	
3	243
<hr/>	
3	81
<hr/>	
3	27
<hr/>	
3	9
<hr/>	
3	3
<hr/>	
	1

$$\begin{aligned}
 \sqrt[3]{64 \times 729} &= \sqrt[3]{64} \times \sqrt[3]{729} \\
 &= \sqrt[3]{4 \times 4 \times 4} \times \sqrt[3]{(3 \times 3 \times 3) \times (3 \times 3 \times 3)} \\
 &= \sqrt[3]{4 \times 4 \times 4} \times \sqrt[3]{9 \times 9 \times 9} \\
 &= \sqrt[3]{64 \times 729} = (4) \times (9) = 36
 \end{aligned}$$

17. Question

Evaluate:

$$\frac{\sqrt[3]{729}}{\sqrt[3]{1000}}$$

Answer

By prime factorization method we get;

3	729	2	1000
3	243	2	500
3	81	2	250
3	27	5	125
3	9	5	25
3	3	5	5
	1		1

$$\begin{aligned}\frac{\sqrt[3]{729}}{\sqrt{1000}} &= \frac{\sqrt[3]{(3 \times 3 \times 3) \times (3 \times 3 \times 3)}}{\sqrt[3]{(2 \times 2 \times 2) \times (5 \times 5 \times 5)}} \\ &= \frac{\sqrt[3]{9 \times 9 \times 9}}{\sqrt[3]{10 \times 10 \times 10}} \\ &= \frac{\sqrt[3]{729}}{\sqrt{1000}} = \frac{9}{10}\end{aligned}$$

18. Question

Evaluate:

$$\frac{\sqrt[3]{-512}}{\sqrt[3]{343}}$$

Answer

The prime factorization of 512:

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

The prime factorization of 343:

$$\begin{array}{r|l}
 7 & 343 \\
 \hline
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

$$\begin{aligned}
 \sqrt[3]{\frac{512}{343}} &= \frac{\sqrt[3]{8 \times 8 \times 8}}{\sqrt[3]{7 \times 7 \times 7}} \\
 &= \sqrt[3]{\frac{-512}{343}} = \frac{-8}{7}
 \end{aligned}$$

Exercise 4D

1. Question

Which of the following number is a perfect cube?

- A. 141
- B. 294
- C. 216
- D. 496

Answer

The prime factorization of 141:

$$\begin{array}{r|l}
 3 & 141 \\
 \hline
 47 & 47 \\
 \hline
 & 1
 \end{array}$$

$$= \sqrt[3]{141} = \sqrt[3]{3 \times 47}$$

The prime factorization of 294:

2	294
<hr/>	
3	147
<hr/>	
7	49
<hr/>	
7	7
<hr/>	
	1

$$= \sqrt[3]{294} = \sqrt[3]{2 \times 3 \times 7 \times 7}$$

The prime factorization of 216:

6	216
<hr/>	
6	36
<hr/>	
6	6
<hr/>	
	1

$$= \sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6}$$

The prime factorization of 496:

2	496
2	248
2	124
2	62
31	31
	1

$$= \sqrt[3]{496} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 31}$$

So, only in case of 216 a perfect pair of three 6's is formed, Hence 216 is a perfect cube.

2. Question

Which of the following numbers is a perfect cube?

- A. 1152
- B. 1331
- C. 2016
- D. 739

Answer

Having prime factors of all the numbers we get,

$$= \sqrt[3]{1152}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3}$$

$$= 4 \sqrt[3]{2 \times 3 \times 3} \text{ (not a perfect cube)}$$

11	1331
11	121
11	11
	1

$$= \sqrt[3]{1331}$$

$$= \sqrt[3]{11 \times 11 \times 11}$$

$$= 11 \text{ (perfect cube)}$$

$$= \sqrt[3]{2016}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7}$$

$$= 2\sqrt[3]{2 \times 2 \times 3 \times 3 \times 7}$$

(not a perfect cube)

$$= \sqrt[3]{739} = \text{no prime factors are possible}$$

Hence, only case of 1331 a perfect triplet pair of 11's is formed, Hence 1331 is a perfect cube.

3. Question

Choose the correct answer: $\sqrt[3]{512} = ?$

- A. 6
- B. 7
- C. 8
- D. 9

Answer

The prime factorization of 512 is :

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$\sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = 8.$$

4. Question

Choose the correct answer: $\sqrt[3]{125 \times 64} = ?$

- A. 100
- B. 40
- C. 20
- D. 30

Answer

The prime factorization of 125 is :

5	125
5	25
5	5
1	

$$125 = 5 \times 5 \times 5$$

The prime factorization of 64 is :

2	64
2	32
2	16
2	8
2	4
2	2
1	

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= \sqrt[3]{125 \times 64} = \sqrt[3]{5 \times 5 \times 5 \times 4 \times 4 \times 4} = \sqrt[3]{5 \times 5 \times 5} \times \sqrt[3]{4 \times 4 \times 4} = 5 \times 4 = 20.$$

5. Question

Choose the correct answer: $\sqrt[3]{\frac{64}{343}} = ?$

A. $\frac{4}{9}$

B. $\frac{4}{7}$

C. $\frac{8}{7}$

D. $\frac{8}{21}$

Answer

2	64
<hr/>	
2	32
<hr/>	
2	16
<hr/>	
2	8
<hr/>	
2	4
<hr/>	
2	2
<hr/>	
	1

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

7	343
7	49
7	7
1	

$$343 = 7 \times 7 \times 7$$

$$= \sqrt[3]{\frac{64}{343}} = \sqrt[3]{\frac{4 \times 4 \times 4}{7 \times 7 \times 7}} = \frac{\sqrt[3]{4 \times 4 \times 4}}{\sqrt[3]{7 \times 7 \times 7}} = \frac{4}{7}$$

6. Question

Choose the correct answer: $\sqrt[3]{\frac{-512}{729}} = ?$

A. $\frac{-7}{9}$

B. $\frac{-8}{9}$

C. $\frac{7}{9}$

D. $\frac{8}{9}$

Answer

The prime factorization of 512 is :

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

The prime factorization of 729 is :

$$\begin{array}{r|l}
 3 & 729 \\
 \hline
 3 & 243 \\
 \hline
 3 & 81 \\
 \hline
 3 & 27 \\
 \hline
 3 & 9 \\
 \hline
 3 & 3 \\
 \hline
 & 1
 \end{array}$$

$$= \sqrt[3]{-\frac{512}{729}} = \sqrt[3]{\frac{-(8 \times 8 \times 8)}{9 \times 9 \times 9}} = \frac{\sqrt[3]{-(8 \times 8 \times 8)}}{\sqrt[3]{9 \times 9 \times 9}} = -\frac{8}{9}$$

7. Question

By what least number should 648 be multiplied to get a perfect cube?

- A. 3
- B. 6
- C. 9
- D. 8

Answer

The prime factorization of 648 is :

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$$648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 3$$

So, we can see that to make it a perfect cube we should complete the pair of three 3's .

Hence, it will be multiplied by $= 3 \times 3 = 9$.

8. Question

By what least number should 1536 be divided to get a perfect cube?

- A. 3
- B. 4
- C. 6
- D. 8

Answer

The prime factorization of 1536 is :

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
1	

$$1536 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$$

So, we can see that to make it a perfect cube we should eliminate one 3 from it.

Hence, it will be divided by = 3.

9. Question

Choose the correct answer: $\left(1\frac{3}{10}\right)^3 = ?$

A. $1\frac{27}{1000}$

B. $2\frac{27}{1000}$

C. $2\frac{197}{1000}$

D. none of these

Answer

We have: $\left(1\frac{3}{10}\right)^3$

$$= \left(\frac{13}{10}\right)^3$$

$$= \frac{2197}{1000}$$

$$= 2\frac{197}{1000}$$

10s. Question

Choose the correct answer: $(0.8)^3 = ?$

A. 51.2

B. 5.12

C. 0.512

D. none of these

Answer

We have:

$$(0.8)^3 = 0.8 \times 0.8 \times 0.8 = 0.512.$$

CCE Test Paper-4

1. Question

Evaluate $\left(1\frac{2}{5}\right)^3$.

Answer

$$\left(1\frac{2}{5}\right)^3 = \left(\frac{7}{5}\right)^3 = \frac{7 \times 7 \times 7}{5 \times 5 \times 5} = \frac{343}{125}.$$

2. Question

Evaluate $\sqrt[3]{4096}$.

Answer

The prime factorization of 4096 is:

$$= 2 \times 2 \times 2 \times 2$$

$$= 1$$

3. Question

Evaluate $\sqrt[3]{216 \times 343}$.

Answer

The prime factorization of 216 is :

2	216
2	108
2	54
3	27
3	9
3	3
	1

The prime factorization of 343 is:

7	343
7	49
7	7
	1

$$343 = 7 \times 7 \times 7$$

$$= \sqrt[3]{216 \times 343}$$

$$= \sqrt[3]{216} \times \sqrt[3]{343}$$

$$= \sqrt[3]{6 \times 6 \times 6} \times \sqrt[3]{7 \times 7 \times 7}$$

$$= 6 \times 7$$

$$= 42$$

4. Question

Evaluate $\sqrt[3]{\frac{-64}{125}}$.

Answer

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

125,

5	125
5	25
5	5
1	

$$= \sqrt[3]{-\frac{64}{125}} = \sqrt[3]{-\frac{(4 \times 4 \times 4)}{5 \times 5 \times 5}} = \frac{\sqrt[3]{-(4 \times 4 \times 4)}}{\sqrt[3]{5 \times 5 \times 5}} = -\frac{4}{5}$$

5. Question

Choose the correct answer: $\left(1\frac{3}{4}\right)^3 = ?$

A. $1\frac{27}{64}$

B. $2\frac{27}{64}$

C. $5\frac{23}{64}$

D. none of these

Answer

$$= \left(1\frac{3}{4}\right)^3$$

$$= \left(\frac{7}{4}\right)^3$$

$$= \frac{7 \times 7 \times 7}{4 \times 4 \times 4}$$

$$= \frac{343}{64}$$

$$= 5\frac{23}{64}$$

6. Question

Which of the following numbers is a perfect cube?

A. 121

B. 169

C. 196

D. 216

Answer

From the prime factorization of 121, we get,

11	121
<hr/>	
11	11
<hr/>	
1	

$$121 = 11 \times 11$$

13	169
<hr/>	
13	13
<hr/>	
1	

$$169 = 13 \times 13$$

2	196
<hr/>	
2	98
<hr/>	
7	49
<hr/>	
7	7
<hr/>	
1	

$$196 = 2 \times 2 \times 7 \times 7 = 14 \times 14$$

2	216
<hr/>	
2	108
<hr/>	
2	54
<hr/>	
3	27
<hr/>	
3	9
<hr/>	
3	3
<hr/>	
	1

$$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} = 2 \times 3 = 6$$

Hence, in case of 216 perfect triplet pairs of 2's and 3's is formed, so 216 is a perfect cube.

7. Question

Choose the correct answer: $\sqrt[3]{216 \times 64} = ?$

- A. 64
- B. 32
- C. 24
- D. 36

Answer

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= \sqrt[3]{216 \times 64}$$

$$= \sqrt[3]{216} \times \sqrt[3]{64}$$

$$= \sqrt[3]{6 \times 6 \times 6} \times \sqrt[3]{4 \times 4 \times 4}$$

$$= 6 \times 4 = 24.$$

8. Question

Choose the correct answer: $\sqrt[3]{\frac{-343}{729}} = ?$

A. $\frac{7}{9}$

B. $\frac{-7}{9}$

C. $\frac{-9}{7}$

D. $\frac{9}{7}$

Answer

$$\begin{array}{r|l}
 7 & 343 \\
 \hline
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

$$343 = 7 \times 7 \times 7$$

$$\begin{array}{r|l}
 3 & 729 \\
 \hline
 3 & 243 \\
 \hline
 3 & 81 \\
 \hline
 3 & 27 \\
 \hline
 3 & 9 \\
 \hline
 3 & 3 \\
 \hline
 & 1
 \end{array}$$

$$= \sqrt[3]{\frac{343}{729}}$$

$$= \frac{\sqrt[3]{-(343)}}{\sqrt[3]{729}}$$

$$= \frac{\sqrt[3]{-(7 \times 7 \times 7)}}{\sqrt[3]{9 \times 9 \times 9}}$$

$$= -\frac{7}{9}$$

9. Question

By what least number should 324 be multiplied to get a perfect cube?

- A. 12
- B. 14
- C. 16
- D. 18

Answer

From the prime factorization of 324 we get,

$$\begin{array}{r|l} 2 & 324 \\ \hline 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$= 324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 = \underline{3 \times 3 \times 3} \times 2 \times 2 \times 3$$

So to make it a perfect cube we must complete the triplet pairs of 2's and 3's.

Hence, it will be multiplied by $2 \times 3 \times 3 = 18$.

10. Question

Choose the correct answer: $\sqrt[3]{\frac{128}{250}} = ?$

- A. $\frac{3}{5}$
- B. $\frac{4}{5}$

C. $\frac{2}{5}$

D. none of these

Answer

From the prime factorization of 128, we get,

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

From the prime factorization of 250, we get,

$$\begin{array}{r|l}
 2 & 250 \\
 \hline
 5 & 125 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

$$= \sqrt[3]{\frac{128}{250}}$$

$$= \frac{\sqrt[3]{128}}{\sqrt[3]{250}}$$

$$= \frac{\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}}{\sqrt[3]{2 \times 5 \times 5 \times 5}}$$

$$= \frac{4\sqrt[3]{2}}{5\sqrt[3]{2}}$$

$$= \frac{4}{5}$$

51. Question

Which of the following is a cube of an odd number?

- A. 216
- B. 512
- C. 343
- D. 1000

Answer

From the prime factorization of 216, we get,

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

From the prime factorization of 512, we get,

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

From the prime factorization of 343, we get,

7	343
<hr/>	
7	49
<hr/>	
7	7
<hr/>	
	1

$$343 = 7 \times 7 \times 7$$

From the prime factorization of 1000, we get,

2	1000
<hr/>	
2	500
<hr/>	
2	250
<hr/>	
5	125
<hr/>	
5	25
<hr/>	
5	5
<hr/>	
	1

$$216 = 6 \times 6 \times 6$$

$$512 = 8 \times 8 \times 8$$

$$343 = 7 \times 7 \times 7$$

$$1000 = 10 \times 10 \times 10$$

Hence, we can see clearly that 343 is a cube of an odd number.

5. Question

Fill in the blanks.

$$(i) = \sqrt[3]{ab} = (\sqrt[3]{a}) \times (\text{_____})$$

$$(ii) \sqrt[3]{\frac{a}{b}} = \text{_____}$$

$$(iii) \sqrt[3]{-x} = \text{_____}$$

$$(iv) (0.5)^3 = \text{_____}$$

Answer

$$(i) = \sqrt[3]{ab} = (\sqrt[3]{a}) \times (\sqrt[3]{b})$$

$$(ii) = \sqrt[3]{\frac{a}{b}}$$

$$= \frac{\sqrt[3]{a}}{\sqrt[3]{b}}.$$

$$(iii) = \sqrt[3]{-x} = -\sqrt[3]{x}.$$

$$(iv) = 0.5^3$$

$$= 0.5 \times 0.5 \times 0.5$$

$$= 0.125.$$