CBSE Test Paper 02

CH-12 Three Dimensional Geometry

1. The direction cosines of the line joining (1 , - 1 , 1) , and (-1 , 1 , 1) are
   a. $<$ 2 , - 2 , 0 $>$
   b. $<$ 1 , - 1 , 1 $>$
   c. $< \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 >$
   d. $<$ 1 , - 1 , 0 $>$
2. The line x = 1 , y = 2 is
   a. parallel to Z – axs
   b. lies in a plane parallel to XY – plane
   c. parallel to X – axs
   d. parallel to Y – axs
3. The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12)
   a. $cos^{-1}(\frac{2}{3})$
   b. $tan^{-1}(-\frac{2}{3})$
   c. none of these
   d. $cos^{-1}(\frac{3}{2})$
4. Perpendicular distance of the point (3,4,5) from the y-axis is,
   a. $\sqrt{34}$
   b. 4
   c. $\sqrt{41}$
   d. 5
5. The points A (0 , 0 , 0) , B (1 , $\sqrt{3}$, 0) , C (2 , 0 , 0) and D (1 , 0 , $\sqrt{3}$) are the vertices of
   a. none of these
   b. parallelogram
   c. square
   d. rhombus
6. Fill in the blanks:

   If the mid-points of the sides of a triangle AB;BC;CA are D(1, 2, -3), E(3, 0, 1) and F(-1, 1,

-4), then the centroid of the triangle ABC is ________.

7. Fill in the blanks:

   If the point P lies on z-axis, then coordinates of P are of the form ________.

8. A point is on the x-axis. What are its y-coordinates and z-coordinates?

9. If a parallelopiped is formed by planes drawn through the points (5,8,10) and (3,6,8) parallel to the coordinate planes, then find the length of diagonal of the parallelopiped.

10. The mid-points of the sides of a triangle ABC are given by (- 2,3,5), (4, -1, 7) and (6,5,3). Find the coordinates of A, B and C.

11. Find the ratio in which the line joining (2,4, 5) and (3,5,4) is divided by the yz - plane.

12. Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, -8) is divided by the YZ-plane.

13. Show that the points A (1, 2, 3), B(-1, -2, -1), C(2, 3, 2) and D(4, 7, 6) are the vertices of a parallelogram ABCD, but it is not a rectangle.

14. Find the equation of the set of points which are equidistance from the points (1, 2, 3) and (3, 2, -1).

15. Prove that the point A (1,3,0), B (- 5,5,2), C (- 9, -1,2) and D (- 3, - 3,0) taken in order are the vertices of a parallelogram. Also, show that ABCD is not a rectangle.

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Solution

1. (c)$< \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 >$

   Explanation:

   The direction ratio of the line joining (x1 , y1 , z1) , and (x2 , y2 , z2) = < x1-x2 , y1-y2 , z1-z2 >

   The direction ratio of the line joining (1 , - 1 , 1) , and (-1 , 1 , 1) = < 1+1 , -1-1 , 1-1> = < 2 , -2 , 0 >

   The direction cosines of the line = <
   $\frac{2}{\sqrt{(-2)^2+(2)^2+(0)^2}}, \frac{-2}{\sqrt{(-2)^2+(2)^2+(0)^2}}, \frac{0}{\sqrt{(-2)^2+(2)^2+(0)^2}}$ > = $< \frac{2}{\sqrt{8}}, \frac{-2}{\sqrt{8}}, \frac{0}{\sqrt{8}} >$ =
   $< \frac{2}{2\sqrt{2}}, \frac{-2}{2\sqrt{2}}, \frac{0}{2\sqrt{2}} > = < \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0 >$

2. (a) parallel to Z – axs

   Explanation: Since z co-ordinate is zero it is parallel to Z axis

   (b) lies in a plane parallel to XY – plane

3. (a) $cos^{-1}(\frac{2}{3})$

   Explanation:

   The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12)

   Direction ratios of the line joining the points A(3, 1, 4), B(7, 2, 12) is <x2-x1 , y2-y1, z2-z1> = < 7-3 , 2-1 , 12-4> = <4 ,1 ,8>

   Now as the angle between two lines having direction ratios <a1,b1,c1> and <a2,b2,c2> is given by

   $\text{Cos}^{-1}\frac{a1a2+b1b2+c1c2}{\sqrt{a1^2+b1^2+c1^2}\sqrt{a2^2+b2^2+c2^2}}$

   Using the vuales we have

   $\cos^{-1}\frac{2\times4+2\times1+1\times8}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}} = \cos^{-1}\frac{18}{27} = \cos^{-1}\frac{2}{3}$

4. (a) $\sqrt{34}$

Explanation:

Distance of (α, β, γ) from y-axis is given by

$\therefore$ Distance(d) of (3,4,5) from y- axis is

d=$\sqrt{3^2+5^2}=\sqrt{9+25}=\sqrt{34}$

5. (a) none of these

   Explanation:

   Direction ratios of AB are $(1-0, \sqrt{3}-0, 0-0)$ i.e $(1, \sqrt{3}, 0)$

   Direction ratios of BC are$(1-0, \sqrt{3}-0, 0-0)$ i.e $(1, -\sqrt{3}, 0)$

   Direction ratios of CD are $(-1-0, 0-0, \sqrt{3}-0)$ i.e $(-1, 0, \sqrt{3})$

   Direction ratios of CD are $(-1-0, 0-0, -\sqrt{3}-0)$ i.e $(-1, 0, \sqrt{3})$

   If ABCD is a parallelogram, then

   $\mathrm{AB} \| \mathrm{CD}$ and $\mathrm{AD} \| \mathrm{BC}$

   Now,$\frac{1}{-1} \neq \frac{\sqrt{3}}{0} \neq \frac{0}{\sqrt{3}}\left[\because \frac{\mathbf{a}_1}{\mathbf{a}_2} \neq \frac{\mathbf{b1}}{\mathbf{b}_2} \neq \frac{\mathbf{c}_1}{\mathbf{c}_2}\right]$

   $\therefore$ AB is not parallel to CD

   Similarly $\frac{-1}{1} \neq \frac{0}{\sqrt{-3}} \neq \frac{-\sqrt{3}}{0}$

   $\Rightarrow$ AD and BC are not parallel

   $\therefore$ ABCD is not a parallelogram and hence it is not a square or rhombus

6. (1, 1, -2)

7. (0, 0, z)

8. We know that coordinates of any point on the x-axis will be (x, 0, 0) . Thus y-coordinate and z-coordinate of the point are zero.

9. Given points are (5, 8,10) and (3, 6, 8).

   $\therefore$ Length of diagonal $=\sqrt{(3-5)^2+(6-8)^2+(8-10)^2}$

   [$\because$ distance = $\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$]

   $=\sqrt{4+4+4}=2\sqrt{3}$

10. Given midpoints D(−2,3,5), E(4,−1,7) and F(6,5,3)

    Assume D is midpoint of AB, E is midpoint of BC

    F is midpoint of CA

    A(x_1, y_1, z_1) B(x_2, y_2, z_2) C(x_3, y_3, z_3)

From midpoint formula, we get following equations

$x_1 + x_2 = -4$; $x_2 + x_3 = 8$; $x_3 + x_1 = 12$

$y_1 + y_2 = 6$; $y_2 + y_3 = -2$; $y_3 + y_1 = 10$

$z_1 + z_2 = 10$; $z_2 + z_3 = 14$; $z_3 + z_1 = 6$

Solving above set of equations we get

A = (0, 9, 1)

B = (-4, -3, 9)

C = (12, 1, 5)

11. Given points are (2,4,5) and (3,5,4)

    In YZ plane, x = 0

    Assume the point P divides the line joining the given points in the ratio m:n. So, lets equate x-term of point P equal to zero. Therefore,

    $0 = \frac{3m+2n}{m+n}$

    3m = -2n

    m:n = -2:3

    which means YZ plane divides the line in 2:3 ratio externally.

12. Let YZ-plane divides the line segment joining the points A(4, 8, 10) and B(6, 10, -8) at P(x, y, z) in the ratio k: 1. Then, the coordinates of P are

    $\left(\frac{4+6k}{k+1}, \frac{8+10k}{k+1}, \frac{10-8k}{k+1}\right)$

    $\left[\because \text{ coordinates of internal division, } \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)\right]$

    Since P lies on the YZ-plane, its x-coordinate is zero,

    i.e., $\frac{4+6k}{k+1} = 0 \quad \Rightarrow \quad k = -\frac{2}{3}$

    Therefore, YZ-plane divides AB externally in the ratio 2:3.

13. To show ABCD is a parallelogram we need to show opposite side are equal

    Note that

$$AB = \sqrt{(-1-1)^2 + (-2-2)^2 + (-1-3)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$
$$BC = \sqrt{(2+1)^2 + (3+2)^2 + (2+1)^2} = \sqrt{9+25+9} = \sqrt{43}$$
$$CD = \sqrt{(4-2)^2 + (7-3)^2 + (6-2)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$DA = \sqrt{(1-4)^2 + (2-7)^2 + (3-6)^2} = \sqrt{9+25+9} = \sqrt{43}$$

Since AB = CD and BC = AD, ABCD is a parallelogram.

Now it is required to prove that ABCD is not a rectangle. For this, we show that diagonals AC and BD are unequal. We have

$$AC = \sqrt{(2-1)^2 + (3-2)^2 + (2-3)^2} = \sqrt{1+1+1} = \sqrt{3}$$

$$BD = \sqrt{(24+1)^2 + (7+2)^2 + (6+1)^2} = \sqrt{25+81+49} = \sqrt{155}$$

Since AB$\neq$BD, ABCD is not a rectangle

14. Let a point P(x, y, z) be equidistant from the points A(1, 2, 3) and P(3, 2, -1).

Then, $PA = \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$

[$\because$ distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$]

$= \sqrt{x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9}$

and $PB = \sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$

[$\because$ distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$]

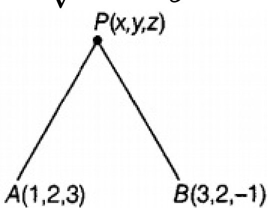
$= \sqrt{x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1}$

$= \sqrt{x^2 + y^2 + z^2 - 6x - 4y + 2z + 14}$

According to the question, PA = PB

$\therefore \sqrt{x^2 + y^2 + z^2 - 2x - 4y - 6z + 14}$

$= \sqrt{x^2 + y^2 + z^2 - 6x - 4y + 2z + 14}$



On squaring both sides, we get

$x^2 + y^2 + z^2 - 2x - 4y - 6z + 14 = x^2 + y^2 + z^2 - 6x - 4y + 2z + 14$

$\Rightarrow$ 4x - 8z = 0

$\Rightarrow$ x - 2z = 0 [dividing both sides by 4]

15. Here,

AB = $\sqrt{(1+5)^2 + (3-5)^2 + (0-2)^2}$

= $\sqrt{36+4+4}$

= $\sqrt{44}$

= $2\sqrt{11}$ units

BC = $\sqrt{(-5+9)^2+(5+1)^2+(2-2)^2}$

= $\sqrt{16+36}$

= $\sqrt{52}$

= $2\sqrt{13}$ units

CD = $\sqrt{(-9+3)^2+(-1+3)^2+(2-0)^2}$

= $\sqrt{36+4+4}$

= $2\sqrt{11}$ units

DA = $\sqrt{(-3-4)^2+(-3-3)^2+0}$

= $\sqrt{16+36}$

= $\sqrt{52}$

= $2\sqrt{13}$ units

AC = $\sqrt{(1+9)^2+(3+1)^2+(0-2)^2}$

= $\sqrt{150+16+4}$

= $\sqrt{120}$

= $4\sqrt{5}$ units

BD = $\sqrt{(-3+5)^2+(-3-5)^2+(0-2)^2}$

= $\sqrt{4+64+4}$

= $\sqrt{72}$

= $6\sqrt{2}$ units

Since,

AB = CD and BC = DA

$\Rightarrow$ ABCD is a parallelogram = BD

but, AC $\neq$ BD

$\Rightarrow$ ABCD is not a rectangle.