CBSE Test Paper 03

Chapter 8 Application of Integrals

1. The larger area bounded by $y^2 = 4x$ and $x^2 + y^2 - 2x - 3 = 0$ is equal to
   a. $2\pi - \frac{4}{3}$
   b. $2\pi + \frac{4}{3}$
   c. $2\pi + \frac{8}{3}$
   d. $2\pi + \frac{2}{3}$

2. The area of the region bounded by y = | x - 1| and y = 1 is
   a. 2
   b. $\frac{1}{2}$
   c. none of these
   d. 1

3. The area bounded by the curves y = |x - 1| and y = 1 is given by
   a. 1
   b. $\frac{1}{2}$
   c. 2
   d. none of these

4. The area bounded by the curve y = 2x - x^2 and the line x + y = 0 is
   a. $\frac{35}{6}$ sq. units
   b. $\frac{19}{6}$ sq. units
   c. none of these
   d. $\frac{9}{2}$ sq. units

5. The area of the plane region bounded by the curves x + y^2 = 0 and x + 3y^2 = 1 is equal to
   a. none of these
   b. $\frac{4}{3} sq.\ units$

c. $\frac{1}{3} sq.\ units$
d. $\frac{5}{3} sq.\ units$

6. Evaluate $\lim_{n \to \infty} \left(\frac{(n+1)(n+2)....3n}{n^{2n}} \right)^{1/n}$.

7. Find the area bounded by the parabolas $y^2 = 4a(x + a)$ and $y^2 = -4a(x - a)$.

8. If the area bounded by the curves y = f(x), the X-axis and the ordinates at x = 1 and x = b is (b - 1)sin (3b + 4), then find f(x).

9. Find the area of two regions $\left\{(x, y) : y^2 \leqslant 4x, 4x^2 + 4y^2 \leqslant 9\right\}$.

10. Find the area of the region $\left\{(x, y) : 0 \leqslant y \leqslant \left(x^2 + 1\right),\right.$ $\left.0 \leqslant y \leqslant (x + 1), 0 \leqslant x \leqslant 2\right\}$

11. Using integration, find the area of the region given below: $\{(x, y) : 0 \leqslant y \leqslant x^2 + 1, 0 \leqslant y \leqslant x + 1, 0 \leqslant x \leqslant 2\}$

12. Find the area of the region bounded by the curve $y^2 = 2x$ and $x^2 + y^2 = 4x$.

13. Find the area of the region included between the parabola $y = \frac{3}{4}x^2$ and the line 3x - 2y + 12 = 0.

14. Find the area under the curve $y = \sqrt{a^2 - x^2}$ between the lines x=0 and x=a.

15. Using integration, find the area of the triangular region whose sides have the equations y = 2x + 1, y = 3x + 1 and x = 4.

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Solution

1. (c) $2\pi+\frac{8}{3}$
   Explanation: We have, $x^2+y^2-2x-3=0 \Rightarrow (x-1)^2+y^2=4.$

   It meets $y^2 = 4x$ at x = 1.
   Required area $=2\int_3^1 \sqrt{4x}\,dx+$area of semi-circle with radius 1. $=\frac{8}{3}+2\pi$. sq. units
2. (d) 1
   Explanation: Required area : $\left|\int_0^1[(x-1)-(1-x)]dx\right|=1$
3. (a) 1
   Explanation: The given curves are : (i) y = x – 1 , x > 1 . (ii) y = - (x – 1) , x < 1. (iii) y = 1 these three lines enclose a triangle whose area is : $\frac{1}{2}$ .base.height = $\frac{1}{2}$ .2 .1 = 1 sq. unit.
4. (d) $\frac{9}{2}$ sq. units
   Explanation: The equation y = $2x - x^2$ i.e.$y-1=-(x-1)^2$ represents a downward parabola with vertex at (1, 1) which meets x – axis where y = 0 .i .e. where x = 0, 2. Also , the line y = - x meets this parabola where -x = $2x - x^2$ i.e. where x = 0, 3. Therefore, required area is:$\int_0^3(y_{parabola}-y_{line})dx=\int_0^3(2x-x^2-(-x))dx$
   $=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3=\frac{27}{2}-9=\frac{9}{2}sq.units$
5. (b) $\frac{4}{3}sq.\ units$
   Explanation: On solving the given curves, we get y = ±1 and x = -2. Required area :
   $\left|\int_{-1}^1(x_1-x_2)dy\right|$
   $=\left|\int_{-1}^1(1-3y^2+2y^2)dy\right|$
   $=\left|2\int_0^1(1-y^2)dy\right|$

$= \left|2\left[y-\frac{y^3}{3}\right]_0^1\right| = \frac{4}{3}\, sq.\, units$

6. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{1/n}$

$= e^{\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n} n\left(1+\frac{r}{n}\right)}$

$= e^{\int_0^2 ln(1+x)dx}$

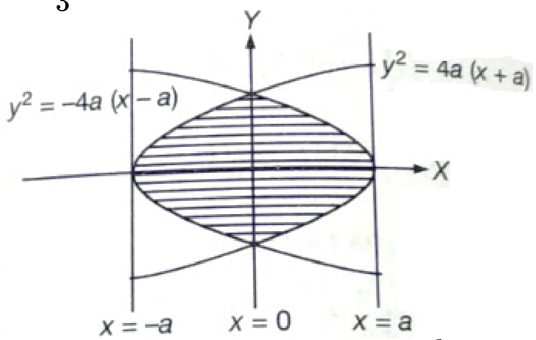
$= e^{[(x+1)(ln(x+1)-1)]_0^2}$

$= e^{3ln3-2} = \frac{27}{e^2}$

7. Required area $=4\int_0^a \sqrt{4a(a-x)}dx$

$= 4.2\sqrt{a}\left[-2\frac{(a-x)^{3/2}}{3}\right]_0^a$

$= \frac{16}{3}a^2$



8. According to the question, $\int_1^b f(x)dx = (b-1)sin(3b+4)$

Differentiating both sides w.r.t b,

f(b) = 3(b - 1).cos(3b + 4)+sin (3b + 4)

∴ f(x) = sin (3x + 4)+3(x - 1) cos(3x + 4)

If the area bounded by the curves y = f(x), the X-axis and the ordinates at x = 1 and x = b is (b - 1)sin (3b + 4), then

f(x) = sin (3x + 4)+3(x - 1) cos(3x + 4)

9. $y^2 = 4x, 4x^2+4y^2 = 9$

Area $= 2\left[\int_0^{1/2} 2\sqrt{x}\,dx + \int_{1/2}^{3/2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2}dx\right]$

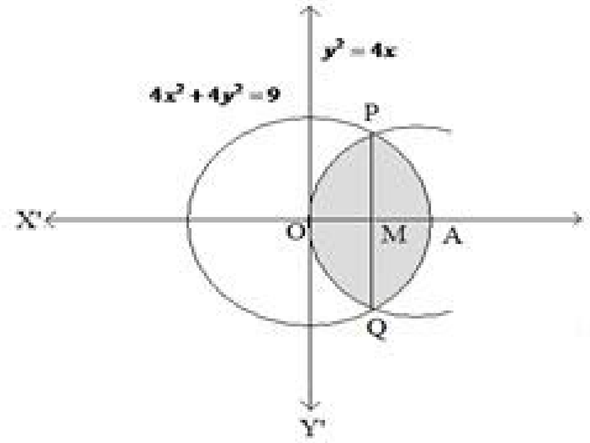
$= 2\left[2\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^{1/2}\right] + 2\left[\frac{x}{2}\sqrt{\frac{9}{4}-x^2}+\frac{\frac{9}{4}}{2}\sin^{-1}\left(\frac{x}{\frac{3}{2}}\right)\right]_{1/2}^{3/2}$

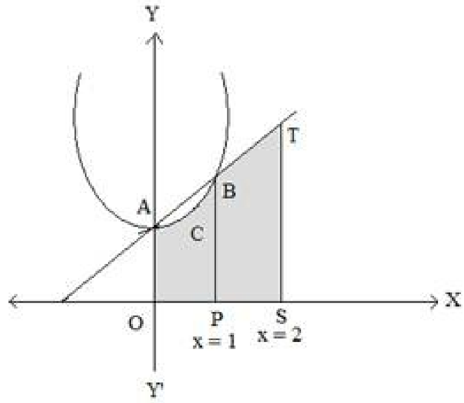
$= \frac{8}{3}\left[\left(\frac{1}{2}\right)^{3/2}-0\right] + \left[\left(\frac{3}{2}(0)+\frac{9}{4}\sin^{-1}(1)\right) - \left(\frac{1}{2}\sqrt{\frac{9}{4}-\frac{1}{4}}+\frac{9}{4}\sin^{-1}\frac{1}{3}\right)\right]$

$= \frac{8}{3}\left(\frac{1}{2\sqrt{2}}\right)+\frac{9}{4}\left(\frac{\pi}{2}\right)-\frac{1}{\sqrt{2}}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$

$= \frac{\sqrt{2}}{6}+\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left[\frac{1}{3}\right]$

$$= \left(\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{2}\right) + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$$
$$= \left[\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right] = \left(\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) + \frac{1}{3\sqrt{2}}\right) sq. units$$



10. 

y = x^2 + 1

y = x + 1

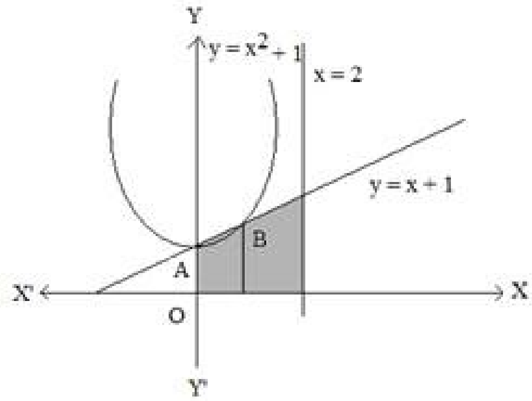
$x \leqslant 2$

Area $= \int_0^1 \left(x^2+1\right) dx + \int_0^1 (x+1)\, dx.$

$= \left[\frac{x^3}{3} + x\right]_0^1 + \left[\frac{x^2}{2} + x\right]_1^2$

$= \frac{4}{3} + \frac{5}{2}$

$= \frac{23}{6}$ sq unit

11. 

y = x^2 + 1

y = x + 1

x = 2

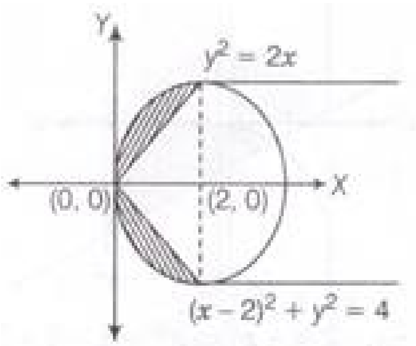
Area $= \int_0^1 \left(x^2+1\right) dx + \int_1^2 \left(x+1\right) dx$

$= \left[\left(\frac{x^3}{3}+x\right)\right]_0^1 + \left[\left(\frac{x^2}{2}+x\right)\right]_1^2$

$= \left[\left(\frac{1}{3}+1\right)-0\right] + \left[(2+2)-\left(\frac{1}{2}+1\right)\right]$

$= \frac{23}{6}$ sq units.

12. We have, $y^2 = 2x$ and $x^2 + y^2 = 4x$



$\Rightarrow x^2 + 2x = 4x$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x\,(x-2) = 0$

$\Rightarrow x = 0, 2$

Also, $x^2 + y^2 = 4x$

$\Rightarrow x^2 - 4x = -y^2$

$\Rightarrow x^2 - 4x + 4 = -y^2 + 4$

$\Rightarrow (x-2)^2 - 2^2 = -y^2$

$\therefore$ Required area $= 2\int_0^2 \left[\sqrt{2^2-(x-2)^2} - \sqrt{2}x\right] dx$

$= 2\left[\left[\frac{x-2}{2}\sqrt{2^2-(x-2)^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x-2}{2}\right)^2\right] - \left[\sqrt{2}.\,\frac{x^{3/2}}{3/2}\right]_0^2\right]$

$= 2\left[\left(0+0-1.0+2.\frac{\pi}{2}\right) - \frac{2\sqrt{2}}{3}\left(2^{3/2}-0\right)\right]$

$= \frac{4\pi}{2} - \frac{8.2}{3} = 2\pi - \frac{16}{3} = 2\left(\pi - \frac{8}{3}\right)$ sq units

13. The equations of given curves are

$y = \frac{3}{4}x^2$............(1)}

and $3x - 2y + 12 = 0$ ....(2)

From (2), $2y = 3x + 12$

$\therefore\ y=\frac{3x+12}{2}$

putting this value of y in (1), we get

$\Rightarrow \frac{3x+12}{2}=\frac{3}{4}x^2$

$\Rightarrow 6x+24=3x^2$

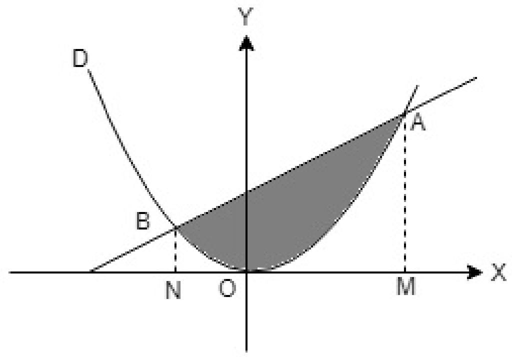
$\Rightarrow x^2-2x-8=0$

$\Rightarrow (x+2)(x-4)=0$

$\Rightarrow x=-2,4$

$\therefore\ y=3,12$

Thus, curves (1) and (2) intersect in points A(4,12) and B(-2,3).



From A, draw AM$\perp$ x-axis and from B, draw BN$\perp$ x-axis.

Required area= the area of the region included between the parabola $y=\frac{3}{4}x^2$ and the line $3x-2y+12=0$

= area of trapezium BNMA - (area BNO + area OMA)

=$\frac{1}{2}(3+12)\times 6-\int_{-2}^{4}\frac{3}{4}x^2\,dx$ [using (1)]

=$45-\frac{3}{4}\int_{-2}^{4}x^2\,dx$

=$45-\frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$

=$45-\frac{1}{4}[x^3]_{-2}^{4}$

=$45-\frac{1}{4}[64+8]$

= 27 sq.units.

14. The equation of given curve is $y=\sqrt{a^2-x^2}$

Required area = the area under the curve $y=\sqrt{a^2-x^2}$ between the lines x=0 and x=a

=$\int_0^a y\,dx$

=$\int_0^a \sqrt{a^2-x^2}\,dx$

put x=a sinθ ,then dx= a cos $\theta\ d\theta$

when x=0 , a sin $\theta=0$,

$sin\theta = 0$

$\theta = 0$

when x= a ,asin θ =a

sin θ=1

θ=$\frac{\pi}{2}$

$\therefore\ I = \int_0^{\pi/2} \sqrt{a^2 - a^2sin^2\theta}.\, acos\theta\, d\theta$

$= a^2 \int_0^{\pi/2} cos^2\theta\, d\theta$

$= \frac{a^2}{2} \int_0^{\pi/2} 2\, cos^2\theta\, d\theta$

$= \frac{a^2}{2} \int_0^{\pi/2} (1 + cos\ 2\theta) d\theta$

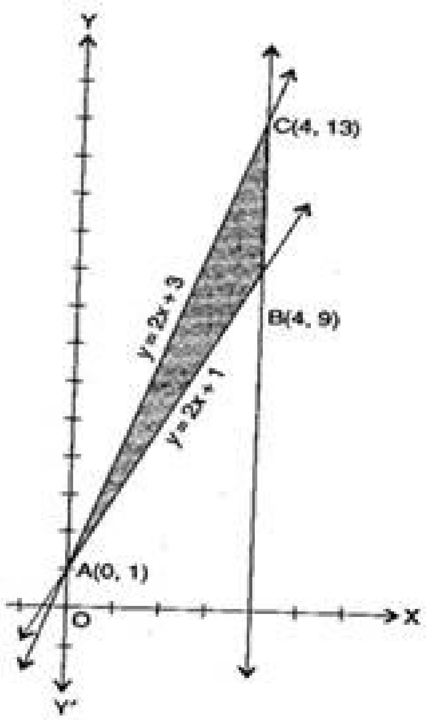
$= \frac{a^2}{2} \left[\theta + \frac{sin2\theta}{2}\right]_0^{\pi/2}$

$= \frac{a^2}{2} \left[\left(\frac{\pi}{2} + \frac{1}{2} sin\pi\right) - \left(0 + \frac{1}{2} sin0\right)\right]$

$= \frac{a^2}{2} \left[\left(\frac{\pi}{2} + 0\right) - (0 - 0)\right]$

$= \frac{\pi a^2}{4}$ sq.units

15. Equations of one side of triangle is



y = 2x + 1 ...(i)

second line of triangle is y = 3x + 1 ...(ii)

third line of triangle is x = 4 ...(iii)

Solving eq. (i) and (ii), we get x = 0 and y = 1

∴ Point of intersection of lines (i) and (ii) is A (0, 1)

Putting x = 4 in eq. (i), we get y = 9

$\therefore$ Point of intersection of lines (i) and (iii) is B (4, 9)

Putting x = 4 in eq. (i), we get y = 13

$\therefore$ Point of intersection of lines (ii) and (iii) is C (4, 13)

$\therefore$ Area between line (ii) i.e., AC and x - axis

$$= \left|\int_0^4 y dx\right| = \left|\int_0^4 (3x+1)\, dx\right| = \left(\frac{3x^2}{2} + x\right)_0^4$$

= 24 + 4 = 28 sq. units ...(iv)

Again Area between line (i) i.e., AB and x - axis

$$= \left|\int_0^4 y dx\right| = \left|\int_0^4 (2x+1)\, dx\right| = \left(x^2 + x\right)_0^4$$

= 16 + 4 = 20 sq. units ...(v)

Therefore, Required area of $\triangle ABC$

= Area given by (iv) - Area given by (v)

= 28 - 20 = 8 sq. units.