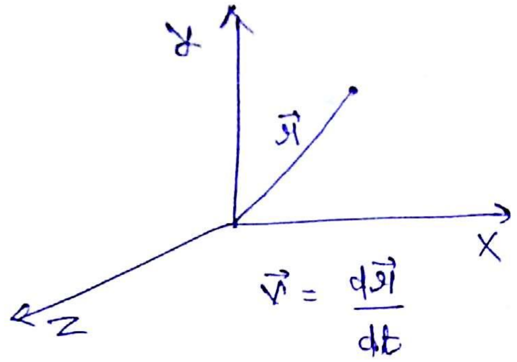
Dynamics

- kinematics ($\vec{r}, \vec{v}, \vec{a}$)
- kinetics ($\vec{r}, \vec{v}, \vec{a}$ & $\vec{F}$)



$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Newton's 2nd law :-

For a particle

if $\vec{F}_R \neq 0$ then $\vec{a} \neq 0$

$$\boxed{\vec{a} = \frac{\vec{F}_R}{m}}$$

$$\underbrace{\vec{F}_R}_{\text{Resultⁿ of force}} = \underbrace{m\vec{a}}_{\text{effect/response}} \quad \text{— 2nd law}$$

For a rigid body if $(\vec{F}_R)_{ext} \neq 0$ then $\vec{a}_{cm} \neq 0$

$$\vec{a}_{cm} = \frac{(\vec{F}_R)_{ext}}{m} \; ; \; \boxed{\underbrace{(\vec{F}_R)_{ext}}_{\text{Rate of actual exte force}} = \underbrace{m\vec{a}_{cm}}_{\text{Response/effect}}} \text{ — 2nd law}$$

Rectilinear translation :—

$$(ds)_1 = (ds)_2 = (ds)_3$$ → during time dt

$$\vec{V}_1 = \vec{V}_2 = \vec{V}_3 = \vec{V}$$

$$\vec{a}_1 = \vec{a}_2 = \vec{a}_3 = \vec{a}$$

relative velocity

$$\left.\begin{aligned} \vec{V}_{1/2} &= \vec{V}_1 - \vec{V}_2 = 0 \\ \vec{a}_{1/2} &= \vec{a}_1 - \vec{a}_2 = 0 \end{aligned}\right\} \text{Rest.}$$

if $\vec{a}$ is uniform

i) $v = u + at$

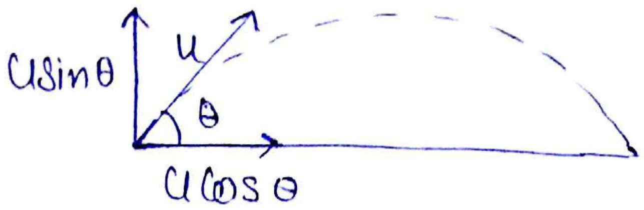
ii) $s = ut + \frac{1}{2}at^2$

iii) $v^2 = u^2 + 2as$

Ex. Freely falling bodies

$u = 0 \; ; \; a = g(-\hat{i})$

Ex – Projectile Motion



$a_x = 0$

$a_y = -g$

P. 2.16
Pg 73
(Gate 2017)

$u = 40 \text{ m/s}$ initial velocity

$a = -0.1 v$

velocity after 3 sec.

Sol^n

$$a = -0.1 v$$

$$\frac{dv}{dt} = -0.1 v$$

$$\int_{40}^{v} \frac{dv}{v} = -0.1 \int_{0}^{3} dt$$

$$\Rightarrow -0.3 = \ln v_f - \ln 40$$

$$v_f = 29.63 \text{ m/s.}$$

Ques: if $s = \frac{t^3}{3} - 36t$ then find

i) a at t = 4 sec.

ii) when does the particle reverse it's dirn and what will be it's accn at that instant.

Soln (i) $S = \frac{t^3}{3} - 36t$

$$v = \frac{ds}{dt} = \frac{\cancel{3}t^2}{\cancel{3}} - 36$$

$$a = \frac{dv}{dt} = 2t$$

$$a|_{t=4} = 2 \times 4 = 8 \text{ m/sec}^2$$

(ii)

$$v = \frac{ds}{dt} = t^2 - 36$$

when particle reverse it's dirn its velocity will be zero

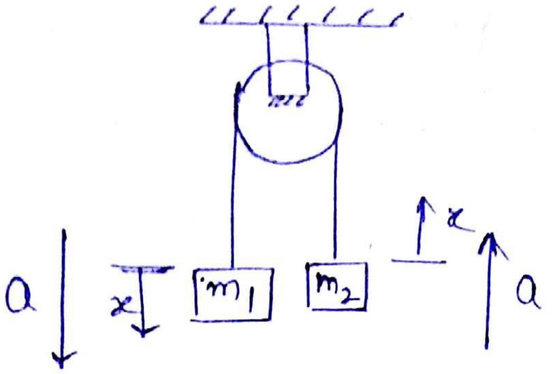
$$t^2 - 36 = 0 \Rightarrow t = 6 \text{ sec.}$$

$$a = 2t$$

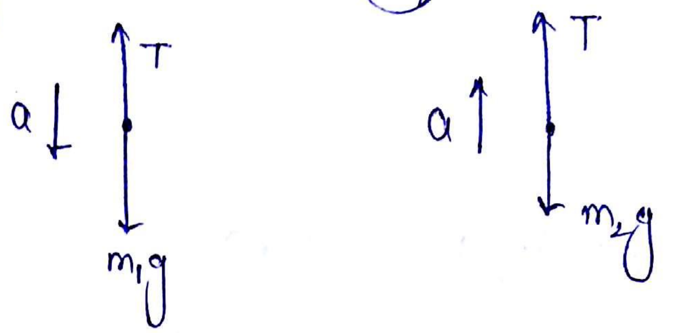
$$a|_{t=6\,se} = 2 \times 6 = 12 \text{ m/sec}^2$$

kinetics of rectilinear translation !–

<u>Case-1</u>



m_1 m_2

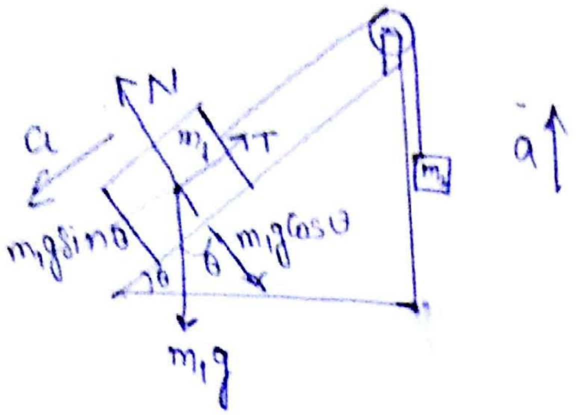


$$\underbrace{m_1 g - T = m_1 a \qquad T - m_2 g = m_2 a}_{\text{effect}} \qquad \text{— 2nd law}$$

$$m_1 g - m_2(a+g) = m_1 a$$

$$m_1 g - m_1 a - m_1 g = m_1 a$$

Case - 2

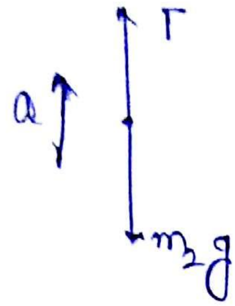


(m_1)

$$\underbrace{N}_{\text{force}} = \underbrace{m_1 g\cos\theta}_{\text{force}} \quad \text{— 1st law}$$

$$\underbrace{m_1 g\sin\theta - T}_{\text{Resultant of force}} = \underbrace{m_1 a}_{\text{effect}} \quad \text{— 2nd law}$$

(m_2)



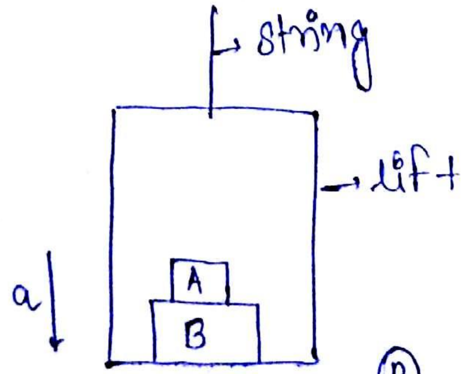
$$T - m_2 g = m_2 a \quad \text{— 2nd law}$$

$$m_1 g\sin\theta - m_2 g - m_2 a = m_1 a$$

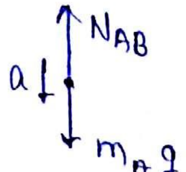
$$a = \frac{m_1 g\sin\theta - m_2 g}{(m_1 + m_2)} \quad ; \; T =$$

Case-3

$a < g$



(A)



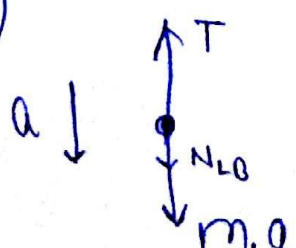
$$m_A g - N_{AB} = m_A a \quad \text{— 2nd law}$$

(B)

$$m_B g + N_{BA} - N_{BL} = m_B a \quad \text{— 2nd law}$$

$$N_{AB} = N_{BA} \rightarrow \text{3rd law}$$

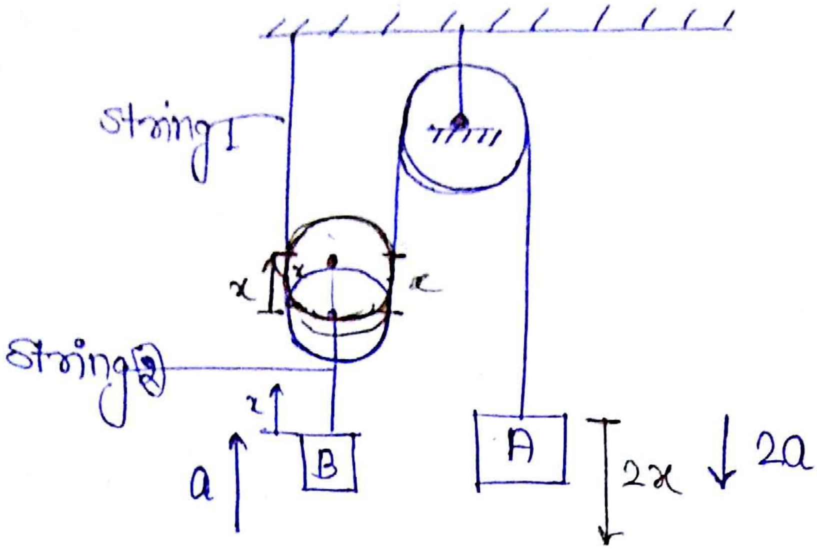
(Lift)

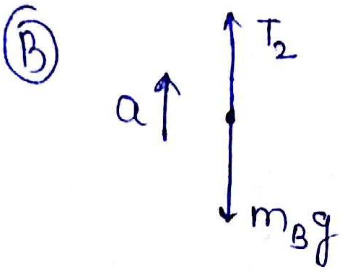


$$m_L g + N_{LB} - T = m_L a \quad \text{— 2nd law}$$

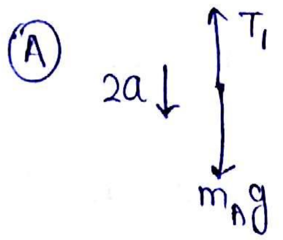
$$N_{BL} = N_{LB} \quad \text{— 3rd law}$$

Case- 4



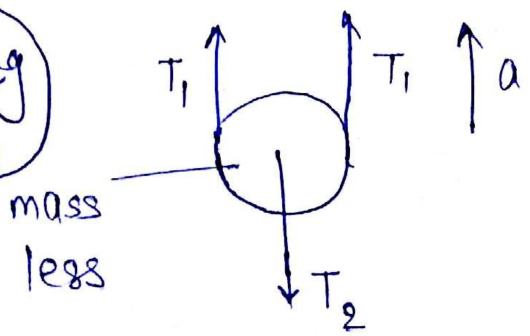


$T_2 - m_B g = m_B a$ - 2nd law - ①



$m_A g - T_1 = m_A (2a)$ - 2nd law - ②

moving Pulley



$2T_1 - T_2 = (0)\,a$ - 2nd law

$2T_1 = T_2$ - ③

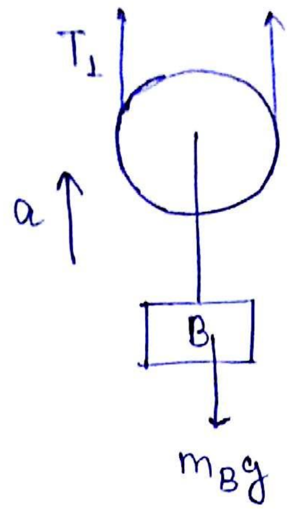
From eq ① & ③

$2T_1 - m_B g = m_B a$

$m_A g - T_1 = m_A (2a)$

moving pully + string 2 + B → Becoz $\vec{a}$ is same for all

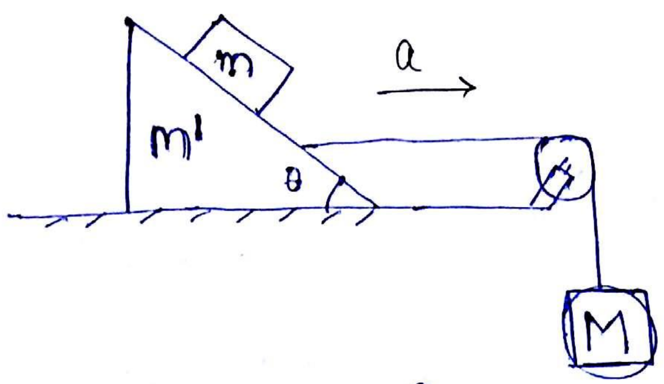
A Single rigid body



$$2T_1 - m_B g = m_B a$$

– 2nd law

Ques:- Find the mass M of the hanging clock as shown in fig. So as to prevent the sliping of smaller block over the triangulated block.



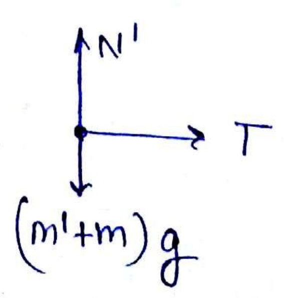
* All the surfaces are smooth

Solⁿ

(M)

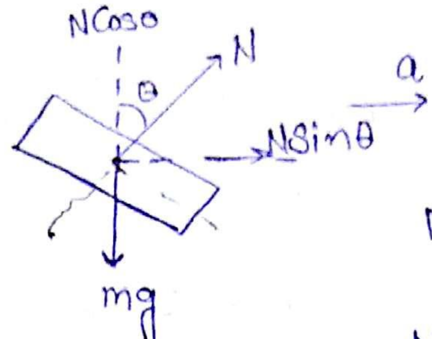
$Mg - T = Ma$ – 2nd law – (1)

(m'+m)



$T = (m'+m)\,a$ – 2nd law – (2)

(m)



$N\cos\theta = mg$ — 1st law

$N\sin\theta = ma$ — 2nd law

$g\tan\theta = a$

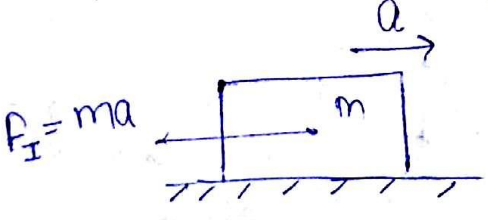
$$Mg - T = Ma$$

$$Mg - (m'+m)g\tan\theta = Mg\tan\theta$$

$$M = \frac{m'+m}{\cot\theta - 1}$$

D'Alembert's Principle :—

It states under the action of effective force and inertia force body will be in dynamic equ^m.



$(\vec{F}_R) = m\vec{a}$ — 2nd law

$(\vec{F}_R) + (\vec{F}_I) = 0$

Applied only when
acce^n ✓
velo. ✗

$\vec{F}_R$: effective force (Resultant of actual force)

$\vec{F}_I$: Inertia force (Pseudo/Imaginary force)

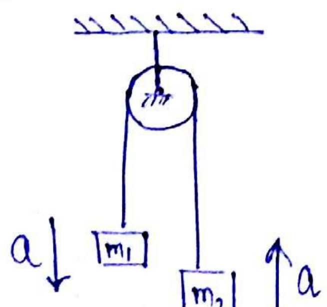
If a body mass m moves rightward with an acc^n a then it means there is a resultant force $F_R = ma$ acting rightward.

Now if you apply a force $F_I = ma$ acting leftwards at the CG of the body then it will bring the body in equ^m called dynamic equ^m.

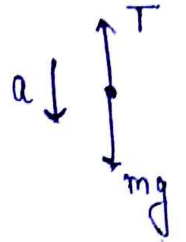
Note – i) F_I is an imaginary force

ii) Inertia force acts opposite to acc^n but not opposite to the motion.

Case 1

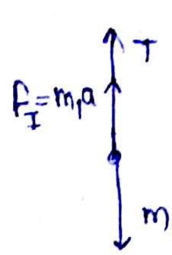


m_1



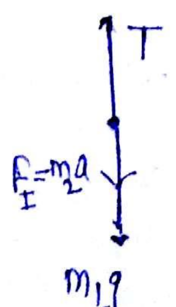
$$\underbrace{m_1 g - T = m_1 a}_{\text{effect}} \quad - 2^{nd}\ \text{law}$$

m_1 D' Ambert



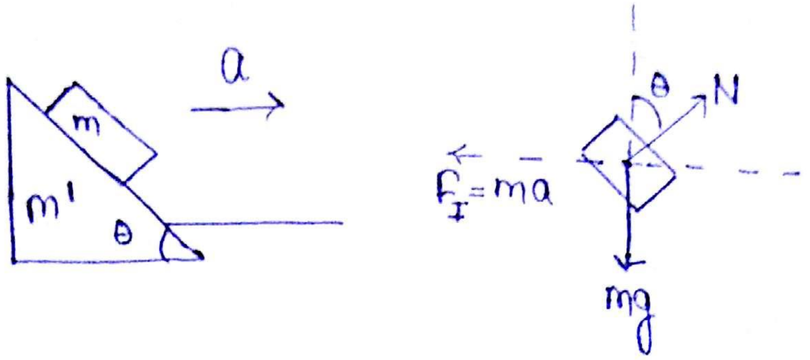
$$T + \underbrace{m_1 a}_{\text{force}} = m_1 g \quad - \text{D'Ambert}$$

m_2



$$T = m_2 a + m_2 g \quad - \text{D'Ambert}$$

Case-2



$N\cos\theta = mg$ - 1st law

$N\sin\theta = \underbrace{ma}_{\text{In. Force.}}$ D'Ambrt

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* if $a = F(s)$

$a = \frac{dv}{dt}$; $v = \frac{ds}{dt}$

$dt = \frac{dv}{a}$; $dt = \frac{ds}{v}$

$$\boxed{v\,dv = a\,ds}$$

Ques if $a = -8s^{-2}$, velocity of the particle at $s = 16$ m is.

$a = -8s^{-2}$

$\int v\,dv = \int -8s^{-2}\,ds$

$\frac{v^2}{2} = -8\left(\frac{s^{-1}}{-1}\right)$

$$V^2 = \frac{16}{S}$$

$$V = \sqrt{\frac{16}{S}}$$

at $S = 16\,m$

$$V = \sqrt{16/16} = 1\ m/s$$