Class IX Session 2024-25
Subject - Mathematics
Sample Question Paper - 12

Time: 3 hours **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case study based questions carrying 4 marks each with subparts of 1, 1, and 2 marks each, respectively.
7. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. Find $512^{\frac{2}{3}}$.

A. 16
B. 64
C. 8
D. 46

2. Express 0.6666... in the form of $\frac{p}{q}$.

A. $\frac{5}{2}$
B. $\frac{2}{5}$
C. $\frac{3}{2}$
D. $\frac{2}{3}$

3. If the measures of the sides of a triangle is given by a, b and c, then the semi-perimeter =
   A. $\frac{a+b+c}{2}$
   B. $\frac{a+b+c}{3}$
   C. $\frac{a+b+c}{4}$
   D. $2(a+b+c)$

4. If the height of a cone is doubled, then % increase in new volume will be
   A. 50%
   B. 100%
   C. 150%
   D. 200%

5. Write the coefficient of x in the polynomial $4x + 6x^2 - 8$.
   A. 0
   B. 1
   C. 6
   D. 4

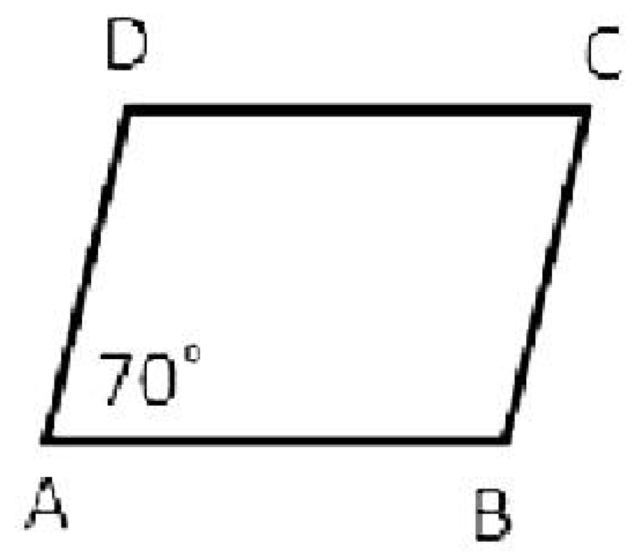
6. $203 \times 205 =$
   A. 41165
   B. 41615
   C. 42615
   D. 41625

7. Which of the following is not the test of congruency for two triangles?
   A. ASA
   B. AAA
   C. SAS
   D. SSS

8. If two lines intersect, vertically opposite angles are ______
   A. equal
   B. supplementary
   C. complementary
   D. right angles

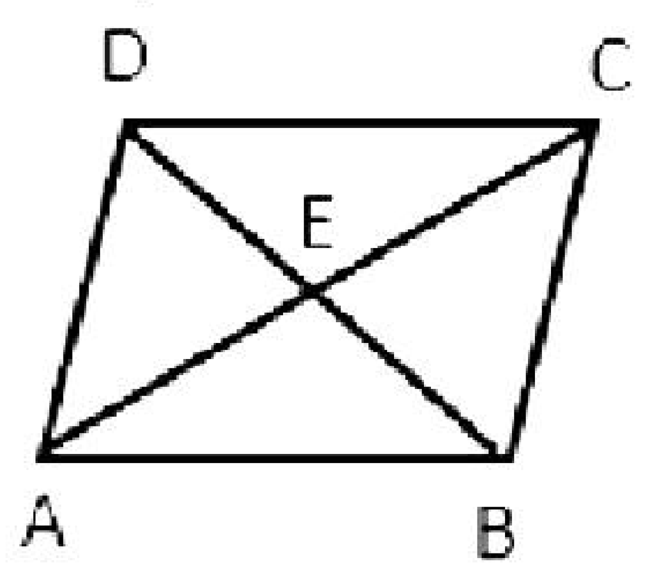
9. Sum of a linear pair of angles is
   A. 45°
   B. 90°
   C. 160°
   D. 180°

10. In the given figure, ABCD is a parallelogram in which $\angle A = 70°$. Calculate $\angle C$.



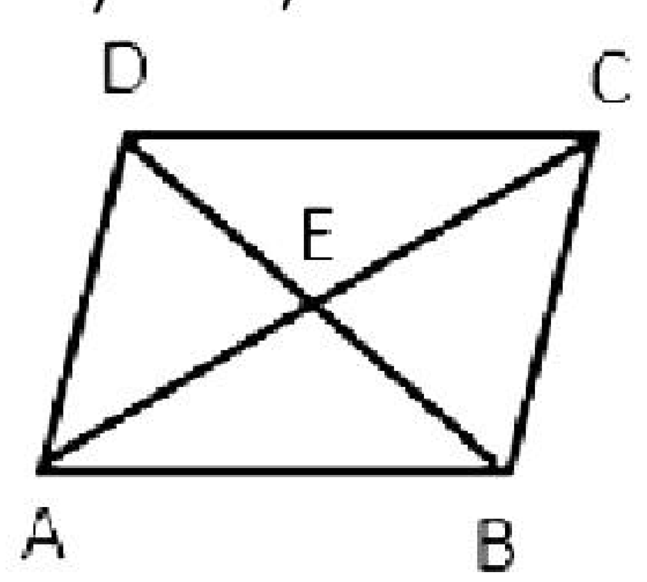
A. 110°
B. 80°
C. 60°
D. 70°

11. Let ABCD be a parallelogram. The diagonals bisect at E. If DE = 5 cm and AE = 7 cm, then find AC.



A. 10 cm
B. 14 cm
C. 5 cm
D. 7 cm

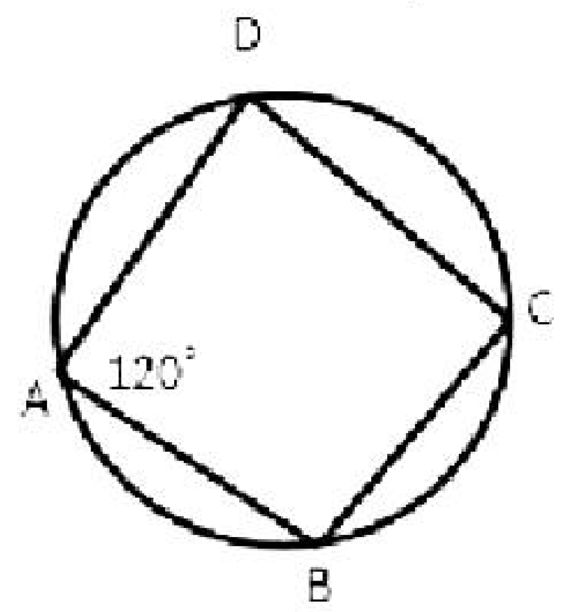
12. Let ABCD be a parallelogram. If AD = x + 2y, BC = 2x + 3, DC = x + 7 and AB = 3y + 2, then find BC.



A. 8 cm
B. 7 cm
C. 6 cm
D. 5 cm

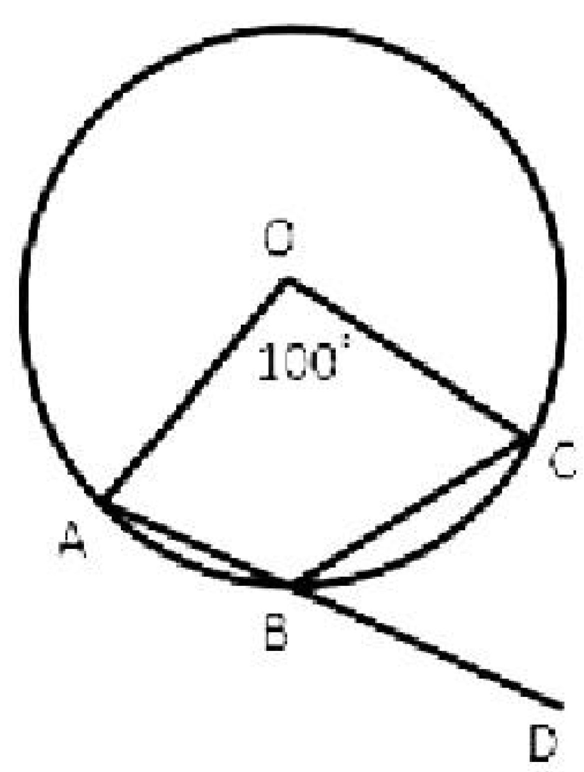
13. Two chords AB and CD are equidistant from the centre, then
A. AB $\perp$ CD
B. AB = CD
C. AB || CD
D. AB < CD

14. ABCD is a cyclic quadrilateral. Find ∠C.



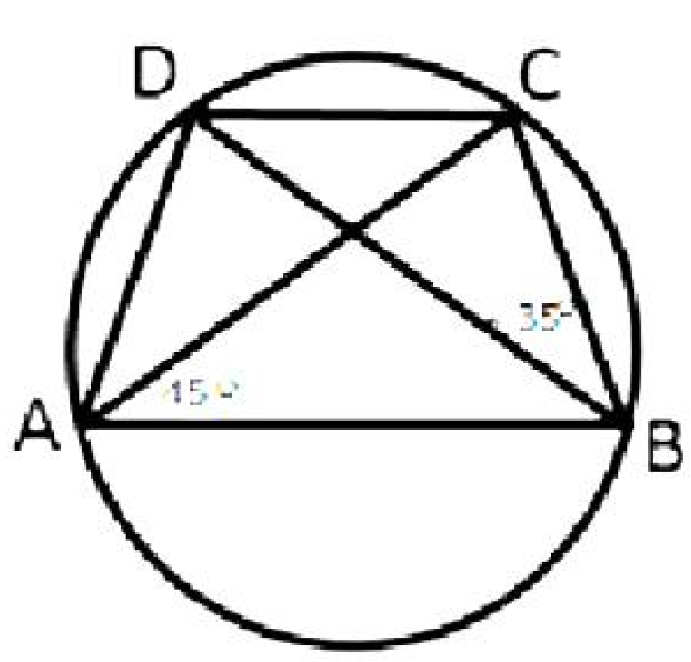
A. 110°
B. 80°
C. 60°
D. 70°

15. In the given figure, O is the centre of a circle in which ∠AOC = 100°. If the side AB of a quadrilateral OABC is produced to D, then find ∠CBD.



A. 50°
B. 80°
C. 90°
D. 40°

16. In the given figure, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If ∠DBC = 35° and ∠BAC = 45°, then find ∠BCD.



A. 100°
B. 45°
C. 55°
D. 80°

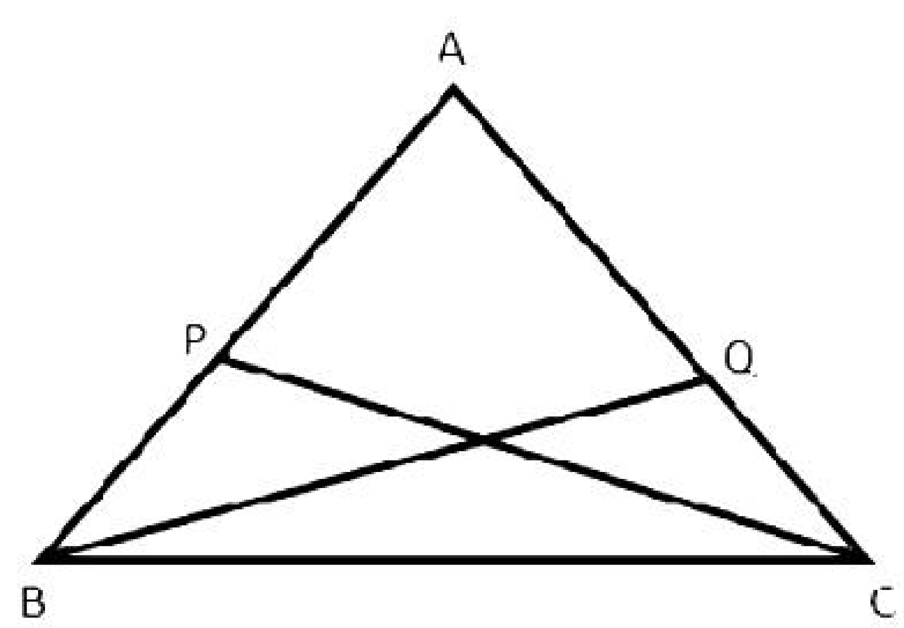
17. In a linear pair, if one angle is equal to twice the other, then find x.
   A. 50°
   B. 60°
   C. 70°
   D. 40°

18. ___________ are assumptions which are obvious universal truths.
   A. Identities
   B. Postulates
   C. Theorems
   D. Axioms

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19. **Statement A (Assertion)**: If $x + y = 2$, then $x + y - z = 2 + z$.
   Statement R (Reason): If equals are subtracted from equals, then the remainders are also equal.
   A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
   B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
   C. Assertion (A) is true but reason (R) is false.
   D. Assertion (A) is false but reason (R) is true.

20. **Statement A (Assertion)**: In ΔABC, $AB = AC$. If P is a point on AB and Q is a point on AC such that $AP = AQ$, then $\Delta APC \cong \Delta AQB$.



   Statement R (Reason): If two corresponding sides and an included angle of two triangles are equal, then by SAS congruence criteria, both triangles are congruent.

   A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
   B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
   C. Assertion (A) is true but reason (R) is false.
   D. Assertion (A) is false but reason (R) is true.

Section B

Section B consists of 5 questions of 2 marks each.

21. Find the value of the polynomial $5x - 4x^2 + 3$ at [2]
(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

22. If $x + \frac{1}{5x} = 3$, find the value of $x^3 + \frac{1}{125x^3}$. [2]

23. Simplify: $\frac{1}{7+4\sqrt{3}} + \frac{1}{2+\sqrt{5}}$ [2]

24. Rationalise the denominator: $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$ [2]

OR

If a and b are rational numbers, then find the values of a and b in equation: $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$

25. If $x = 3$ and $y = 2$ is a solution of the equation $5x - 7y = k$, then find the value of k and write the resultant equation. [2]

OR

If (0, a) and (b, 0) are the solutions of the linear equation $3x = 7y - 21$. Find a and b.

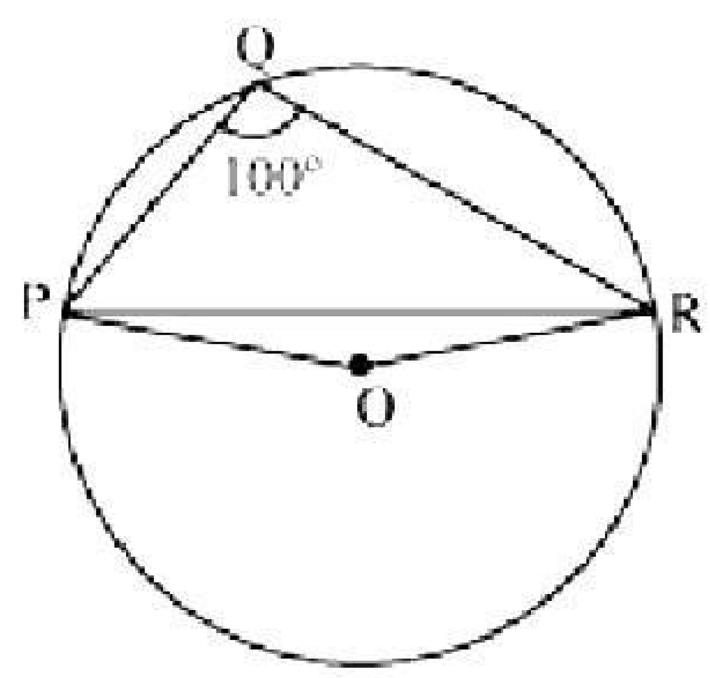
Section C

Section C consists of 6 questions of 3 marks each.

26. Sides of a triangle are in the ratio of 12:17:25 and its perimeter is 540 cm. Find its area. [3]

27. Determine which of the following polynomials has $(x + 1)$ as a factor: [3]
    (i) $x^3 + x^2 + x + 1$
    (ii) $x^4 + x^3 + x^2 + x + 1$
    (iii) $x^4 + 3x^3 + 3x^2 + x + 1$

28. In the given figure, $\angle PQR = 100^\circ$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$. [3]



OR

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.

29. Is it possible to construct a frequency polygon without histogram? If yes, construct a frequency polygon for the below data. [3]

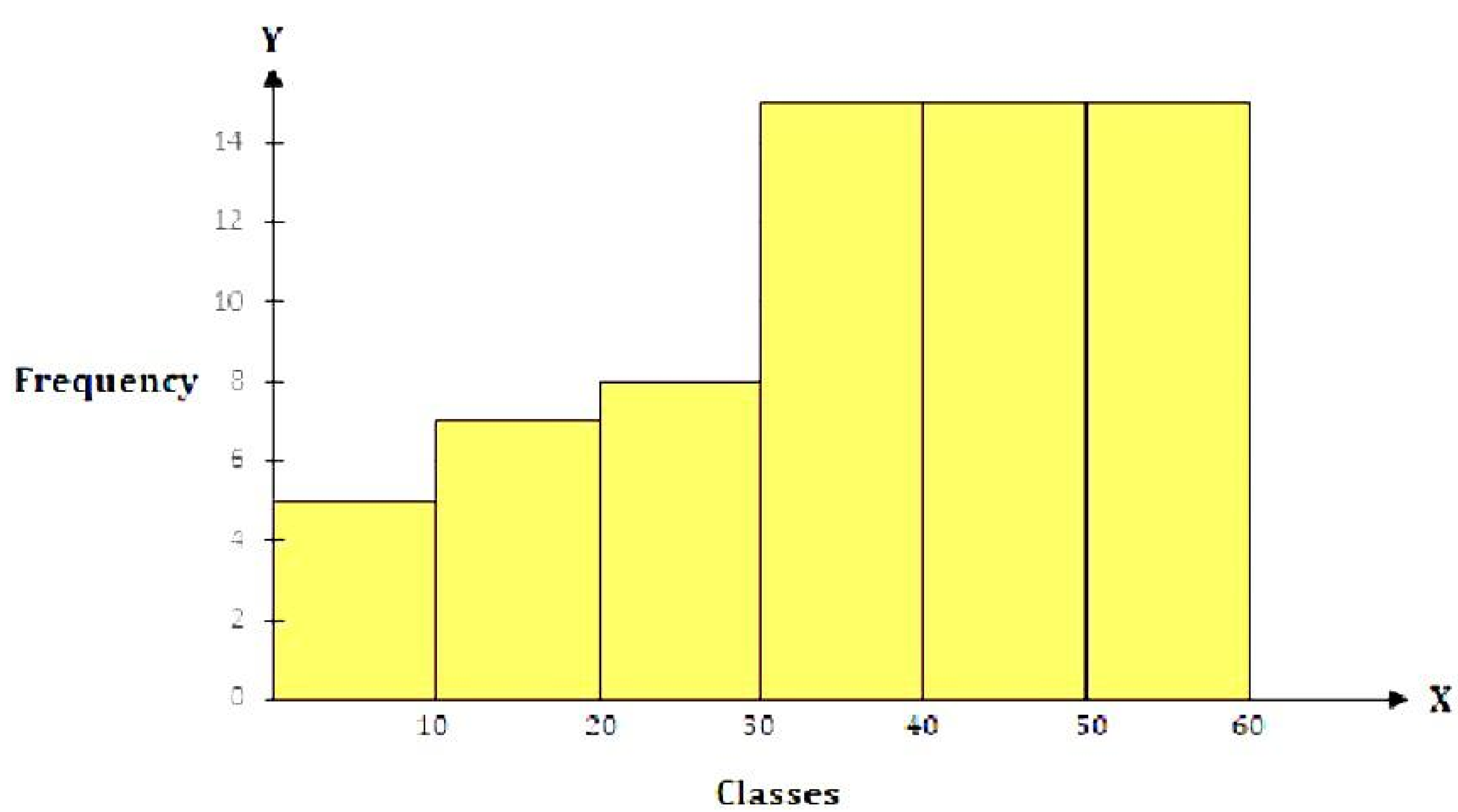
Classes	Frequency
Under 10	7
Under 20	19
Under 30	34
Under 40	44
Under 50	50

OR

The below data is given for constructing histogram.

Classes	Frequency
0-10	5
10-20	7
20-30	8
30-60	15

Anvi represented this data as below:



Teacher said that there is an error in this histogram.
What do you think? Support your answer with proper reason and graph.

30. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone. [3]

31. Distribution of weight (in kg) of 100 people is given below: [3]

Weight in Kg	Frequency
40–45	13
45–50	25
50–55	28
55–60	15
60–65	12
65–70	5
70–75	2

Construct a histogram for the above distribution.

Section D
Section D consists of 4 questions of 5 marks each.

32. Use suitable identities to find the following products: [5]

i) $(x + 4)(x + 10)$

ii) $(x + 8)(x - 10)$

iii) $(3x + 4)(3x - 5)$

iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

v) $(3 - 2x)(3 + 2x)$

OR

Expand each of the following, using suitable identities:

i) $(x + 2y + 4z)^2$

ii) $(2x - y + z)^2$

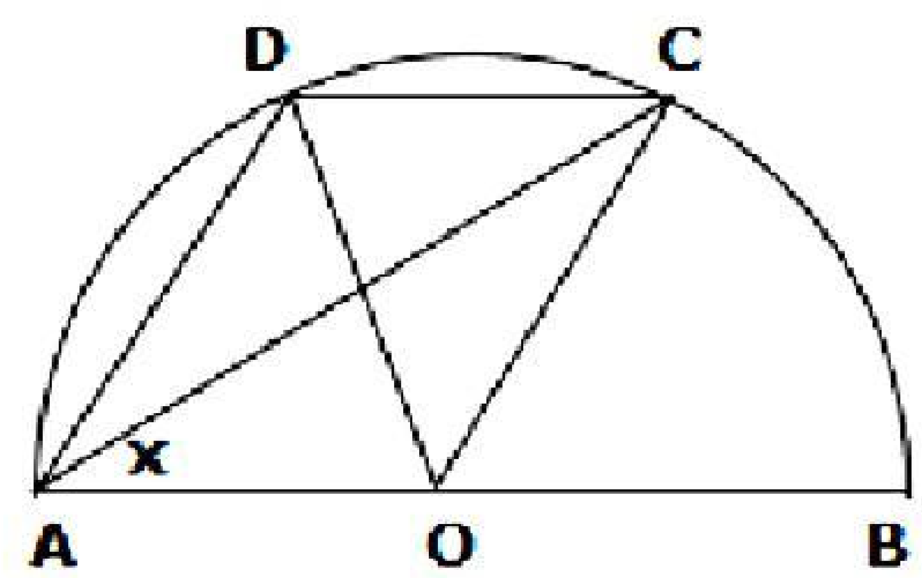
iii) $(-2x + 3y + 2z)^2$

iv) $(3a - 7b - c)^2$

v) $(-2x + 5y - 3z)^2$

33. If each diagonal of a quadrilateral separates it into two triangles of equal area, then show that the quadrilateral is a parallelogram. [5]

34. In the given figure, AOB is a diameter and DC is parallel to AB. If ∠CAB = x, then find (in terms of x) the values of ∠COB, ∠DOC, ∠DAC and ∠ADC. [5]



OR

AB and CD are two chords on the same side of a circle. AB = 12 cm and CD = 24 cm. The distance between two parallel chords is 4 cm. Find the radius of the circle.

35. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to the corresponding segments of the other chord. [5]

Section E

Case study-based questions are compulsory.

36. Real numbers are broadly classified as rational and irrational numbers. Rational numbers are the ones that can be expressed in p/q form, where q is not equal to zero and p and q are co-prime. Whereas irrational numbers can't be expressed in p/q from. The decimal expansion of a rational number can either be recurring or terminating. But the decimal expansion of an irrational number is neither recurring nor terminating. Based on the given information, answer the following questions.

i. The decimal expansion of $\frac{532}{17}$ is_____ [2]

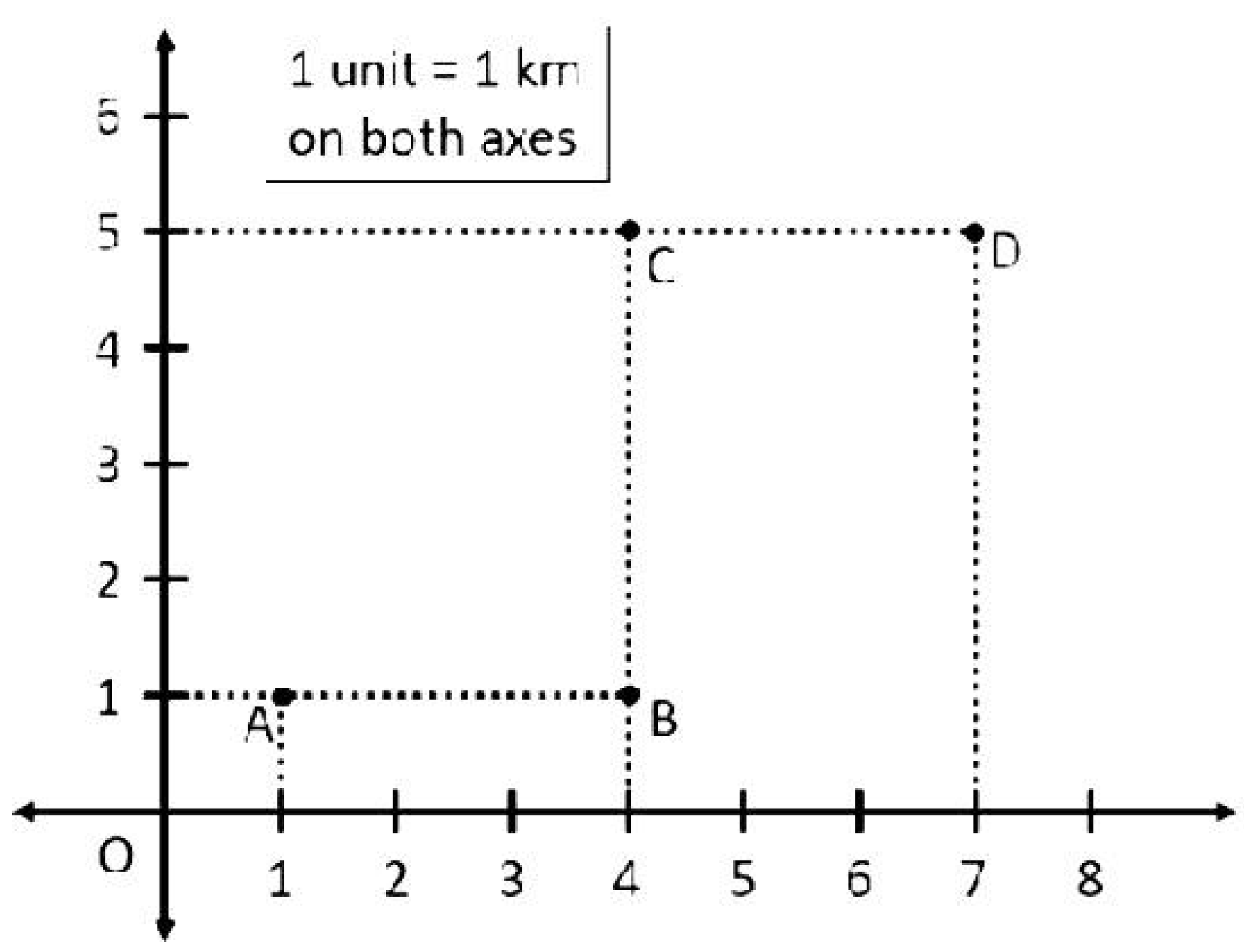
OR

Convert 1.235235235.... in p/q form [2]

ii. π is a/an _______(rational/irrational) number [1]

iii. Irrational numbers can't be represented on number line, true or false. [1]

37. The map showing the locations of houses of Amy (A), Betty (B), Clair (C) and Diana (D), is shown below. Using the details given, answer the following questions.



i. The co-ordinates of point A is _____ [1]

ii. The co-ordinates of point B is _____ [1]

iii. The distance between points A and B is_______ [2]

OR

The distance between points C and B is_______ [2]

38. A shopkeeper sells two varieties of rice: Type A and Type B. The cost incurred for the farmer to grow Type A and Type B rice were Rs. 12 per kg and Rs. 10 per kg respectively. And the selling price of Type A rice and Type B rice were Rs. 18 per kg and Rs. 14 per kg respectively. Now if the farmer has sold 'x' kg of Type A rice and 'y' kg of Type B rice, then answer the following questions.

i. The total cost price for the farmer is given by _____ [1]

ii. The total selling price for the farmer is given by _____ [1]

iii. The total profit earned by the farmer is given by _____ [2]

OR

The profit percent is given by _____ [2]

Solution

Section A

1. Correct option: B

Explanation:

$$512^{\frac{2}{3}} = \left(8^3\right)^{\frac{2}{3}} = \left(8\right)^{3\times\frac{2}{3}} = 8^2 = 64$$

2. Correct option: D

Explanation:

Let x = 0.6666 ....(1)

Then, 10x = 6.6666 ....(2)

Subtracting (1) from (2),

9x = 6

$\Rightarrow x = \dfrac{6}{9} = \dfrac{2}{3}$

3. Correct option: A

Explanation:

Sides of a triangle are a, b and c respectively.

Then, semi-perimeter = $\dfrac{a+b+c}{2}$

4. Correct option: B

Explanation:

Let the original height of a cone be h.

Then, its volume $= \dfrac{1}{3}\pi r^2 h$

New height = 2h

New volume $= \dfrac{1}{3}\pi r^2 2h = \dfrac{2}{3}\pi r^2 h$

$\therefore$ Increase in volume $= \dfrac{2}{3}\pi r^2 h - \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi r^2 h$

$\therefore$ %Increase $= \dfrac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h} \times 100 = 100\%$

5. Correct option: D

Explanation:

Given polynomial is $4x + 6x^2 - 8$

Hence, the coefficient of x is 4.

6. Correct option: B
Explanation:
$203 \times 205 = (200 + 3)(200 + 5)$
$= 200^2 + (3 + 5) \times 200 + 3 \times 5$
$= 40000 + 1600 + 15$
$= 41615$

7. Correct option: B
Explanation:
AAA is not the test of congruency of two triangles.

8. Correct option: A
Explanation:
If two lines intersect, vertically opposite angles are equal.

9. Correct option: D
Explanation:
Sum of a linear pair of angles is 180°.

10. Correct option: D
Explanation:
In the parallelogram, opposite angles are equal.
$\therefore \angle A = \angle C = 70^\circ$

11. Correct option: B
Explanation:
AE = 7 cm and since diagonals bisect at E.
$\therefore AC = 2AE = 2 \times 7 = 14$ cm

12. Correct option: D
Explanation:
Opposite sides of a parallelogram are equal.
AD = BC
$\therefore x + 2y = 2x + 3$
$\therefore x - 2y = -3$ ....(1)
Also, AB = DC
$\therefore 3y + 2 = x + 7$
$\therefore x - 3y = -5$ ....(2)
Solving equations (1) and (2), we get
$x = 1$ and $y = 2$
Hence, $BC = 2x + 3 = 2(1) + 3 = 5$ cm

13. Correct option: B
Explanation:
Equal chords of a circle are equidistant from the centre.

14. Correct option: C

Explanation:

ABCD is a cyclic quadrilateral and opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \angle A + \angle C = 180°$

$\Rightarrow 120° + \angle C = 180°$

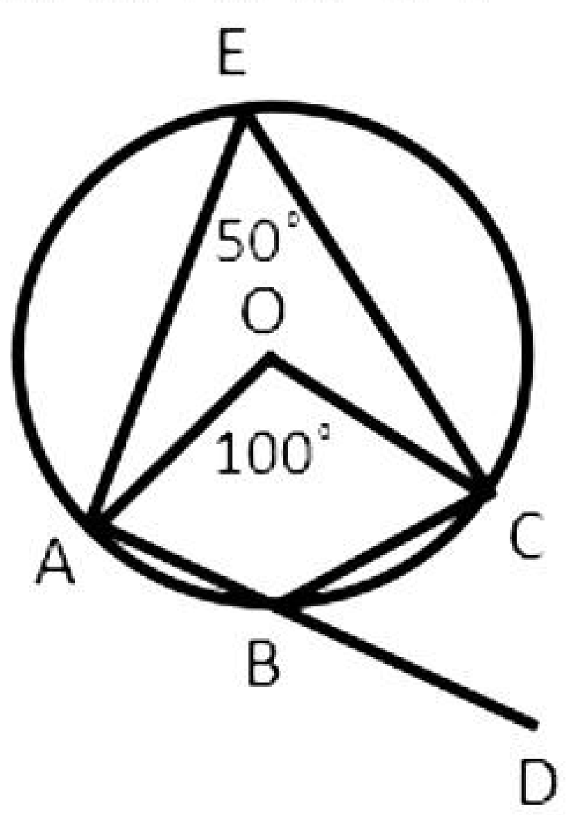
$\Rightarrow \angle C = 60°$

15. Correct option: A

Explanation:

Take a point E on the remaining part of the circumference of the circle.

Join AE and CE.



Angle made by the arc at the centre is twice the angle subtended by the same arc at any point on the circumference of the circle.

$\Rightarrow \angle AEC = \frac{1}{2} \times \angle AOC = \frac{1}{2} \times 100° = 50°$

Now, side AB of a cyclic quadrilateral ABCE has been produced to D.

$\Rightarrow \angle AEC + \angle CBA = 180°$ ...(opposite angles of cyclic quadrilateral are supplementary)

$\Rightarrow 50° + \angle CBA = 180°$

$\Rightarrow \angle CBA = 130°$

Now, $\angle CBA + \angle CBD = 180°$ ... (linear pair)

$\Rightarrow 130° + \angle CBD = 180°$

$\Rightarrow \angle CBD = 50°$

16. Correct option: A

Explanation:

Since the angles in the same segment of a circle are equal.

$\Rightarrow \angle CAD = \angle DBC = 35°$

Now,

$\angle DAB = \angle CAD + \angle BAC = 35° + 45° = 80°$

But, $\angle DAB + \angle BCD = 180°$ ... (opposite angles of a cyclic quadrilateral)

$\Rightarrow \angle BCD = 180° - 80° = 100°$

17. Correct option: B
Explanation:
Let the smaller angle = x
Then, larger angle = 2x
Now, $x + 2x = 180°$
$\Rightarrow 3x = 180°$
$\Rightarrow x = 60°$

18. Correct option: D
Explanation:
Axioms are assumptions which are obvious universal truths.

19. Correct option: D
Explanation:
Given: $x + y = 2$
If equals are subtracted from equals, then the remainders are also equal.
So, the reason is true.
$\Rightarrow x + y - z = 2 - z$
Hence, the assertion is false.

20. Correct option: A
Explanation: In ΔAPC and ΔAQB,
$AP = AQ$ (given)
$AC = AB$ (given)
$\angle CAP = \angle BAQ$ (common)
$\therefore \Delta APC \cong \Delta AQB$ (SAS congruence criteria)
Hence, both assertion and reason are true and reason is the correct explanation for assertion.

Section B

21.

(i) $p(x) = 5x - 4x^2 + 3$

$$p(0) = 5(0) - 4(0)^2 + 3 = 3$$

(ii) $p(x) = 5x - 4x^2 + 3$

$$p(-1) = 5(-1) - 4(-1)^2 + 3$$
$$= -5 - 4 + 3$$
$$= -6$$

(iii) $p(x) = 5x - 4x^2 + 3$

$$p(2) = 5(2) - 4(2)^2 + 3$$
$$= 10 - 16 + 3$$
$$= -3$$

22. Given, $x + \frac{1}{5x} = 3$

$$\Rightarrow \left(x + \frac{1}{5x}\right)^3 = 3^3$$

$$\Rightarrow x^3 + \frac{1}{125x^3} + 3 \times x \times \frac{1}{5x}\left(x + \frac{1}{5x}\right) = 27$$

$$\Rightarrow x^3 + \frac{1}{125x^3} + \frac{3}{5} \times 3 = 27$$

$$\Rightarrow x^3 + \frac{1}{125x^3} + \frac{9}{5} = 27$$

$$\Rightarrow x^3 + \frac{1}{125x^3} = 27 - \frac{9}{5} = \frac{126}{5}$$

$$\Rightarrow x^3 + \frac{1}{125x^3} = 25\frac{1}{5}$$

23.

$$\frac{1}{7 + 4\sqrt{3}} + \frac{1}{2 + \sqrt{5}} = \frac{1}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}} + \frac{1}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}}$$

$$= \frac{7 - 4\sqrt{3}}{7^2 - \left(4\sqrt{3}\right)^2} + \frac{2 - \sqrt{5}}{2^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{7 - 4\sqrt{3}}{49 - 48} + \frac{2 - \sqrt{5}}{4 - 5}$$

$$= 7 - 4\sqrt{3} - \left(2 - \sqrt{5}\right)$$

$$= 7 - 4\sqrt{3} - 2 + \sqrt{5}$$

$$= 5 - 4\sqrt{3} + \sqrt{5}$$

24.

$$\frac{\sqrt{2}}{\sqrt{2} + \sqrt{3} - \sqrt{5}} = \frac{\sqrt{2}}{\left(\sqrt{2} + \sqrt{3}\right) - \sqrt{5}} \times \frac{\left(\sqrt{2} + \sqrt{3}\right) + \sqrt{5}}{\left(\sqrt{2} + \sqrt{3}\right) + \sqrt{5}}$$

$$= \frac{\sqrt{2}\left(\sqrt{2} + \sqrt{3} + \sqrt{5}\right)}{\left(\sqrt{2} + \sqrt{3}\right)^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{\sqrt{2}\left(\sqrt{2} + \sqrt{3} + \sqrt{5}\right)}{2 + 3 + 2\sqrt{2 \times 3} - 5}$$

$$= \frac{\sqrt{2}\left(\sqrt{2} + \sqrt{3} + \sqrt{5}\right)}{2\sqrt{6}}$$

$$= \frac{\sqrt{2}\left(\sqrt{2} + \sqrt{3} + \sqrt{5}\right)}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{\sqrt{24} + \sqrt{36} + \sqrt{60}}{2 \times 6}$$

$$= \frac{\sqrt{6 \times 4} + 6 + \sqrt{15 \times 4}}{12}$$

$$= \frac{2\sqrt{6} + 6 + 2\sqrt{15}}{12}$$

$$= \frac{2\left(\sqrt{6} + 3 + \sqrt{15}\right)}{12}$$

$$= \frac{\sqrt{6} + 3 + \sqrt{15}}{6}$$

OR

$$\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = a + b\sqrt{6}$$

$$\Rightarrow \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = a + b\sqrt{6}$$

$$\Rightarrow \frac{\left(\sqrt{3} + \sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2} = a + b\sqrt{6}$$

$$\Rightarrow \frac{\left(\sqrt{3}\right)^2 + 2\sqrt{3}\sqrt{2} + \left(\sqrt{2}\right)^2}{3 - 2} = a + b\sqrt{6}$$

$$\Rightarrow 3 + 2\sqrt{6} + 2 = a + b\sqrt{6}$$

$$\Rightarrow 5 + 2\sqrt{6} = a + b\sqrt{6}$$

$\therefore$ a = 5 and b = 2

25. x = 3 and y = 2 is a solution of the equation 5x – 7y = k.

$\Rightarrow$ 5 × 3 – 7 × 2 = k

$\Rightarrow$ 15 – 14 = k

$\Rightarrow$ k = 1

Therefore, the resultant equation is 5x – 7y = 1.

OR

The given linear equation is 3x = 7y – 21.

(0, a) is the solution of given linear equation.

$\Rightarrow$ 0 = 7a − 21

$\Rightarrow$ 7a = 21

$\Rightarrow$ a = 3

(b, 0) is the solution of given linear equation.

$\Rightarrow$ 3b = 0 − 21

$\Rightarrow$ 3b = −21

$\Rightarrow$ b = −7

Section C

26. Let the common ratio between the sides of a given triangle be x.

So, sides of the triangle will be 12x, 17x, and 25x.

Perimeter of this triangle = 540 cm

12x + 17x + 25x = 540 cm

54x = 540 cm

x = 10 cm

Thus, the sides of triangle are 120 cm, 170 cm, and 250 cm.

$$s = \frac{\text{perimeter of triangle}}{2} = \frac{540\text{ cm}}{2} = 270\text{ cm}$$

By Heron's formula,

$$\begin{aligned}
\text{Area of a triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\
&= \left[\sqrt{270(270-120)(270-170)(270-250)}\right]\text{cm}^2 \\
&= \left[\sqrt{270\times150\times100\times20}\right]\text{cm}^2 \\
&= \left[\sqrt{30\times9\times30\times5\times100\times4\times5}\right]\text{cm}^2 \\
&= \left[30\times3\times5\times10\times2\right]\text{cm}^2 \\
&= 9000\text{ cm}^2
\end{aligned}$$

So, the area of this triangle will be 9000 cm^2.

27.

(i) If (x + 1) is a factor of $p(x) = x^3 + x^2 + x + 1$, $p(-1)$ must be zero.

Here, $p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

Hence, (x + 1) is a factor of this polynomial.

(ii) If (x + 1) is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$, $p(-1)$ must be zero.

Here, $p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 - 1 + 1 - 1 + 1 = 1$

As, $p(-1) \neq 0$

So, (x + 1) is not a factor of this polynomial.

(iii) If (x + 1) is a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$, $p(-1)$ must be 0.

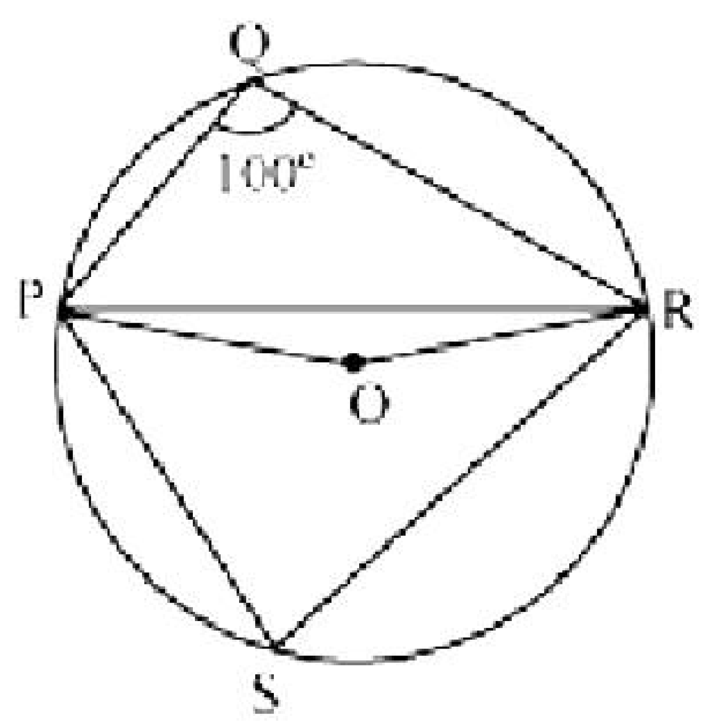
Here, $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1 = 1 - 3 + 3 - 1 + 1 = 1$

As, $p(-1) \neq 0$

So, (x + 1) is not a factor of this polynomial.

28.



Consider PR as a chord of a circle.

Take any point S on the major arc of a circle.

Now PQRS is a cyclic quadrilateral.

$\angle PQR + \angle PSR = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$\Rightarrow \angle PSR = 180^\circ - 100^\circ = 80^\circ$

Now, that the angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$\therefore \angle POR = 2\angle PSR = 2(80^\circ) = 160^\circ$

In ΔPOR

OP = OR (radii of the same circle)

$\therefore \angle OPR = \angle ORP$ (Angles opposite to equal sides of a triangle)

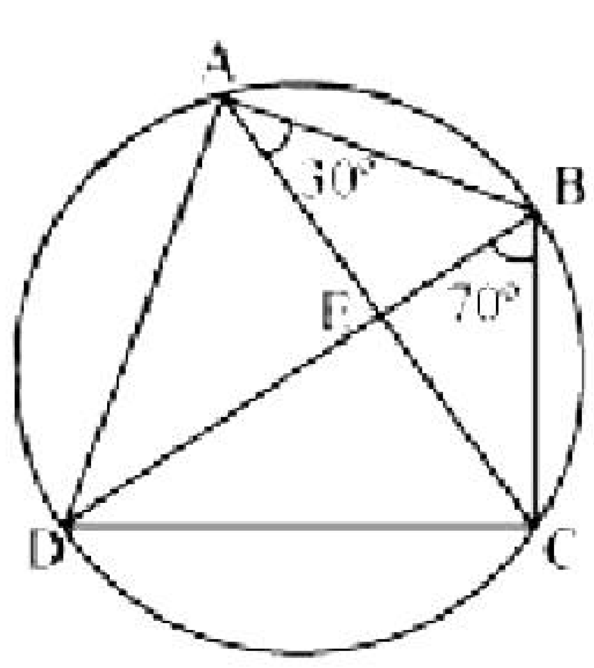
$\angle OPR + \angle ORP + \angle POR = 180^\circ$ (Angle sum property of a triangle)

$2\angle OPR + 160^\circ = 180^\circ$

$2\angle OPR = 180^\circ - 160^\circ = 20^\circ$

$\angle OPR = 10^\circ$

OR



For chord CD

$\angle CBD = \angle CAD$ (Angles in the same segment)

$\angle CAD = 70^\circ$

$\angle BAD = \angle BAC + \angle CAD = 30^\circ + 70^\circ = 100^\circ$

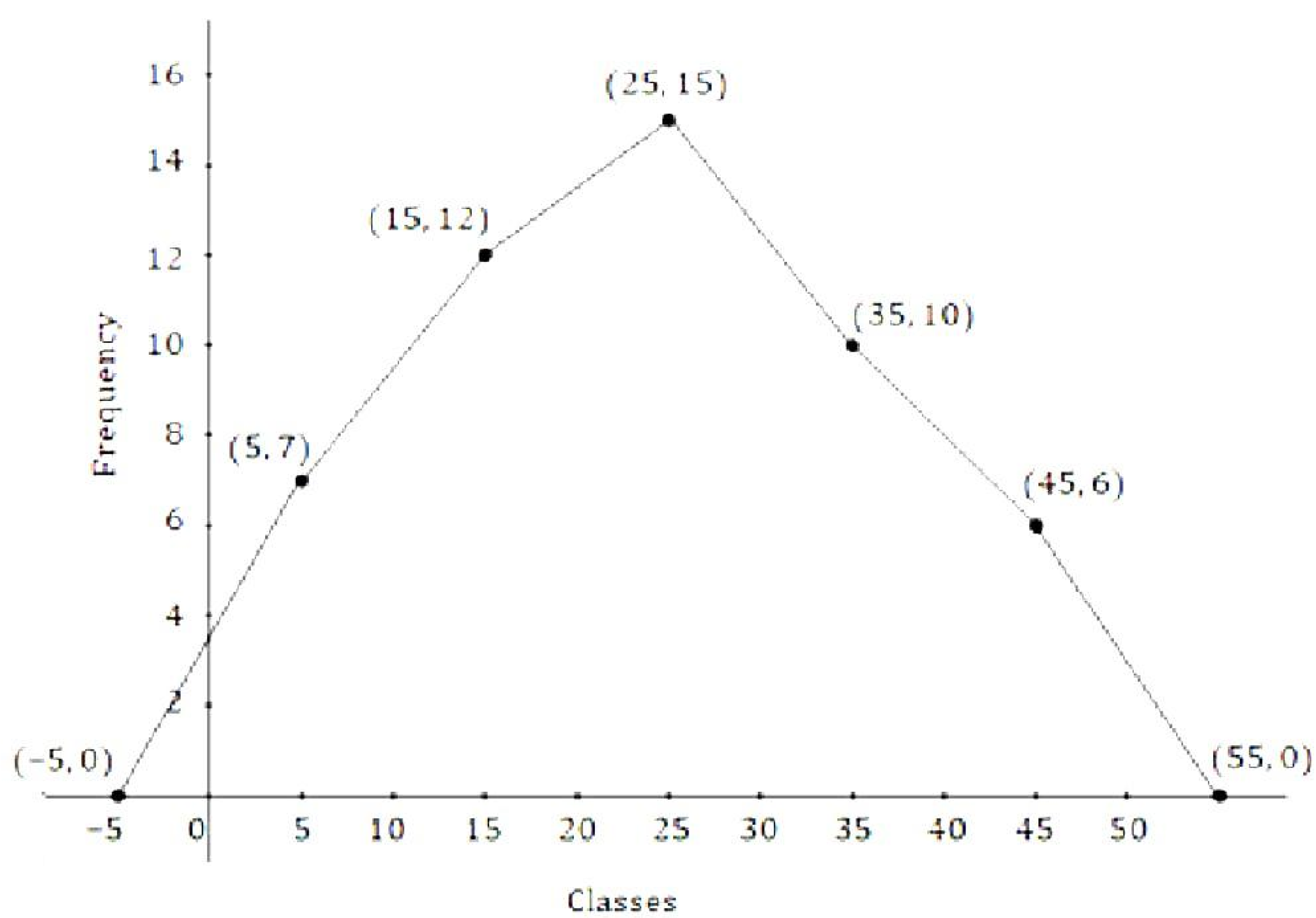
$\angle BCD + \angle BAD = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$\angle BCD + 100^\circ = 180^\circ$
$\angle BCD = 80^\circ$
In ΔABC
AB = BC (given)
$\therefore \angle BCA = \angle CAB$ (Angles opposite to equal sides of a triangle)
$\Rightarrow \angle BCA = 30^\circ$
We have $\angle BCD = 80^\circ$
$\Rightarrow \angle BCA + \angle ACD = 80^\circ$
$30^\circ + \angle ACD = 80^\circ$
$\Rightarrow \angle ACD = 50^\circ$
$\Rightarrow \angle ECD = 50^\circ$

29. Yes, it is possible to construct a frequency polygon without histogram.
The given data is represented as follows:

Classes	Frequency	Class Mark
0-10	7	5
10-20	19 – 7 = 12	15
20-30	34 – 19 = 15	25
30-40	44 – 34 = 10	35
40-50	50 – 44 = 6	45

The frequency polygon is as follows:



OR

Yes, teacher is correct. There is an error in the histogram created by Anvi as she showed frequency 15 for classes 30-40, 40-50 and 50-60 each.

We find that the class width of classes 0-10, 10-20, 20-30 is 10 each. But the class width of class 30-60 is 30. So, here, we need to adjust frequency of this class.

And, Adjusted frequency of a class

$$= \frac{\text{Minimum class-size}}{\text{Class-size}} \times \text{Frequency of the class}$$

Classes	Frequency	Adjusted Frequency
0-10	5	$\frac{10}{10} \times 5 = 5$
10-20	7	$\frac{10}{10} \times 7 = 7$
20-30	8	$\frac{10}{10} \times 8 = 8$
30-60	15	$\frac{10}{30} \times 15 = 5$

Then, the correct histogram is as follows:

30.

(i) Slant height of a cone = 14 cm

Let the radius of the base of the cone be r.

Curved surface area of the cone = πrl

$$308\ \text{cm}^2 = \left(\frac{22}{7} \times r \times 14\right)\ \text{cm}$$

$$\Rightarrow r = \left(\frac{308}{44}\right)\ \text{cm} = 7\ \text{cm}$$

Thus, the radius of the base of the cone is 7 cm.

(ii) Total surface area of the cone = Curved surface area of the cone + Area of the base = $\pi rl + \pi r^2$

$$= \left[308 + \frac{22}{7} \times (7)^2\right]\ \text{cm}^2$$

$$= (308 + 154)\ \text{cm}^2$$

$$= 462\ \text{cm}^2$$

Thus, the total surface area of the cone is 462 cm^2.

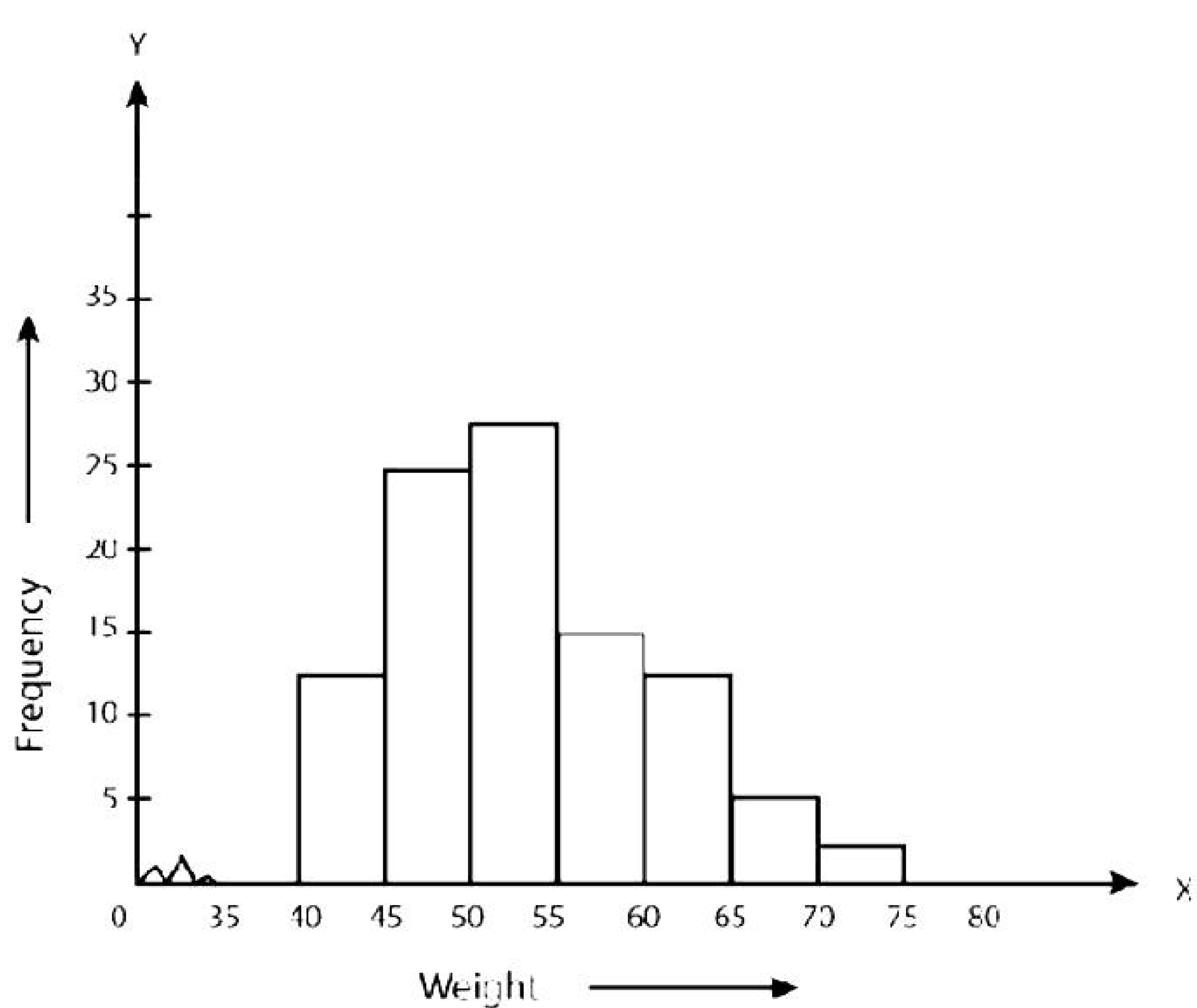
31. Steps of construction:

a. The weights are represented on the horizontal axis. The scale on the horizontal axis as 1 cm = 5 kg. Also, since the first class interval is starting from 35 and not zero, we show it on the graph by marking a kink or a break on the axis.

b. The number of people (frequency) are represented on the vertical axis. Since the maximum frequency is 28, we choose the scale as 1 cm = 5 people.

c. We now draw rectangles (or rectangular bars) of width equal to the class-size and lengths according to the frequencies of the corresponding class intervals.

d.



Section D

32.

(i) By using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(x+4)(x+10) = x^2 + (4+10)x + 4\times 10$$
$$= x^2 + 14x + 40$$

(ii) By using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$(x+8)(x-10) = x^2 + (8-10)x + (8)(-10)$$
$$= x^2 - 2x - 80$$

(iii) $(3x+4)(3x-5) = 9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)$

By using the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right) = 9\left[x^2 + \left(\frac{4}{3}-\frac{5}{3}\right)x + \left(\frac{4}{3}\right)\left(-\frac{5}{3}\right)\right]$$
$$= 9\left[x^2 - \frac{1}{3}x - \frac{20}{9}\right]$$
$$= 9x^2 - 3x - 20$$

(iv) By using identity $(x+y)(x-y) = x^2 - y^2$

$$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right) = \left(y^2\right)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4}$$

(v) By using identity $(x+y)(x-y) = x^2 - y^2$

$$(3-2x)(3+2x) = (3)^2 - (2x)^2$$
$$= 9 - 4x^2$$

OR

We know that, $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i)

$$(x+2y+4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$$
$$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$$

(ii)

$$(2x-y+z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$$
$$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$$

(iii)

$$(-2x+3y+2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$$
$$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$$

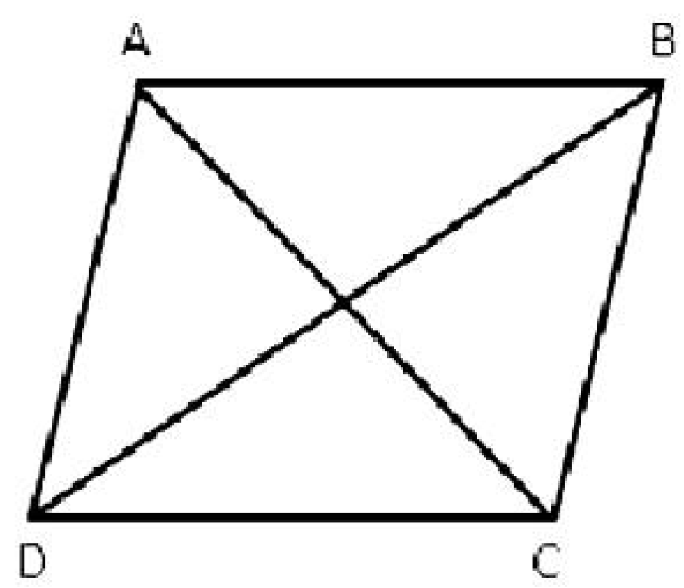
(iv)

$$(3a-7b-c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$$
$$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$$

(v)

$$(-2x+5y-3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$$
$$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$$

33.



Each diagonal of a quadrilateral separated it into two triangles of equal area.

So, we have

A(ΔABD) = A(ΔBDC) ....(1)

And, A(ΔABD) + A(ΔBDC) = A(quadrilateral ABCD)

⇒ 2A(ΔABD) = A(quadrilateral ABCD) ....(2) [From (1)]

Also, A(ΔACB) = A(ΔADC) ....(3)

And, A(ΔACB) + A(ΔADC) = A(quadrilateral ABCD)

⇒ 2A(ΔACB) = A(quadrilateral ABCD) ....(4) [from (3)]

From equation (2) and (4),

2A(ΔABD) = 2A(ΔACB)

⇒ A(ΔABD) = A(ΔACB)

Since, ΔABD and ΔACB are on the same base AB, they must have equal corresponding altitudes.

So, altitude from D of ΔABD = altitude from C of ΔACB

DC || AB

Similarly, AD || BC.

Hence, ABCD is a parallelogram.

34. Angle at the centre is twice the angle at the circumference of a circle subtended by the same chord.

⇒ ∠COB = 2∠CAB

⇒ ∠COB = 2x

∠OCD = ∠COB = 2x ....(alternate angles)

In ΔOCD,

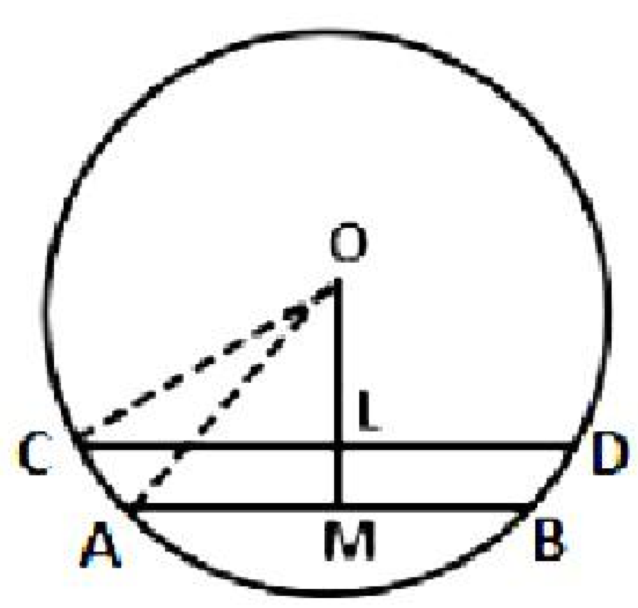
OC = OD ....(radii of the same circle)

$\angle ODC = \angle OCD = 2x$
$\Rightarrow \angle DOC = 180° - 2x - 2x$
$\Rightarrow \angle DOC = 180° - 4x$

Again, angle at the centre is twice the angle at the circumference subtended by the same chord.
$\Rightarrow \angle DAC = ½ \angle DOC = ½(180° - 4x)$
$\Rightarrow \angle DAC = 90° - 2x$

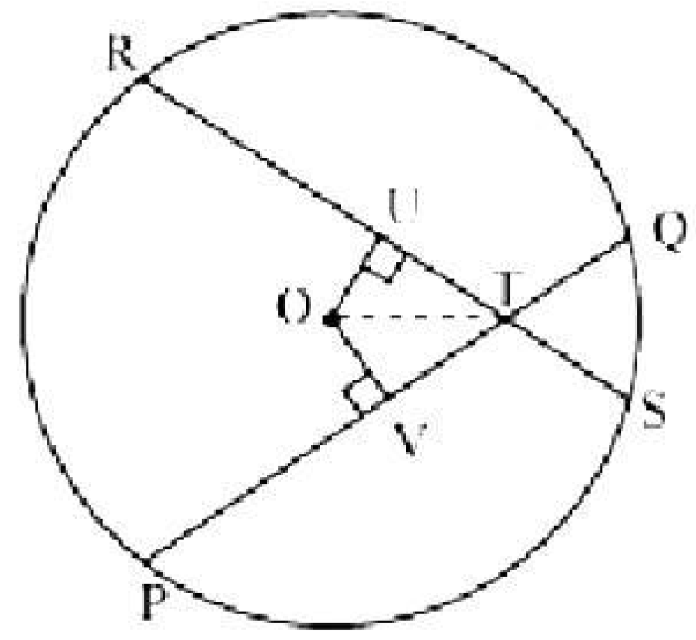
Since DC || AB,
DC || AO
$\Rightarrow \angle ACD = \angle OAC = x$ (alternate angles)
Now, $\angle ADC = 180° - \angle DAC - \angle ACD$
$= 180° - (90° - 2x) - x$
$\Rightarrow \angle ADC = 90° + x$

OR



AB = 12 cm, CD = 24 cm and LM = 4 cm
Let OL = x cm
Then OM = (x + 4) cm
Join OA and OC.
Then OA = OC = r
Since the perpendicular from the centre to a chord bisects the chord,
AM = MB = 6 cm and CL = LD = 12 cm
In triangles OAM and OCL,
$OA^2 = OM^2 + AM^2$ and $OC^2 = OL^2 + CL^2$
$r^2 = (x + 4)^2 + 6^2$ and $r^2 = x^2 + 12^2$
$(x + 4)^2 + 6^2 = x^2 + 12^2$
$x^2 + 8x + 16 + 36 = x^2 + 144$
$8x = 92$
$\Rightarrow x = \frac{92}{8} = 11.5$ cm
Then, $r^2 = (11.5)^2 + 12^2 = 276.25$
$r = \sqrt{276.25} = \sqrt{\frac{27625}{100}} = \sqrt{\frac{1105}{4}}$
$\Rightarrow r = \frac{\sqrt{1105}}{2}$ cm

35.



Let PQ and RS be two equal chords of a given circle intersecting each other at point T.

Draw OV $\perp$ chord PQ and OU $\perp$ chord RS.

In ΔOVT and ΔOUT,

OV = OU	(Equal chords of a circle are equidistant from the centre)	
$\angle OVT = \angle OUT$	(Each 90°)	
OT = OT	(common)	
$\therefore \Delta OVT \cong \Delta OUT$	(RHS congruence rule)	
$\therefore$ VT = UT	(CPCT)	... (1)
Given, PQ = RS		... (2)

$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}RS$

$\Rightarrow$ PV = RU ... (3)

On adding equations (1) and (3), we have

PV + VT = RU + UT

$\Rightarrow$ PT = RT ... (4)

On subtracting equation (4) from equation (2), we have

PQ – PT = RS – RT

$\Rightarrow$ QT = ST ... (5)

Equations (4) and (5) show that the corresponding segments of chords PQ and RS are congruent to each other.

Section E

36.

i. The decimal expansion of $\frac{532}{17}$ is 31.2941176471…. which is non-terminating.

OR

Let x = 1.235235…

1000x = 1235.235235…

1000x − x = 1234

999x = 1234

$x = \frac{1234}{999}$

ii. π is an irrational number as its value is 3.14159…..

iii. False, Irrational numbers can be represented on the number line.

37.

i. The co-ordinates of point A are (1, 1).

ii. The co-ordinates of point B are (4, 1).

iii. The distance between points A and B is 4 − 1 = 3 (difference of x- coordinates for horizontal lines)

OR

The distance between points C and B is 5 − 1 = 4 (difference of y- coordinates for vertical lines)

38.

i. The cost incurred for the farmer to grow Type A rice and Type B rice were Rs. 12 per kg and Rs. 10 per kg.
Hence, total cost = Rs. (12x + 10y)

ii. The selling price of Type A rice and Type B rice were Rs. 18 per kg and Rs. 14 per kg.
Hence, total selling price = Rs. (18x + 14y)

iii. Cost price = Rs. (12x + 10y)
Selling price = Rs. (18x + 14y)
Total profit = [(18x + 14y) − (12x + 10y)] = Rs. (6x + 4y)

OR

Cost price = Rs. (12x + 10y)

Selling price = Rs. (18x + 14y)

Total profit = Rs. (6x + 4y)

$$P\% = \left(\frac{6x+4y}{12x+10y} \times 100\right)\% = \left(\frac{3x+2y}{6x+5y} \times 100\right)\%$$