9. Integration

- Integration is the inverse process of differentiation. If $\frac{d}{dx}f(x) = g(x)$, then we can write $\int g(x)\,dx = f(x) + C$. This is called the general or the indefinite integral and C is called the constant of integration.
- Some standard indefinite integrals are given as follows:
  - $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
  - $\int dx = x + C$
  - $\int \sin x dx = -\cos x + C$
  - $\int \cos x dx = \sin x + C$
  - $\int \sec^2 x dx = \tan x + C$
  - $\int \operatorname{cosec}^2 x dx = -\cot x + C$
  - $\int \sec x \tan x dx = \sec x + C$
  - $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$
  - $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$ or $-\cos^{-1} x + C$
  - $\int \frac{dx}{\sqrt{1-x^2}} = \tan^{-1} x + C$ or $-\cot^{-1} x + C$
  - $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$ or $-\operatorname{cosec}^{-1} x + C$
  - $\int e^x dx = e^x + C$
  - $\int a^x dx = \frac{a^x}{\log a} + C$
  - $\int \frac{1}{x} dx = \log |x| + C$
  - $\int e^{ax} dx = \frac{e^{ax}}{a} + C$
- Properties of indefinite integrals:
  - $\frac{d}{dx}\int f(x)\,dx = f(x)$ and $\int f'(x)\,dx = f(x) + C$
  - If the derivative of two indefinite integrals is the same, then they belong to same family of curves and hence they are equivalent.
  - $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
  - $\int k f(x)\,dx = k\int f(x)\,dx$, where *k* is any constant