Derivatives

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. Find $f'(x)$, if $f(x) = 2x^2 + 3x - 5$, at $x = -1$.
(a) 0 (b) 1 (c) –1 (d) –2

2. Find $f'(x)$, if $f(x) = \sin x$, at $x = 0$.
(a) 1 (b) 5 (c) 3 (d) 2

3. Find $f'(x)$, if $f(x) = 3$, at $x = 0$ and at $x = 3$.
(a) 0, 0 (b) 1, 2 (c) 5, 1 (d) 2, 8

4. Find $f'(x)$, if $f(x) = x^2$
(a) x (b) $2x$ (c) $4x$ (d) $x^2/2$

5. Find $f'(x)$, if $f(x) = a$, where a is a real number.
(a) 4 (b) 1 (c) –1 (d) 0

6. Find $f'(x)$, if $f(x) = \frac{1}{x}$
(a) $\frac{1}{x^2}$ (b) $-\frac{1}{x^2}$ (c) 0 (d) 1

7. Find $f'(x)$, if $f(x) = \frac{2x+3}{x-2}$
(a) $\frac{-7}{(x-2)}$ (b) $\frac{-7}{(x-2)^2}$
(c) $\frac{7}{x-2}$ (d) None of these

8. Compute the derivative of $6x^{100} - x^{55} + x$.
(a) $x^{99} - x^{54} + 1$
(b) $100x^{99} - 55x^4 + 1$
(c) $600x^{99} - 55x^{54} + 1$
(d) None of these

9. Find the derivative of $f(x) = 1 + x + x^2 + x^3 + \ldots + x^{50}$ at $x = 1$.
(a) 1275 (b) 1200 (c) 1326 (d) 1542

10. Find the derivative of $f(x) = \frac{x+1}{x}$.
(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) 0 (d) $-\frac{1}{x^2}$

11. Compute the derivative of $f(x) = \sin^2 x$.
(a) $\sin 2x$ (b) $2 \sin x$ (c) $2x \sin 2x$ (d) $\sin x$

12. Find the derivative of $\frac{x^5 - \cos x}{\sin x}$.
(a) $\frac{x^5 \cos x}{\sin^2 x}$ (b) $\frac{1}{\sin x} - \frac{x^5 \cos x}{\sin^2 x}$
(c) $\frac{x}{\sin^2 x}$ (d) None of these

13. Find the derivative of $(x^2 + 1) \cos x$.
(a) $2x \cos x - (x^2 + 1) \sin x$
(b) $2x \sin x - x^2 \cos x$
(c) $x^2(\cos x - \sin x)$
(d) $2x(\sin x + \cos x)$

14. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx} =$
(a) $\frac{x+1}{x}$ (b) $\frac{1}{1+x}$
(c) $\frac{-1}{(1+x)^2}$ (d) $\frac{x}{1+x}$

15. If $y = f(x) = -\text{cosec } x \cdot \cos x$, then $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}}$ is equal to
(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 3

16. If $h(x) = \frac{2+x^2}{2-x^2}, h'(1) =$
(a) 2 (b) 4 (c) 6 (d) 8

17. If $f(x) = 1 - \sqrt{x} + (1+\sqrt{x})^2, f'(1)$ is equal to
(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2

18. If $y = x^2 + \sin x + \frac{1}{x^2}$, then $\frac{dy}{dx}$ is equal to
(a) $2x - \cos x + 2$ (b) $x - 2x^2 + \cos x$
(c) $2x + \cos x - (2/x^3)$ (d) $(2/x^3) - \cos x$

19. If $y = x \tan x$, then dy/dx is equal to

(a) $\dfrac{\tan x}{x - x^2 - y^2}$ (b) $\dfrac{y}{x - x^2 - y^2}$

(c) $\dfrac{\tan x}{y - x}$ (d) $\dfrac{\cos x \sin x + x}{\cos^2 x}$

20. If $y = (x + 1)(x + 2)(x + 3)(x + 4)(x + 5)$, then the value of $\dfrac{dy}{dx}$ at $x = 0$ is equal to

(a) 374 (b) 742 (c) 472 (d) 274

21. Find the derivative of $2x^4 + x$.

(a) $x^3 + 1$ (b) $8x^3 + 1$

(c) $x^3 - 1$ (d) None of these

22. If $f(x) = x \sin x$, then $f'\left(\dfrac{\pi}{2}\right)$ is equal to

(a) 0 (b) 1 (c) –1 (d) $\dfrac{1}{2}$

23. If $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$, then $\dfrac{dy}{dx}$ at $x = 1$ is

(a) 1 (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) 0

24. If $y = \dfrac{1 + \dfrac{1}{x^2}}{1 - \dfrac{1}{x^2}}$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{-4x}{(x^2 - 1)^2}$ (b) $\dfrac{-4x}{x^2 - 1}$

(c) $\dfrac{1 - x^2}{4x}$ (d) $\dfrac{4x}{x^2 - 1}$

25. If $y = \dfrac{\sin(x + 9)}{\cos x}$, then $\dfrac{dy}{dx}$ at $x = 0$ is

(a) cos 9 (b) sin 9 (c) 0 (d) 1

26. If $f(x) = 1 + \dfrac{x}{2} + \dfrac{x^2}{3} + \ldots + \dfrac{x^{100}}{101}$, then $f'(0)$ is equal to

(a) 1/100 (b) 100

(c) Does not exist (d) $\dfrac{1}{2}$

27. If $y = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots.$, then $\dfrac{dy}{dx} =$ ______ .

(a) $2y$ (b) y (c) $-y$ (d) $-2y$

28. If $f(x) = \dfrac{x - 4}{2\sqrt{x}}$, then $f'(1)$ is

(a) $\dfrac{3}{4}$ (b) $\dfrac{-1}{4}$ (c) $\dfrac{5}{4}$ (d) $\dfrac{1}{4}$

29. Find the derivative of $(\sec x - 1)(\sec x + 1)$

(a) $2 \tan x \sec^2 x$ (b) $\tan x \sec x$

(c) $\tan x \sec^2 x$ (d) $\tan x$

30. Find the derivative of $4\sqrt{x} - 2$

(a) $\dfrac{2}{x}$ (b) $\dfrac{2}{x^2}$ (c) $\dfrac{2}{\sqrt{x}}$ (d) $\dfrac{1}{\sqrt{x}}$

31. Find the derivative of $(x^2 + 1) \cos x$

(a) $2x\cos x - x\sin x$

(b) $2x \cos x - x^2 \sin x - \sin x$

(c) $2x\cos x$

(d) $x\cos x - \sin x$

Case Based MCQs

Case I : Read the following passage and answer the questions from 32 to 36.

Let f be a real valued function, the function defined by $f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$ whenever the limit exists is defined to be the derivative of f at x.

For a function $f(x) = \tan x$, answer the following questions.

32. The value of $f(x + h)$ is

(a) $\sin (x + h)$ (b) $\tan^2 (x + h)$

(c) $\tan (x + h)$ (d) $\tan (x - h)$

33. The value of $f(x + h) - f(x)$ is

(a) $\dfrac{\sin h}{\cos (x + h) \cos x}$ (b) $\dfrac{\tan h}{\cos (x + h) \cos x}$

(c) $\dfrac{\cos h}{\sin(x + h) \cos x}$ (d) $\dfrac{\cos(x + h) \cos x}{\sin(x + h)}$

34. The value of $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is

(a) $\cos^2 x$ (b) $\tan x$

(c) $\frac{1}{\cos^2 x}$ (d) 1

35. The value of $f'(0)$ is

(a) 0 (b) 1 (c) 2 (d) 3

36. Find the value of $f'(60°)$.

(a) $\sqrt{3}$ (b) 2 (c) 4 (d) $6\sqrt{3}$

Case II : Read the following passage and answer the questions from 37 to 41.

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ be a polynomial function, where a_i's are all real numbers and $a_n \neq 0$. Then the derivative of function $f(x)$ is given by

$$\frac{d}{dx}f(x) = na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + 2a_2 x + a_1$$

For a function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$, answer the following questions.

37. The derivative of $f(x)$ w.r.t x is

(a) $x^{100} + x^{99} + x^{98} + ... + x + 1$

(b) $x^{99} + x^{98} + ... + x + 1$

(c) $100\,x^{99} + 99x^{98} + ... + x + 1$

(d) None of these.

38. The value of $f'(0)$ is

(a) 2 (b) 0 (c) 1 (d) –1

39. The value of $f'(1)$ is

(a) 100 (b) 0

(c) 200 (d) 10

40. Which of the following condition satisfies?

(a) $f'(1) = 99\,f'(0)$

(b) $f'(1) = 100\,f'(0)$

(c) $f'(0) = f'(1)$

(d) $f'(0) = 100\,f'(1)$

41. The value of $f'(0)$ is always

(a) coefficient of x^2 in $f(x)$

(b) 0

(c) constant term of $f(x)$

(d) None of these

Case III : Read the following passage and answer the questions from 42 to 46.

Let f and g be two functions such that their derivatives are defined in a common domain. We define the derivative of product of two functions is given by the product rule *i.e.,*

$$\frac{d}{dx}[f(x)\cdot g(x)] = g(x)\frac{d}{dx}f(x) + f(x)\cdot\frac{d}{dx}g(x)$$

The derivative of quotient of two functions is given by quotient rule *i.e.,*

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x).\frac{d}{dx}g(x)}{(g(x))^2}; g(x) \neq 0$$

42. Find the derivative of $x(x + 2)$.

(a) $x + 2$ (b) $2x + 1$

(c) $3x + 4$ (d) $2(x + 1)$

43. The value of $f'(x)$, if $f(x) = \sin x \cdot \cos x$ is

(a) $\cos 2x$ (b) $\sin 2x$

(c) 1 (d) $\cos x$

44. The value of $\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$ is

(a) $(x - 1)^2$ (b) $\frac{-2}{(x-1)^2}$

(c) $\frac{1}{(x-1)^2}$ (d) $\frac{-1}{(x-1)^2}$

45. Find the value of derivative of $\frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$ at $x = \frac{\pi}{2}$.

(a) $\frac{\pi}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\pi}{\sqrt{2}}$ (d) $\frac{\pi}{3}$

46. The product rule given above is also referred as

(a) Sandwich theorem

(b) Leibnitz product rule

(c) Chain rule

(d) None of these

Assertion & Reasoning Based MCQs

Directions (Q.-47 to 50) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.

(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.

(c) Assertion is correct statement but Reason is wrong statement.

(d) Assertion is wrong statement but Reason is correct statement.

47. Let $u = f(x)$ and $v = g(x)$. Then,

Assertion : $(uv)' = u'v + uv'$ is a Leibnitz rule or product rule.

Reason : $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v'}$ is a Leibnitz rule or quotient rule.

48. Assertion : The derivative of $f(x) = x^3$ is x^2.

Reason : The derivative of $f(x) = x^n$ is nx^{n-1}

49. Assertion : The derivative of $y = 2x - \frac{3}{4}$ is 2.

Reason : The derivative of $y = cx$ is c.

50. Assertion : The derivative of $h(x) = \frac{x + \cos x}{\tan x}$ is $\frac{(1 - \sin x)\tan x - (x + \cos x)\sec^2 x}{(\tan x)^2}$.

Reason : $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{(v)^2}$.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Write the derivative of $x^3 \sin x$ w.r.t. x.

2. Write the derivative of $x^2 \tan x$ w.r.t. x.

3. Evaluate : $\frac{d}{dx}(\sec^2 x)$

4. Differentiate $f(x) = (3x + 5)(1 + \tan x)$ with respect to x.

5. Find the derivative of

$f(x) = \cos x - \sin x$ at $x = \frac{2\pi}{3}$.

6. Differentiate $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ w.r.t. x.

7. Find the derivative of $x^3 \sec x$ w.r.t. x.

8. Differentiate $f(x) = \frac{x^3 + x^2 + 1}{x}$ with respect to x.

9. Find the derivative of $x^{-3}(5 + 3x)$.

10. Find the derivative of $x^3 - 3x^2 + 2x^{-4}$

Short Answer Type Questions (SA-I)

11. Find the derivative of $\sqrt{\frac{1 + \sin x}{1 - \sin x}}$.

12. If $f(x) = \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2$, then find $f'\left(\frac{\pi}{4}\right)$.

13. Differentiate $\tan 2x$ using first principle.

14. For the function f, given by

$f(x) = x^2 - 6x + 8$, prove that $f(5) - 3f(2) = f(8)$.

15. Find the derivative of

$f(x) = (ax^2 + \sin x)(p + q\cos x)$

16. Find the derivative of $f(x) = 10x$ using first principle.

17. Find the derivative of $\frac{1}{ax^2+b}$, with respect to x.

18. If $f(x) = 1 - x + x^2 - x^3 \ldots -x^{99} + x^{100}$, then find $f'(1)$.

19. Find the derivative of $x^5(3 - 6x^{-9})$ with respect to x.

20. Find the derivative of $\frac{3x+4}{5x^2-7x+9}$.

Short Answer Type Questions (SA-II)

21. Find the derivative of $y = \frac{x+\sin x}{(x^2-1)}$.

22. Using the method of first principle find the derivative of $f(x) = \frac{2x+7}{x+2}$.

23. Find the derivative of $\tan(2x + 3)$ by first principle method.

24. Find the derivative of

(i) $(6x^3 + 9x)(5x + 10)$ (ii) $\frac{5x+4}{x-3}$

25. Differentiate $\frac{2x^3-\sin x}{\cot x}$ w.r.t. x.

26. Find the derivative of $f(x) = \sqrt{\cos x}$ using first principle.

27. Find $f'(x)$ using first principle, where $f(x) = x - \frac{1}{x}$.

28. Find the derivative of $\cos x$ using first principle.

29. Find the derivative of $\sin 2x$ by first principle.

30. Differentiate $f(x) = \frac{3-x}{3+4x}$ with respect to x by first principle of derivative.

31. Differentiate $x^{2/3}$ by using first principle.

32. Differentiate : $\frac{a+b\sin x}{c+d\cos x}$

33. Differentiate : $\frac{4x+5\sin x}{3x+7\cos x}$

34. If $y = \frac{\sin x+\cos x}{\sin x-\cos x}$, then find the value of $\frac{dy}{dx}$ at $x = 0$.

35. Find the derivative of the following functions :

(i) $5 \sin x - 6 \cos x + 7$

(ii) $3 \cot x + 5 \operatorname{cosec} x$

(iii) $2 \tan x - 7 \sec x$.

Long Answer Type Questions (LA)

36. Differentiate $\cot\sqrt{x}$ w.r.t. x from first principle method.

37. Find the derivative of $\sin x + \cos x$ from first principle.

38. (i) Find the derivative of $\cot x$ using first principle method.

(ii) Find the derivative of $f(x) = \frac{x}{1+\tan x}$.

39. Find the derivative of $\operatorname{cosec} x$ with respect to x, from first principle.

40. If $f(x) = \frac{x^n - a^n}{x - a}$ for some constant 'a', then prove that $f'(a)$ does not exists.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(c) :** We have, $f(x) = 2x^2 + 3x - 5$

$\therefore \quad f'(-1) = \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h}$

$= \lim_{h\to 0} \frac{[2(-1+h)^2 + 3(-1+h) - 5] - [2(-1)^2 + 3(-1) - 5]}{h}$

$= \lim_{h\to 0} \frac{2h^2 - h}{h} = \lim_{h\to 0}(2h-1) = 2(0) - 1 = -1$

2. **(a) :** Let $f(x) = \sin x$. Then

$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0} \frac{\sin(0+h) - \sin(0)}{h} = \lim_{h\to 0} \frac{\sin h}{h} = 1$

3. **(a) :** We have, $f(x) = 3$

$\Rightarrow \quad f'(x) = 0 \qquad \left(\because \frac{d}{dx}(c) = 0\right)$

$\therefore \quad f'(0) = 0$ and $f'(3) = 0$

4. **(b) :** We have, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^2 - (x)^2}{h} = \lim_{h\to 0}(h + 2x) = 2x$

5. **(d)**

6. **(b) :** We have, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{(x+h)} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{x - (x+h)}{x(x+h)}\right]$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-h}{x(x+h)}\right] = \lim_{h\to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$

7. **(b) :** We have, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{2(x+h)+3}{x+h-2} - \frac{2x+3}{x-2}}{h}$

$= \lim_{h\to 0} \frac{(2x+2h+3)(x-2) - (2x+3)(x+h-2)}{h(x-2)(x+h-2)}$

$= \lim_{h\to 0} \frac{(2x+3)(x-2) + 2h(x-2) - (2x+3)(x-2) - h(2x+3)}{h(x-2)(x+h-2)}$

$= \lim_{h\to 0} \frac{-7}{(x-2)(x+h-2)} = -\frac{7}{(x-2)^2}$

8. **(c) :** Let $f(x) = 6x^{100} - x^{55} + x$

$\therefore \quad f'(x) = 6 \times 100x^{99} - 55x^{54} + 1 = 600x^{99} - 55x^{54} + 1$

9. **(a) :** We have, $f(x) = 1 + x + x^2 + x^3 + \ldots + x^{50}$

$\therefore \quad f'(x) = 0 + 1 + 2x + 3x^2 + \ldots + 50x^{49}$

So, $f'(1) = 1 + 2 + 3 + \ldots + 50 = \frac{50(50+1)}{2} = 25 \times 51 = 1275$

10. **(d) :** Clearly this function is defined everywhere except at $x = 0$.

$\therefore \quad \frac{d}{dx}[f(x)] = \frac{d}{dx}\left(\frac{x+1}{x}\right) = \frac{x\frac{d}{dx}(x+1) - (x+1)\cdot\frac{d}{dx}(x)}{x^2}$

$= \frac{1\cdot(x) - (x+1)\cdot 1}{x^2} = -\frac{1}{x^2}$

11. **(a) :** We have, $f(x) = \sin^2 x$

$\therefore \quad \frac{d}{dx}[f(x)] = \frac{d}{dx}(\sin x \times \sin x)$

$= (\sin x)' \sin x + \sin x(\sin x)'$ (Using product rule)

$= (\cos x) \sin x + \sin x(\cos x) = 2\sin x \cos x = \sin 2x$

12. **(d) :** Let $f(x) = \frac{x^5 - \cos x}{\sin x}$.

$\therefore \quad f'(x) = \frac{(x^5 - \cos x)' \sin x - (x^5 - \cos x)(\sin x)'}{(\sin x)^2}$

{Using quotient rule}

$= \frac{(5x^4 + \sin x)\sin x - (x^5 - \cos x)\cos x}{\sin^2 x}$

$= \frac{-x^5 \cos x + 5x^4 \sin x + 1}{\sin^2 x} \qquad [\because \sin^2 x + \cos^2 x = 1]$

13. **(a) :** Let $f(x) = (x^2 + 1) \cos x$

$\therefore \quad f'(x) = (x^2 + 1)' \cos x + (\cos x)' (x^2 + 1)$

$= (2x + 0)\cos x + (-\sin x)(x^2 + 1)$

$= 2x \cos x - (x^2 + 1)\sin x$

14. **(c) :** Given $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow \quad x\sqrt{1+y} = -y\sqrt{1+x} \qquad \ldots(i)$

Squaring both the sides, we get

$x^2(1 + y) = y^2(1 + x) \Rightarrow x^2 - y^2 + x^2y - xy^2 = 0$

$\Rightarrow \quad (x - y)(x + y + xy) = 0 \Rightarrow y = x$ or $y(1 + x) = -x$

$\Rightarrow \quad y = x$ or $y = -\dfrac{x}{1+x}$

Note that $y = x$ does not satisfy (i) in general.

$$\Rightarrow \quad y = -\frac{x}{1+x} \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{(1+x)\cdot 1 - x}{(1+x)^2} = \frac{-1}{(1+x)^2}$$

15. **(c) :** We have, $y = -\text{cosec } x \cos x$

$$\Rightarrow \quad y = -\frac{\cos x}{\sin x} = -\cot x$$

$$\therefore \quad \frac{dy}{dx} = -(-\text{cosec}^2 x) = \text{cosec}^2 x$$

$$\therefore \quad \left.\frac{dy}{dx}\right|_{\text{at } x=\frac{\pi}{2}} = \left(\text{cosec}\frac{\pi}{2}\right)^2 = (1)^2 = 1$$

16. **(d) :** We have, $h(x) = \dfrac{2+x^2}{2-x^2}$

$$\therefore \quad h'(x) = \frac{(2-x^2)(2x)-(2+x^2)(-2x)}{(2-x^2)^2}$$

$$= \frac{2x(2-x^2+2+x^2)}{(2-x^2)^2} = \frac{8x}{(2-x^2)^2} \quad \therefore \; h'(1) = \frac{8}{1} = 8.$$

17. **(c) :** We have, $f(x) = 1-\sqrt{x}+(1+\sqrt{x})^2$

$= 1-\sqrt{x}+1+x+2\sqrt{x}$

$$\Rightarrow \quad f(x) = 2+x+\sqrt{x} \quad \Rightarrow \quad f'(x) = 1+\frac{1}{2\sqrt{x}}$$

Now, $f'(1) = 1+\dfrac{1}{2} = \dfrac{3}{2}$.

18. **(c) :** We have, $y = x^2 + \sin x + \dfrac{1}{x^2}$

$$\therefore \quad \frac{dy}{dx} = 2x + \cos x + (-2)x^{-3} = 2x + \cos x - \frac{2}{x^3}$$

19. **(d) :** We have, $y = x \tan x$

$$\therefore \quad \frac{dy}{dx} = x(\tan x)' + \tan x \cdot (x)' = x\sec^2 x + \tan x \cdot 1$$

$$= \frac{x}{\cos^2 x} + \frac{\sin x}{\cos x} = \frac{x + \sin x \cos x}{\cos^2 x}$$

20. **(d) :** $y = (x + 1)(x + 2)(x + 3)(x + 4)(x + 5)$

$$\Rightarrow \quad \frac{dy}{dx} = (x + 2)(x + 3)(x + 4)(x + 5) + (x + 1)(x + 3)(x + 4)(x + 5) + \ldots$$

(Using product rule)

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=0} = 120 + 60 + 40 + 30 + 24 = 274.$$

21. **(b) :** Let $y = 2x^4 + x$.

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(2x^4) + \frac{d}{dx}(x) = 2 \times 4x^3 + 1 = 8x^3 + 1$$

22. **(b) :** As $f'(x) = (x)' \sin x + x (\sin x)' = x \cos x + \sin x$

So, $f'\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}\cos\dfrac{\pi}{2} + \sin\dfrac{\pi}{2} = 1$

23. **(d) :** We have, $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) + \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x}} + \left(\frac{-1}{2}\right) \times \frac{1}{x^{3/2}} = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$$

$$\left.\frac{dy}{dx}\right|_{\text{at } x=1} = \frac{1}{2} - \frac{1}{2} = 0$$

24. **(a) :** We have, $y = \dfrac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}} \Rightarrow y = \dfrac{x^2+1}{x^2-1}$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(x^2-1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2}$$

$$= \frac{2x^3-2x-2x^3-2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

25. **(a) :** We have, $y = \dfrac{\sin(x+9)}{\cos x}$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}(\sin(x+9)) - \sin(x+9)\frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos x(\cos(x+9)) - \sin(x+9)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos x \cos(x+9) + \sin x \sin(x+9)}{\cos^2 x}$$

$$= \frac{\cos(x-x-9)}{\cos^2 x} = \frac{\cos(-9)}{\cos^2 x} = \frac{\cos 9}{\cos^2 x} \quad (\because \cos(-x) = \cos x)$$

$$\therefore \quad \left.\frac{dy}{dx}\right|_{\text{at } x=0} = \frac{\cos 9}{(\cos 0)^2} = \frac{\cos 9}{1} = \cos 9$$

26. **(d) :** We have, $f(x) = 1+\dfrac{x}{2}+\dfrac{x^2}{3}+\dfrac{x^3}{4}+\ldots+\dfrac{x^{100}}{101}$ ...(i)

Differentiating (i) w.r.t. x, we get

$$f'(x) = 0 + \frac{1}{2} + \frac{2x}{3} + \frac{3x^2}{4} + ... + \frac{100x^{99}}{101}$$

$\therefore \quad f'(0) = \frac{1}{2}$

27. (b) : We have,

$$y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$$

Now, $\frac{dy}{dx} = 0 + 1 + \frac{2x}{2} + \frac{3x^2}{6} + \frac{4x^3}{24} + ...$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + ...$$

$$= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ... = y$$

28. (c) : We have, $f(x) = \frac{x-4}{2\sqrt{x}}$

$$\Rightarrow f'(x) = \frac{2\sqrt{x} - (x-4)\cdot 2\cdot\frac{1}{2\sqrt{x}}}{4x}$$

$$= \frac{2x - (x-4)}{4x^{3/2}} = \frac{2x - x + 4}{4x^{3/2}} = \frac{x+4}{4x^{3/2}}$$

$\therefore$ At $x = 1$, $f'(1) = \frac{1+4}{4\times(1)^{3/2}} = \frac{5}{4}$

29. (a) : Let $y = (\sec x - 1)(\sec x + 1)$

$\Rightarrow y = (\sec^2 x - 1)$

$= \tan^2 x$

Now, $\frac{dy}{dx} = 2\tan x \cdot \frac{d}{dx}\tan x = 2\tan x \cdot \sec^2 x$

30. (c) : Let $f(x) = 4\sqrt{x} - 2 \Rightarrow f(x) = 4x^{1/2} - 2$ ... (i)

Differentiating (i) with respect to x, we get

$$\frac{d}{dx}(f(x)) = \frac{1}{2}\times 4x^{\frac{1}{2}-1} - 0$$

$$= \frac{1}{2}\times 4x^{-\frac{1}{2}} = 2x^{-\frac{1}{2}} = \frac{2}{x^{1/2}}$$

31. (b) : Let $f(x) = (x^2 + 1)\cos x$

$\Rightarrow f(x) = x^2\cos x + \cos x$ ... (i)

Differentiating (i) with respect to x, we get

$$\frac{d}{dx}[f(x)] = 2x\cos x + x^2(-\sin x) + (-\sin x)$$

$$= 2x\cos x - x^2\sin x - \sin x$$

32. (c) : Since $f(x) = \tan x$.

Replace x by $x + h$ in $f(x)$, we get

$f(x + h) = \tan(x + h)$

33. (a) : $f(x + h) - f(x) = \tan(x + h) - \tan x$

$$= \frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}$$

$$= \frac{\sin(x+h)\cos x - \sin x\cos(x+h)}{\cos x\cdot\cos(x+h)}$$

$$= \frac{\sin(x+h-x)}{\cos x\cdot\cos(x+h)} = \frac{\sin h}{\cos x\cos(x+h)}$$

34. (c) : $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{\sin h}{h}\cdot\lim_{h\to 0}\frac{1}{\cos x\cdot\cos(x+h)}$$

$$= \lim_{h\to 0}\frac{1}{\cos x\cdot\cos(x+h)} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= \frac{1}{\cos^2 x}$$

35. (b) : Since, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \frac{1}{\cos^2 x} = \sec^2 x$

$\Rightarrow f'(0) = \sec^2(0) = 1$

36. (c) : $f'(60°) = \sec^2 60° = (2)^2 = 4$.

37. (b) : $\because \quad f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$

$\therefore \quad f'(x) = 100.\frac{x^{99}}{100} + 99\frac{x^{98}}{99} + ... + 2.\frac{x}{2} + 1$

$\Rightarrow \quad f'(x) = x^{99} + x^{98} + ... + x + 1$

38. (c) : $f'(0) = 0 + 0 + ... + 0 + 1 = 1$

39. (a) : $f'(1) = \underbrace{1+1+...+1+1}_{100 \text{ times}} = 100$

40. (b) : Since, $100 = 100 \times 1$

$\Rightarrow \quad f'(1) = 100 f'(0)$

41. (c)

42. (d) : $\frac{d}{dx}(x(x+2)) = \frac{d}{dx}(x)\cdot(x+2) + x\frac{d}{dx}(x+2)$

$= 1\cdot(x + 2) + x\cdot 1 = 2x + 2 = 2(x + 1)$

43. (a) : $f(x) = \sin x\cdot\cos x$

$$\Rightarrow \quad f'(x) = \cos x\cdot\frac{d}{dx}(\sin x) + \sin x\cdot\frac{d}{dx}(\cos x)$$

$= \cos x\cdot\cos x + \sin x(-\sin x)$

$= \cos^2 x - \sin^2 x = \cos 2x$

44. (b) : $\frac{d}{dx}\left(\frac{x+1}{x-1}\right) = \frac{\frac{d}{dx}(x+1).(x-1) - (x+1).\frac{d}{dx}(x-1)}{(x-1)^2}$

$$= \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

45. **(c) :** $\because$ $\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$, then $f(x)=\frac{x^2}{\sqrt{2}\sin x}$

$$\Rightarrow f'(x)=\frac{\frac{d}{dx}(x^2)\cdot(\sqrt{2}\sin x)-x^2\frac{d}{dx}(\sqrt{2}\sin x)}{(\sqrt{2}\sin x)^2}$$

$$=\frac{2x\cdot\sqrt{2}\sin x-\sqrt{2}x^2\cos x}{2\sin^2 x}=\frac{2x\sin x-x^2\cos x}{\sqrt{2}\sin^2 x}$$

$$\Rightarrow f'\left(\frac{\pi}{2}\right)=\frac{2\times\frac{\pi}{2}.1-0}{\sqrt{2}\cdot(1)^2}=\frac{\pi}{\sqrt{2}}$$

46. **(b)**

47. **(c) :** Let $u = f(x)$ and $v = g(x)$.

Then, $(uv)' = u'v + uv'$

This is referred as Leibnitz rule or the product rule for differentiating product of functions. Similarly, the quotient rule is $\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$.

48. **(d) :** We have, $f(x) = x^3$

$\Rightarrow$ $f'(x) = 3x^2 \neq x^2$

$\therefore$ Assertion is wrong.

49. **(a) :** We have, $y = 2x-\frac{3}{4}$

$$\Rightarrow \frac{dy}{dx}=(2\times 1)-0=2$$

$\therefore$ Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

50. **(a) :** We have, $h(x) = \frac{x+\cos x}{\tan x}$ ...(i)

Differentiating both sides of (i) w.r.t. 'x', we get

$$h'(x)=\frac{(x+\cos x)'\tan x-(x+\cos x)(\tan x)'}{(\tan x)^2}$$
$$=\frac{(1-\sin x)\tan x-(x+\cos x)\sec^2 x}{(\tan x)^2}$$

$\therefore$ Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

SUBJECTIVE TYPE QUESTIONS

1. Let $f(x) = x^3 \sin x$

$$\Rightarrow f'(x)=\frac{d}{dx}(x^3\sin x)=3x^2\cdot\sin x+x^3\cdot\cos x$$

$= x^2 (3 \sin x + x \cos x)$

2. We have, $\frac{d}{dx}(x^2\cdot\tan x)$

$$= x^2\frac{d}{dx}(\tan x)+\tan x\cdot\frac{d}{dx}(x^2)$$

$= x^2 \cdot \sec^2 x + \tan x \cdot (2x)$

$= x(x \cdot \sec^2 x + 2\tan x)$

3. $\frac{d}{dx}(\sec^2 x)=\frac{d}{dx}(\sec x\cdot\sec x)$

$= \sec x\cdot(\sec x\tan x)+\sec x(\sec x\tan x)$

$\therefore$ $\frac{d}{dx}(\sec^2 x)=2\sec^2 x\tan x$

4. We have, $f(x) = (3x + 5)(1 + \tan x)$

$$f'(x)=\frac{d}{dx}[(3x+5)(1+\tan x)]$$

$$=(3x+5)\frac{d}{dx}(1+\tan x)+(1+\tan x)\frac{d}{dx}(3x+5)$$

$= (3x + 5)(\sec^2 x) + (1 + \tan x)(3)$

$= 3x\sec^2 x + 5\sec^2 x + 3 + 3\tan x$

$= 3(1 + \tan x + x\sec^2 x) + 5\sec^2 x$.

5. We have, $f(x) = \cos x - \sin x$

$$\Rightarrow f'(x)=\frac{d}{dx}(\cos x)-\frac{d}{dx}(\sin x)=-\sin x-\cos x$$

$$\therefore\ f'\left(\frac{2\pi}{3}\right)=-\left(\sin\frac{2\pi}{3}+\cos\frac{2\pi}{3}\right)$$

$$=-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$$

$$\therefore\ f'\left(\frac{2\pi}{3}\right)=\frac{1-\sqrt{3}}{2}$$

6. We have, $\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$

$$=\frac{d}{dx}\left(x+\frac{1}{x}+2\right)$$

$$=\frac{d}{dx}(x)+\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{d}{dx}(2)$$

$$=1+\left(\frac{-1}{x^2}\right)+0=1-\frac{1}{x^2}$$

7. $\frac{d}{dx}(x^3\sec x)=x^3\cdot\frac{d}{dx}(\sec x)+\sec x\cdot\frac{d}{dx}(x^3)$

$= x^3 \cdot \sec x \tan x + \sec x \cdot 3x^2$

$= x^2 \sec x\ (x \tan x + 3)$

8. $f(x) = \dfrac{x^3 + x^2 + 1}{x} = x^2 + x + \dfrac{1}{x}$

$\therefore \quad f'(x) = \dfrac{d}{dx}(x^2) + \dfrac{d}{dx}(x) + \dfrac{d}{dx}\left(\dfrac{1}{x}\right)$

$= 2x + 1 - \dfrac{1}{x^2} = \dfrac{2x^3 + x^2 - 1}{x^2}$

9. Let $y = x^{-3}(5 + 3x)$

$\Rightarrow \quad y = 5 \cdot x^{-3} + 3x^{-2}$

$\therefore \quad \dfrac{dy}{dx} = 5(-3) \cdot x^{-4} + 3(-2) \cdot x^{-3}$

$= -15x^{-4} - 6x^{-3}$

$= -3x^{-3}(5x^{-1} + 2)$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{-3}{x^3}\left(\dfrac{5}{x} + 2\right) = \dfrac{-3}{x^4}(5 + 2x)$

10. $\dfrac{d}{dx}\left(x^3 - 3x^2 + 2x^{-4}\right)$

$= \dfrac{d}{dx}(x^3) - 3\dfrac{d}{dx}(x^2) + 2\dfrac{d}{dx}(x^{-4})$

$= 3x^2 - 3 \times 2x + 2(-4x^{-5})$

$= 3x^2 - 6x - 8x^{-5}$

11. Let $y = \sqrt{\dfrac{1 + \sin x}{1 - \sin x}} = \sqrt{\dfrac{(1 + \sin x)^2}{1 - \sin^2 x}}$

$= \dfrac{1 + \sin x}{\cos x} = \sec x + \tan x$

Now $\dfrac{dy}{dx} = \dfrac{d}{dx}(\sec x + \tan x) = \sec x \tan x + \sec^2 x$

12. $f(x) = \left(\cos\dfrac{x}{2} + \sin\dfrac{x}{2}\right)^2$

$= \cos^2\dfrac{x}{2} + \sin^2\dfrac{x}{2} + 2\sin\dfrac{x}{2}\cos\dfrac{x}{2}$

$\Rightarrow \quad f(x) = 1 + \sin x$

$\Rightarrow \quad f'(x) = 0 + \cos x = \cos x$

$\therefore \quad f'\left(\dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

13. Let $f(x) = \tan 2x$

$\therefore$ By first principle of differentiation, we have,

$f'(x) = \lim\limits_{h \to 0} \dfrac{\tan(2x + 2h) - \tan 2x}{h}$

$= \lim\limits_{h \to 0} \dfrac{2 \tan 2h}{2h} \cdot [1 + \tan(2x + 2h)\tan 2x]$

$\left[\text{Using } \tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}\right]$

$= 2 \cdot (1 + \tan^2 2x) = 2\sec^2 2x.$

14. We have, $f(x) = x^2 - 6x + 8 \Rightarrow f'(x) = 2x - 6$

$\Rightarrow \quad f'(5) = 10 - 6 = 4;\ f'(2) = 4 - 6 = -2;$

$f'(8) = 16 - 6 = 10.$

$\therefore \quad f'(5) - 3f'(2) = 4 - 3(-2) = 10 = f'(8)$

15. Given, $f(x) = (ax^2 + \sin x)(p + q\cos x)$

$\Rightarrow \quad f'(x) = \dfrac{d}{dx}\{(ax^2 + \sin x)(p + q\cos x)\}$

$= (ax^2 + \sin x).\dfrac{d}{dx}(p + q\cos x) + (p + q\cos x).\dfrac{d}{dx}(ax^2 + \sin x)$

$= (ax^2 + \sin x) \cdot (-q \sin x) + (p + q \cos x) \cdot (2ax + \cos x)$

$= -aqx^2 \sin x - q\sin^2 x + 2apx + p\cos x + 2aqx \cos x$

$+ q \cos^2 x$

16. Given, $f(x) = 10x$

From first principle, $f'(x) = \lim\limits_{h \to 0} \dfrac{10(x + h) - 10x}{h}$,

$\Rightarrow \quad f'(x) = \lim\limits_{h \to 0} \dfrac{10h}{h} = \lim\limits_{h \to 0} 10 = 10$

17. Let $y = \dfrac{1}{ax^2 + b}$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{(ax^2 + b)\dfrac{d}{dx}(1) - 1 \cdot \dfrac{d}{dx}(ax^2 + b)}{(ax^2 + b)^2}$

$= \dfrac{(ax^2 + b)0 - (2ax)}{(ax^2 + b)^2} = \dfrac{-2ax}{(ax^2 + b)^2}$

18. We have,

$f(x) = 1 - x + x^2 - x^3 + \ldots\ldots - x^{99} + x^{100}$

$\Rightarrow \quad f'(x) = 0 - 1 + 2x - 3x^2 + \ldots\ldots - 99x^{98} + 100x^{99}$

$= -1 + 2x - 3x^2 + \ldots\ldots - 99x^{98} + 100x^{99}$

Now, for $x = 1$, $f'(1) = -1 + 2 - 3 + \ldots\ldots -99 + 100$

$= (-1 - 3 - 5 - \ldots\ldots - 99) + (2 + 4 + \ldots\ldots + 100)$

$= -\frac{50}{2}[(2\times 1)+(50-1)2]+\frac{50}{2}[(2\times 2)+(50-1)2]$

$= -25\,[2 + 49 \times 2] + 25[4 + 49 \times 2]$

$= -25\,(2 + 98) + 25\,(4 + 98) = -2500 + 2550 = 50$

19. $\frac{d}{dx}\{x^5(3-6x^{-9})\} = \frac{d}{dx}\{3\cdot x^5 - 6x^{-4}\}$

$= \frac{d}{dx}(3x^5) - \frac{d}{dx}(6x^{-4})$

$= 3\,(5x^4) - 6\,(-4)x^{-5}$

$= 15x^4 + 24\,x^{-5} = 15x^4 + \frac{24}{x^5}$

20. Let $y = \frac{3x+4}{5x^2-7x+9}$

$\Rightarrow \frac{dy}{dx} = \frac{(5x^2-7x+9)\frac{d}{dx}(3x+4)-(3x+4)\frac{d}{dx}(5x^2-7x+9)}{(5x^2-7x+9)^2}$

$= \frac{(5x^2-7x+9)3-(3x+4)(10x-7)}{(5x^2-7x+9)^2}$

$= \frac{15x^2-21x+27-30x^2+21x-40x+28}{(5x^2-7x+9)^2}$

$= \frac{55-15x^2-40x}{(5x^2-7x+9)^2}$

21. We have, $y = \frac{x+\sin x}{(x^2-1)} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{\frac{x+\sin x}{x^2-1}\right\}$

$= \frac{(x^2-1)\frac{d}{dx}(x+\sin x)-(x+\sin x)\cdot\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$

$= \frac{(x^2-1)(1+\cos x)-(x+\sin x)(2x)}{(x^2-1)^2}$

$= \frac{x^2+x^2\cos x-1-\cos x-2x^2-2x\sin x}{(x^2-1)^2}$

$= \frac{-x^2+x^2\cos x-1-\cos x-2x\sin x}{(x^2-1)^2}$

$= \frac{-(x^2+1)+\cos x(x^2-1)-2x\sin x}{(x^2-1)^2}$

22. Here, $f(x) = \frac{2x+7}{x+2}$

Then by using first principle, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{\frac{2(x+h)+7}{x+h+2}-\frac{2x+7}{x+2}}{h}$

$= \lim_{h\to 0}\left(\frac{1}{h}\right)\left[\frac{(2x+2h+7)(x+2)-(2x+7)(x+h+2)}{(x+2)(x+h+2)}\right]$

$= \lim_{h\to 0}\frac{1}{h}\left[\frac{2x^2+4x+2hx+4h+7x+14-(2x^2+2xh+4x+7x+7h+14)}{(x+2)(x+h+2)}\right]$

$= \lim_{h\to 0}\frac{1}{h}\left[\frac{4h-7h}{(x+2)(x+h+2)}\right] = \lim_{h\to 0}\frac{-3h}{h(x+2)(x+h+2)}$

$= \frac{-3}{(x+2)^2}$

23. Let $f(x) = \tan(2x + 3)$

Then, using first principle

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{\tan(2(x+h)+3)-\tan(2x+3)}{h}$

$= \lim_{h\to 0}\frac{\tan(2x+2h+3)-\tan(2x+3)}{h}$

$= \lim_{h\to 0}\frac{\tan(2x+2h+3-2x-3)(1+\tan(2x+2h+3)\tan(2x+3))}{h}$

$= \lim_{h\to 0} 2\times\frac{\tan 2h}{2h}\cdot[1+\tan(2x+2h+3)\tan(2x+3)]$

$= 2\cdot[1+\tan^2(2x+3)] = 2\cdot\sec^2(2x+3)$

24. (i) We have, $\frac{d}{dx}\left[(6x^3+9x)(5x+10)\right]$

$= \frac{d}{dx}\{30x^4+60x^3+45x^2+90x\}$

$= 30 \times 4x^3 + 60 \times 3x^2 + 45 \times 2x + 90 \times 1$

$= 120x^3 + 180x^2 + 90x + 90$

$= 30\,(4x^3 + 6x^2 + 3x + 3)$

(ii) We have, $\frac{d}{dx}\left(\frac{5x+4}{x-3}\right)$

$= \frac{(x-3)\cdot\frac{d}{dx}(5x+4)-(5x+4)\frac{d}{dx}(x-3)}{(x-3)^2}$

$= \frac{(x-3)(5)-(5x+4)\cdot 1}{(x-3)^2}$

$= \frac{5x-15-5x-4}{(x-3)^2} = \frac{-19}{(x-3)^2}$

25. Let $y=\frac{2x^3-\sin x}{\cot x}$

$\Rightarrow \quad \frac{dy}{dx}=\frac{d}{dx}\left\{\frac{2x^3-\sin x}{\cot x}\right\}$

$$=\frac{\cot x\cdot\frac{d}{dx}(2x^3-\sin x)-(2x^3-\sin x)\cdot\frac{d}{dx}(\cot x)}{(\cot x)^2}$$

$$=\frac{\cot x\cdot(6x^2-\cos x)-(2x^3-\sin x)(-\text{cosec}^2 x)}{\cot^2 x}$$

$$=\frac{6x^2\cot x-\cot x\cdot\cos x+2x^3\text{cosec}^2 x-\sin x\cdot\text{cosec}^2 x}{\cot^2 x}$$

$$=\frac{6x^2\cdot\frac{\cos x}{\sin x}-\frac{\cos x}{\sin x}\cdot\cos x+2x^3\frac{1}{\sin^2 x}-\sin x\cdot\frac{1}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}}$$

$$=\frac{(6x^2\cos x-\cos^2 x)\sin x+(2x^3-\sin x)}{\cos^2 x}$$

26. Given, $f(x)=\sqrt{\cos x}$

From first principle, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{\sqrt{\cos(x+h)}-\sqrt{\cos x}}{h}$$

$$=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h[\sqrt{\cos(x+h)}+\sqrt{\cos x}]}$$

$$=\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right).\sin\frac{h}{2}}{h[\sqrt{\cos(x+h)}+\sqrt{\cos x}]}$$

$$=\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right)}{[\sqrt{\cos(x+h)}+\sqrt{\cos x}]}\cdot\frac{\sin\frac{h}{2}}{\frac{h}{2}\cdot 2}$$

$$=\frac{-\sin x}{\sqrt{\cos x}+\sqrt{\cos x}}\times 1=\frac{-\sin x}{2\sqrt{\cos x}}$$

27. Given, $f(x)=x-\frac{1}{x}$

From first principle, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{\left\{(x+h)-\frac{1}{(x+h)}\right\}-\left\{x-\frac{1}{x}\right\}}{h}$$

$$=\lim_{h\to 0}\frac{(x+h-x)-\left\{\frac{1}{(x+h)}-\frac{1}{x}\right\}}{h}$$

$$=\lim_{h\to 0}\frac{h-\left\{\frac{x-x-h}{x(x+h)}\right\}}{h}=\lim_{h\to 0}\left[1+\frac{h}{hx(x+h)}\right]$$

$$=\lim_{h\to 0}\left[1+\frac{1}{x(x+h)}\right]=1+\frac{1}{x^2}$$

28. Let $f(x) = \cos x$

$\therefore$ By first principle, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$=\lim_{h\to 0}\frac{2\sin\left(\frac{x+h+x}{2}\right)\cdot\sin\left(\frac{x-x-h}{2}\right)}{h}$$

$$=\lim_{h\to 0}\frac{2\sin\left(x+\frac{h}{2}\right)\cdot\sin\left(-\frac{h}{2}\right)}{h}$$

$$=\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right)\cdot\sin\frac{h}{2}}{2\left(\frac{h}{2}\right)}$$

$$=\frac{-2\sin(x+0)}{2}=-\sin x$$

29. Let $f(x) = \sin 2x$

Then, using first principle,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sin 2(x+h)-\sin 2x}{h}$$

$$=\lim_{h\to 0}\frac{2\cos\frac{2x+2h+2x}{2}\cdot\sin\frac{2x+2h-2x}{2}}{h}$$

$$=\lim_{h\to 0}2\cos(2x+h)\frac{\sin h}{h}$$

$= 2\cos(2x + 0) \times 1 = 2\cos 2x$

30. Here, $f(x)=\frac{3-x}{3+4x}$

From first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3-(x+h)}{3+4(x+h)} - \left(\frac{3-x}{3+4x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{(3+4x)(3-x-h)-(3-x)(3+4(x+h))}{h(3+4x)(3+4(x+h))}$$

$$= \lim_{h\to 0} \frac{3(3+4x)-x(3+4x)-h(3+4x)-3(3+4x) + x(3+4x)-12h+4xh}{h(3+4x)(3+4(x+h))}$$

$$= \lim_{h\to 0} \frac{-3h-4xh-12h+4xh}{h(3+4x)(3+4(x+h))}$$

$$= \lim_{h\to 0} \frac{-15h}{h(3+4x)[3+4(x+h)]} = \frac{-15}{(3+4x)^2}$$

31. Let $f(x) = x^{2/3}$

We have, $\frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{1}{h}[(x+h)^{2/3} - x^{2/3}]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[x^{2/3}\left(1+\frac{h}{x}\right)^{2/3} - x^{2/3}\right]$$

$$= \lim_{h\to 0} \frac{1}{h} x^{2/3}\left[\left(1+\frac{h}{x}\cdot\frac{2}{3}+\frac{2}{3}\left(\frac{2}{3}-1\right)\frac{h^2}{x^2}\cdot\frac{1}{2}+\ldots\right)-1\right]$$

$$= \lim_{h\to 0} \frac{x^{2/3}}{h}\cdot\frac{2h}{3x}\left(1-\frac{1}{6}\cdot\frac{h}{x}+\ldots\ldots\right)$$

$$= \frac{2}{3}x^{(2/3)-1} = \frac{2}{3}x^{-1/3}$$

32. Let $y = \frac{a+b\sin x}{c+d\cos x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(c+d\cos x)\frac{d}{dx}(a+b\sin x) - (a+b\sin x)\frac{d}{dx}(c+d\cos x)}{(c+d\cos x)^2}$$

$$= \frac{(c+d\cos x)(b\cos x)-(a+b\sin x)(-d\sin x)}{(c+d\cos x)^2}$$

$$= \frac{bc\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c+d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd(\cos^2 x+\sin^2 x)}{(c+d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd}{(c+d\cos x)^2}$$

33. Let $f(x) = \frac{4x+5\sin x}{3x+7\cos x}$ ... (i)

Differentiating (i) with respect to x, we get

$$\frac{d}{dx}\{f(x)\} = \frac{(3x+7\cos x)(4x+5\sin x)' - (4x+5\sin x)(3x+7\cos x)'}{(3x+7\cos x)^2}$$

$$= \frac{(3x+7\cos x)(4+5\cos x) - (4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$$

$$= \frac{12x+15x\cos x+28\cos x+35[\cos^2 x+\sin^2 x] - 12x+28x\sin x - 15\sin x}{(3x+7\cos x)^2}$$

$$= \frac{35+15x\cos x+28\cos x+28x\sin x-15\sin x}{(3x+7\cos x)^2}$$

34. We have, $y = \frac{\sin x+\cos x}{\sin x-\cos x}$

Now, $$\frac{dy}{dx} = \frac{(\sin x-\cos x)(\cos x-\sin x) - (\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$$

$$= \frac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}$$

$$= \frac{-[(\sin x-\cos x)^2+(\sin x+\cos x)^2]}{(\sin x-\cos x)^2}$$

$$= \frac{-2}{(\sin x-\cos x)^2}$$

Hence, $\left(\frac{dy}{dx}\right)_{x=0} = -2$

35. (i) Let $f(x) = 5\sin x - 6\cos x + 7$ ... (1)

Differentiating (1) with respect to x, we get

$f'(x) = 5\cos x - 6(-\sin x) + 0$

$\therefore \quad f'(x) = 5\cos x + 6\sin x.$

(ii) Let $f(x) = 3\cot x + 5\operatorname{cosec} x$ ... (1)

Differentiating (1) with respect to x, we get

$\Rightarrow \quad f'(x) = -3\operatorname{cosec}^2 x - 5\cot x \cdot \operatorname{cosec} x$

$= -\operatorname{cosec} x[3\operatorname{cosec} x + 5\cot x].$

(iii) Let $f(x) = 2\tan x - 7\sec x$ ... (1)

Differentiating (1) with respect to x, we get

$f'(x) = 2\sec^2 x - 7\sec x\tan x$

36. Let $f(x)=\cot\sqrt{x}$

From first principle,

$$\frac{d}{dx}f(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\cot\sqrt{x+h}-\cot\sqrt{x}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\cos\sqrt{x+h}}{\sin\sqrt{x+h}}-\frac{\cos\sqrt{x}}{\sin\sqrt{x}}}{h}$$

$$=\lim_{h\to 0}\frac{\sin\sqrt{x}\cos\sqrt{x+h}-\cos\sqrt{x}\sin\sqrt{x+h}}{h(\sin\sqrt{x+h}\sin\sqrt{x})}$$

$$=\lim_{h\to 0}\frac{-\sin(\sqrt{x+h}-\sqrt{x})}{h\sin\sqrt{x+h}\sin\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{-\sin(\sqrt{x+h}-\sqrt{x})}{(x+h-x)\sin\sqrt{x+h}\sin\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{-\sin(\sqrt{x+h}-\sqrt{x})}{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})\sin\sqrt{x+h}\sin\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{\sin(\sqrt{x+h}-\sqrt{x})}{(\sqrt{x+h}-\sqrt{x})}\times\frac{-1}{(\sqrt{x+h}+\sqrt{x})\sin\sqrt{x+h}\cdot\sin\sqrt{x}}$$

$$=\frac{-1}{2\sqrt{x}\sin\sqrt{x}\sin\sqrt{x}}=\frac{-\operatorname{cosec}^2\sqrt{x}}{2\sqrt{x}}$$

37. Let $f(x)=\sin x+\cos x$

By first principle, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$f'(x)=\lim_{h\to 0}\frac{\sin(x+h)+\cos(x+h)-\sin x-\cos x}{h}$$

$$=\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h+\cos x\cos h-\sin x\sin h-\sin x-\cos x}{h}$$

$$=\lim_{h\to 0}\frac{\sin h(\cos x-\sin x)+\sin x(\cos h-1)+\cos x(\cos h-1)}{h}$$

$$=\lim_{h\to 0}\frac{\sin h}{h}(\cos x-\sin x)+\lim_{h\to 0}\frac{\sin x(\cos h-1)}{h}+\lim_{h\to 0}\frac{\cos x(\cos h-1)}{h}$$

$= \cos x - \sin x + \sin x\ (0) + \cos x\ (0)$

$= \cos x - \sin x$

38. (i) Consider $f(x)=\cot x$

From first principle, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{\cot(x+h)-\cot x}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\cos(x+h)}{\sin(x+h)}-\frac{\cos x}{\sin x}}{h}$$

$$=\lim_{h\to 0}\frac{\sin x\cos(x+h)-\cos x\sin(x+h)}{h\cdot\sin(x+h)\sin x}$$

$$=\lim_{h\to 0}\frac{\sin(x-x-h)}{h\cdot\sin(x+h)\sin x}$$

$$=\lim_{h\to 0}\frac{-\sin h}{h}\cdot\frac{1}{\sin(x+h)\sin x}$$

$$\Rightarrow f'(x)=\frac{-1}{\sin x\cdot\sin x}=-\operatorname{cosec}^2 x$$

(ii) Given, $f(x)=\frac{x}{1+\tan x}$

Now, $f'(x)=\frac{d}{dx}\left(\frac{x}{1+\tan x}\right)$

$$=\frac{(1+\tan x)\frac{d(x)}{dx}-x\cdot\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)-x\cdot\sec^2 x}{(1+\tan x)^2}=\frac{1+\tan x-x\sec^2 x}{(1+\tan x)^2}$$

39. Let $f(x)=\operatorname{cosec} x$

Using first principle, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{\operatorname{cosec}(x+h)-\operatorname{cosec}x}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{\sin(x+h)}-\frac{1}{\sin x}}{h}$$

$$=\lim_{h\to 0}\frac{\sin x-\sin(x+h)}{\sin x\cdot\sin(x+h)\cdot h}$$

$$= \lim_{h \to 0} \frac{2.\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin x \cdot \sin(x+h) \cdot h}$$

$$= \lim_{h \to 0} \frac{-2 \cdot \cos\left(x + \frac{h}{2}\right)}{\sin x \cdot \sin(x+h)} \cdot \frac{\sin \frac{h}{2}}{\frac{h}{2} \cdot 2}$$

$$= \lim_{h \to 0} \frac{-\cos\left(x + \frac{h}{2}\right)}{\sin x \cdot \sin(x+h)} \cdot \lim_{h \to 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}}$$

$$= \frac{-\cos(x+0)}{\sin x \cdot \sin(x+0)} \times 1 = \frac{-\cos x}{\sin x \sin x}$$

$\therefore \quad f'(x) = -\operatorname{cosec} x \cdot \cot x.$

40. We have,

$$f(x) = \frac{x^n - a^n}{x - a}$$

$$\Rightarrow f'(x) = \frac{(x-a)\frac{d}{dx}(x^n - a^n) - (x^n - a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$$

$$\Rightarrow f'(x) = \frac{(x-a)nx^{n-1} - (x^n - a^n)(1)}{(x-a)^2}$$

$$\Rightarrow f'(x) = \frac{nx^{n-1}(x-a) - x^n + a^n}{(x-a)^2}$$

At $x = a$, $f'(a) = \frac{na^{n-1}(0) - a^n + a^n}{(a-a)^2}$

$$\Rightarrow \quad f'(a) = \frac{0}{0}$$

Therefore, $f'(a)$ does not exist.

Because, $f(x)$ is not defined at $x = a$.

So, $f'(x)$ at $x = a$ does not exist.