Complex Numbers & Quadratic Equations

Exercise 5A

Q. 1. Evaluate:
(i) i^{19}
(ii) i^{62}
(ii) i^{373}.

Answer : We all know that i = √(-1) .

And i^{4n} = 1

i^{4n+1} = i (where n is any positive integer)

$i^{4n+2} = -1$

i^{4n+3} = -1

So,

(i) L.H.S = i^{19}

= $i^{4\times4+3}$

= i^{4n+3}

Since it is of the form i^{4n+3} so the solution would be simply $-i$

Hence the value of i^{19} is $-i$.

(ii) $\mathrm{L.H.S} = i^{62}$

$\Rightarrow i^{4\times15+2}$

$\Rightarrow i^{4n+2} \Rightarrow i^2 = -1$

so it is of the form i^{4n+2} so its solution would be -1

(iii) $L.H.S. = i^{373}$

$\Rightarrow i^{4\times 93+1}$

$\Rightarrow i^{4n+1}$

⇒ i

So, it is of the form of i^{4n+1} so the solution would be i.

Q. 2. Evaluate:

(i) $\left(\sqrt{-1}\right)^{192}$

(ii) $\left(\sqrt{-1}\right)^{93}$

(iii) $\left(\sqrt{-1}\right)^{30}$ **.**

Answer : Since i = $\sqrt{-1}$ so

(i) L.H.S. = $\left(\sqrt{-1}\right)^{192}$

$\Rightarrow i^{192}$

$\Rightarrow i^{4\times 48} = 1$

Since it is of the form i^{4n} = 1 so the solution would be 1

(ii) L.H.S.= $\left(\sqrt{-1}\right)^{93}$

$\Rightarrow i^{4\times 23+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow i^1 = i$

Since it is of the form of i^{4n+1} = i so the solution would be simply i.

(iii) L.H.S = $\left(\sqrt{-1}\right)^{30}$

$\Rightarrow i^{4\times7+2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow i^2 = -1$

Since it is of the form i^{4n+2} so the solution would be -1

Q. 3. Evaluate:
(i) i^{-50}
(ii) i^{-9}
(ii) i^{-131}.

Answer : (i) $\mathrm{L.H.S.} = i^{-50}$

$\Rightarrow i^{-4\times13+2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow -1$

Since it is of the form i^{4n+2} so the solution would be -1

(ii) $\mathrm{L.H.S.} = i^{-9}$

$\Rightarrow i^{-4\times3+3}$

$\Rightarrow i^{4n+3}$

$\Rightarrow i^3 = -i$

Since it is of the form of i^{4n+3} so the solution would be simply -i.

(iii) $\mathrm{L.H.S.} = i^{-131}$

$\Rightarrow i^{-4\times33+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow i^1 = i$

Since it is of the form i^{4n+1}. so the solution would be i

Q. 4. Evaluate:

(i) $\left(i^{41} + \frac{1}{i^{71}}\right)$

(ii) $\left(i^{53} + \frac{1}{i^{53}}\right)$

Answer :

(i) $\left(i^{41} + \frac{1}{i^{71}}\right) = i^{41} + i^{-71}$

$\Rightarrow i^{4\times10+1} + i^{-4\times18+1}$ (Since $i^{4n+1} = i$)

$\Rightarrow i^1 + i^1$

$\Rightarrow$ 2i

Hence, $\left(i^{41} + \frac{1}{i^{71}}\right)$ = 2i

(ii) $\left(i^{53} + \frac{1}{i^{53}}\right)$

$\Rightarrow i^{53} + i^{-53}$

$\Rightarrow$ $i^{4\times13+1} + i^{-4\times14+3}$ (Since i^{4n+1} = i

$\Rightarrow$ $i^{1} + i^{3}$ i^{4n+3} = -1)

$\Rightarrow$ 0

Hence, $\left(i^{53} + \frac{1}{i^{53}}\right)$ = 0

Q. 5. Prove that 1 + i^2 + i^4 + i^6 = 0

Answer : L.H.S.= 1 + i^2 + i^4 + i^6

To Prove: 1 + i^2 + i^4 + i^6 = 0

$\Rightarrow$ 1 + (-1) +1 + i^2

Since, i^{4n} = 1

(Where n is any positive integer)

$\Rightarrow i^{4n+2}$

$\Rightarrow i^2 = -1$

$\Rightarrow$1 + -1 + 1 + -1=0

⇒L.H.S = R.H.S

Hence proved.

Q. 6. Prove that 6i^{50} + 5i^{33} – 2i^{15} + 6i^{48} = 7i.

Answer : Given: 6i^{50} + 5i^{33} – 2i^{15} + 6i^{48}

To prove: 6i^{50} + 5i^{33} – 2i^{15} + 6i^{48} = 7i

$\Rightarrow$ 6$i^{4\times12+2}$ + 5$i^{4\times8+1}$ – 2$i^{4\times3+3}$ + 6$i^{4\times12}$

$\Rightarrow 6i^2 + 5i^1 - 2i^3 + 6i^0$

$\Rightarrow$ -6+5i+2i+6

$\Rightarrow$ 7i

$\Rightarrow \text{L.H.S} = \text{R.H.S}$

Hence proved.

Q. 7. Prove that $\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}$ = 0.

Answer :

Given: $\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}$

To prove : $\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}$ = 0.

$\Rightarrow$ L.H.S. = $i^{-1} - i^{-2} + i^{-3} - i^{-4}$

$\Rightarrow i^{-4\times1+3} - i^{-4\times1+2} + i^{-4\times1+3} - i^{-4\times1}$

Since i^{4n} = 1

$\Rightarrow i^{4n+1}$ = i

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3}$ = -1

So,

$\Rightarrow i^1 - i^2 + i^3 - 1$

$\Rightarrow$ i +1 – i -1

$\Rightarrow$ 0

⇒L.H.S = R.H.S

Hence Proved

Q. 8. Prove that (1 + i^{10} + i^{20} + i^{30}) is a real number.

Answer : L.H.S = (1 + i^{10} + i^{20} + i^{30})

= (1 + $i^{4\times2+2} + i^{4\times5} + i^{4\times7+2}$)

Since $\Rightarrow i^{4n}$ = 1

$\Rightarrow i^{4n+1}$ = i

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3}$ = -1

= 1 + $i^2 + 1 + i^2$

= 1 + -1 +1 + -1

= 0, which is a real no.

Hence, (1 + i^{10} + i^{20} + i^{30}) is a real number.

Q. 9. Prove that $\left\{i^{21} - \left(\frac{1}{i}\right)^{46}\right\}^2$ = 2i.

Answer : L.H.S.= $\left\{i^{21} - \left(\frac{1}{i}\right)^{46}\right\}^2$

$= \left\{i^{4\times5+1} - i^{-4\times12+2}\right\}^2$

Since i^{4n} = 1

i^{4n+1} = i

$i^{4n+2} = i^2 = -1$

$i^{4n+3} = i^3$ = -1

$= \left\{i^1 - i^2\right\}^2$

$= \left\{i + 1\right\}^2$

Now, applying the formula $(a+b)^2 = a^2 + b^2 +$ 2ab

$= i^2 + 1 + 2i$.

= -1 + 1 + 2i

= 2i

L.H.S = R.H.S

Hence proved.

Q. 10. $\left\{i^{18} + \frac{1}{i^{25}}\right\}^3$ **= 2(1 – i).**

Answer : L.H.S = $\left\{i^{18} + \frac{1}{i^{25}}\right\}^3$

$\Rightarrow \left\{i^{4\times4+2} + i^{-4\times7+3}\right\}^3$

Since i^{4n} = 1

i^{4n+1} = i

i^{4n+2} = -1

i^{4n+3} = -1

= $\left\{i^2 + i^3\right\}^3$.

= $\left(-1-i\right)^3$.

Applying the formula $\left(a+b\right)^3 = a^3 + b^3 + 3ab\left(a+b\right)$

We have,

+ $3i^2 + 3i + 1)$

i + 3 - 3i - 1

= 2(1-i)

L.H.S = R.H.S

Hence proved.

Q. 11. Prove that (1 – i)^n $\left(1-\frac{1}{i}\right)^n$ = 2^n for all values of n N

Answer : L.H.S = $(1 - i)^n \left(1 - \frac{1}{i}\right)^n$

= $(1-i)^n \left(1-i^{-4*1+3}\right)^n$

= $(1-i)^n \left(1-i^3\right)^n$

Since, i^{4n+3} . = -1

= $(1-i)^n (1+i)^n$.

Applying $a^n b^n = (ab)^n$

= $((1-i)(1+i))^n$.

= $\left(1-i^2\right)^n$

= 2^n

L.H.S = R.H.S

Q. 12. Prove that $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$ **= 0.**

Answer : L.H.S = $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

Since we know that i = $\sqrt{-1}$.

So,

= $\sqrt{16}$ i + 3$\sqrt{25}$ i + $\sqrt{36}$ i − $\sqrt{625}$ i

$=4i + 15i + 6i - 25i$

$= 0$

L.H.S = R.H.S

Hence proved.

Q. 13. Prove that $(1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}) = 1$.

Answer : L.H.S = $(1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20})$

$= \sum_{n=0}^{n=20} i^n$

$= 1 + -1 +1 + -1 + \ldots\ldots\ldots.. + 1$

As there are 11 times 1 and 6 times it is with positive sign as i^0 =1 as this is the extra term and there are 5 times 1 with negative sign.

So, these 5 cancel out the positive one leaving one positive value i.e. 1

$= \sum_{n=0}^{20} i^n = 1$

L.H.S = R.H.S

Hence proved.

Q. 14. Prove that $i^{53} + i^{72} + i^{93} + i^{102} = 2i$.

Answer : L.H.S = $i^{53} + i^{72} + i^{93} + i^{102}$

$= i^{4\times13+1} + i^{4\times18} + i^{4\times23+1} + i^{4\times25+2}$

Since $i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$ (where n is any positive integer)

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3}$ = -1

= i + 1 + i + i^2

= i+1+i-1

=2i

L.H.S = R.H.S

Hence proved.

Q. 15. Prove that $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right) = (-1 + i),$ **n N.**

Answer : L.H.S = $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$

= $i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + \ldots + i^{13} + i^{14}$

Since i^{4n} = 1

$\Rightarrow i^{4n+1}$ = i

$\Rightarrow i^{4n+2}$ = -1

$\Rightarrow i^{4n+3}$ = -1

= i – 1 – i + 1 + i - 1……+i-1

As, all terms will get cancel out consecutively except the first two terms. So that will get remained will be the answer.

= i - 1

L.H.S = R.H.S

Hence proved.

Exercise 5B

Q. 1. A. Simplify each of the following and express it in the form a + ib :

2(3 + 4i) + i(5 – 6i)

Answer : Given: 2(3 + 4i) + i(5 – 6i)

Firstly, we open the brackets

$2 \times 3 + 2 \times 4i + i \times 5 - i \times 6i$

$= 6 + 8i + 5i - 6i^2$

$= 6 + 13i - 6(-1)$ [∵, $i^2 = -1$]

$= 6 + 13i + 6$

$= \underbrace{12}_{\text{Real part}} + \underbrace{13i}_{\text{Imaginary part}}$

Q. 1. B. Simplify each of the following and express it in the form a + ib :

$$\left(3+\sqrt{-16}\right)-\left(4-\sqrt{-9}\right)$$

Answer : Given: $\left(3+\sqrt{-16}\right)-\left(4-\sqrt{-9}\right)$

We re – write the above equation

$$\left(3+\sqrt{(-1)\times 16}\right)(-1)\left(4-\sqrt{(-1)\times 9}\right)$$

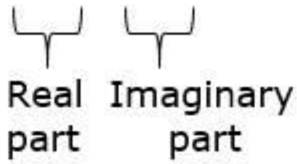
$= \left(3+\sqrt{16i^2}\right)-\left(4-\sqrt{9i^2}\right)$ [∵ $i^2 = -1$]

= (3 + 4i) – (4 – 3i)

Now, we open the brackets, we get

3 + 4i – 4 + 3i

= -1 + 7i



Q. 1. C. Simplify each of the following and express it in the form a + ib :

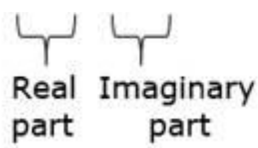
(–5 + 6i) – (–2 + i)

Answer : Given: (–5 + 6i) – (–2 + i)

Firstly, we open the brackets

-5 + 6i + 2 – i

= -3 + 5i



Q. 1. D. Simplify each of the following and express it in the form a + ib :

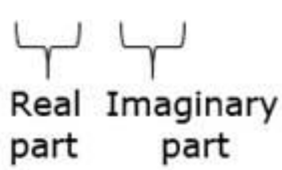
(8 – 4i) – (- 3 + 5i)

Answer : Given: (8 – 4i) – (- 3 + 5i)

Firstly, we open the brackets

8 – 4i + 3 – 5i

= 11 – 9i



Q. 1. E. Simplify each of the following and express it in the form a + ib :

$(1 - i)^2 (1 + i) - (3 - 4i)^2$

Answer : Given: $(1 - i)^2 (1 + i) - (3 - 4i)^2$

$= (1 + i^2 - 2i)(1 + i) - (9 + 16i^2 - 24i)$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$= (1 - 1 - 2i)(1 + i) - (9 - 16 - 24i)$ $[\because i^2 = -1]$

$= (-2i)(1 + i) - (- 7 - 24i)$

Now, we open the brackets

$-2i \times 1 - 2i \times i + 7 + 24i$

$= -2i - 2i^2 + 7 + 24i$

$= -2(-1) + 7 + 22i$ $[\because, i^2 = -1]$

$= 2 + 7 + 22i$

$= 9 + 22i$

$\underbrace{\quad}_{\text{Real part}} \; \underbrace{\quad}_{\text{Imaginary part}}$

Q. 1. F. Simplify each of the following and express it in the form a + ib :

$$\left(5+\sqrt{-3}\right)\left(5-\sqrt{-3}\right)$$

Answer : Given: $(5 + \sqrt{-3})(5 - \sqrt{-3})$

We re – write the above equation

$$(5 + \sqrt{(-1) \times 3})(5 - \sqrt{(-1) \times 3})$$

$= (5 + \sqrt{3i^2})(5 - \sqrt{3i^2})$ $[\because, i^2 = -1]$

$= (5 + i\sqrt{3})(5 - i\sqrt{3})$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

Here, $a = 5$ and $b = i\sqrt{3}$

$= (5)^2 - (i\sqrt{3})^2$

$= 25 - (3i^2)$

$= 25 - [3 \times (-1)]$

$= 25 + 3$

$= 28 + 0$

$= \underbrace{28}_{\text{Real part}} + \underbrace{0i}_{\text{Imaginary part}}$

Q. 1. G. Simplify each of the following and express it in the form a + ib :

(3 + 4i) (2 – 3i)

Answer : Given: $(3 + 4i)(2 - 3i)$

Firstly, we open the brackets

$3 \times 2 + 3 \times (-3i) + 4i \times 2 - 4i \times 3i$

$= 6 - 9i + 8i - 12i^2$

$= 6 - i - 12(-1)$ [$\because$, $i^2 = -1$]

$= 6 - i + 12$

$= 18 - i$

$\underbrace{\qquad}_{\text{Real part}} \quad \underbrace{\qquad}_{\text{Imaginary part}}$

Q. 1. H. Simplify each of the following and express it in the form a + ib :

$$\left(-2+\sqrt{-3}\right)\left(-3+2\sqrt{-3}\right)$$

Answer : Given: $(-2+\sqrt{-3})(-3+2\sqrt{-3})$

We re – write the above equation

$$(-2+\sqrt{(-1)\times 3})(-3+2\sqrt{(-1)\times 3})$$

$= (-2+\sqrt{3i^2})(-3+2\sqrt{3i^2})$ [∵, $i^2 = -1$]

$$= (-2+i\sqrt{3})(-3+2i\sqrt{3})$$

Now, open the brackets,

= -2 × (-3) + (-2) × 2i√3 + i√3 × (-3) + i√3 × 2i√3

= 6 – 4i√3 – 3i√3 + 6i^2

= 6 – 7i√3 + [6 × (-1)] [∵, i^2 = -1]

= 6 – 7i√3 – 6

$= \underbrace{0}_{\text{Real part}} \underbrace{- 7i\sqrt{3}}_{\text{Imaginary part}}$

Q. 2. A. Simplify each of the following and express it in the form (a + ib) :

$$\left(2+\sqrt{-3}\right)^2$$

Answer : Given: $(2 - \sqrt{-3})^2$

We know that,

$(a - b)^2 = a^2 + b^2 - 2ab$ …(i)

So, on replacing a by 2 and b by $\sqrt{-3}$ in eq. (i), we get

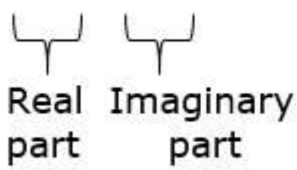
$(2)^2 + (\sqrt{-3})^2 - 2(2)(\sqrt{-3})$

$= 4 + (-3) - 4\sqrt{-3}$

$= 4 - 3 - 4\sqrt{-3}$

$= 1 - 4\sqrt{3i^2}$ [$\because$ $i^2 = -1$]

$= 1 - 4i\sqrt{3}$



Q. 2. B. Simplify each of the following and express it in the form (a + ib) :

$(5 - 2i)^2$

Answer : Given: $(5 - 2i)^2$

We know that,

$(a - b)^2 = a^2 + b^2 - 2ab$ …(i)

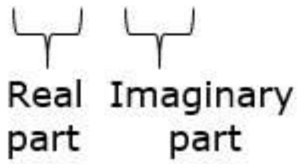
So, on replacing a by 5 and b by 2i in eq. (i), we get

$(5)^2 + (2i)^2 - 2(5)(2i)$

$= 25 + 4i^2 - 20i$

$= 25 - 4 - 20i$ [$\because$ $i^2 = -1$]

$= 21 - 20i$



Q. 2. C. Simplify each of the following and express it in the form (a + ib) :

$(-3 + 5i)^3$

Answer : Given: $(-3 + 5i)^3$

We know that,

$(-a + b)^3 = -a^3 + 3a^2b - 3ab^2 + b^3$ …(i)

So, on replacing a by 3 and b by 5i in eq. (i), we get

$-(3)^3 + 3(3)^2(5i) - 3(3)(5i)^2 + (5i)^3$

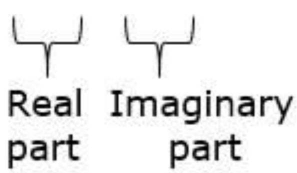
$= -27 + 3(9)(5i) - 3(3)(25i^2) + 125i^3$

$= -27 + 135i - 225i^2 + 125i^3$

$= -27 + 135i - 225 \times (-1) + 125i \times i^2$

$= -27 + 135i + 225 - 125i$ [$\because i^2 = -1$]

$= 198 + 10i$



Q. 2. D. Simplify each of the following and express it in the form (a + ib) :

$$\left(-2 - \frac{1}{3}i\right)^3$$

Answer : Given: $\left(-2 - \frac{1}{3}i\right)^3$

We know that,

$(-a-b)^3 = -a^3 - 3a^2b - 3ab^2 - b^3$ …(i)

So, on replacing a by 2 and b by 1/3i in eq. (i), we get

$$-(2)^3 - 3(2)^2\left(\frac{1}{3}i\right) - 3(2)\left(\frac{1}{3}i\right)^2 - \left(\frac{1}{3}i\right)^3$$

$$= -8 - 4i - 6\left(\frac{1}{9}i^2\right) - \left(\frac{1}{27}i^3\right)$$

$$= -8 - 4i - \frac{2}{3}i^2 - \frac{1}{27}i(i^2)$$

$$= -8 - 4i - \frac{2}{3}(-1) - \frac{1}{27}i(-1)$$ [∵ $i^2 = -1$]

$$= -8 - 4i + \frac{2}{3} + \frac{1}{27}i$$

$$= \left(-8 + \frac{2}{3}\right) + \left(-4i + \frac{1}{27}i\right)$$

$$= \left(\frac{-24+2}{3}\right) + \left(\frac{-108i + i}{27}\right)$$

$$= -\frac{22}{3} + \left(-\frac{107}{27}i\right)$$

$$= \underbrace{-\frac{22}{3}}_{\text{Real part}} \underbrace{-\frac{107}{27}}_{\text{Imaginary part}} i$$

Q. 2. E. Simplify each of the following and express it in the form (a + ib) :

$(4-3i)^{-1}$

Answer : Given: $(4-3i)^{-1}$

We can re- write the above equation as

$$= \frac{1}{4 - 3i}$$

Now, rationalizing

$$= \frac{1}{4 - 3i} \times \frac{4 + 3i}{4 + 3i}$$

$$= \frac{4+3i}{(4-3i)(4+3i)} \quad \ldots\text{(i)}$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{4 + 3i}{(4)^2 - (3i)^2}$$

$$= \frac{4 + 3i}{16 - 9i^2}$$

$$= \frac{4+3i}{16-9(-1)} \quad [\because i^2 = -1]$$

$$= \frac{4 + 3i}{16 + 9}$$

$$= \frac{4 + 3i}{25}$$

$$= \underbrace{\frac{4}{25}}_{\text{Real part}} + \underbrace{\frac{3}{25}i}_{\text{Imaginary part}}$$

Q. 2. F. Simplify each of the following and express it in the form (a + ib) :

$$\left(-2 + \sqrt{-3}\right)^{-1}$$

Answer : Given: $(-2 + \sqrt{-3})^{-1}$

We can re- write the above equation as

$$= \frac{1}{-2+\sqrt{-3}}$$

$$= \frac{1}{-2+\sqrt{3i^2}}$$ [∵ $i^2 = -1$]

$$= \frac{1}{-2+i\sqrt{3}}$$

Now, rationalizing

$$= \frac{1}{-2+i\sqrt{3}} \times \frac{-2-i\sqrt{3}}{-2-i\sqrt{3}}$$

$$= \frac{-2-i\sqrt{3}}{(-2+i\sqrt{3})(-2-i\sqrt{3})}$$ …(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{-2-i\sqrt{3}}{(-2)^2-\left(i\sqrt{3}\right)^2}$$

$$= \frac{-2-i\sqrt{3}}{4-(3i^2)}$$

$$= \frac{-2-i\sqrt{3}}{4-3(-1)}$$ [∵ $i^2 = -1$]

$$= \frac{-2-i\sqrt{3}}{4+3}$$

$$= \frac{-2 - i\sqrt{3}}{7}$$

$$= -\frac{2 + i\sqrt{3}}{7}$$

$$= \underbrace{-\frac{2}{7}}_{\text{Real part}} \underbrace{-\frac{\sqrt{3}}{7}i}_{\text{Imaginary part}}$$

Q. 2. G. Simplify each of the following and express it in the form (a + ib) :

$(2 + i)^{-2}$

Answer : Given: $(2 + i)^{-2}$

Above equation can be re – written as

$$= \frac{1}{(2 + i)^2}$$

Now, rationalizing

$$= \frac{1}{(2 + i)^2} \times \frac{(2 - i)^2}{(2 - i)^2}$$

$$= \frac{(2 - i)^2}{(2 + i)^2(2 - i)^2}$$

$= \frac{4 + i^2 - 4i}{(4 + i^2 + 4i)(4 + i^2 - 4i)}$ [$\because$ $(a – b)^2 = a^2 + b^2 – 2ab$]

$= \frac{4 - 1 - 4i}{(4 - 1 + 4i)(4 - 1 - 4i)}$ [$\because i^2 = -1$]

$= \frac{3 - 4i}{(3 + 4i)(3 - 4i)}$ …(i)

Now, we know that,

(a + b)(a – b) = (a^2 – b^2)

So, eq. (i) become

$$= \frac{3-4i}{(3)^2-(4i)^2}$$

$$= \frac{3-4i}{9-16i^2}$$

$$= \frac{3-4i}{9-16(-1)}$$

$$= \frac{3-4i}{25}$$

$$= \underbrace{\frac{3}{25}}_{\text{Real part}} - \underbrace{\frac{4}{25}i}_{\text{Imaginary part}}$$

Q. 2. H. Simplify each of the following and express it in the form (a + ib) :

$(1 + 2i)^{-3}$

Answer : Given: $(1 + 2i)^{-3}$

Above equation can be re – written as

$$= \frac{1}{(1+2i)^3}$$

Now, rationalizing

$$= \frac{1}{(1+2i)^3} \times \frac{(1-2i)^3}{(1-2i)^3}$$

$$= \frac{(1-2i)^3}{(1+2i)^3(1-2i)^3}$$

We know that,

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$= \frac{(1)^3 - 3(1)^2(2i) + 3(1)(2i)^2 - (2i)^3}{[(1)^3 + 3(1)^2(2i) + 3(1)(2i)^2 + (2i)^3][(1)^3 - 3(1)^2(2i) + 3(1)(2i)^2 - (2i)^3]}$$

$$= \frac{1 - 6i + 6i^2 - 8i^3}{[1 + 6i + 6i^2 + 8i^3][1 - 6i + 6i^2 - 8i^3]}$$

$$= \frac{1-6i+6(-1)-8i(-1)}{[1+6i+6(-1)+8i(-1)][1-6i+6(-1)-8i(-1)]}$$ [∵$i^2 = -1$]

$$= \frac{1 - 6i - 6 + 8i}{[1 + 6i - 6 - 8i][1 - 6i - 6 + 8i]}$$

$$= \frac{-5 + 2i}{[-5 - 2i][-5 + 2i]}$$

$$= \frac{-5 + 2i}{-5(-5) - 5(2i) - 2i(-5) - 2i(2i)}$$

$$= \frac{-5 + 2i}{25 - 10i + 10i - 4i^2}$$

$$= \frac{-5+2i}{25-4(-1)}$$ [∵$i^2 = -1$]

$$= \frac{-5 + 2i}{29}$$

$$= \underbrace{-\frac{5}{29}}_{\text{Real part}} + \underbrace{\frac{2}{29}i}_{\text{Imaginary part}}$$

Q. 2. I. Simplify each of the following and express it in the form (a + ib) :

$(1 + i)^3 – (1 – i)^3$

Answer : Given: $(1 + i)^3 – (1 – i)^3$ …(i)

We know that,

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$

By applying the formulas in eq. (i), we get

$(1)^3 + 3(1)^2(i) + 3(1)(i)^2 + (i)^3 – [(1)^3 – 3(1)^2(i) + 3(1)(i)^2 – (i)^3]$

$= 1 + 3i + 3i^2 + i^3 – [1 – 3i + 3i^2 – i^3]$

$= 1 + 3i + 3i^2 + i^3 – 1 + 3i – 3i^2 + i^3$

$= 6i + 2i^3$

$= 6i + 2i(i^2)$

$= 6i + 2i(-1)$ [$\because i^2 = -1$]

$= 6i – 2i$

$= 4i$

$= \underbrace{0}_{\text{Real part}} + \underbrace{4i}_{\text{Imaginary part}}$

Q. 3. A. Express each of the following in the form (a + ib):

$$\frac{1}{(4+3i)}$$

Answer : Given: $\frac{1}{4+3i}$

Now, rationalizing

$$= \frac{1}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{4-3i}{(4+3i)(4-3i)} \quad \ldots(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{4-3i}{(4)^2-(3i)^2}$$

$$= \frac{4-3i}{16-9i^2}$$

$$= \frac{4-3i}{16-9(-1)} \quad [\because i^2 = -1]$$

$$= \frac{4-3i}{16+9}$$

$$= \frac{4-3i}{25}$$

$$= \underbrace{\frac{4}{25}}_{\text{Real part}} - \underbrace{\frac{3}{25}i}_{\text{Imaginary part}}$$

Q. 3. B. Express each of the following in the form (a + ib):

$$\frac{(3+4i)}{(4+5i)}$$

Answer : Given: $\frac{3+4i}{4+5i}$

Now, rationalizing

$$= \frac{3+4i}{4+5i} \times \frac{4-5i}{4-5i}$$

$$= \frac{(3+4i)(4-5i)}{(4+5i)(4-5i)} \quad \ldots(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(3+4i)(4-5i)}{(4)^2 - (5i)^2}$$

$$= \frac{3(4) + 3(-5i) + 4i(4) + 4i(-5i)}{16 - 25i^2}$$

$$= \frac{12 - 15i + 16i - 20i^2}{16 - 25(-1)} \quad [\because i^2 = -1]$$

$$= \frac{12 + i - 20(-1)}{16 + 25}$$

$$= \frac{12 + i + 20}{41}$$

$$= \frac{32 + i}{41}$$

$$= \underbrace{\frac{32}{41}}_{\text{Real part}} + \underbrace{\frac{1}{41}}_{\text{Imaginary part}} i$$

Q. 3. C. Express each of the following in the form (a + ib):

$$\frac{\left(5 + \sqrt{2}i\right)}{\left(1 - \sqrt{2}i\right)}$$

Answer : Given: $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$

Now, rationalizing

$$= \frac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \frac{1+\sqrt{2}i}{1+\sqrt{2}i}$$

$= \frac{(5+\sqrt{2}i)(1+\sqrt{2}i)}{(1-\sqrt{2}i)(1+\sqrt{2}i)}$ …(i)

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(5+\sqrt{2}i)(1+\sqrt{2}i)}{(1)^2 - \left(\sqrt{2}i\right)^2}$$

$$= \frac{5(1) + 5\left(\sqrt{2}i\right) + \sqrt{2}i(1) + \sqrt{2}i(\sqrt{2}i)}{1 - 2i^2}$$

$= \frac{5+5\sqrt{2}i+\sqrt{2}i+2i^2}{1-2(-1)}$ [$\because i^2 = -1$]

$$= \frac{5 + 6i\sqrt{2} + 2(-1)}{1+2}$$

$$= \frac{3 + 6i\sqrt{2}}{3}$$

$$= \frac{3(1 + 2i\sqrt{2})}{3}$$

$\underbrace{\qquad}_{\text{Real part}} \underbrace{\qquad}_{\text{Imaginary part}}$ $= 1 + 2i\sqrt{2}$

Q. 3. D. Express each of the following in the form (a + ib):

$$\frac{(-2+5i)}{(3-5i)}$$

Answer : Given: $\frac{-2+5i}{3-5i}$

Now, rationalizing

$$= \frac{-2+5i}{3-5i} \times \frac{3+5i}{3+5i}$$

$$= \frac{(-2+5i)(3+5i)}{(3-5i)(3+5i)} \quad \text{…(i)}$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(-2+5i)(3+5i)}{(3)^2-(5i)^2}$$

$$= \frac{-2(3)+(-2)(5i)+5i(3)+5i(5i)}{9-25i^2}$$

$$= \frac{-6-10i+15i+25i^2}{9-25(-1)} \quad [\because i^2 = -1]$$

$$= \frac{-6+5i+25(-1)}{9+25}$$

$$= \frac{-31+5i}{34}$$

$$= \underbrace{-\frac{31}{34}}_{\text{Real part}} + \underbrace{\frac{5}{34}i}_{\text{Imaginary part}}$$

Q. 3. E. Express each of the following in the form (a + ib):

$$\frac{(3-4i)}{(4-2i)(1+i)}$$

Answer : Given: $\frac{3-4i}{(4-2i)(1+i)}$

Solving the denominator, we get

$$\frac{3-4i}{(4-2i)(1+i)} = \frac{3-4i}{4(1)+4(i)-2i(1)-2i(i)}$$

$$= \frac{3-4i}{4+4i-2i-2i^2}$$

$$= \frac{3-4i}{4+2i-2(-1)}$$

$$= \frac{3-4i}{6+2i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 6 + 2i

$$= \frac{3-4i}{6+2i} \times \frac{6-2i}{6-2i}$$

$$= \frac{(3-4i)(6-2i)}{(6+2i)(6-2i)} \quad \ldots\text{(i)}$$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{(3-4i)(6-2i)}{(6)^2-(2i)^2}$$

$$= \frac{3(6)+3(-2i)+(-4i)(6)+(-4i)(-2i)}{36-4i^2}$$

$= \frac{18-6i-24i+8i^2}{36-4(-1)}$ [$\because i^2 = -1$]

$$= \frac{18-30i+8(-1)}{36+4}$$

$$= \frac{18-30i-8}{40}$$

$$= \frac{10-30i}{40}$$

$$= \frac{10(1-3i)}{40}$$

$$= \frac{1-3i}{4}$$

$$= \underbrace{\frac{1}{4}}_{\text{Real part}} - \underbrace{\frac{3}{4}i}_{\text{Imaginary part}}$$

Q. 3. F. Express each of the following in the form (a + ib):

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

Answer : Given: $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

Firstly, we solve the given equation

$$= \frac{3(2)+3(3i)-2i(2)+(-2i)(3i)}{(1)(2)+1(-i)+2i(2)+2i(-i)}$$

$$= \frac{6 + 9i - 4i - 6i^2}{2 - i + 4i - 2i^2}$$

$$= \frac{6 + 5i - 6(-1)}{2 + 3i - 2(-1)}$$

$$= \frac{6 + 6 + 5i}{2 + 3i + 2}$$

$$= \frac{12 + 5i}{4 + 3i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 4 + 3i

$$= \frac{12 + 5i}{4 + 3i} \times \frac{4 - 3i}{4 - 3i}$$

$$= \frac{(12+5i)(4-3i)}{(4+3i)(4-3i)} \quad \ldots(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(12 + 5i)(4 - 3i)}{(4)^2 - (3i)^2}$$

$$= \frac{12(4) + 12(-3i) + 5i(4) + 5i(-3i)}{16 - 9i^2}$$

$$= \frac{48-36i+20i-15i^2}{19-9(-1)} \quad [\because i^2 = -1]$$

$$= \frac{48-16i-15(-1)}{16+9} \quad [\because i^2 = -1]$$

$$= \frac{48 - 16i + 15}{25}$$

$$= \frac{63 - 16i}{25}$$

$$\underbrace{\frac{63}{25}}_{\text{Real part}} - \underbrace{\frac{16}{25}i}_{\text{Imaginary part}} = \frac{63}{25} - \frac{16}{25}i$$

Q. 3. G. Express each of the following in the form (a + ib):

$$\frac{(2+3i)^2}{(2-i)}$$

Answer : Given: $\frac{(2+3i)^2}{(2-i)}$

Now, we rationalize the above equation by multiply and divide by the conjugate of (2 – i)

$$= \frac{(2+3i)^2}{(2-i)} \times \frac{(2+i)}{(2+i)}$$

$$= \frac{(2+3i)^2(2+i)}{(2-i)(2+i)}$$

$$= \frac{(4+9i^2+12i)(2+i)}{(2)^2-(i)^2}$$

[∵(a + b)(a – b) = $(a^2 – b^2)$]

$$= \frac{[4+9(-1)+12i](2+i)}{4-i^2}$$ [∵$i^2 = -1$]

$$= \frac{[4-9+12i](2+i)}{4-(-1)}$$

$$= \frac{(-5+12i)(2+i)}{5}$$

$$= \frac{-10 - 5i + 24i + 12i^2}{5}$$

$$= \frac{-10 + 19i + 12(-1)}{5}$$

$$= \frac{-10 - 12 + 19i}{5}$$

$$= \frac{-22 + 19i}{5}$$

$$= \underbrace{-\frac{22}{5}}_{\text{Real part}} + \underbrace{\frac{19}{5}i}_{\text{Imaginary part}}$$

Q. 3. H. Express each of the following in the form (a + ib):

$$\frac{(1-i)^3}{(1-i^3)}$$

Answer : Given: $\frac{(1-i)^3}{(1-i^3)}$

The above equation can be re-written as

$$= \frac{(1)^3 - (i)^3 - 3(1)^2(i) + 3(1)(i)^2}{(1 - i \times i^2)}$$

[$\because$(a – b)^3 = a^3 – b^3 – 3a^{2}b + 3ab^2]

$= \frac{1 - i^3 - 3i + 3i^2}{[1 - i(-1)]}$ [$\because$i^2 = -1]

$$= \frac{1 - i \times i^2 - 3i + 3(-1)}{(1 + i)}$$

$$= \frac{1 - i(-1) - 3i - 3}{1 + i}$$

$$= \frac{-2 + i - 3i}{1 + i}$$

$$= \frac{-2 - 2i}{1 + i}$$

$$= \frac{-2(1 + i)}{1 + i}$$

$$= \underbrace{-2}_{\text{Real part}} + \underbrace{0i}_{\text{Imaginary part}}$$

Q. 3. I. Express each of the following in the form (a + ib):

$$\frac{(1+2i)^3}{(1+i)(2-i)}$$

Answer : Given: $\frac{(1+2i)^3}{(1+i)(2-i)}$

We solve the above equation by using the formula

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$$= \frac{(1)^3 + (2i)^3 + 3(1)^2(2i) + 3(1)(2i)^2}{1(2) + 1(-i) + i(2) + i(-i)}$$

$$= \frac{1 + 8i^3 + 6i + 12i^2}{2 - i + 2i - i^2}$$

$= \frac{1+8i\times i^2+6i+12(-1)}{2+i-(-1)}$ [$\because i^2 = -1$]

$$= \frac{1 + 8i(-1) + 6i - 12}{2 + i + 1}$$

$$= \frac{1 - 8i + 6i - 12}{3 + i}$$

$$= \frac{-11 - 2i}{3 + i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 3 + i

$$= \frac{-11 - 2i}{3 + i} \times \frac{3 - i}{3 - i}$$

$$= \frac{(-11-2i)(3-i)}{(3+i)(3-i)} \quad \ldots(i)$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(-11 - 2i)(3 - i)}{(3)^2 - (i)^2}$$

$$= \frac{-11(3) + (-11)(-i) + (-2i)(3) + (-2i)(-i)}{9 - i^2}$$

$$= \frac{-33+11i-6i+2i^2}{9-(-1)} \quad [\because i^2 = -1]$$

$$= \frac{-33+5i+2(-1)}{9+1} \quad [\because i^2 = -1]$$

$$= \frac{-33 + 5i - 2}{10}$$

$$= \frac{-35 + 5i}{10}$$

$$= \frac{5(-7 + i)}{10}$$

$$= \frac{-7 + i}{2}$$

Real part Imaginary part

$$= \frac{-7}{2} + \frac{1}{2}i$$

Q. 4. Simplify each of the following and express it in the form (a + ib):

(i) $\left(\frac{5}{-3+2i} + \frac{2}{1-i}\right)\left(\frac{4-5i}{3+2i}\right)$

(ii) $\left(\frac{1}{1+4i} - \frac{2}{1+i}\right)\left(\frac{1-i}{5+3i}\right)$

Answer : Given:

$$\left(\frac{5}{-3+2i} + \frac{2}{1-i}\right)\left(\frac{4-5i}{3+2i}\right)$$

$$= \left[\frac{5(1-i)+2(-3+2i)}{(-3+2i)(1-i)}\right]\left(\frac{4-5i}{3+2i}\right)$$ [Taking the LCM]

$$= \left[\frac{5-5i-6+4i}{(-3)(1-i)+2i(1-i)}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left[\frac{-1-i}{-3+3i+2i-2i^2}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left[\frac{-(1+i)}{-3+5i-2(-1)}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left(\frac{-(1+i)}{-1+5i}\right)\left(\frac{4-5i}{3+2i}\right)$$

$$= \frac{-1(4-5i)-i(4-5i)}{-1(3+2i)+5i(3+2i)}$$

$$= \frac{-4+5i-4i+5i^2}{-3-2i+15i+10i^2}$$

$= \frac{-4+i+5(-1)}{-3+13i+10(-1)}$ [Putting $i^2 = -1$]

$$= \frac{-9+i}{-13+13i}$$

$$= \frac{-(9-i)}{-(13-13i)}$$

$$= \frac{9-i}{13-13i}$$

Now, rationalizing by multiply and divide by the conjugate of (13 – 13i)

$$= \frac{9-i}{13-13i} \times \frac{13+13i}{13+13i}$$

$$= \frac{(9-i)(13+13i)}{(13-13i)(13+13i)}$$

$= \frac{117+117i-13i-13i^2}{(13)^2-(13i)^2}$ [∵ $(a - b)(a + b) = (a^2 - b^2)$]

$= \frac{117+104i-13(-1)}{169-169i^2}$ [∵ $i^2 = -1$]

$$= \frac{130+104i}{169(1-i^2)}$$

$= \frac{13(10+8i)}{169[1-(-1)]}$ [Taking 13 common]

$$= \frac{10+8i}{13 \times 2}$$

$$= \frac{5+4i}{13}$$

$$= \frac{5}{13} + \frac{4}{13}i$$

(ii) Given:

$$\left(\frac{1}{1+4i}-\frac{2}{1+i}\right)\left(\frac{1-i}{5+3i}\right)$$

$$=\left[\frac{1(1+i)-2(1+4i)}{(1+4i)(1+i)}\right]\left(\frac{1-i}{5+3i}\right)$$ [Taking the LCM]

$$=\left[\frac{1+i-2-8i}{(1)(1+i)+4i(1+i)}\right]\left(\frac{1-i}{5+3i}\right)$$

$$=\left[\frac{-1-7i}{1+i+4i+4i^2}\right]\left(\frac{1-i}{5+3i}\right)$$

$$=\left[\frac{-1-7i}{1+5i+4(-1)}\right]\left(\frac{1-i}{5+3i}\right)$$

$$=\left(\frac{-1-7i}{-3+5i}\right)\left(\frac{1-i}{5+3i}\right)$$

$$=\frac{-1(1-i)-7i(1-i)}{-3(5+3i)+5i(5+3i)}$$

$$=\frac{-1+i-7i+7i^2}{-15-9i+25i+15i^2}$$

$$=\frac{-1-6i+7(-1)}{-15+16i+15(-1)}$$

$$=\frac{-6i-8}{16i-30}$$

$$=\frac{-2(4+3i)}{-2(15-8i)}$$

$$=\frac{4+3i}{15-8i}$$

Now, rationalizing by multiply and divide by the conjugate of (15 + 8i)

$$= \frac{4+3i}{15-8i} \times \frac{15+8i}{15+8i}$$

$$= \frac{(4+3i)(15+8i)}{(15)^2-(8i)^2}$$ [∵ (a – b)(a + b) = (a^2 – b^2)]

$$= \frac{4(15+8i)+3i(15+8i)}{225-64i^2}$$

$$= \frac{60+32i+45i+24i^2}{225-64(-1)}$$ [∵ i^2 = -1]

$$= \frac{60+77i+24(-1)}{225+64}$$

$$= \frac{36+77i}{289}$$

$$= \frac{36}{289} + \frac{77}{289}i$$

Q. 5. Show that

(i) $\left\{\frac{(3+2i)}{(2-3i)} + \frac{(3-2i)}{(2+3i)}\right\}$ **is purely real,**

(ii) $\left\{\frac{(\sqrt{7}+i\sqrt{3})}{(\sqrt{7}-i\sqrt{3})} + \frac{(\sqrt{7}-i\sqrt{3})}{(\sqrt{7}+i\sqrt{3})}\right\}$ **is purely real.**

Answer : Given: $\frac{3+2i}{2-3i} + \frac{3-2i}{2+3i}$

Taking the L.C.M, we get

$$= \frac{(3+2i)(2+3i)+(3-2i)(2-3i)}{(2-3i)(2+3i)}$$

$$= \frac{3(2) + 3(3i) + 2i(2) + 2i(3i) + 3(2) + 3(-3i) - 2i(2) + (-2i)(-3i)}{(2)^2 - (3i)^2}$$

[∵ (a + b)(a – b) = ($a^2 - b^2$)]

$$= \frac{6 + 9i + 4i + 6i^2 + 6 - 9i - 4i + 6i^2}{4 - 9i^2}$$

$$= \frac{12 + 12i^2}{4 - 9i^2}$$

Putting $i^2 = -1$

$$= \frac{12 + 12(-1)}{4 - 9(-1)}$$

$$= \frac{12 - 12}{4 + 9}$$

= 0 + 0i

Hence, the given equation is purely real as there is no imaginary part.

(ii) Given: $\frac{\sqrt{7}+i\sqrt{3}}{\sqrt{7}-i\sqrt{3}} + \frac{\sqrt{7}-i\sqrt{3}}{\sqrt{7}+i\sqrt{3}}$

Taking the L.C.M, we get

$$= \frac{(\sqrt{7} + i\sqrt{3})(\sqrt{7} + i\sqrt{3}) + (\sqrt{7} - i\sqrt{3})(\sqrt{7} - i\sqrt{3})}{(\sqrt{7} - i\sqrt{3})(\sqrt{7} + i\sqrt{3})}$$

$$= \frac{(\sqrt{7}+i\sqrt{3})^2+(\sqrt{7}-i\sqrt{3})^2}{(\sqrt{7})^2-(i\sqrt{3})^2} \quad \ldots(i)$$

[∵ (a + b)(a – b) = ($a^2 - b^2$)]

Now, we know that,

$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

So, by applying the formula in eq. (i), we get

$$=\frac{2\left[\left(\sqrt{7}\right)^2+\left(i\sqrt{3}\right)^2\right]}{7-3i^2}$$

$$=\frac{2[7+3i^2]}{7-3(-1)}$$

Putting i^2 = -1

$$=\frac{2[7+3(-1)]}{7+3}$$

$$=\frac{2[7-3]}{10}$$

$$=\frac{8}{10}+0i$$

$$=\frac{4}{5}+0i$$

Hence, the given equation is purely real as there is no imaginary part.

Q. 6. Find the real values of θ for which $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real.

Answer : Since $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real

Firstly, we need to solve the given equation and then take the imaginary part as 0

$$\frac{1+i\cos\theta}{1-2i\cos\theta}$$

We rationalize the above by multiply and divide by the conjugate of (1 -2i cos θ)

$$=\frac{1+i\cos\theta}{1-2i\cos\theta}\times\frac{1+2i\cos\theta}{1+2i\cos\theta}$$

$$=\frac{(1+i\cos\theta)(1+2i\cos\theta)}{(1-2i\cos\theta)(1+2i\cos\theta)}$$

We know that,

(a – b)(a + b) = $(a^2 – b^2)$

$$=\frac{1(1)+1(2i\cos\theta)+i\cos\theta(1)+i\cos\theta(2i\cos\theta)}{(1)^2-(2i\cos\theta)^2}$$

$$=\frac{1+2i\cos\theta+i\cos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$$

$=\frac{1+3i\cos\theta+2(-1)\cos^2\theta}{1-4(-1)\cos^2\theta}$ [∵ $i^2 = -1$]

$$=\frac{1+3i\cos\theta-2\cos^2\theta}{1+4\cos^2\theta}$$

$$=\frac{1-2\cos^2\theta}{1+4\cos^2\theta}+i\frac{3\cos\theta}{1+4\cos^2\theta}$$

Since $\frac{1+\mathrm{i}\cos\theta}{1-2i\cos\theta}$ is purely real [given]

Hence, imaginary part is equal to 0

i.e. $\frac{3\cos\theta}{1+4\cos^2\theta}=0$

⇒ 3 cos θ = 0 × (1 + 4 $\cos^2θ$)

⇒ 3 cos θ = 0

⇒ cos θ = 0

⇒ cos θ = cos 0

Since, cos θ = cos y

Then $\theta = (2n+1)\frac{\pi}{2}\pm y$ where n Є Z

Putting y = 0

$\theta = (2n + 1)\frac{\pi}{2} \pm 0$

$\theta = (2n + 1)\frac{\pi}{2}$ where n Є Z

Hence, for $\theta = (2n + 1)\frac{\pi}{2}$. where n Є Z $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real.

Q. 7. If |z + i| = |z – i|, prove that z is real.

Answer : Let z = x + iy

Consider, |z + i| = |z – i|

⇒ |x + iy + i| = |x + iy – i|

⇒ |x + i(y +1)| = |x + i(y – 1)|

$\Rightarrow \sqrt{(x)^2 + (y+1)^2} = \sqrt{(x)^2 + (y-1)^2}$

$\left[\because |z| = modulus = \sqrt{a^2 + b^2}\right]$

$\Rightarrow \sqrt{x^2 + y^2 + 1 + 2y} = \sqrt{x^2 + y^2 + 1 - 2y}$

Squaring both the sides, we get

⇒ $x^2 + y^2 + 1 + 2y = x^2 + y^2 + 1 - 2y$

⇒ $x^2 + y^2 + 1 + 2y - x^2 - y^2 - 1 + 2y = 0$

⇒ 2y + 2y = 0

⇒ 4y = 0

⇒ y = 0

Putting the value of y in eq. (i), we get

z = x + i(0)

⇒ z = x

Hence, z is purely real.

Q. 8. Give an example of two complex numbers z_1 and z_2 such that $z_1 \neq z_2$ and $|z_1| = |z_2|$.

Answer : Let $z_1 = 3 - 4i$ and $z_2 = 4 - 3i$

Here, $z_1 \neq z_2$

Now, calculating the modulus, we get,

$|z_1| = \sqrt{3^2 + (4)^2} = \sqrt{25} = 5$

$|z_2| = \sqrt{4^2 + (3)^2} = \sqrt{25} = 5$

Q. 9. A. Find the conjugate of each of the following:

(–5 – 2i)

Answer : Given: z = (-5 – 2i)

Here, we have to find the conjugate of (-5 – 2i)

So, the conjugate of (- 5 – 2i) is (-5 + 2i)

Q. 9. B. Find the conjugate of each of the following:

$$\frac{1}{(4+3i)}$$

Answer : Given: $\frac{1}{4+3i}$

First, we calculate $\frac{1}{4+3i}$ and then find its conjugate

Now, rationalizing

$$= \frac{1}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{4-3i}{(4+3i)(4-3i)} \quad \ldots\text{(i)}$$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{4-3i}{(4)^2-(3i)^2}$$

$$= \frac{4-3i}{16-9i^2}$$

$= \frac{4-3i}{16-9(-1)}$ [∵ $i^2 = -1$]

$$= \frac{4-3i}{16+9}$$

$$= \frac{4-3i}{25}$$

$$= \frac{4}{25} - \frac{3}{25}i$$

Hence, $\frac{1}{4+3i} = \frac{4}{25} - \frac{3}{25}i$

So, a conjugate of $\frac{1}{4+3i}$ is $\frac{4}{25} + \frac{3}{25}i$

Q. 9. C. Find the conjugate of each of the following:

$$\frac{(1+i)^2}{(3-i)}$$

Answer : Given: $\frac{(1+i)^2}{(3-i)}$

Firstly, we calculate $\frac{(1+i)^2}{(3-i)}$ and then find its conjugate

$\frac{(1+i)^2}{(3-i)} = \frac{1+i^2+2i}{(3-i)}$ [∵ $(a + b)^2 = a^2 + b^2 + 2ab$]

$= \frac{1+(-1)+2i}{3-i}$ [∵ $i^2 = -1$]

$$= \frac{2i}{3-i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 3 – i

$$= \frac{2i}{3-i} \times \frac{3+i}{3+i}$$

$= \frac{(2i)(3+i)}{(3+i)(3-i)}$ …(i)

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{(2i)(3+i)}{(3)^2-(i)^2}$$

$$= \frac{2i(3)+2i(i)}{9-i^2}$$

$= \frac{6i+2i^2}{9-(-1)}$ [∵ $i^2 = -1$]

$= \frac{6i+2(-1)}{9+1}$ [∵ $i^2 = -1$]

$$= \frac{6i-2}{10}$$

$$= \frac{2(3i-1)}{10}$$

$$= \frac{(-1+3i)}{5}$$

$= -\frac{1}{5} + \frac{3}{5}i$

Hence, $\frac{(1+i)^2}{(3-i)} = -\frac{1}{5} + \frac{3}{5}i$

So, the conjugate of $\frac{(1+i)^2}{(3-i)}$ is $-\frac{1}{5} - \frac{3}{5}i$

Q. 9. D. Find the conjugate of each of the following:

$$\frac{(1+i)(2+i)}{(3+i)}$$

Answer : Given: $\frac{(1+i)(2+i)}{(3+i)}$

Firstly, we calculate $\frac{(1+i)(2+i)}{(3+i)}$ and then find its conjugate

$$\frac{(1+i)(2+i)}{(3+i)} = \frac{1(2) + 1(i) + i(2) + i(i)}{(3+i)}$$

$$= \frac{2 + i + 2i + i^2}{3 + i}$$

$= \frac{2+3i-1}{3+i}$ [∵ $i^2 = -1$]

$$= \frac{1 + 3i}{3 + i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 3 + i

$$= \frac{1 + 3i}{3 + i} \times \frac{3 - i}{3 - i}$$

$= \frac{(1+3i)(3-i)}{(3+i)(3-i)}$ …(i)

Now, we know that,

(a + b)(a – b) = (a^2 – b^2)

So, eq. (i) become

$$= \frac{(1+3i)(3-i)}{(3)^2-(i)^2}$$

$$= \frac{1(3)+1(-i)+3i(3)+3i(-i)}{9-i^2}$$

$$= \frac{3-i+9i-3i^2}{9-(-1)}$$ [∵ i^2 = -1]

$$= \frac{3+8i-3(-1)}{9+1}$$ [∵ i^2 = -1]

$$= \frac{3+8i+3}{10}$$

$$= \frac{6+8i}{10}$$

$$= \frac{2(3+4i)}{10}$$

$$= \frac{3+4i}{5}$$

$$= \frac{3}{5}+\frac{4}{5}i$$

Hence, $\frac{(1+i)(2+i)}{(3+i)} = \frac{3}{5}+\frac{4}{5}i$

So, the conjugate of $\frac{(1+i)^2}{(3-i)}$ is $\frac{3}{5}-\frac{4}{5}i$

Q. 9. E. Find the conjugate of each of the following:

$\sqrt{-3}$

Answer : Given: z = √-3

The above can be re – written as

$$z = \sqrt{(-1) \times 3}$$

$z = \sqrt{3i^2}$ [∵ $i^2 = -1$]

z = 0 + i√3

So, the conjugate of z = 0 + i√3 is

$$\bar{z} = 0 - i\sqrt{3}$$

Or $\bar{z} = -i\sqrt{3} = -\sqrt{-3}$

Q. 9. F. Find the conjugate of each of the following:

$$\sqrt{2}$$

Answer : Given: z = √2

The above can be re – written as

z = √2 + 0i

Here, the imaginary part is zero

So, the conjugate of z = √2 + 0i is

$$\bar{z} = \sqrt{2} - 0i$$

Or $\bar{z} = \sqrt{2}$

Q. 9. G. Find the conjugate of each of the following:

$$-\sqrt{-1}$$

Answer : Given: z = -√-1

The above can be re – written as

$z = -\sqrt{i^2}$ [∵ i^2 = -1]

z = 0 – i

So, the conjugate of z = (0 – i) is

$\bar{z} = 0 + i$

Or $\bar{z} = i$

Q. 9. H. Find the conjugate of each of the following:

$(2 – 5i)^2$

Answer : Given: z = $(2 – 5i)^2$

First we calculate $(2 – 5i)^2$ and then we find the conjugate

$(2 – 5i)^2 = (2)^2 + (5i)^2 – 2(2)(5i)$

= $4 + 25i^2 – 20i$

= 4 + 25(-1) – 20i [∵ i^2 = -1]

= 4 – 25 – 20i

= -21 – 20i

Now, we have to find the conjugate of (-21 – 20i)

So, the conjugate of (- 21 – 20i) is (-21 + 20i)

Q. 10. A. Find the modulus of each of the following:

$\left(3+\sqrt{-5}\right)$

Answer : Given: z = (3 + √-5)

The above can be re – written as

$z = 3 + \sqrt{(-1) \times 5}$

z = 3 + i√5 [∵ i^2 = -1]

Now, we have to find the modulus of (3 + i√5)

So, $|z| = |3 + i\sqrt{5}| = \sqrt{(3)^2 + (\sqrt{5})^2} = \sqrt{9 + 5} = \sqrt{14}$

Hence, the modulus of (3 + √-5) is √14

Q. 10. B. Find the modulus of each of the following:

(–3 – 4i)

Answer : Given: z = (-3 – 4i)

Now, we have to find the modulus of (-3 – 4i)

So, $|z| = |-3 - 4i| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Hence, the modulus of (-3 – 4i) is 5

Q. 10. C. Find the modulus of each of the following:

(7 + 24i)

Answer : Given: z = (7 + 24i)

Now, we have to find the modulus of (7 + 24i)

So, $|z| = |7 + 24i| = \sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25$

Hence, the modulus of (7 + 24i) is 25

Q. 10. D. Find the modulus of each of the following:

3i

Answer : Given: z = 3i

The above equation can be re – written as

z = 0 + 3i

Now, we have to find the modulus of (0 + 3i)

So, $|z| = |0 + 3i| = \sqrt{(0)^2 + (3)^2} = \sqrt{9} = 3$

Hence, the modulus of (3i) is 3

Q. 10. E. Find the modulus of each of the following:

$$\frac{(3+2i)^2}{(4-3i)}$$

Answer : Given: $\frac{(3+2i)^2}{(4-3i)}$

Firstly, we calculate $\frac{(3+2i)^2}{(4-3i)}$ and then find its modulus

$\frac{(3+2i)^2}{(4-3i)} = \frac{9+4i^2+12i}{(4-3i)}$ [∵ $(a + b)^2 = a^2 + b^2 + 2ab$]

$= \frac{9+4(-1)+12i}{4-3i}$ [∵ $i^2 = -1$]

$$= \frac{5 + 12i}{4 - 3i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 4 + 3i

$$= \frac{5 + 12i}{4 - 3i} \times \frac{4 + 3i}{4 + 3i}$$

$= \frac{(5+12i)(4+3i)}{(4-3i)(4+3i)}$ …(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{5(4) + (5)(3i) + 12i(4) + 12i(3i)}{(4)^2 - (3i)^2}$$

$$= \frac{20 + 15i + 48i + 36i^2}{16 - 9i^2}$$

$= \frac{20+63i+36(-1)}{16-9(-1)}$ [∵ $i^2 = -1$]

$= \frac{20-36+63i}{16+9}$ [∵ $i^2 = -1$]

$$= \frac{-16 + 63i}{25}$$

$$= -\frac{16}{25} + \frac{63}{25}i$$

Now, we have to find the modulus of $\left(-\frac{16}{25} + \frac{63}{25}i\right)$

So, $|z| = \left|-\frac{16}{25} + \frac{63}{25}i\right| = \sqrt{\left(-\frac{16}{25}\right)^2 + \left(\frac{63}{25}\right)^2}$

$$= \sqrt{\frac{256}{625} + \frac{3969}{625}}$$

$$= \sqrt{\frac{4225}{625}}$$

$$= \frac{65}{25}$$

$$= \frac{13}{5}$$

Hence, the modulus of $\frac{(3+2i)^2}{(4-3i)}$ is $\frac{13}{5}$

Q. 10. F. Find the modulus of each of the following:

$$\frac{(2-i)(1+i)}{(1+i)}$$

Answer : Given: $\frac{(2-i)(1+i)}{(1+i)}$

Firstly, we calculate $\frac{(2-i)(1+i)}{(1+i)}$ and then find its modulus

$$\frac{(2-i)(1+i)}{(1+i)} = \frac{2(1)+2(i)+(-i)(1)+(-i)(i)}{(1+i)}$$

$$= \frac{2+2i-i-i^2}{1+i}$$

$= \frac{2+i-(-1)}{1+i}$ [∵ $i^2 = -1$]

$$= \frac{3+i}{1+i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 1 + i

$$= \frac{3+i}{1+i} \times \frac{1-i}{1-i}$$

$= \frac{(3+i)(1-i)}{(1+i)(1-i)}$ …(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{3(1-i)+i(1-i)}{(1)^2-(i)^2}$$

$$= \frac{3(1) + 3(-i) + i(1) + i(-i)}{1 - i^2}$$

$= \frac{3-3i+i-i^2}{1-(-1)}$ [$\because$ i^2 = -1]

$= \frac{3-2i-(-1)}{1+1}$ [$\because$ i^2 = -1]

$$= \frac{3 - 2i + 1}{2}$$

$$= \frac{4 - 2i}{2}$$

= 2 – i

Now, we have to find the modulus of (2 – i)

So, $|z| = |2 - i| = |2 + (-1)i| = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$

Q. 10. G. Find the modulus of each of the following:

5

Answer : Given: z = 5

The above equation can be re – written as

z = 5 + 0i

Now, we have to find the modulus of (5 + 0i)

So, $|z| = |5 + 0i| = \sqrt{(5)^2 + (0)^2} = 5$

Q. 10. H. Find the modulus of each of the following:

(1 + 2i) (i – 1)

Answer : Given: z = (1 + 2i)(i – 1)

Firstly, we calculate the (1 + 2i)(i – 1) and then find the modulus

So, we open the brackets,

1(i – 1) + 2i(i – 1)

= 1(i) + (1)(-1) + 2i(i) + 2i(-1)

= i – 1 + 2i^2 – 2i

= - i – 1 + 2(-1) [∵ i^2 = - 1]

= - i – 1 – 2

= - i – 3

Now, we have to find the modulus of (-3 - i)

So, $|z| = |-3 - i| = |-3 + (-1)i| = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10}$

Q. 11. A. Find the multiplicative inverse of each of the following:

$$\left(1 - \sqrt{3}i\right)$$

Answer : Given: (1 - i√3)

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of z = z^{-1}

$$= \frac{1}{z}$$

Putting z = 1 - i√3

$$\text{So, Multiplicative inverse of } 1 - i\sqrt{3} = \frac{1}{1 - i\sqrt{3}}$$

Now, rationalizing by multiply and divide by the conjugate of (1 - i√3)

$$= \frac{1}{1 - i\sqrt{3}} \times \frac{1 + i\sqrt{3}}{1 + i\sqrt{3}}$$

$$= \frac{1 + i\sqrt{3}}{(1 - i\sqrt{3})(1 + i\sqrt{3})}$$

Using (a – b)(a + b) = (a^2 – b^2)

$$= \frac{1 + i\sqrt{3}}{(1)^2 - \left(i\sqrt{3}\right)^2}$$

$$= \frac{1 + i\sqrt{3}}{1 - 3i^2}$$

$= \frac{1+i\sqrt{3}}{1-3(-1)}$ [∵ i^2 = -1]

$$= \frac{1 + i\sqrt{3}}{1 + 3}$$

$$= \frac{1 + i\sqrt{3}}{4}$$

$$= \frac{1}{4} + \frac{\sqrt{3}}{4}i$$

Hence, Multiplicative Inverse of $(1 - i\sqrt{3})$ is $\frac{1}{4} + \frac{\sqrt{3}}{4}i$

Q. 11. B. Find the multiplicative inverse of each of the following:

(2 + 5i)

Answer : Given: 2 + 5i

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of z = z^{-1}

$$= \frac{1}{z}$$

Putting z = 2 + 5i

$$\text{So, Multiplicative inverse of } 2+5\mathrm{i} = \frac{1}{2+5\mathrm{i}}$$

Now, rationalizing by multiply and divide by the conjugate of (2+5i)

$$= \frac{1}{2+5i} \times \frac{2-5i}{2-5i}$$

$$= \frac{2-5i}{(2+5i)(2-5i)}$$

Using (a – b)(a + b) = $(a^2 - b^2)$

$$= \frac{2-5i}{(2)^2-(5i)^2}$$

$$= \frac{2-5i}{4-25i^2}$$

$= \frac{2-5i}{4-25(-1)}$ [∵ $i^2 = -1$]

$$= \frac{2-5i}{4+25}$$

$$= \frac{2-5i}{29}$$

$$= \frac{2}{29} - \frac{5}{29}i$$

Hence, Multiplicative Inverse of (2+5i) is $\frac{2}{29} - \frac{5}{29}\mathrm{i}$

Q. 11. C. Find the multiplicative inverse of each of the following:

$$\frac{(2+3\mathrm{i})}{(1+\mathrm{i})}$$

Answer : Given: $\frac{2+3i}{1+i}$

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of z = z^{-1}

$$= \frac{1}{z}$$

$$\text{Putting z } = \frac{2+3i}{1+i}$$

$$\text{So, Multiplicative inverse of } \frac{2+3i}{1+i} = \frac{1}{\frac{2+3i}{1+i}} = \frac{1+i}{2+3i}$$

Now, rationalizing by multiply and divide by the conjugate of (2+3i)

$$= \frac{1+i}{2+3i} \times \frac{2-3i}{2-3i}$$

$$= \frac{(1+i)(2-3i)}{(2+3i)(2-3i)}$$

Using (a – b)(a + b) = ($a^2 - b^2$)

$$= \frac{1(2-3i)+i(2-3i)}{(2)^2-(3i)^2}$$

$$= \frac{2-3i+2i-3i^2}{4-9i^2}$$

$$= \frac{2-i-3(-1)}{4-9(-1)}$$ [∵ $i^2 = -1$]

$$= \frac{5-i}{4+9}$$

$$= \frac{5 - i}{13}$$

$$= \frac{5}{13} - \frac{1}{13}i$$

$$\text{Hence, Multiplicative Inverse of } \frac{(2+3\mathrm{i})}{1+i} \text{ is} \frac{5}{13} - \frac{1}{13}i$$

Q. 11. D. Find the multiplicative inverse of each of the following:

$$\frac{(1+\mathrm{i})(1+2\mathrm{i})}{(1+3\mathrm{i})}$$

Answer : Given: $\frac{(1+i)(1+2i)}{(1+3i)}$

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of z = z^{-1}

$$= \frac{1}{z}$$

$$\text{Putting z } = \frac{(1+\mathrm{i})(1+2\mathrm{i})}{(1+3\mathrm{i})}$$

$$\text{So, Multiplicative inverse of } \frac{(1+i)(1+2i)}{(1+3i)} = \frac{1}{\frac{(1+i)(1+2i)}{(1+3i)}}$$

$$= \frac{(1+3i)}{(1+i)(1+2i)}$$

We solve the above equation

$$= \frac{1+3i}{1(1)+1(2i)+i(1)+i(2i)}$$

$$= \frac{1+3i}{1+2i+i+2i^2}$$

$$= \frac{1+3i}{1+3i+2(-1)}$$ [∵ $i^2 = -1$]

$$= \frac{1+3i}{-1+3i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of (-1 + 3i)

$$= \frac{1+3i}{-1+3i} \times \frac{-1-3i}{-1-3i}$$

$$= \frac{(1+3i)(-1-3i)}{(-1+3i)(-1-3i)}$$ …(i)

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{1(-1-3i)+3i(-1-3i)}{(-1)^2-(3i)^2}$$

$$= \frac{-1-3i-3i-9i^2}{1-9i^2}$$

$$= \frac{-1-6i-9(-1)}{1-9(-1)}$$ [∵ $i^2 = -1$]

$$= \frac{-1-6i+9}{1+9}$$

$$= \frac{8-6i}{10}$$

$$= \frac{2(4-3i)}{10}$$

$$= \frac{4 - 3i}{5}$$

$$= \frac{4}{5} - \frac{3}{5}i$$

$$\text{Hence, Multiplicative inverse of } \frac{(1+i)(1+2i)}{(1+3i)} = \frac{4}{5} - \frac{3}{5}i$$

Q. 12. If $\left(\frac{1-i}{1+i}\right)^{100}$ = (a + ib), find the values of a and b.

Answer : Given: $a + ib = \left(\frac{1-i}{1+i}\right)^{100}$

Consider the given equation,

$$a + ib = \left(\frac{1-i}{1+i}\right)^{100}$$

Now, we rationalize

$$= \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{100}$$

[Here, we multiply and divide by the conjugate of 1 + i]

$$= \left(\frac{(1-i)^2}{(1+i)(1-i)}\right)^{100}$$

$$= \left(\frac{1 + i^2 - 2i}{(1+i)(1-i)}\right)^{100}$$

Using (a + b)(a – b) = ($a^2 - b^2$)

$$= \left(\frac{1 + (-1) - 2i}{(1)^2 - (i)^2}\right)^{100}$$

$$= \left(\frac{-2i}{1-i^2}\right)^{100}$$

$$= \left(\frac{-2i}{1-(-1)}\right)^{100}$$ [$\because$ $i^2 = -1$]

$$= \left(\frac{-2i}{2}\right)^{100}$$

$= (-i)^{100}$

$= [(-i)^4]^{25}$

$= (i^4)^{25}$

$= (1)^{25}$

[$\because$ $i^4 = i^2 \times i^2 = -1 \times -1 = 1$]

$(a + ib) = 1 + 0i$

On comparing both the sides, we get

a = 1 and b = 0

Hence, the value of a is 1 and b is 0

Q. 13. If $\left(\frac{1+i}{1-i}\right)^{93} - \left(\frac{1-i}{1+i}\right)^{3}$ **= x + iy, find x and y.**

Answer : Consider,

$$x + iy = \left(\frac{1+i}{1-i}\right)^{93} - \left(\frac{1-i}{1+i}\right)^{3}$$

Now, rationalizing

$$x + iy = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^{93} - \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{3}$$

$$= \left(\frac{(1+i)^2}{(1-i)(1+i)}\right)^{93} - \left(\frac{(1-i)^2}{(1+i)(1-i)}\right)^3$$

In denominator, we use the identity

$(a - b)(a + b) = a^2 - b^2$

$$= \left(\frac{1+i^2+2i}{(1)^2-(i)^2}\right)^{93} - \left(\frac{1+i^2-2i}{(1)^2-(i)^2}\right)^3$$

$$= \left(\frac{1+(-1)+2i}{1-i^2}\right)^{93} - \left(\frac{1+(-1)-2i}{1-i^2}\right)^3$$

$$= \left(\frac{2i}{1-(-1)}\right)^{93} - \left(\frac{-2i}{1-(-1)}\right)^3$$

$$= \left(\frac{2i}{2}\right)^{93} - \left(\frac{-2i}{2}\right)^3$$

$= (i)^{93} - (-i)^3$

$= (i)^{92+1} - [-(i)^3]$

$= [(i)^{92}(i)] - [-(i^2 \times i)]$

$= [(i^4)^{23}(i)] - [- (-i)]$

$= [(1)^{23}(i)] - i$

$= i - i$

$x + iy = 0$

$\therefore x = 0$ and $y = 0$

Q. 14. If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$.

Answer : Consider the given equation,

$$x + iy = \frac{a + ib}{a - ib}$$

Now, rationalizing

$$x + iy = \frac{a + ib}{a - ib} \times \frac{a + ib}{a + ib}$$

$$= \frac{(a + ib)(a + ib)}{(a - ib)(a + ib)}$$

$$= \frac{a(a + ib) + ib(a + ib)}{(a)^2 - (ib)^2}$$

[(a – b)(a + b) = a^2 – b^2]

$$= \frac{a^2 + iab + iab + i^2 b^2}{a^2 - i^2 b^2}$$

$$= \frac{a^2 + iab + iab + (-1)b^2}{a^2 - (-1)b^2}$$ [i^2 = -1]

$$x + iy = \frac{a^2 + 2iab - b^2}{a^2 + b^2}$$

$$x + iy = \frac{(a^2 - b^2)}{a^2 + b^2} + i\frac{2ab}{a^2 + b^2}$$

On comparing both the sides, we get

$$x = \frac{(a^2 - b^2)}{a^2 + b^2} \;\&\; y = \frac{2ab}{a^2 + b^2}$$

Now, we have to prove that $x^2 + y^2 = 1$

Taking LHS,

$x^2 + y^2$

Putting the value of x and y, we get

$$\left[\frac{(a^2-b^2)}{a^2+b^2}\right]^2+\left[\frac{2ab}{a^2+b^2}\right]^2$$

$$=\frac{1}{(a^2+b^2)^2}[(a^2-b^2)^2+(2ab)^2]$$

$$=\frac{1}{(a^2+b^2)^2}[a^4+b^4-2a^2b^2+4a^2b^2]$$

$$=\frac{1}{(a^2+b^2)^2}[a^4+b^4+2a^2b^2]$$

$$=\frac{1}{(a^2+b^2)^2}[(a^2+b^2)^2]$$

= 1

= RHS

Q. 15. If $(a+ib)=\frac{c+i}{c-i}$ **, where c is real, prove that** $a^2+b^2=1$ **and** $\frac{b}{a}=\frac{2c}{c^2-1}$ **.**

Answer : Consider the given equation,

$$a+ib=\frac{c+i}{c-i}$$

Now, rationalizing

$$a+ib=\frac{c+i}{c-i}\times\frac{c+i}{c+i}$$

$$=\frac{(c+i)(c+i)}{(c-i)(c+i)}$$

$$=\frac{(c+i)^2}{(c)^2-(i)^2}$$

$[(a-b)(a+b)=a^2-b^2]$

$$= \frac{c^2 + 2ic + i^2}{c^2 - i^2}$$

$a + ib = \frac{c^2 + 2ic + (-1)}{c^2 - (-1)}$ [i^2 = -1]

$$a + ib = \frac{c^2 + 2ic - 1}{c^2 + 1}$$

$$a + ib = \frac{(c^2 - 1)}{c^2 + 1} + i\frac{2c}{c^2 + 1}$$

On comparing both the sides, we get

$$a = \frac{(c^2 - 1)}{c^2 + 1} \ \& \ b = \frac{2c}{c^2 + 1}$$

Now, we have to prove that $a^2 + b^2 = 1$

Taking LHS,

$a^2 + b^2$

Putting the value of a and b, we get

$$\left[\frac{(c^2 - 1)}{c^2 + 1}\right]^2 + \left[\frac{2c}{c^2 + 1}\right]^2$$

$$= \frac{1}{(c^2 + 1)^2}[(c^2 - 1)^2 + (2c)^2]$$

$$= \frac{1}{(c^2 + 1)^2}[c^4 + 1 - 2c^2 + 4c^2]$$

$$= \frac{1}{(c^2 + 1)^2}[c^4 + 1 + 2c^2]$$

$$= \frac{1}{(c^2 + 1)^2}[(c^2 + 1)^2]$$

= 1

= RHS

Now, we have to prove $\frac{b}{a} = \frac{2c}{c^2-1}$

Taking LHS, $\frac{b}{a}$

Putting the value of a and b, we get

$$\frac{b}{a} = \frac{\frac{2c}{c^2+1}}{\frac{(c^2-1)}{c^2+1}} = \frac{2c}{c^2+1} \times \frac{c^2+1}{c^2-1} = \frac{2c}{c^2-1} = RHS$$

Hence Proved

Q. 16. Show that $(1-i)^n\left(1-\frac{1}{i}\right)^n = 2^n$ **for all n N.**

Answer : To show: $(1-i)^n\left(1-\frac{1}{i}\right)^n = 2^n$

Taking LHS,

$$(1-i)^n\left(1-\frac{1}{i}\right)^n$$

$= (1-i)^n\left(1-\frac{1}{i}\times\frac{i}{i}\right)^n$ [rationalize]

$$= (1-i)^n\left(1-\frac{i}{i^2}\right)^n$$

$= (1-i)^n\left(1-\frac{i}{-1}\right)^n$ [$\because$ $i^2 = -1$]

$= (1-i)^n(1+i)^n$

$= [(1-i)(1+i)]^n$

$= [(1)^2 – (i)^2]^n$ $[(a + b)(a – b) = a^2 – b^2]$

$= (1 – i^2)^n$

$= [1 – (-1)]^n [\because i^2 = -1]$

$= (2)^n$

$= 2^n$

= RHS

Hence Proved

Q. 17. Find the smallest positive integer n for which $(1 + i)^{2n} = (1 – i)^{2n}$.

Answer :

Given: $(1 + i)^{2n} = (1 – i)^{2n}$

Consider the given equation,

$(1 + i)^{2n} = (1 – i)^{2n}$

$$\Rightarrow \frac{(1+i)^{2n}}{(1-i)^{2n}} = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1$$

Now, rationalizing by multiply and divide by the conjugate of (1 – i)

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^{2n} = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{(1-i)(1+i)}\right)^{2n} = 1$$

$$\Rightarrow \left[\frac{1+i^2+2i}{(1)^2-(i)^2}\right]^{2n} = 1$$

$[(a + b)^2 = a^2 + b^2 + 2ab \;\&\; (a – b)(a + b) = (a^2 – b^2)]$

$\Rightarrow \left[\frac{1+(-1)+2i}{1-(-1)}\right]^{2n} = 1$ [$i^2 = -1$]

$\Rightarrow \left[\frac{2i}{2}\right]^{2n} = 1$

⇒ $(i)^{2n} = 1$

Now, $i^{2n} = 1$ is possible if n = 2 because $(i)^{2(2)} = i^4 = (-1)^4 = 1$

So, the smallest positive integer n = 2

Q. 18. Prove that (x + 1 + i) (x + 1 – i) (x – 1 – i) (x – 1 – i) = (x^4 + 4).

Answer : To Prove:

(x + 1 + i) (x + 1 – i) (x – 1 + i) (x – 1 – i) = (x^4 + 4)

Taking LHS

(x + 1 + i) (x + 1 – i) (x – 1 + i) (x – 1 – i)

= [(x + 1) + i][(x + 1) – i][(x – 1) + i][(x – 1) – i]

Using $(a – b)(a + b) = a^2 – b^2$

$$\underbrace{[(x+1)+i][(x+1)-i]}_{a\,=\,x+1\ \&\ b\,=\,i}\underbrace{[(x-1)+i][(x-1)-i]}_{a\,=\,x-1\ \&\ b\,=\,i}$$

$= [(x + 1)^2 – (i)^2] [(x – 1)^2 – (i)^2]$

$= [x^2 + 1 + 2x – i^2](x^2 + 1 – 2x – i^2]$

$= [x^2 + 1 + 2x – (-1)](x^2 + 1 – 2x – (-1)]$ [∵ $i^2 = -1$]

$= [x^2 + 2 + 2x][x^2 + 2 – 2x]$

Again, using $(a – b)(a + b) = a^2 – b^2$

Now, $a = x^2 + 2$ and $b = 2x$

$= [(x^2 + 2)^2 – (2x)^2]$

$= [x^4 + 4 + 2(x^2)(2) – 4x^2]$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

$= [x^4 + 4 + 4x^2 – 4x^2]$

$= x^4 + 4$

= RHS

∴ LHS = RHS

Hence Proved

Q. 19. If a = (cosθ + i sinθ), prove that $\frac{1+a}{1-a} = \left(\cot\frac{\theta}{2}\right)i$**.**

Answer : Given: a = cosθ + isinθ

To prove: $\frac{1+a}{1-a} = \left(\cot\frac{\theta}{2}\right)i$

Taking LHS,

$$\frac{1+a}{1-a}$$

Putting the value of a, we get

$$= \frac{1 + \cos\theta + i\sin\theta}{1 - (\cos\theta + i\sin\theta)}$$

$$= \frac{1 + \cos\theta + i\sin\theta}{1 - \cos\theta - i\sin\theta}$$

We know that,

$1 + \cos 2\theta = 2\cos^2\theta$

Or $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

And $1-\cos\theta = 2\sin^2\frac{\theta}{2}$

Using the above two formulas

$$=\frac{2\cos^2\dfrac{\theta}{2}+i\sin\theta}{2\sin^2\dfrac{\theta}{2}-i\sin\theta}$$

Using, $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$=\frac{2\cos^2\dfrac{\theta}{2}+i2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{2\sin^2\dfrac{\theta}{2}-2i\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}$$

$$=\frac{2\cos\dfrac{\theta}{2}\left[\cos\dfrac{\theta}{2}+i\sin\dfrac{\theta}{2}\right]}{2\sin\dfrac{\theta}{2}\left[\sin\dfrac{\theta}{2}-i\cos\dfrac{\theta}{2}\right]}$$

$$=\cot\frac{\theta}{2}\left[\frac{\cos\dfrac{\theta}{2}+i\sin\dfrac{\theta}{2}}{\sin\dfrac{\theta}{2}-i\cos\dfrac{\theta}{2}}\right]\left[\because\frac{\cos\theta}{\sin\theta}=\cot\theta\right]$$

Rationalizing by multiply and divide by the conjugate of $\sin\frac{\theta}{2}-i\cos\frac{\theta}{2}$

$$=\left(\cot\frac{\theta}{2}\right)\left[\frac{\cos\dfrac{\theta}{2}+i\sin\dfrac{\theta}{2}}{\sin\dfrac{\theta}{2}-i\cos\dfrac{\theta}{2}}\times\frac{\sin\dfrac{\theta}{2}+i\cos\dfrac{\theta}{2}}{\sin\dfrac{\theta}{2}+i\cos\dfrac{\theta}{2}}\right]$$

$$=\left(\cot\frac{\theta}{2}\right)\frac{\left(\cos\dfrac{\theta}{2}+i\sin\dfrac{\theta}{2}\right)\left(\sin\dfrac{\theta}{2}+i\cos\dfrac{\theta}{2}\right)}{\left(\sin\dfrac{\theta}{2}-i\cos\dfrac{\theta}{2}\right)\left(\sin\dfrac{\theta}{2}+i\cos\dfrac{\theta}{2}\right)}$$

$$=\left(\cot\frac{\theta}{2}\right)\frac{\left(\cos\dfrac{\theta}{2}\right)\left(\sin\dfrac{\theta}{2}+i\cos\dfrac{\theta}{2}\right)+i\sin\dfrac{\theta}{2}\left(\sin\dfrac{\theta}{2}+i\cos\dfrac{\theta}{2}\right)}{\left(\sin\dfrac{\theta}{2}\right)^2-\left(i\cos\dfrac{\theta}{2}\right)^2}$$

$$= \left(\cot\frac{\theta}{2}\right)\frac{\cos\frac{\theta}{2}\sin\frac{\theta}{2} + i\cos^2\frac{\theta}{2} + i\sin^2\frac{\theta}{2} + i^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} - i^2\cos^2\frac{\theta}{2}}$$

Putting $i^2 = -1$, we get

$$= \left(\cot\frac{\theta}{2}\right)\frac{\cos\frac{\theta}{2}\sin\frac{\theta}{2} + i\cos^2\frac{\theta}{2} + i\sin^2\frac{\theta}{2} + (-1)\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} - (-1)\cos^2\frac{\theta}{2}}$$

$$= \left(\cot\frac{\theta}{2}\right)\frac{\cos\frac{\theta}{2}\sin\frac{\theta}{2} + i\,(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}) - \sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}$$

We know that,

$\cos^2\theta + \sin^2\theta = 1$

$$= \left(\cot\frac{\theta}{2}\right)\left[\frac{i\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)}{1}\right]$$

$$= \cot\frac{\theta}{2}\,(i)$$

= RHS

Hence Proved

Q. 20. If $z_1 = (2 - i)$ and $z_2 = (1 + i)$, find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$.

Answer :

Given: $z_1 = (2 - i)$ and $z_2 = (1 + i)$

To find: $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$

Consider,

$$\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$$

Putting the value of z_1 and z_2, we get

$$= \left|\frac{2 - i + 1 + i + 1}{2 - i - (1 + i) + i}\right|$$

$$= \left|\frac{4}{2 - i - 1 - i + i}\right|$$

$$= \left|\frac{4}{1 - i}\right|$$

Now, rationalizing by multiply and divide by the conjugate of 1 – i

$$= \left|\frac{4}{1 - i} \times \frac{1 + i}{1 + i}\right|$$

$$= \left|\frac{4(1 + i)}{(1 - i)(1 + i)}\right|$$

$= \left|\frac{4(1+i)}{(1)^2-(i)^2}\right|$ [$(a – b)(a + b) = a^2 – b^2$]

$$= \left|\frac{4(1 + i)}{1 - i^2}\right|$$

$= \left|\frac{4(1+i)}{1-(-1)}\right|$ [Putting $i^2 = -1$]

$$= \left|\frac{4(1 + i)}{2}\right|$$

= |2(1 + i)|

= |2 + 2i|

Now, we have to find the modulus of (2 + 2i)

So, $|z| = |2 + 2i| = \sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

Hence, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right| = 2\sqrt{2}$

Q. 21. A. Find the real values of x and y for which:

(1 – i) x + (1 + i) y = 1 – 3i

Answer :

(1 – i) x + (1 + i) y = 1 – 3i

⇒ x – ix + y + iy = 1 – 3i

⇒ (x + y) – i(x – y) = 1 – 3i

Comparing the real parts, we get

x + y = 1 …(i)

Comparing the imaginary parts, we get

x – y = -3 …(ii)

Solving eq. (i) and (ii) to find the value of x and y

Adding eq. (i) and (ii), we get

x + y + x – y = 1 + (-3)

⇒ 2x = 1 – 3

⇒ 2x = -2

⇒ x = -1

Putting the value of x = -1 in eq. (i), we get

(-1) + y = 1

⇒ y = 1 + 1

⇒ y = 2

Q. 21. B. Find the real values of x and y for which:

$(x + iy) (3 – 2i) = (12 + 5i)$

Answer : $x(3 – 2i) + iy(3 – 2i) = 12 + 5i$

$\Rightarrow 3x – 2ix + 3iy – 2i^2y = 12 + 5i$

$\Rightarrow 3x + i(-2x + 3y) – 2(-1)y = 12 + 5i$ [$\because i^2 = -1$]

$\Rightarrow 3x + i(-2x + 3y) + 2y = 12 + 5i$

$\Rightarrow (3x + 2y) + i(-2x + 3y) = 12 + 5i$

Comparing the real parts, we get

$3x + 2y = 12$ …(i)

Comparing the imaginary parts, we get

$–2x + 3y = 5$ …(ii)

Solving eq. (i) and (ii) to find the value of x and y

Multiply eq. (i) by 2 and eq. (ii) by 3, we get

$6x + 4y = 24$ …(iii)

$–6x + 9y = 15$ …(iv)

Adding eq. (iii) and (iv), we get

$6x + 4y – 6x + 9y = 24 + 15$

$\Rightarrow 13y = 39$

$\Rightarrow y = 3$

Putting the value of y = 3 in eq. (i), we get

$3x + 2(3) = 12$

$\Rightarrow 3x + 6 = 12$

$\Rightarrow 3x = 12 – 6$

$\Rightarrow 3x = 6$

$\Rightarrow x = 2$

Hence, the value of x = 2 and y = 3

Q. 21. A. Find the real values of x and y for which:

$(1 – i) x + (1 + i) y = 1 – 3i$

Answer : $(1 – i) x + (1 + i) y = 1 – 3i$

$x – ix + y + iy = 1 – 3i$

$\Rightarrow (x + y) – i(x – y) = 1 – 3i$

Comparing the real parts, we get

$x + y = 1$ …(i)

Comparing the imaginary parts, we get

$x – y = -3$ …(ii)

Solving eq. (i) and (ii) to find the value of x and y

Adding eq. (i) and (ii), we get

$x + y + x – y = 1 + (-3)$

$\Rightarrow 2x = 1 – 3$

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$

Putting the value of x = -1 in eq. (i), we get

$(-1) + y = 1$

$\Rightarrow y = 1 + 1$

$\Rightarrow y = 2$

Q. 21. B. Find the real values of x and y for which:

(x + iy) (3 – 2i) = (12 + 5i)

Answer : $x(3 - 2i) + iy(3 - 2i) = 12 + 5i$

$\Rightarrow 3x - 2ix + 3iy - 2i^2y = 12 + 5i$

$\Rightarrow 3x + i(-2x + 3y) - 2(-1)y = 12 + 5i$ [$\because i^2 = -1$]

$\Rightarrow 3x + i(-2x + 3y) + 2y = 12 + 5i$

$\Rightarrow (3x + 2y) + i(-2x + 3y) = 12 + 5i$

Comparing the real parts, we get

$3x + 2y = 12$ …(i)

Comparing the imaginary parts, we get

$-2x + 3y = 5$ …(ii)

Solving eq. (i) and (ii) to find the value of x and y

Multiply eq. (i) by 2 and eq. (ii) by 3, we get

$6x + 4y = 24$ …(iii)

$-6x + 9y = 15$ …(iv)

Adding eq. (iii) and (iv), we get

$6x + 4y - 6x + 9y = 24 + 15$

$\Rightarrow 13y = 39$

$\Rightarrow y = 3$

Putting the value of y = 3 in eq. (i), we get

$3x + 2(3) = 12$

$\Rightarrow 3x + 6 = 12$

$\Rightarrow 3x = 12 - 6$

$\Rightarrow 3x = 6$

$\Rightarrow x = 2$

Hence, the value of $x = 2$ and $y = 3$

Q. 21. C. Find the real values of x and y for which:

$x + 4yi = ix + y + 3$

Answer : Given: $x + 4yi = ix + y + 3$

or $x + 4yi = ix + (y + 3)$

Comparing the real parts, we get

$x = y + 3$

Or $x – y = 3$ …(i)

Comparing the imaginary parts, we get

$4y = x$ …(ii)

Putting the value of $x = 4y$ in eq. (i), we get

$4y – y = 3$

$\Rightarrow 3y = 3$

$\Rightarrow y = 1$

Putting the value of $y = 1$ in eq. (ii), we get

$x = 4(1) = 4$

Hence, the value of $x = 4$ and $y = 1$

Q. 21. D. Find the real values of x and y for which:

$(1 + i) y^2 + (6 + i) = (2 + i)x$

Answer : Given: $(1 + i) y^2 + (6 + i) = (2 + i)x$

Consider, $(1 + i) y^2 + (6 + i) = (2 + i)x$

$\Rightarrow y^2 + iy^2 + 6 + i = 2x + ix$

$\Rightarrow (y^2 + 6) + i(y^2 + 1) = 2x + ix$

Comparing the real parts, we get

$y^2 + 6 = 2x$

$\Rightarrow 2x - y^2 - 6 = 0$ …(i)

Comparing the imaginary parts, we get

$y^2 + 1 = x$

$\Rightarrow x - y^2 - 1 = 0$ …(ii)

Subtracting the eq. (ii) from (i), we get

$2x - y^2 - 6 - (x - y^2 - 1) = 0$

$\Rightarrow 2x - y^2 - 6 - x + y^2 + 1 = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

Putting the value of $x = 5$ in eq. (i), we get

$2(5) - y^2 - 6 = 0$

$\Rightarrow 10 - y^2 - 6 = 0$

$\Rightarrow -y^2 + 4 = 0$

$\Rightarrow - y^2 = -4$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \sqrt{4}$

$\Rightarrow y = \pm 2$

Hence, the value of $x = 5$ and $y = \pm 2$

Q. 21. E. Find the real values of x and y for which:

$\frac{(x+3i)}{(2+iy)}$ **= (1 – i)**

Answer : Given:

$$\frac{x+3i}{2+iy} = (1-i)$$

⇒ x + 3i = (1 – i)(2 + iy)

⇒ x + 3i = 1(2 + iy) – i(2 + iy)

⇒ x + 3i = 2 + iy – 2i – i^2y

⇒ x + 3i = 2 + i(y – 2) – (-1)y [i^2 = -1]

⇒ x + 3i = 2 + i(y – 2) + y

⇒ x + 3i = (2 + y) + i(y – 2)

Comparing the real parts, we get

x = 2 + y

⇒ x – y = 2 …(i)

Comparing the imaginary parts, we get

3 = y – 2

⇒ y = 3 + 2

⇒ y = 5

Putting the value of y = 5 in eq. (i), we get

x – 5 = 2

⇒ x = 2 + 5

⇒ x = 7

Hence, the value of x = 7 and y = 5

Q. 21. F. Find the real values of x and y for which:

$$\frac{(1+i)x-2i}{(3+i)}+\frac{(2-3i)y+i}{(3-i)}=i$$

Answer : Consider,

$$\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$$

$$=\frac{x+xi-2i}{3+i}+\frac{2y-3iy+i}{3-i}=i$$

Taking LCM

$$\Rightarrow \frac{(x+xi-2i)(3-i)+(2y-3iy+i)(3+i)}{(3+i)(3-i)}=i$$

$$\Rightarrow \frac{3x+3xi-6i-xi-xi^2+2i^2+6y-9iy+3i+2iy-3i^2y+i^2}{(3)^2-(i)^2}=i$$

Putting $i^2 = -1$

$$\Rightarrow \frac{3x+2xi-6i-x(-1)+2(-1)+6y-7iy+3i-3(-1)y+(-1)}{9-(-1)}=i$$

$$\Rightarrow \frac{3x+2xi-6i+x-2+6y-7iy+3i+3y-1}{9+1}=i$$

$$\Rightarrow \frac{4x+2xi-3i-3+9y-7iy}{10}=i$$

$\Rightarrow 4x + 2xi – 3i – 3 + 9y – 7iy = 10i$

$\Rightarrow (4x – 3 + 9y) + i(2x – 3 – 7y) = 10i$

Comparing the real parts, we get

$4x – 3 + 9y = 0$

$\Rightarrow 4x + 9y = 3$ …(i)

Comparing the imaginary parts, we get

$2x – 3 – 7y = 10$

$\Rightarrow 2x – 7y = 10 + 3$

$\Rightarrow 2x – 7y = 13$ …(ii)

Multiply the eq. (ii) by 2, we get

$4x – 14y = 26$ …(iii)

Subtracting eq. (i) from (iii), we get

$4x – 14y – (4x + 9y) = 26 – 3$

$\Rightarrow 4x – 14y – 4x – 9y = 23$

$\Rightarrow -23y = 23$

$\Rightarrow y = -1$

Putting the value of $y = -1$ in eq. (i), we get

$4x + 9(-1) = 3$

$\Rightarrow 4x – 9 = 3$

$\Rightarrow 4x = 12$

$\Rightarrow x = 3$

Hence, the value of $x = 3$ and $y = -1$

Q. 22

Find the real values of x and y for which (x – iy) (3 + 5i) is the conjugate of (-6 – 24i).

Answer : Given: $(x – iy) (3 + 5i)$ is the conjugate of $(-6 – 24i)$

We know that,

Conjugate of $– 6 – 24i = - 6 + 24i$

∴ According to the given condition,

$(x - iy)(3 + 5i) = -6 + 24i$

$\Rightarrow x(3 + 5i) - iy(3 + 5i) = -6 + 24i$

$\Rightarrow 3x + 5ix - 3iy - 5i^2y = -6 + 24i$

$\Rightarrow 3x + i(5x - 3y) - 5(-1)y = -6 + 24i$ [$\because i^2 = -1$]

$\Rightarrow 3x + i(5x - 3y) + 5y = -6 + 24i$

$\Rightarrow (3x + 5y) + i(5x - 3y) = -6 + 24i$

Comparing the real parts, we get

$3x + 5y = -6$ …(i)

Comparing the imaginary parts, we get

$5x - 3y = 24$ …(ii)

Solving eq. (i) and (ii) to find the value of x and y

Multiply eq. (i) by 5 and eq. (ii) by 3, we get

$15x + 25y = -30$ …(iii)

$15x - 9y = 72$ …(iv)

Subtracting eq. (iii) from (iv), we get

$15x - 9y - 15x - 25y = 72 - (-30)$

$\Rightarrow -34y = 72 + 30$

$\Rightarrow -34y = 102$

$\Rightarrow y = -3$

Putting the value of $y = -3$ in eq. (i), we get

$3x + 5(-3) = -6$

$\Rightarrow 3x - 15 = -6$

$\Rightarrow 3x = -6 + 15$

$\Rightarrow 3x = 9$

$\Rightarrow x = 3$

Hence, the value of $x = 3$ and $y = -3$

Q. 23. Find the real values of x and y for which the complex number ($-3 + iyx^2$) and ($x^2 + y + 4i$) are conjugates of each other.

Answer : Let $z_1 = -3 + iyx^2$

So, the conjugate of z_1 is

$$\bar{z_1} = -3 - iyx^2$$

And $z_2 = x^2 + y + 4i$

So, the conjugate of z_2 is

$$\bar{z}_2 = x^2 + y - 4i$$

Given that: $\bar{z_1} = z_2 \ \& \ z_1 = \bar{z}_2$

Firstly, consider $\bar{z_1} = z_2$

$- 3 - iyx^2 = x^2 + y + 4i$

$\Rightarrow x^2 + y + 4i + iyx^2 = -3$

$\Rightarrow x^2 + y + i(4 + yx^2) = -3 + 0i$

Comparing the real parts, we get

$x^2 + y = -3$ …(i)

Comparing the imaginary parts, we get

$4 + yx^2 = 0$

$\Rightarrow x^2y = -4$ …(ii)

Now, consider $z_1 = \bar{z}_2$

$-3 + iyx^2 = x^2 + y - 4i$

$\Rightarrow x^2 + y - 4i - iyx^2 = -3$

$\Rightarrow x^2 + y + i(-4i - yx^2) = -3 + 0i$

Comparing the real parts, we get

$x^2 + y = -3$

Comparing the imaginary parts, we get

$-4 - yx^2 = 0$

$\Rightarrow x^2y = -4$

Now, we will solve the equations to find the value of x and y

From eq. (i), we get

$x^2 = -3 - y$

Putting the value of x^2 in eq. (ii), we get

$(-3 - y)(y) = -4$

$\Rightarrow -3y - y^2 = -4$

$\Rightarrow y^2 + 3y = 4$

$\Rightarrow y^2 + 3y - 4 = 0$

$\Rightarrow y^2 + 4y - y - 4 = 0$

$\Rightarrow y(y + 4) - 1(y + 4) = 0$

$\Rightarrow (y - 1)(y + 4) = 0$

$\Rightarrow y - 1 = 0$ or $y + 4 = 0$

$\Rightarrow y = 1$ or $y = -4$

When $y = 1$, then

$x^2 = -3 - 1$

$= -4$ [It is not possible]

When $y = -4$, then

$x^2 = -3 -(-4)$

$= -3 + 4$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \sqrt{1}$

$\Rightarrow x = \pm 1$

Hence, the values of $x = \pm 1$ and $y = -4$

Q. 24. If $z = (2 - 3i)$, prove that $z^2 - 4z + 13 = 0$ and hence deduce that $4z^3 - 3z^2 + 169 = 0$.

Answer : Given: $z = 2 - 3i$

To Prove: $z^2 - 4z + 13 = 0$

Taking LHS, $z^2 - 4z + 13$

Putting the value of $z = 2 - 3i$, we get

$(2 - 3i)^2 - 4(2 - 3i) + 13$

$= 4 + 9i^2 - 12i - 8 + 12i + 13$

$= 9(-1) + 9$

$= -9 + 9$

$= 0$

$=$ RHS

Hence, $z^2 - 4z + 13 = 0$ …(i)

Now, we have to deduce $4z^3 - 3z^2 + 169$

Now, we will expand $4z^3 - 3z^2 + 169$ in this way so that we can use the above equation i.e. $z^2 - 4z + 13$

$= 4z^3 - 16z^2 + 13z^2 + 52z - 52z + 169$

Re – arrange the terms,

$= 4z^3 - 16z^2 + 52z + 13z^2 - 52z + 169$

$= 4z(z^2 - 4z + 13) + 13(z^2 - 4z + 13)$

$= 4z(0) + 13(0)$ [from eq. (i)]

$= 0$

= RHS

Hence Proved

Q. 25. If $(1 + i)z = (1 - i)\bar{z}$ then prove that $z = -i\bar{z}$.

Answer : Let $z = x + iy$

Then,

$\bar{z} = x - iy$

Now, Given: $(1 + i)z = (1 - i)\bar{z}$

Therefore,

$(1 + i)(x + iy) = (1 - i)(x - iy)$

$x + iy + xi + i^2y = x - iy - xi + i^2y$

We know that $i^2 = -1$, therefore,

$x + iy + ix - y = x - iy - ix - y$

$2xi + 2yi = 0$

$x = -y$

Now, as $x = -y$

$z = -\bar{z}$

Hence, Proved.

Q. 26. If $\left(\frac{z-1}{z+1}\right)$ is purely an imaginary number and z ≠ -1 then find the value of |z|.

Answer : Given: $\frac{z-1}{z+1}$ is purely imaginary number

Let z = x + iy

So, $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$

$$= \frac{(x-1)+iy}{(x+1)+iy}$$

Now, rationalizing the above by multiply and divide by the conjugate of [(x + 1) + iy]

$$= \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)-iy]}$$

Using (a – b)(a + b) = ($a^2 - b^2$)

$$= \frac{(x-1)[(x+1)-iy]+iy[(x+1)-iy]}{(x+1)^2-(iy)^2}$$

$$= \frac{(x-1)(x+1)+(x-1)(-iy)+iy(x+1)+(iy)(-iy)}{x^2+1+2x-i^2y^2}$$

$$= \frac{x^2-1-ixy+iy+ixy+iy-i^2y^2}{x^2+1+2x-i^2y^2}$$

Putting $i^2 = -1$

$$= \frac{x^2 - 1 + 2iy - (-1)y^2}{x^2 + 1 + 2x - (-1)y^2}$$

$$= \frac{x^2 - 1 + 2iy + y^2}{x^2 + 1 + 2x + y^2}$$

$$= \frac{x^2 - 1 + y^2}{x^2 + 1 + 2x + y^2} + i\frac{2y}{x^2 + 1 + 2x + y^2}$$

Since, the number is purely imaginary it means real part is 0

$$\therefore \frac{x^2 - 1 + y^2}{x^2 + 1 + 2x + y^2} = 0$$

$\Rightarrow x^2 + y^2 - 1 = 0$

$\Rightarrow x^2 + y^2 = 1$

$$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{1}$$

$$\Rightarrow \sqrt{x^2 + y^2} = 1$$

$\therefore |z| = 1$

Q. 27. Solve the system of equations, Re(z^2) = 0, |z| = 2.

Answer : Given: Re(z^2) = 0 and |z| = 2

Let z = x + iy

$$\therefore |z| = \sqrt{x^2 + y^2}$$

$\Rightarrow 2 = \sqrt{x^2 + y^2}$ [Given]

Squaring both the sides, we get

$x^2 + y^2 = 4$ …(i)

Since, z = x + iy

$\Rightarrow z^2 = (x + iy)^2$

$\Rightarrow z^2 = x^2 + i^2y^2 + 2ixy$

$\Rightarrow z^2 = x^2 + (-1)y^2 + 2ixy$

$\Rightarrow z^2 = x^2 - y^2 + 2ixy$

It is given that $Re(z^2) = 0$

$\Rightarrow x^2 - y^2 = 0$ …(ii)

Adding eq. (i) and (ii), we get

$x^2 + y^2 + x^2 - y^2 = 4 + 0$

$\Rightarrow 2x^2 = 4$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

Putting the value of $x^2 = 2$ in eq. (i), we get

$2 + y^2 = 4$

$\Rightarrow y^2 = 2$

$\Rightarrow y = \pm\sqrt{2}$

Hence, $z = \sqrt{2} \pm i\sqrt{2}, -\sqrt{2} \pm i\sqrt{2}$

Q. 28. Find the complex number z for which |z| = z + 1 + 2i.

Answer : Given: $|z| = z + 1 + 2i$

Consider,

$|z| = (z + 1) + 2i$

Squaring both the sides, we get

$|z|^2 = [(z + 1) + (2i)]^2$

$\Rightarrow |z|^2 = |z + 1|^2 + 4i^2 + 2(2i)(z + 1)$

$\Rightarrow |z|^2 = |z|^2 + 1 + 2z + 4(-1) + 4i(z + 1)$

$\Rightarrow 0 = 1 + 2z - 4 + 4i(z + 1)$

$\Rightarrow 2z - 3 + 4i(z + 1) = 0$

Let $z = x + iy$

$\Rightarrow 2(x + iy) - 3 + 4i(x + iy + 1) = 0$

$\Rightarrow 2x + 2iy - 3 + 4ix + 4i^2y + 4i = 0$

$\Rightarrow 2x + 2iy - 3 + 4ix + 4(-1)y + 4i = 0$

$\Rightarrow 2x - 3 - 4y + i(4x + 2y + 4) = 0$

Comparing the real part, we get

$2x - 3 - 4y = 0$

$\Rightarrow 2x - 4y = 3 \quad \ldots(i)$

Comparing the imaginary part, we get

$4x + 2y + 4 = 0$

$\Rightarrow 2x + y + 2 = 0$

$\Rightarrow 2x + y = -2 \quad \ldots(ii)$

Subtracting eq. (ii) from (i), we get

$2x - 4y - (2x + y) = 3 - (-2)$

$\Rightarrow 2x - 4y - 2x - y = 3 + 2$

$\Rightarrow -5y = 5$

$\Rightarrow y = -1$

Putting the value of $y = -1$ in eq. (i), we get

$2x - 4(-1) = 3$

$\Rightarrow 2x + 4 = 3$

$\Rightarrow 2x = 3 - 4$

$\Rightarrow$ 2x = - 1

$\Rightarrow x = -\frac{1}{2}$

Hence, the value of z = x + iy

$= -\frac{1}{2} + i(-1)$

$z = -\frac{1}{2} - i$

Exercise 5C

Q. 1. Express each of the following in the form (a + ib) and find its conjugate.

(i) $\frac{1}{(4+3i)}$

(ii) $(2 + 3i)^2$

(iii) $\frac{(2-i)}{(1-2i)^2}$

(iv) $\frac{(1+i)(1+2i)}{(1+3i)}$

(v) $\left(\frac{1+2i}{2+i}\right)^2$

(vi) $\frac{(2+i)}{(3-i)(1+2i)}$

Answer :

(i) Let $z = \frac{1}{4+3i} = \frac{1}{4+3i} \times \frac{4-3i}{4-3i}$

$= \frac{4-3i}{16+9} = \frac{4}{25} - \frac{3}{25}i$

$\Rightarrow \bar{z} = \frac{4}{25} + \frac{3}{25}i$

(ii) Let z = $(2 + 3i)^2$ = (2 + 3i)(2 + 3i)

= 4 + 6i + 6i + $9i^2$

= 4 + 12i + $9i^2$

= 4 + 12i - 9

= - 5 + 12i

$\bar{z} = -5 - 12i$

(iii) Let $z = \frac{(2-i)}{(1-2i)^2} = \frac{(2-i)}{1+4i^2-4i}$

$= \frac{(2-i)}{1-4i-4} = \frac{2-i}{-3-4i}$

$= \frac{2-i}{-3-4i} \times \frac{-3+4i}{-3+4i} = \frac{(2-i)(-3+4i)}{9+16}$

$= \frac{-6+11i-4i^2}{25} = \frac{-2+11i}{25}$

$= \frac{-2}{25} + \frac{11}{25}i$

$\bar{z} = \frac{-2}{25} - \frac{11}{25}i$

(iv) Let $z = \frac{(1+i)(1+2i)}{(1+3i)} = \frac{1+i+2i+2i^2}{(1+3i)}$

$$= \frac{1 + 3i - 2}{1 + 3i} = \frac{-1 + 3i}{1 + 3i}$$

$$= \frac{-1 + 3i}{1 + 3i} \times \frac{1 - 3i}{1 - 3i} = \frac{-1 + 3i + 3i - 9i^2}{1 - 9i^2} = \frac{-1 + 6i + 9}{1 + 9} = \frac{8 + 6i}{10}$$

$$= \frac{8}{10} + \frac{6}{10}i$$

$$\bar{z} = \frac{8}{10} - \frac{6}{10}i$$

(v) Let $z = \left(\frac{1 + 2i}{2 + i}\right)^2 = \frac{1 + 4i^2 + 2i}{4 + i^2 + 4i} = \frac{1 - 4 + 2i}{4 - 1 + 4i} = \frac{-3 + 2i}{3 + 4i}$

$$= \frac{-3 + 2i}{3 + 4i} \times \frac{3 - 4i}{3 - 4i}$$

$$= \frac{-9 + 12i + 6i - 8i^2}{9 + 16} = \frac{-9 + 18i + 8}{25} = \frac{-1 + 18i}{25}$$

$$= \frac{-1}{25} + \frac{18}{25}i$$

$$\bar{z} = \frac{-1}{25} - \frac{18}{25}i$$

(vi) Let $z = \frac{(2 + i)}{(3 - i)(1 + 2i)} = \frac{2 + i}{3 + 6i - 1 - 2i^2}$

$$= \frac{2 + i}{3 + 6i - 1 + 2} = \frac{2 + i}{4 + 6i}$$

$$= \frac{2 + i}{4 + 6i} \times \frac{4 - 6i}{4 - 6i}$$

$$= \frac{8 - 12i + 4i - 6i^2}{16 + 36}$$

$$= \frac{8 - 8i + 6}{52}$$

$$= \frac{14 - 8i}{52}$$

$$= \frac{14}{52} - \frac{8}{52}i$$

$$\bar{z} = \frac{14}{52} + \frac{8}{52}i$$

Q. 2. Express each of the following in the form (a + ib) and find its multiplicative inverse:

(i) $\frac{1+2i}{1-3i}$

(ii) $\frac{(1+7i)}{(2-i)^2}$

(iii) $\frac{-4}{\left(1+i\sqrt{3}\right)}$

Answer :

(i) Let $z = \frac{1+2i}{1-3i}$

$$= \frac{1 + 2i}{1 - 3i} \times \frac{1 + 3i}{1 + 3i} = \frac{1 + 3i + 2i + 6i^2}{1 - 9i^2}$$

$$= \frac{1 + 5i + 6i^2}{1 + 9} = \frac{-5 + 5i}{10}$$

$$z = \frac{-1}{2} + \frac{1}{2}i$$

$\Rightarrow \bar{z} = \frac{-1}{2} - \frac{1}{2}i$

$\Rightarrow |z|^2 = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

∴ The multiplicative inverse of $\frac{1+2i}{1-3i}$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\frac{-1}{2} - \frac{1}{2}i}{\frac{1}{2}} = -1 - i$$

(ii) Let $z = \frac{1+7i}{(2-i)^2}$

$$= \frac{1 + 7i}{4 + i^2 - 4i} = \frac{1 + 7i}{4 - 1 - 4i} = \frac{1 + 7i}{3 - 4i}$$

$$= \frac{1 + 7i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i}$$

$$= \frac{3 + 4i + 21i + 28i^2}{9 + 16} = \frac{3 + 25i - 28}{25} = \frac{-25 + 25i}{25}$$

z = - 1 + i

$\Rightarrow \bar{z} = -1 - i$

$\Rightarrow |z|^2 = (-1)^2 + (1)^2 = 1 + 1 = 2$

∴ The multiplicative inverse of $\frac{1+7i}{(2-i)^2}$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{-1 - i}{2} = \frac{-1}{2} - \frac{1}{2}i$$

(iii) Let $z = \frac{-4}{(1 + i\sqrt{3})}$

$$= \frac{-4}{1 + i\sqrt{3}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$$

$$= \frac{-4 + i4\sqrt{3}}{1 + 3} = \frac{-4 + i4\sqrt{3}}{4}$$

$$= -1 + i\sqrt{3}$$

Z = - 1 + i√3

$\Rightarrow \overline{z} = -1 - i\sqrt{3}$

$\Rightarrow |z|^2 = (-1)^2 + (\sqrt{3})^2 = 1 + 3 = 4$

∴ The multiplicative inverse of $\frac{-4}{(1 + i\sqrt{3})}$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{-1 + i\sqrt{3}}{4} = \frac{-1}{4} + \frac{i\sqrt{3}}{4}$$

Q. 3. If $(x + iy)^3 = (u + iv)$ then prove that $\left(\frac{u}{x} + \frac{v}{y}\right) = 4(x^2 - y^2)$.

Answer : Given that, $(x + iy)^3 = (u + iv)$

$\Rightarrow x^3 + (iy)^3 + 3x^2iy + 3xi^2y^2 = u + iv$

$\Rightarrow x^3 - iy^3 + 3x^2iy - 3xy^2 = u + iv$

$\Rightarrow x^3 - 3xy^2 + i(3x^2y - y^3) = u + iv$

On equating real and imaginary parts, we get

$U = x^3 - 3xy^2$ and $v = 3x^2y - y^3$

Now , $\frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$

$$= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$$

$= x^2 - 3y^2 + 3x^2 - y^2$

$= 4x^2 - 4y^2$

$= 4(x^2 - y^2)$

Hence, $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

Q. 4. If $(x + iy)^{1/3} = (a + ib)$ then prove that $\left(\frac{x}{a} + \frac{y}{b}\right) = 4(a^2 - b^2)$.

Answer : Given that, $(x + iy)^{1/3} = (a + ib)$

$\Rightarrow (x + iy) = (a + ib)^3$

$\Rightarrow (a + ib)^3 = x + iy$

$\Rightarrow a^3 + (ib)^3 + 3a^2ib + 3ai^2b^2 = x + iy$

$\Rightarrow a^3 - ib^3 + 3a^2ib - 3ab^2 = x + iy$

$\Rightarrow a^3 - 3ab^2 + i(3a^2b - b^3) = x + iy$

On equating real and imaginary parts, we get

$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

Now, $\frac{x}{a} + \frac{y}{b} = \frac{a^3 - 3ab^2}{a} + \frac{3a^2b - b^3}{b}$

$$= \frac{a(a^2 - 3b^2)}{a} + \frac{b(3a^2 - b^2)}{b}$$

$= a^2 - 3b^2 + 3a^2 - b^2$

$= 4a^2 - 4b^2$

$= 4(a^2 - b^2)$

Hence, $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

Q. 5. Express $(1 - 2i)^{-3}$ in the form $(a + ib)$.

Answer : We have, $(1 - 2i)^{-3}$

$$\Rightarrow \frac{1}{(1-2i)^3} = \frac{1}{1 - 8i^3 - 6i + 12i^2} = \frac{1}{1 + 8i - 6i - 12} = \frac{1}{2i - 11}$$

$$\Rightarrow \frac{1}{-11 + 2i}$$

$$= \frac{1}{-11 + 2i} \times \frac{-11 - 2i}{-11 - 2i}$$

$$= \frac{-11 - 2i}{(-11)^2 - (2i)^2} = \frac{-11 - 2i}{121 + 4}$$

$$= \frac{-11 - 2i}{125}$$

$$= \frac{-11}{125} - \frac{2i}{125}$$

Q. 6. Find real values of x and y for which

$(x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)$.

Answer : We have, $(x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)$.

$\Rightarrow x^4 + 2xi - 3x^2 + iy = 3 - 5i + 1 + 2iy$

$\Rightarrow (x^4 - 3x^2) + i(2x - y) = 4 + i(2y - 5)$

On equating real and imaginary parts, we get

$x^4 - 3x^2 = 4$ and $2x - y = 2y - 5$

$\Rightarrow x^4 - 3x^2 - 4 = 0$ eq(i) and $2x - y - 2y + 5 = 0$ eq(ii)

Now from eq (i), $x^4 - 3x^2 - 4 = 0$

$\Rightarrow x^4 - 4x^2 + x^2 - 4 = 0$

$\Rightarrow x^2 (x^2 - 4) + 1(x^2 - 4) = 0$

$\Rightarrow (x^2 - 4)(x^2 + 1) = 0$

⇒ $x^2 - 4 = 0$ and $x^2 + 1 = 0$

⇒ $x = \pm 2$ and $x = \sqrt{-1}$

Real value of $x = \pm 2$

Putting $x = 2$ in eq (ii), we get

$2x - 3y + 5 = 0$

⇒ $2\times 2 - 3y + 5 = 0$

⇒ $4 - 3y + 5 = 0 = 9 - 3y = 0$

⇒ $y = 3$

Putting $x = -2$ in eq (ii), we get

$2x - 3y + 5 = 0$

⇒ $2\times -2 - 3y + 5 = 0$

⇒ $-4 - 3y + 5 = 0 = 1 - 3y = 0$

⇒ $y = \frac{1}{3}$

Q. 7. If $z^2 + |z|^2 = 0$, show that z is purely imaginary.

Answer : Let $z = a + ib$

⇒ $|z| = \sqrt{(a^2 + b^2)}$

Now, $z^2 + |z|^2 = 0$

⇒ $(a + ib)^2 + a^2 + b^2 = 0$

⇒ $a^2 + 2abi + i^2b^2 + a^2 + b^2 = 0$

⇒ $a^2 + 2abi - b^2 + a^2 + b^2 = 0$

⇒ $2a^2 + 2abi = 0$

⇒ $2a(a + ib) = 0$

Either $a = 0$ or $z = 0$

Since $z \neq 0$

$a = 0 \Rightarrow z$ is purely imaginary.

Q. 8. If $\frac{z-1}{z+1}$ is purely imaginary and $z = -1$, show that $|z| = 1$.

Answer : Let $z = a + ib$

Now, $\frac{z-1}{z+1} = \frac{a+ib-1}{a+ib+1}$

$$= \frac{(a-1)+ib}{(a+1)+ib}$$

$$\Rightarrow \frac{(a-1)+ib}{(a+1)+ib} \times \frac{(a+1)-ib}{(a+1)-ib}$$

$$= \frac{a^2 + a - iab - a - 1 + ib + iab + ib - i^2b^2}{(a+1)^2 + b^2}$$

$$= \frac{a^2 + -1 + ib + ib + b^2}{(a+1)^2 + b^2} = \frac{a^2 + b^2 - 1 + 2ib}{(a+1)^2 + b^2}$$

Given that $\frac{z-1}{z+1}$ is purely imaginary $\Rightarrow$ real part = 0

$$\Rightarrow \frac{a^2 + b^2 - 1}{(a+1)^2 + b^2} = 0$$

$\Rightarrow a^2 + b^2 - 1 = 0$

$\Rightarrow a^2 + b^2 = 1$

$\Rightarrow |z| = 1$

Hence proved.

Q. 9. If z_1 is a complex number other than –1 such that $|z_1| = 1$ and $z_2 = \frac{z_1 - 1}{z_1 + 1}$ then show that z2 is purely imaginary.

Answer : Let $z_1 = a + ib$ such that $|z_1| = \sqrt{(a^2 + b^2)} = 1$

Now, $z_2 = \frac{z_1 - 1}{z_1 + 1} = \frac{a + ib - 1}{a + ib + 1} = \frac{(a-1) + ib}{(a+1) + ib}$

$$\Rightarrow \frac{(a-1) + ib}{(a+1) + ib} \times \frac{(a+1) - ib}{(a+1) - ib}$$

$$= \frac{a^2 + a - iab - a - 1 + ib + iab + ib - i^2b^2}{(a+1)^2 + b^2}$$

$$= \frac{a^2 + -1 + ib + ib + b^2}{(a+1)^2 + b^2} = \frac{a^2 + b^2 - 1 + 2ib}{(a+1)^2 + b^2}$$

$$= \frac{(a^2 + b^2) - 1 + 2ib}{(a+1)^2 + b^2} = \frac{1 - 1 + 2ib}{(a+1)^2 + b^2} \; [\because a^2 + b^2 = 1]$$

$$= 0 + \frac{2ib}{(a+1)^2 + b^2}$$

Thus, the real part of z_2 is 0 and z_2 is purely imaginary.

Q. 10. For all z C, prove that

(i) $\frac{1}{2}\left(z + \bar{z}\right) = \mathrm{Re}(z)$

(ii) $\frac{1}{2}\left(z + \bar{z}\right) = \mathrm{Re}(z)$.

(iii) $z\bar{z}$ **= $|z|^2$**

(iv) $\left(z + \bar{z}\right)$ **is real**

(v) $\left(z - \bar{z}\right)$ **is 0 or imaginary.**

Answer :

Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$$Now\,, \frac{z + \bar{z}}{2} = \frac{(a + ib) + (a - ib)}{2} = \frac{2a}{2} = a = Re(z)$$

Hence Proved.

(ii) Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$$w\,, \frac{z + \bar{z}}{2}$$

$$= \frac{(a + ib) + (a - ib)}{2}$$

$$= \frac{2a}{2} = \frac{a}{1} = Re\,(z)$$

Hence, Proved.

(iii) Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$$Now, z\bar{z} = (a + ib)(a - ib) = a^2 - (ib)^2 = a^2 + b^2 = |z|^2$$

Hence Proved.

(iv) Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$Now, z + \bar{z} = (a + ib) + (a - ib) = 2a = 2Re(z)$

Hence, $z + \bar{z}$ is real.

(v) Case 1. Let z = a + 0i

$\Rightarrow \bar{z} = a - 0i$

$Now, z - \bar{z} = (a + 0\mathrm{i}) - (a - 0\mathrm{i}) = 0$

Case 2. Let z = 0 + bi

$\Rightarrow \bar{z} = 0 - bi$

$Now, z - \bar{z} = (0 + ib) - (0 - ib) = 2ib = 2iIm(z) = Imaginary$

Case 2. Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$Now, z - \bar{z} = (a + ib) - (a - ib) = 2ib = 2iIm(z) = Imaginary$

Thus, $\left(z - \bar{z}\right)$ is 0 or imaginary.

Q. 11. If z_1 = (1 + i) and z_2 = (–2 + 4i), prove that Im $\left(\frac{z_1 z_2}{\bar{z}_1}\right) = 2$

Answer : We have, z_1 = (1 + i) and z_2 = (–2 + 4i)

Now, $\frac{z_1 z_2}{\bar{z}_1} = \frac{(1+\mathrm{i})(-2+4\mathrm{i})}{\overline{(1+\mathrm{i})}}$

$$= \frac{-2 + 4i - 2i + 4i^2}{(1 - \mathrm{i})} = \frac{-2 + 4i - 2i - 4}{(1 - \mathrm{i})} = \frac{-6 + 2i}{(1 - \mathrm{i})}$$

$$= \frac{-6 + 2i}{(1 - \mathrm{i})} \times \frac{(1 + i)}{(1 + i)}$$

$$= \frac{-6 - 6i + 2i + 2i^2}{1 + 1}$$

$$= \frac{-6 - 4i - 2}{2} = \frac{-8 - 4i}{2}$$

= - 4 - 2i

Hence, $Im\left(\frac{z_1 z_2}{z_2}\right) = -2$

Q. 12. If a and b are real numbers such that $a^2 + b^2 = 1$ then show that a real value of x satisfies the equation, $\frac{1-ix}{1+ix} = (a - ib)$

Answer : We have,

$$\frac{1 - ix}{1 + ix} = (a - ib) = \frac{a - ib}{1}$$

Applying componendo and dividendo, we get

$$\frac{(1 - ix) + (1 + ix)}{(1 - ix) - (1 + ix)} = \frac{a - ib + 1}{a - ib - 1}$$

$$\Rightarrow \frac{1 - ix + 1 + ix}{1 - ix - 1 + ix} = \frac{a - ib + 1}{a - ib - 1}$$

$$\Rightarrow \frac{2}{-2ix} = \frac{a - ib + 1}{-(-a + ib + 1)}$$

$$\Rightarrow ix = \frac{1 - a + ib}{1 + a - ib} \times \frac{1 + a + ib}{1 + a + ib}$$

$$= \frac{1 + a + ib - a - a^2 - aib + ib + aib + i^2 b^2}{(1 + a)^2 - i^2 b^2}$$

$$\Rightarrow ix = \frac{1 - a^2 - b^2 + 2ib}{(1 + a)^2 - i^2 b^2} = \frac{1 - a^2 - b^2 + 2ib}{(1 + a)^2 + b^2} = \frac{1 - (a^2 + b^2) + 2ib}{1 + a^2 + 2a + b^2}$$

$$\Rightarrow ix = \frac{1-(a^2 + b^2) + 2ib}{1 + 2a + (a^2 + b^2)}$$

$$\Rightarrow ix = \frac{1-1 + 2ib}{1 + 2a + 1} \left[\because a^2 + b^2 = 1\right]$$

$$\Rightarrow ix = \frac{2ib}{2 + 2a}$$

$$\Rightarrow x = \frac{2b}{2 + 2a} = Real\ value$$

Exercise 5D

Q. 1. Find the modulus of each of the following complex numbers and hence express each of them in polar form: 4

Answer : Let Z = 4 = r(cosθ + isinθ)

Now, separating real and complex part, we get

4 = rcosθ……….eq.1

0 = rsinθ…………eq.2

Squaring and adding eq.1 and eq.2, we get

16 = r^2

Since r is always a positive no., therefore,

r = 4,

Hence its modulus is 4.

Now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{0}{4}$$

Tanθ = 0

Since cosθ = 1, sinθ = 0 and tanθ = 0. Therefore the θ lies in first quadrant.

Tanθ = 0, therefore θ = 0°

Representing the complex no. in its polar form will be

Z = 4(cos0° + isin0°)

Q. 2. Find the modulus of each of the following complex numbers and hence express each of them in polar form: –2

Answer : Let Z = -2 = r(cosθ + isinθ)

Now, separating real and complex part, we get

-2 = rcosθ………. eq.1

0 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no, therefore,

r = 2,

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{0}{-2}$$

Tanθ = 0

Since cosθ = -1, sinθ = 0 and tanθ = 0. Therefore the θ lies in second quadrant.

Tanθ = 0, therefore θ = π

Representing the complex no. in its polar form will be

Z = 2(cosπ + isinπ)

Q. 3. Find the modulus of each of the following complex numbers and hence express each of them in polar form: –i

Answer : Let Z = -i = r(cosθ + isinθ)

Now, separating real and complex part, we get

0 = rcosθ……….eq.1

-1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

Hence its modulus is 1.

Now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-1}{0}$$

Tanθ = -∞

Since cosθ = 0, sinθ = -1 and tanθ = -∞ . Therefore the θ lies in fourth quadrant.

Tanθ = -∞, therefore $\theta = -\frac{\pi}{2}$

Representing the complex no. in its polar form will be

$$Z = 1\{\cos\left(-\frac{\pi}{2}\right) + \text{i}\sin(-\frac{\pi}{2})\}$$

Q. 4. Find the modulus of each of the following complex numbers and hence express each of them in polar form: 2i

Answer : Let Z = 2i = r(cosθ + isinθ)

Now, separating real and complex part, we get

0 = rcosθ ……….eq.1

2 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no., therefore,

r = 2,

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{2}{0}$$

Tanθ = ∞

Since cosθ = 0, sinθ = 1 and tanθ = ∞. Therefore the θ lies in first quadrant.

tanθ = ∞, therefore $\theta = \frac{\pi}{2}$

Representing the complex no. in its polar form will be

$$Z = 2\{\cos\left(\frac{\pi}{2}\right) + i\sin(\frac{\pi}{2})\}$$

Q. 5. Find the modulus of each of the following complex numbers and hence express each of them in polar form: 1 – i

Answer : Let Z = 1 - i = r(cosθ + isinθ)

Now , separating real and complex part , we get

1 = rcosθ ……….eq.1

-1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

r = √2,

Hence its modulus is √2.

Now , dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-1}{1}$$

Tanθ = -1

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = -\frac{1}{\sqrt{2}}$ and tanθ = -1 . Therefore the θ lies in fourth quadrant.

Tanθ = -1, therefore $\theta = -\frac{\pi}{4}$

Representing the complex no. in its polar form will be

$$Z = \sqrt{2}\{\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\}$$

Q. 6. Find the modulus of each of the following complex numbers and hence express each of them in polar form: –1 + i

Answer : Let Z = 1 - i = r(cosθ + isinθ)

Now, separating real and complex part, we get

-1 = rcosθ ……….eq.1

1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

$r = \sqrt{2}$,

Hence its modulus is √2.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{1}{-1}$$

Tanθ = -1

Since $\cos\theta = -\frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and tanθ = -1. Therefore the θ lies in second quadrant.

Tanθ = -1, therefore $\theta = \frac{3\pi}{4}$

Representing the complex no. in its polar form will be

$$Z = \sqrt{2}\{\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4})\}$$

Q. 7. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\sqrt{3} + i$

Answer : Let Z = $\sqrt{3}$ + i = r(cosθ + isinθ)

Now , separating real and complex part , we get

$\sqrt{3}$ = rcosθ ……….eq.1

1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

4 = r^2

Since r is always a positive no., therefore,

r =2,

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\text{Tan}\theta = \frac{1}{\sqrt{3}}$$

Since $\cos\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = \frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$. Therefore the θ lies in first quadrant.

$\mathrm{Tan}\theta = \frac{1}{\sqrt{3}}$, therefore $\theta = \frac{\pi}{6}$

Representing the complex no. in its polar form will be

$Z = 2\{\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})\}$

Q. 8. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $-1+\sqrt{3}i$

Answer : Let Z = $\sqrt{3}i$ - 1 = r(cosθ + isinθ)

Now , separating real and complex part , we get

-1 = rcosθ ……….eq.1

$\sqrt{3}$ = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no., therefore,

r = 2,

Hence its modulus is 2.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{\sqrt{3}}{-1}$$

$$\mathrm{Tan}\theta = -\frac{\sqrt{3}}{1}$$

Since $\cos\theta = -\frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = -\frac{\sqrt{3}}{1}$. therefore the θ lies in second quadrant.

Tanθ = $-\sqrt{3}$, therefore $\theta = \frac{2\pi}{3}$

Representing the complex no. in its polar form will be

$$Z = 2\{\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3})\}$$

Q. 9. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $1-\sqrt{3}i$

Answer : Let Z = $-\sqrt{3}i$ + 1 = r(cosθ + isinθ)

Now, separating real and complex part, we get

1 = rcosθ ……….eq.1

$-\sqrt{3}$ = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no., therefore,

r = 2,

Hence its modulus is 2.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{-\sqrt{3}}{1}$$

$$\text{Tan}\theta = -\frac{\sqrt{3}}{1}$$

Since $\cos\theta = \frac{1}{2}$, $\sin\theta = -\frac{\sqrt{3}}{2}$ and $\tan\theta = -\frac{\sqrt{3}}{1}$. Therefore the θ lies in the fourth quadrant.

Tanθ = $-\sqrt{3}$, therefore θ = $-\frac{\pi}{3}$

Representing the complex no. in its polar form will be

Z = 2{cos$\left(-\frac{\pi}{3}\right)$ + isin$\left(-\frac{\pi}{3}\right)$}

Q. 10. Find the modulus of each of the following complex numbers and hence express each of them in polar form: 2 – 2i

Answer : Let Z = 2 - 2i = r(cosθ + isinθ)

Now , separating real and complex part , we get

2 = rcosθ ……….eq.1

-2 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

8 = r^2

Since r is always a positive no. therefore,

r = 2$\sqrt{2}$,

Hence its modulus is 2√2.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-2}{2}$$

Tanθ = -1

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = -\frac{1}{\sqrt{2}}$ and tanθ = -1 . Therefore the θ lies in the fourth quadrant.

Tanθ = -1, therefore $\theta = -\frac{\pi}{4}$

Representing the complex no. in its polar form will be

$Z = 2\sqrt{2}\{\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\}$

Q. 11. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $-4 + 4\sqrt{3}i$

Answer : Let Z = 4√2i - 4 = r(cosθ + isinθ)

Now, separating real and complex part, we get

-4 = rcosθ ……….eq.1

4√3 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$64 = r^2$

Since r is always a positive no., therefore,

r = 8

Hence its modulus is 8.

Now, dividing eq.2 by eq.1, we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{4\sqrt{3}}{-4}$$

$$\tan\theta = -\frac{\sqrt{3}}{1}$$

Since $\cos\theta = -\frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = -\frac{\sqrt{3}}{1}$. Therefore the θ lies in second the quadrant.

Tanθ = -√3, therefore θ= $\frac{2\pi}{3}$.

Representing the complex no. in its polar form will be

$$Z = 8\{\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3})\}$$

Q. 12. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $-3\sqrt{2}+3\sqrt{2}i$

Answer : Let Z = 3√2i - 3√2 = r(cosθ + isinθ)

Now, separating real and complex part, we get

-3√2 = rcosθ ……….eq.1

3√2 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

36 = r^2

Since r is always a positive no., therefore,

r = 6

Hence its modulus is 6.

Now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{3\sqrt{2}}{-3\sqrt{2}}$$

$$\text{Tan}\theta = -\frac{1}{1}$$

Since $\cos\theta = -\frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and tanθ = -1 . therefore the θ lies in secothe nd quadrant.

Tanθ = -1 , therefore θ = $\frac{3\pi}{4}$.

Representing the complex no. in its polar form will be

$Z = 6\{\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4})\}$

Q. 13. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\frac{1+i}{1-i}$

Answer : $= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$$= \frac{1+i^2+2i}{1-i^2}$$

$$= \frac{2i}{2}$$

= i

Let Z = i = r(cosθ + isinθ)

Now , separating real and complex part , we get

0 = rcosθ ……….eq.1

1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

Hence its modulus is 1.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{1}{0}$$

tanθ = ∞

Since cosθ = 0, sinθ = 1 and tanθ = ∞. Therefore the θ lies in first quadrant.

tanθ = ∞, therefore $\theta = \frac{\pi}{2}$

Representing the complex no. in its polar form will be

$$Z = 1\{\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2})\}$$

Q. 14. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\frac{1-i}{1+i}$

Answer : $= \frac{1-i}{1+i} \times \frac{1-i}{1-i}$

$$= \frac{1 + i^2 - 2i}{1 - i^2}$$

$$= -\frac{2i}{2}$$

= -i

Let Z = -i = r(cosθ + isinθ)

Now, separating real and complex part, we get

0 = rcosθ……….eq.1

-1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

Hence its modulus is 1.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{r sin\theta}{r cos\theta} = \frac{-1}{0}$$

Tanθ = -∞

Since cosθ = 0, sinθ = -1 and tanθ = -∞, therefore the θ lies in fourth quadrant.

Tanθ = -∞, therefore $\theta = -\frac{\pi}{2}$

Representing the complex no. in its polar form will be

$Z = 1\{\cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})\}$

Q. 15. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\frac{1+3i}{1-2i}$

Answer :

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+6i^2+5i}{1-4i^2}$$

$$= \frac{5i-5}{5}$$

= i - 1

Let Z = 1 - i = r(cosθ + isinθ)

Now , separating real and complex part , we get

-1 = rcosθ ……….eq.1

1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

r =√2,

Hence its modulus is √2.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{1}{-1}$$

Tanθ = -1

Since $\cos\theta = -\frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and tanθ = -1 . Therefore the θ lies in second quadrant.

Tanθ = -1 , therefore $\theta = \frac{3\pi}{4}$

Representing the complex no. in its polar form will be

$$Z = \sqrt{2}\{\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4})\}$$

Q. 16. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\frac{1-3i}{1+2i}$

Answer :

$$\frac{1 - 3i}{1 + 2i} \times \frac{1 - 2i}{1 - 2i}$$

$$= \frac{1 + 6i^2 - 5i}{1 - 4i^2}$$

$$= \frac{-5i - 5}{5}$$

= -i - 1

Let Z = -1 - i = r(cosθ + isinθ)

Now , separating real and complex part , we get

-1 = rcosθ ……….eq.1

-1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

2 = r^2

Since r is always a positive no., therefore,

r = √2,

Hence its modulus is √2.

Now , dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-1}{-1}$$

tanθ = 1

Since $\cos\theta = -\frac{1}{\sqrt{2}}$, $\sin\theta = -\frac{1}{\sqrt{2}}$ and tanθ = 1 . Therefore the θ lies in third quadrant.

Tanθ = 1, therefore $\theta = -\frac{3\pi}{4}$

Representing the complex no. in its polar form will be

$$Z = \sqrt{2}\{\cos(-\frac{3\pi}{4}) + i\sin(-\frac{3\pi}{4})\}$$

Q. 17. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\frac{5-i}{2-3i}$

Answer :

$$= \frac{5 - i}{2 - 3i} \times \frac{2 + 3i}{2 + 3i}$$

$= \frac{10 - 3i^2 + 13i}{4 - 9i^2}$

$= \frac{+13i + 13}{13}$

= i + 1

Let Z = 1 + i = r(cosθ + isinθ)

Now , separating real and complex part , we get

1 = rcosθ ……….eq.1

1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

2 = r^2

Since r is always a positive no., therefore,

r = √2,

Hence its modulus is √2.

Now , dividing eq.2 by eq.1 , we get,

$\frac{rsin\theta}{rcos\theta} = \frac{1}{1}$

Tanθ = 1

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and tanθ = 1. Therefore the θ lies in first quadrant.

Tanθ = 1, therefore $\theta = \frac{\pi}{4}$

Representing the complex no. in its polar form will be

$Z = \sqrt{2}\{\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})\}$

Q. 18. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\frac{-16}{1+\sqrt{3}i}$

Answer :

$$= \frac{-16}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{-16+16\sqrt{3}i}{1-3i^2}$$

$$= \frac{16\sqrt{3}i-16}{4}$$

= $4\sqrt{3}$i - 4

Let Z = $4\sqrt{3}$i - 4 = r(cosθ + isinθ)

Now , separating real and complex part , we get

-4 = rcosθ ……….eq.1

$4\sqrt{3}$ = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

64 = r^2

Since r is always a positive no., therefore,

r = 8,

Hence its modulus is 8.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{4\sqrt{3}}{-4}$$

$\tan\theta = -\sqrt{3}$

Since $\cos\theta = -\frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = -\sqrt{3}$. Therefore the θ lies in second quadrant.

$\tan\theta = -\sqrt{3}$, therefore $\theta = \frac{2\pi}{3}$

Representing the complex no. in its polar form will be

$$Z = 8\{\cos(\tfrac{2\pi}{3}) + i\sin(\tfrac{2\pi}{3})\}$$

Q. 19. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

Answer :

$$= \frac{2 + 6\sqrt{3}i}{5 + \sqrt{3}i} \times \frac{5 - \sqrt{3}i}{5 - \sqrt{3}i}$$

$$= \frac{10 + 28\sqrt{3}i - 18i^2}{25 - 3i^2}$$

$$= \frac{28\sqrt{3}i + 28}{28}$$

$= \sqrt{3}i + 1$

Let $Z = \sqrt{3}i + 1 = r(\cos\theta + i\sin\theta)$

Now , separating real and complex part , we get

$1 = r\cos\theta$ ……….eq.1

$\sqrt{3} = r\sin\theta$ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no., therefore,

r = 2,

Hence its modulus is 2.

Now , dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{\sqrt{3}}{1}$$

tanθ = √3

Since $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and tanθ = $\sqrt{3}$. therefore the θ lies in first quadrant.

Tanθ = √3, therefore $\theta = \frac{\pi}{3}$

Representing the complex no. in its polar form will be

$$Z = 2\{\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})\}$$

Q. 20

Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\sqrt{\frac{1+i}{1-i}}$

Answer :

$$= \sqrt{\frac{1+i}{1-i}} \times \sqrt{\frac{1+ i}{1+ i}}$$

$$= \sqrt{\frac{(1+i)^2}{1-i^2}}$$

$$= \frac{1+i}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

Let $Z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = r(\cos\theta + i\sin\theta)$

Now, separating real and complex part , we get

$\frac{1}{\sqrt{2}} = r\cos\theta$ ……….eq.1

$\frac{1}{\sqrt{2}} = r\sin\theta$ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

hence its modulus is 1.

now , dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{\frac{i}{\sqrt{2}}}{\frac{i}{\sqrt{2}}}$$

$\tan\theta = 1$

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and $\tan\theta = 1$. therefore the θ lies in first quadrant.

Tanθ = 1, therefore $\theta = \frac{\pi}{4}$

Representing the complex no. in its polar form will be

$Z = 1\{\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})\}$

Q. 20. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\sqrt{\frac{1+i}{1-i}}$

Answer :

$$= \sqrt{\frac{1+i}{1-i}} \times \sqrt{\frac{1+i}{1+i}}$$

$$= \sqrt{\frac{(1+i)^2}{1-i^2}}$$

$$= \frac{1+i}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

Let $Z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = r(\cos\theta + i\sin\theta)$

Now, separating real and complex part, we get

$\frac{1}{\sqrt{2}} = r\cos\theta$ ……….eq.1

$\frac{1}{\sqrt{2}} = \text{rsin}\theta$ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

Hence its modulus is 1.

Now , dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{\frac{i}{\sqrt{2}}}{\frac{i}{\sqrt{2}}}$$

tanθ = 1

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and tanθ =1. Therefore the θ lies in first quadrant.

Tanθ = 1, therefore $\theta = \frac{\pi}{4}$

Representing the complex no. in its polar form will be

$$Z = 1\{\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})\}$$

Q. 21. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $-\sqrt{3} - \mathrm{i}$

Answer : Let Z = -i - √3 = r(cosθ + isinθ)

Now, separating real and complex part, we get

-√3 = rcosθ ……….eq.1

-1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no., therefore,

r = 2

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

$$\frac{r sin\theta}{r cos\theta} = \frac{-1}{-\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

Since $\cos\theta = -\frac{\sqrt{3}}{2}$, $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$. Therefore the θ lies in third quadrant.

$\tan\theta = \frac{1}{\sqrt{3}}$, therefore $\theta = -\frac{5\pi}{6}$.

Representing the complex no. in its polar form will be

$$Z = 2\{\cos(-\frac{5\pi}{6}) + i\sin(-\frac{5\pi}{6})\}$$

Q. 22. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $(i^{25})^3$

Answer : $= i^{75}$

$= i^{4n+3}$ where n = 18

Since $i^{4n+3} = -i$

$i^{75} = -i$

Let Z = -i = r(cosθ + isinθ)

Now , separating real and complex part , we get

0 = rcosθ ……….eq.1

-1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

Hence its modulus is 1.

Now , dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-1}{0}$$

tanθ = -∞

Since cosθ = 0 , sinθ = -1 and tanθ = -∞ . therefore the θ lies in fourth quadrant.

Tanθ = -∞ , therefore $\theta = -\frac{\pi}{2}$

Representing the complex no. in its polar form will be

$$Z = 1\{\cos(-\tfrac{\pi}{2}) + i\sin(-\tfrac{\pi}{2})\}$$

Q. 23. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $\frac{(1-i)}{\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$

Answer :

$$= \frac{1 - i}{\frac{1}{2} + i\frac{\sqrt{3}}{2}}$$

$$= \frac{2 - 2i}{1 + i\sqrt{3}}$$

$$= \frac{2 - 2i}{1 + \sqrt{3}i} \times \frac{1 - \sqrt{3}i}{1 - \sqrt{3}i}$$

$$= \frac{2 - 2\sqrt{3}i - 2i + 2\sqrt{3}i^2}{1 - 3i^2}$$

$$= \frac{\left(2 - 2\sqrt{3}\right) + i(2\sqrt{3} + 2)}{4}$$

$$= \frac{\left(1 - \sqrt{3}\right) + i(\sqrt{3} + 1)}{2}$$

Let $Z = \frac{\left(1-\sqrt{3}\right) + i(\sqrt{3}+1)}{2} = \mathrm{r}(\cos\theta + i\sin\theta)$

Now, separating real and complex part , we get

$\frac{1-\sqrt{3}}{2} = r\cos\theta$ ……….eq.1

$\frac{1+\sqrt{3}}{2} = r\sin\theta$ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

$r = \sqrt{2}$,

Hence its modulus is $\sqrt{2}$.

Now, dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{\frac{1+\sqrt{3}}{2}}{\frac{1-\sqrt{3}}{2}}$$

$$\tan\theta = \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

Since $\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}}$, $\sin\theta = \frac{1+\sqrt{3}}{2\sqrt{2}}$ and tanθ = $\frac{1+\sqrt{3}}{1-\sqrt{3}}$. Therefore the θ lies in second quadrant. As

$\text{Tan}\theta = \frac{1+\sqrt{3}}{1-\sqrt{3}}$, therefore $\theta = \frac{7\pi}{12}$

Representing the complex no. in its polar form will be

$$Z = \sqrt{2}\{\cos(\frac{7\pi}{12}) + i\sin(\frac{7\pi}{12})\}$$

Q. 24. Find the modulus of each of the following complex numbers and hence express each of them in polar form: (sin 120° – i cos 120°)

Answer : = sin(90° + 30°) - icos(90° + 30°)

= cos30° + isin30°

Since, sin(90°+ α) = cosα

And cos(90° + α) = -sinα

$$= \frac{\sqrt{3}}{2} + i\frac{1}{2}$$

Hence it is of the form

$$Z = \frac{\sqrt{3}}{2} + i\frac{1}{2} = r(\cos\theta + i\sin\theta)$$

Therefore r = 1

Hence its modulus is 1 and argument is $\frac{\pi}{6}$.

Exercise 5E

Q. 1. $x^2 + 2 = 0$

Answer : This equation is a quadratic equation.

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Given:

$\Rightarrow x^2 + 2 = 0$

$\Rightarrow x^2 = -2$

$\Rightarrow x = \pm \sqrt{(-2)}$

But we know that $\sqrt{(-1)} = i$

$\Rightarrow x = \pm\sqrt{2}\, i$

Ans: $x = \pm\sqrt{2}\, i$

Q. 2. $x^2 + 5 = 0$

Answer : Given:

$x^2 + 5 = 0$

$\Rightarrow x^2 = -5$

$\Rightarrow x = \pm\sqrt{(-5)}$

$\Rightarrow x = \pm\sqrt{5}\, i$

Ans: $x = \pm\sqrt{5}\, i$

Q. 3. $2x^2 + 1 = 0$

Answer : $2x^2 + 1 = 0$

$\Rightarrow 2x^2 = -1$

$\Rightarrow x^2 = -\frac{1}{2}$

$\Rightarrow x = \pm\sqrt{-\frac{1}{2}}$

$\Rightarrow x = \pm\sqrt{\frac{1}{2}}i$

$\Rightarrow x = \pm\frac{i}{\sqrt{2}}$

Ans: $x = \pm\frac{i}{\sqrt{2}}$

Q. 4. $x^2 + x + 1 = 0$

Answer : Given:

$x^2 + x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-1\pm\sqrt{1^2-(4\times1\times1)}}{2\times1}$

$\Rightarrow x = \frac{-1\pm\sqrt{1-4}}{2}$

$\Rightarrow x = \frac{-1\pm\sqrt{-3}}{2}$

$\Rightarrow x = \frac{-1\pm\sqrt{3}i}{2}$

$\Rightarrow x = -\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$

Ans: $x = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $x = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Q. 5. $x^2 - x + 2 = 0$

Answer : Given:

$x^2 - x + 2 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - (4 \times 1 \times 2)}}{2 \times 1}$

$\Rightarrow x = \frac{1 \pm \sqrt{1-8}}{2}$

$\Rightarrow x = \frac{1 \pm \sqrt{-7}}{2}$

$\Rightarrow x = \frac{1 \pm \sqrt{7}i}{2}$

$\Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{7}}{2}i$

Ans: $x = \frac{1}{2} + \frac{\sqrt{7}}{2}i$ and $x = \frac{1}{2} - \frac{\sqrt{7}}{2}i$

Q. 6. $x^2 + 2x + 2 = 0$

Answer : Given:

$x^2 + 2x + 2 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-2 \pm \sqrt{(2)^2 - (4 \times 1 \times 2)}}{2 \times 1}$

$\Rightarrow x = \frac{-2 \pm \sqrt{4-8}}{2}$

$\Rightarrow x = \frac{-2\pm\sqrt{-4}}{2}$

$\Rightarrow x = \frac{-2\pm 2i}{2}$

$\Rightarrow x = -\frac{2}{2} \pm \frac{2}{2}i$

$\Rightarrow x = -1 \pm i$

Ans: x = -1 + i and x = -1-i

Q. 7. $2x^2 - 4x + 3 = 0$

Answer : Given:

$2x^2 - 4x + 3 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-(-4)\pm\sqrt{(-4)^2-(4\times2\times3)}}{2\times2}$

$\Rightarrow x = \frac{4\pm\sqrt{16-24}}{4}$

$\Rightarrow x = \frac{4\pm\sqrt{-8}}{4}$

$\Rightarrow x = \frac{4\pm 2\sqrt{2}i}{4}$

$\Rightarrow x = \frac{4}{4} \pm \frac{2\sqrt{2}}{4}i$

$\Rightarrow x = 1 \pm \frac{i}{\sqrt{2}}$

Ans: $x = 1 + \frac{i}{\sqrt{2}}$ and $x = 1 - \frac{i}{\sqrt{2}}$

Q. 8. $x^2 + 3x + 5 = 0$

Answer : Given:

$x^2 + 3x + 5 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-3 \pm \sqrt{(3)^2 - (4 \times 1 \times 5)}}{2 \times 1}$

$\Rightarrow x = \frac{-3 \pm \sqrt{9 - 20}}{2}$

$\Rightarrow x = \frac{-3 \pm \sqrt{-11}}{2}$

$\Rightarrow x = \frac{-3 \pm \sqrt{11}i}{2}$

$\Rightarrow x = -\frac{3}{2} \pm \frac{\sqrt{11}}{2} i$

Ans: $x = -\frac{3}{2} + \frac{\sqrt{11}}{2} i$ and $x = -\frac{3}{2} - \frac{\sqrt{11}}{2} i$

Q. 9. $\sqrt{5}x^2 + x + \sqrt{5} = 0$

Answer : Given:

$\sqrt{5}x^2 + x + \sqrt{5} = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-1 \pm \sqrt{(1)^2-(4\times\sqrt{5}\times\sqrt{5})}}{2\times\sqrt{5}}$

$\Rightarrow x = \frac{-1\pm\sqrt{1-20}}{2\sqrt{5}}$

$\Rightarrow x = \frac{-1\pm\sqrt{-19}}{2\sqrt{5}}$

$\Rightarrow x = \frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$

$\Rightarrow x = -\frac{1}{2\sqrt{5}} \pm \frac{\sqrt{19}}{2\sqrt{5}}i$

Ans: $x = -\frac{\sqrt{5}}{10} + \frac{\sqrt{\frac{19}{5}}}{2}i$ and $x = -\frac{\sqrt{5}}{10} - \frac{\sqrt{\frac{19}{5}}}{2}i$

Q. 10. $25x^2$ – 30x + 11 = 0

Answer : Given:

$25x^2 - 30x + 11 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-(-30)\pm\sqrt{(-30)^2-(4\times25\times11)}}{2\times25}$

$\Rightarrow x = \frac{30\pm\sqrt{900-1100}}{50}$

$\Rightarrow x = \frac{30\pm\sqrt{-200}}{50}$

$\Rightarrow x = \frac{30\pm10\sqrt{2}i}{50}$

$\Rightarrow x = -\frac{30}{50} \pm \frac{10\sqrt{2}}{50} i$

Ans: $x = -\frac{3}{5} + \frac{\sqrt{2}}{5} i$ and $x = -\frac{3}{5} - \frac{\sqrt{2}}{5} i$

Q. 11. 8x^2 + 2x + 1 = 0

Answer : Given:

$8x^2 + 2x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-2 \pm \sqrt{(2)^2 - (4 \times 8 \times 1)}}{2 \times 8}$

$\Rightarrow x = \frac{-2 \pm \sqrt{4 - 32}}{16}$

$\Rightarrow x = \frac{-2 \pm \sqrt{-28}}{16}$

$\Rightarrow x = \frac{-2 \pm 2\sqrt{7} i}{16}$

$\Rightarrow x = -\frac{2}{16} \pm \frac{2\sqrt{7}}{16} i$

Ans: $x = -\frac{1}{8} + \frac{\sqrt{7}}{8} i$ and $x = -\frac{1}{8} - \frac{\sqrt{7}}{8} i$

Q. 12. 27x^2 + 10x + 1 = 0

Answer :

Given:

$27x^2 + 10x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-10 \pm \sqrt{(10)^2 - (4 \times 27 \times 1)}}{2 \times 27}$$

$$\Rightarrow x = \frac{-10 \pm \sqrt{100 - 108}}{54}$$

$$\Rightarrow x = \frac{-10 \pm \sqrt{-8}}{54}$$

$$\Rightarrow x = \frac{-10 \pm 2\sqrt{2}i}{54}$$

$$\Rightarrow x = -\frac{10}{54} \pm \frac{2\sqrt{2}}{54}i$$

Ans: $x = -\frac{5}{27} + \frac{\sqrt{2}}{27}i$ and $x = -\frac{5}{27} - \frac{\sqrt{2}}{27}i$

Q. 13. $2x^2 - \sqrt{3}x + 1 = 0$

Answer : Given:

$$2x^2 - \sqrt{3}x + 1 = 0$$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-\sqrt{3}) \pm \sqrt{(-\sqrt{3})^2 - (4 \times 2 \times 1)}}{2 \times 2}$$

$\Rightarrow x = \frac{\sqrt{3} \pm \sqrt{3-8}}{4}$

$\Rightarrow x = \frac{\sqrt{3} \pm \sqrt{-5}}{4}$

$\Rightarrow x = \frac{\sqrt{3} \pm \sqrt{5}i}{4}$

$\Rightarrow x = \frac{\sqrt{3}}{4} \pm \frac{\sqrt{5}}{4}i$

Ans: $x = \frac{\sqrt{3}}{4} + \frac{\sqrt{5}}{4}i$ and $x = \frac{\sqrt{3}}{4} - \frac{\sqrt{5}}{4}i$

Q. 14. $17x^2 - 8x + 1 = 0$

Answer : Given:

$17x^2 - 8x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - (4 \times 17 \times 1)}}{2 \times 17}$

$\Rightarrow x = \frac{8 \pm \sqrt{64-68}}{34}$

$\Rightarrow x = \frac{8 \pm \sqrt{-4}}{34}$

$\Rightarrow x = \frac{8 \pm 2i}{34}$

$\Rightarrow x = \frac{8}{34} \pm \frac{2}{34}i$

Ans: $x = \frac{4}{17} + \frac{1}{17}i$ and $x = \frac{4}{17} - \frac{1}{17}i$

Q. 15. $3x^2 + 5 = 7x$

Answer : Given:

$3x^2 + 5 = 7x$

$\Rightarrow 3x^2 - 7x + 5 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-7) \pm \sqrt{(-7)^2 - (4 \times 3 \times 5)}}{2 \times 3}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 60}}{6}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{-11}}{6}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{11}i}{6}$$

$$\Rightarrow x = \frac{7}{6} \pm \frac{\sqrt{11}}{6}i$$

Ans: $x = \frac{7}{6} + \frac{\sqrt{11}}{6}i$ and $x = \frac{7}{6} - \frac{\sqrt{11}}{6}i$

Q. 16. $3x^2 - 4x + \frac{20}{3} = 0$

Answer : Given:

$$3x^2 - 4x + \frac{20}{3} = 0$$

Multiplying both the sides by 3 we get,

$$9x^2 - 12x + 20 = 0$$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-(-12) \pm \sqrt{(-12)^2 - (4 \times 9 \times 20)}}{2 \times 9}$

$\Rightarrow x = \frac{12 \pm \sqrt{144 - 720}}{18}$

$\Rightarrow x = \frac{12 \pm \sqrt{-576}}{18}$

$\Rightarrow x = \frac{12 \pm 24i}{18}$

$\Rightarrow x = \frac{12}{18} \pm \frac{24}{18} i$

$\Rightarrow x = \frac{2}{3} \pm \frac{4}{3} i$

Ans: $x = \frac{2}{3} + \frac{4}{3} i$ and $x = \frac{2}{3} - \frac{4}{3} i$

Q. 17. $3x^2 + 7ix + 6 = 0$

Answer : Given:

$3x^2 + 7ix + 6 = 0$

$\Rightarrow 3x^2 + 9ix - 2ix + 6 = 0$

$\Rightarrow 3x(x + 3i) - 2i\left(x - \frac{6}{2i}\right) = 0$

$\Rightarrow 3x(x + 3i) - 2i\left(x - \frac{3 \times i}{i \times i}\right) = 0$ …($i^2 = -1$)

$\Rightarrow 3x(x + 3i) - 2i(x - \frac{3 \times i}{-1}) = 0$

$\Rightarrow 3x(x + 3i) - 2i(x + 3i) = 0$

$\Rightarrow (x + 3i)(3x - 2i) = 0$

$\Rightarrow$x + 3i = 0 & 3x-2i = 0

$\Rightarrow$ x = 3i & $x = \frac{2}{3}i$

Ans: x = 3i and $x = \frac{2}{3}i$

Q. 18. $21x^2 - 28x + 10 = 0$

Answer : Given:

$21x^2 - 28x + 10 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-(-28) \pm \sqrt{(-28)^2 - (4 \times 21 \times 10)}}{2 \times 21}$

$\Rightarrow x = \frac{28 \pm \sqrt{784 - 840}}{42}$

$\Rightarrow x = \frac{28 \pm \sqrt{-56}}{42}$

$\Rightarrow x = \frac{28 \pm 2\sqrt{14}i}{42}$

$\Rightarrow x = \frac{28}{42} \pm \frac{2\sqrt{14}}{42}i$

Ans: $x = \frac{2}{3} + \frac{\sqrt{14}}{21}i$ and $x = \frac{2}{3} - \frac{\sqrt{14}}{21}i$

Q. 19. $x^2 + 13 = 4x$

Answer : Given:

$x^2 + 13 = 4x$

$\Rightarrow x^2 - 4x + 13 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 1 \times 13)}}{2 \times 1}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{-36}}{2}$$

$$\Rightarrow x = \frac{4 \pm 6i}{2}$$

$$\Rightarrow x = \frac{4}{2} \pm \frac{6}{2} i$$

$$\Rightarrow x = 2 \pm 3i$$

Ans: $x = 2 + 3i$ & $x = 2 - 3i$

Q. 20. $x^2 + 3ix + 10 = 0$

Answer : Given:

$x^2 + 3ix + 10 = 0$

$\Rightarrow x^2 + 5ix - 2ix + 10 = 0$

$$\Rightarrow x(x + 5i) - 2i\left(x - \frac{10}{2i}\right) = 0$$

$$\Rightarrow x(x + 5i) - 2i\left(x - \frac{5 \times i}{i \times i}\right) = 0$$

$\Rightarrow x(x + 5i) - 2i(x - \frac{5\times i}{-1})$ = 0

$\Rightarrow x(x + 5i) - 2i(x + 5i) = 0$

$\Rightarrow (x + 5i)(x - 2i) = 0$

⇒x + 5i = 0 & x-2i = 0

⇒x = -5i & x = 2i

Ans: x = -5i & x = 2i

Q. 21. $2x^2 + 3ix + 2 = 0$

Answer : Given:

$2x^2 + 3ix + 2 = 0$

⇒ $2x^2 + 4ix - ix + 2 = 0$

$\Rightarrow 2x(x + 2i) - i\left(x - \frac{2}{i}\right) = 0$

$\Rightarrow 2x(x + 2i) - i\left(x - \frac{2\times i}{i\times i}\right) = 0$

$\Rightarrow 2x(x + 2i) - i(x - \frac{2\times i}{-1})$ = 0

$\Rightarrow 2x(x + 2i) - i(x + 2i) = 0$

$\Rightarrow (x + 2i)(2x - i) = 0$

⇒x + 2i = 0 & 2x-i = 0

⇒ x = -2i & $x = \frac{i}{2}$

Ans: x = -2i and $x = \frac{i}{2}$

Exercise 5F

Q. 1. $\sqrt{5+12i}$

Answer : Let, $(a + ib)^2 = 5 + 12i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 5 + 12i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 5 + 12i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 5$…………..eq.1

$\Rightarrow 2ab = 12$……..eq.2

$\Rightarrow a = \frac{6}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{6}{b}\right)^2 - b^2 = 5$

$\Rightarrow 36 - b^4 = 5b^2$

$\Rightarrow b^4 + 5b^2 - 36 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = -9$ or $b^2 = 4$

As b is real no. so, $b^2 = 4$

$b = 2$ or $b = -2$

Therefore, a = 3 or a= -3

Hence the square root of the complex no. is 3 + 2i and -3 -2i.

Q. 2. $\sqrt{-7+24i}$

Answer : Let, $(a + ib)^2 = -7 + 24i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -7 + 24i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -7 + 24i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = -7$………….eq.1

$\Rightarrow 2ab = 24$…….. eq.2

$\Rightarrow a = \frac{12}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{12}{b}\right)^2 - b^2 = -7$

$\Rightarrow 144 - b^4 = -7b^2$

$\Rightarrow b^4 - 7b^2 - 144 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = -9$ or $b^2 = 16$

As b is real no. so, $b^2 = 16$

b= 4 or b= -4

Therefore, a= 3 or a= -3

Hence the square root of the complex no. is 3 + 4i and -3 -4i.

Q. 3. $\sqrt{-2+2\sqrt{3}i}$

Answer : Let, $(a + ib)^2 = -2 + 2\sqrt{3}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -2 + 2\sqrt{3}i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -2 + 2\sqrt{3}i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = -2$…………..eq.1

$\Rightarrow 2ab = 2\sqrt{3}$……..eq.2

$\Rightarrow a = \frac{\sqrt{3}}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{\sqrt{3}}{b}\right)^2 - b^2 = -2$

$\Rightarrow 3 - b^4 = -2b^2$

$\Rightarrow b^4 - 2b^2 - 3 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = -1$ or $b^2 = 3$

As b is real no. so, $b^2 = 3$

b= $\sqrt{3}$ or b = $-\sqrt{3}$

Therefore, a= 1 or a= -1

Hence the square root of the complex no. is 1 + $\sqrt{3}$i and -1 -$\sqrt{3}$i.

Q. 4. $\sqrt{1+4\sqrt{-3}}$

Answer : Let, $(a + ib)^2 = 1 + 4\sqrt{3}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 1 + 4\sqrt{3}i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 1 + 4\sqrt{3}i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 1$…………..eq.1

$\Rightarrow 2ab = 4\sqrt{3}$…….eq.2

$\Rightarrow a = \frac{2\sqrt{3}}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{2\sqrt{3}}{b}\right)^2 - b^2 = 1$

$\Rightarrow 12 - b^4 = b^2$

$\Rightarrow b^4 + b^2 - 12 = 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow$ $b^2 = -4$ or $b^2 = 3$

As b is real no. so, $b^2 = 3$

b= $\sqrt{3}$ or b= $-\sqrt{3}$

Therefore, a= 2 or a= -2

Hence the square root of the complex no. is 2 + $\sqrt{3}$i and -2 -$\sqrt{3}$i.

Q. 5. $\sqrt{i}$

Answer : Let, $(a + ib)^2 = 0 + i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ $a^2 + (bi)^2 + 2abi = 0 + i$

Since $i^2 = -1$

$\Rightarrow$ $a^2 - b^2 + 2abi = 0 + i$

Now, separating real and complex parts, we get

$\Rightarrow$ $a^2 - b^2 = 0$ …………..eq.1

$\Rightarrow$ 2ab =1…….. eq.2

$\Rightarrow$ a = $\frac{1}{2b}$

Now, using the value of a in eq.1, we get

$\Rightarrow$ $\left(\frac{1}{2b}\right)^2 - b^2 = 0$

$\Rightarrow$ $1 - 4b^4 = 0$

$\Rightarrow$ $4b^2 = 1$

Simplify and get the value of b^2 , we get,

$\Rightarrow$ $b^2 = -\frac{1}{2}$ or $b^2 = \frac{1}{2}$

As b is real no. so, $b^2 = 3$

b= $\frac{1}{\sqrt{2}}$ or b= $-\frac{1}{\sqrt{2}}$

Therefore , a= $\frac{1}{\sqrt{2}}$ or a= $-\frac{1}{\sqrt{2}}$

Hence the square root of the complex no. is $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ and $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$.

Q. 6. $\sqrt{4i}$

Answer : Let, $(a + ib)^2 = 0 + 4i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ $a^2 + (bi)^2 + 2abi = 0 + 4i$

Since $i^2 = -1$

$\Rightarrow$ $a^2 - b^2 + 2abi = 0 + 4i$

Now, separating real and complex parts, we get

$\Rightarrow$ $a^2 - b^2 = 0$ …………..eq.1

$\Rightarrow$ $2ab = 4$…….. eq.2

$\Rightarrow$ $a = \frac{2}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow$ $\left(\frac{2}{b}\right)^2 - b^2 = 0$

$\Rightarrow$ $4 - b^4 = 0$

$\Rightarrow$ $b^4 = 4$

Simplify and get the value of b^2 , we get,

$\Rightarrow$ $b^2 = -2$ or $b^2 = 2$

As b is real no. so, $b^2 = 2$

b= $\sqrt{2}$ or b= $-\sqrt{2}$

Therefore , a= $\sqrt{2}$ or a= -$\sqrt{2}$

Hence the square root of the complex no. is $\sqrt{2}$ + $\sqrt{2}$i and -$\sqrt{2}$ - $\sqrt{2}$i.

Q. 7. $\sqrt{3+4\sqrt{-7}}$

Answer : Let, $(a + ib)^2 = 3 + 4\sqrt{7}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ $a^2 + (bi)^2 + 2abi = 3 + 4\sqrt{7}i$

Since $i^2 = -1$

$\Rightarrow$ $a^2 - b^2 + 2abi = 3 + 4\sqrt{7}i$

now, separating real and complex parts, we get

$\Rightarrow$ $a^2 - b^2 = 3$ …………..eq.1

$\Rightarrow$ $2ab = 4\sqrt{7}$…….. eq.2

$\Rightarrow$ $a = \frac{2\sqrt{7}}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{2\sqrt{7}}{b}\right)^2 - b^2 = 3$

$\Rightarrow 12 - b^4 = 3b^2$

$\Rightarrow b^4 + 3b^2 - 28 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = -7$ or $b^2 = 4$

as b is real no. so, $b^2 = 4$

b= 2 or b= -2

Therefore , a= $\sqrt{7}$ or a= -$\sqrt{7}$

Hence the square root of the complex no. is $\sqrt{7}$ + 2i and -$\sqrt{7}$ -2i.

Q. 8. $\sqrt{16-30i}$

Answer : Let, $(a + ib)^2 = 16 - 30i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 16 - 30i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 16 - 30i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 16$…………..eq.1

$\Rightarrow 2ab = -30$…….. eq.2

$\Rightarrow a = -\frac{15}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(-\frac{15}{b}\right)^2 - b^2 = 16$

$\Rightarrow 225 - b^4 = 16b^2$

$\Rightarrow b^4 + 16b^2 - 225 = 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = -25$ or $b^2 = 9$

As b is real no. so, $b^2 = 9$

b= 3 or b= -3

Therefore, a= -5 or a= 5

Hence the square root of the complex no. is -5 + 3i and 5 - 3i.

Q. 9. $\sqrt{-4-3i}$

Answer : Let, $(a + ib)^2 = -4 - 3i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -4 -3i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -4 - 3i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = -4$…………..eq.1

$\Rightarrow 2ab = -3$…….. eq.2

$\Rightarrow a = -\frac{3}{2b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(-\frac{3}{2b}\right)^2 - b^2 = -4$

$\Rightarrow 9 - 4b^4 = -16b^2$

$\Rightarrow 4b^4 - 16b^2 - 9 = 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = \frac{9}{2}$ or $b^2 = -2$

As b is real no. so, $b^2 = \frac{9}{2}$

$b = \frac{3}{\sqrt{2}}$ or $b = -\frac{3}{\sqrt{2}}$

Therefore, $a = -\frac{1}{\sqrt{2}}$ or $a = \frac{1}{\sqrt{2}}$

Hence the square root of the complex no. is $-\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}}i$ and $\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}}i$.

Q. 10. $\sqrt{-15 - 8i}$

Answer : Let, $(a + ib)^2 = -15 - 8i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -15 - 8i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -15 - 8i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = -15$…………..eq.1

$\Rightarrow$ 2ab = -8…….. eq.2

$\Rightarrow$ a = $-\frac{4}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow$ $\left(-\frac{4}{b}\right)^2$ – b^2 = -15

$\Rightarrow$ 16 – b^4 = -15b^2

$\Rightarrow$ b^4 - 15b^2 - 16= 0

Simplify and get the value of b^2 , we get,

$\Rightarrow$ b^2 = 16 or b^2 = -1

As b is real no. so, b^2 = 16

b= 4 or b= -4

Therefore, a= -1 or a= 1

Hence the square root of the complex no. is -1 + 4i and 1 - 4i.

Q. 11. $\sqrt{-11-60i}$

Answer : Let, $(a + ib)^2$ = -11 - 60i

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ $a^2 + (bi)^2 + 2abi = -11 - 60i$

Since $i^2 = -1$

$\Rightarrow$ $a^2 - b^2 + 2abi = -11 - 60i$

Now, separating real and complex parts, we get

$\Rightarrow$ $a^2 - b^2$ = -11…………..eq.1

$\Rightarrow$ 2ab = -60…….. eq.2

$\Rightarrow$ a = $-\frac{30}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow$ $\left(-\frac{30}{b}\right)^2$ – b^2 = -11

$\Rightarrow$ 900 – b^4 = -11b^2

$\Rightarrow$ b^4- 11b^2 - 900= 0

Simplify and get the value of b^2, we get,

$\Rightarrow$ b^2 = 36 or b^2 = -25

as b is real no. so, b^2 = 36

b= 6 or b= -6

Therefore , a= -5 or a= 5

Hence the square root of the complex no. is -5 + 6i and 5 – 6i.

Q. 12. $\sqrt{7-30\sqrt{-2}}$

Answer : Let, $(a + ib)^2 = 7 - 30\sqrt{2}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ $a^2 + (bi)^2 + 2abi = 7 - 30\sqrt{2}i$

Since $i^2 = -1$

$\Rightarrow$ $a^2 - b^2 + 2abi = 7 - 30\sqrt{2}i$

Now, separating real and complex parts, we get

$\Rightarrow$ $a^2 - b^2 = 7$ …………..eq.1

$\Rightarrow$ $2ab = 30\sqrt{2}$........ eq.2

$\Rightarrow$ $a = \frac{15\sqrt{2}}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow$ $\left(\frac{15\sqrt{2}}{b}\right)^2 - b^2 = 7$

$\Rightarrow$ $450 - b^4 = 7b^2$

$\Rightarrow$ $b^4 + 7b^2 - 450 = 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow$ $b^2 = -25$ or $b^2 = 18$

As b is real no. so, $b^2 = 18$

$b = 3\sqrt{2}$ or $b = -3\sqrt{2}$

Therefore , a= 5 or a= - 5

Hence the square root of the complex no. is $5 + 3\sqrt{2}i$ and $-5 - 3\sqrt{2}i$.

Q. 13. $\sqrt{-8i}$

Answer : Let, $(a + ib)^2 = 0 - 8i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ $a^2 + (bi)^2 + 2abi = 0 - 8i$

Since $i^2 = -1$

$\Rightarrow$ $a^2 - b^2 + 2abi = 0 - 8i$

Now, separating real and complex parts, we get

$\Rightarrow$ $a^2 - b^2 = 0$ …………..eq.1

$\Rightarrow$ $2ab = -8$…….. eq.2

$\Rightarrow$ a = $-\frac{4}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow$ $\left(-\frac{4}{b}\right)^2 - b^2 = 0$

$\Rightarrow$ $16 - b^4 = 0$

$\Rightarrow$ $b^4 = 16$

Simplify and get the value of b^2, we get,

$\Rightarrow$ $b^2 = -4$ or $b^2 = 4$

As b is real no. so, $b^2 = 4$

b= 2 or b= -2

Therefore , a= -2 or a= 2

Hence the square root of the complex no. is -2 + 2i and 2 - 2i.

Q. 14. $\sqrt{1-i}$

Answer : Let, $(a + ib)^2 = 1 - i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ $a^2 + (bi)^2 + 2abi = 1 - i$

Since $i^2 = -1$

$\Rightarrow$ $a^2 - b^2 + 2abi = 1 - i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 1$…………..eq.1

$\Rightarrow 2ab = -1$…….. eq.2

$\Rightarrow a = -\frac{1}{2b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(-\frac{1}{2b}\right)^2 - b^2 = 1$

$\Rightarrow 1 - 4b^4 = 4b^2$

$\Rightarrow 4b^4 + 4b^2 - 1 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = \frac{-4 \pm \sqrt{32}}{8}$

As b is real no. so, $b^2 = \frac{-4 + 4\sqrt{2}}{8}$

$b^2 = \frac{-1 + \sqrt{2}}{2}$

$b = \sqrt{\frac{-1 + \sqrt{2}}{2}}$ or $b = -\sqrt{\frac{-1 + \sqrt{2}}{2}}$

Therefore , $a = -\sqrt{\frac{1 + \sqrt{2}}{2}}$ or $a = \sqrt{\frac{1 + \sqrt{2}}{2}}$

Hence the square root of the complex no. is $-\sqrt{\frac{1 + \sqrt{2}}{2}} + \sqrt{\frac{-1 + \sqrt{2}}{2}}\, i$ and $\sqrt{\frac{1 + \sqrt{2}}{2}} - \sqrt{\frac{-1 + \sqrt{2}}{2}}\, i$.

Exercise 5G

Q. 1. Evaluate $\frac{1}{i^{78}}$.

Answer : we have, $\frac{1}{i^{78}}$

$$= \frac{1}{(i^4)^{19}.i^2}$$

We know that, $i^4 = 1$

$$\Rightarrow \frac{1}{1^{19}.i^2}$$

$$\Rightarrow \frac{1}{i^2} = \frac{1}{-1}$$

$$\Rightarrow \frac{1}{i^{78}} = -1$$

Q. 2. Evaluate ($i^{57} + i^{70} + i^{91} + i^{101} + i^{104}$).

Answer : We have, $i^{57} + i^{70} + i^{91} + i^{101} + i^{104}$

$= (i^4)^{14}.i + (i^4)^{17}.i^2 + (i^4)^{22}.i^3 + (i^4)^{25}.i + (i^4)^{26}$

We know that, $i^4 = 1$

$\Rightarrow (1)^{14}.i + (1)^{17}.i^2 + (1)^{22}.i^3 + (1)^{25}.i + (1)^{26}$

$= i + i^2 + i^3 + i +1$

$= i -1 -i + i +1$

$= i$

Q. 3. Evaluate

$$\left(\frac{i^{180} + i^{178} + i^{176} + i^{174} + i^{172}}{i^{170} + i^{168} + i^{166} + i^{164} + i^{162}}\right)$$

Answer :

We have, $\left(\frac{i^{180}+i^{178}+i^{176}+i^{174}+i^{172}}{i^{170}+i^{168}+i^{166}+i^{164}+i^{162}}\right)$

$$=\left(\frac{i^{180}+i^{178}+i^{176}+i^{174}+i^{172}}{i^{170}+i^{168}+i^{166}+i^{164}+i^{162}}\right)$$

$$=\left(\frac{(i^4)^{45}+(i^4)^{44}.i^2+(i^4)^{44}+(i^4)^{43}.i^2+(i^4)^{43}}{(i^4)^{42}.i^2+(i^4)^{42}+(i^4)^{41}.i^2+(i^4)^{41}+(i^4)^{40}.i^2}\right)$$

$$=\left(\frac{(1)^{45}+(1)^{44}.i^2+(1)^{44}+(1)^{43}.i^2+(1)^{43}}{(1)^{42}.i^2+(1)^{42}+(1)^{41}.i^2+(1)^{41}+(1)^{40}.i^2}\right)$$

$$=\left(\frac{1+i^2+1+.i^2+1}{i^2+1+i^2+1+i^2}\right)$$

$$=\left(\frac{1-1+1-1+1}{-1+1-1+1-1}\right)$$

$$=\left(\frac{1}{-1}\right)$$

= -1

Q. 4. Evaluate ($i^{4n+1} - i^{4n-1}$)

Answer : We have, $i^{4n+1} - i^{4n-1}$

$= i^{4n}.i - i^{4n}.i^{-1}$

$= (i^4)^n.i - (i^4)^n.\ i^{-1}$

$= (1)^n.i - (1)^n.\ i^{-1}$

$= i - i^{-1}$

$$= i - \frac{1}{i}$$

$= \frac{i^2 - 1}{i}$

$= \frac{-1 - 1}{i}$

$= \frac{-2}{i} \times \frac{i}{i}$

$= \frac{-2i}{i^2} = \frac{-2i}{-1}$

= 2i

Q. 5. Evaluate $\left(\sqrt{36} \times \sqrt{-25}\right)$**.**

Answer : We have, $\left(\sqrt{36} \times \sqrt{-25}\right)$

$= 6 \times \sqrt{-1 \times 25}$

$= 6 \times \left(\sqrt{-1} \times \sqrt{25}\right)$

$= 6 \times \left(\sqrt{-1} \times 5\right)$

= 6×5i = 30i

Q. 6. Find the sum ($i^n + i^{n+1} + i^{n+2} + i^{n+3}$), where n N.

Answer : We have $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$= i^n + i^n.i + i^n.i^2 + i^n.i^3$

$= i^n (1 + i + i^2 + i^3)$

$= i^n (1 + i - 1 - i)$

$= i^n (0) = 0$

Q. 7. Find the sum ($i + i^2 + i^3 + i^4 + \dots$ up to 400 terms)., where n N.

Answer : We have, $i + i^2 + i^3 + i^4 + \ldots$ up to 400 terms

We know that given series is GP where a=i , r = i and n = 400

Thus, $S = \frac{a(1-r^n)}{1-r}$

$$= \frac{i(1-(i)^{400})}{1-i}$$

$$= \frac{i(1-(i^4)^{100})}{1-i}$$

$$= \frac{i(1-1^{100})}{1-i} \; [\because i^4 = 1]$$

$$= \frac{i(1-1)}{1-i} = 0$$

Q. 8. Evaluate $(1 + i^{10} + i^{20} + i^{30})$.

Answer : We have, $1 + i^{10} + i^{20} + i^{30}$

$= 1 + (i^4)^2.i^2 + (i^4)^5 + (i^4)^7.i^2$

We know that, $i^4 = 1$

$\Rightarrow 1 + (1)^2.i^2 + (1)^5 + (1)^7.i^2$

$= 1 + i^2 + 1 + i^2$

$= 1 -1 + 1 -1$

$= 0$

Q. 9. Evaluate: $\left(i^{41} + \frac{1}{i^{71}}\right)$.

Answer : We have, $\left(i^{41} + \frac{1}{i^{71}}\right)$

$i^{41} = i^{40} . i = i$

$i^{71} = i^{68} . i^3 = -i$

Therefore,

$$\left(i^{41}+\frac{1}{i^{71}}\right)=i-\frac{1}{i}=\frac{i^2-1}{i}$$

$$\left(i^{41}+\frac{1}{i^{71}}\right)= -\frac{2}{i}\times\frac{i}{i}$$

$$\left(i^{41}+\frac{1}{i^{71}}\right)= -\frac{2i}{i^2}=2i$$

Hence, $\left(i^{41}+\frac{1}{i^{71}}\right)=2i$

Q. 10. Find the least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n=1$ **.**

Answer : We have, $\left(\frac{1+\mathrm{i}}{1-\mathrm{i}}\right)^{\mathrm{n}}=1$

Now, $\frac{1+\mathrm{i}}{1-\mathrm{i}}= \frac{1+\mathrm{i}}{1-\mathrm{i}}\times\frac{1+\mathrm{i}}{1+\mathrm{i}}$

$$=\frac{(1+\mathrm{i})^2}{1^2-\mathrm{i}^2}$$

$$=\frac{1^2+2\mathrm{i}+\mathrm{i}^2}{1-(-1)}$$

$$=\frac{1+2\mathrm{i}-1}{2}$$

$$=\mathrm{i}$$

$\therefore\left(\frac{1+\mathrm{i}}{1-\mathrm{i}}\right)^{\mathrm{n}}=(\mathrm{i})^{\mathrm{n}}=1\Rightarrow \mathrm{n}$ is multiple of 4

∴ The least positive integer n is 4

Q. 11. Express $(2-3i)^3$ in the form (a + ib).

Answer : We have, $(2 - 3i)^3$

$= 2^3 - 3 \times 2^2 \times 3i - 3 \times 2 \times (3i)^2 - (3i)^3$

$= 8 - 36i + 54 + 27i$

$= 46 - 9i.$

Q. 12. Express $\frac{\left(3+i\sqrt{5}\right)\left(3-\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-\sqrt{2}i\right)}$ **in the form (a + ib).**

Answer : We have, $\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2i}\right)-\left(\sqrt{3}-\sqrt{2i}\right)}$

$$= \frac{(3)^2 - \left(i\sqrt{5}\right)^2}{\sqrt{3} + \sqrt{2i} - \sqrt{3} + \sqrt{2i}} \ [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{9+5}{2\sqrt{2i}} \times \frac{\sqrt{2i}}{\sqrt{2i}}$$

$$= \frac{14\sqrt{2i}}{2\left(\sqrt{2i}\right)^2}$$

$$= \frac{7\sqrt{2i}}{-2}$$

$$= \frac{-7\sqrt{2i}}{2}$$

Q. 13. Express $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$ **in the form (a + ib).**

Answer : We have, $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$

We know that $\sqrt{-1} = i$

Therefore,

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}}=\frac{3-4i}{1-3i}$$

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}}=\frac{3-4i}{1-3i}\times\frac{1+3i}{1+3i}$$

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}}=\frac{3+9i-4i-12\,i^2}{(1)^2-(3i)^2}$$

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}}=\frac{15+5i}{1+9}=\frac{15}{10}+\frac{5i}{10}=\frac{3}{2}+\frac{1}{2}i$$

Hence,

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}}=\frac{3}{2}+\frac{i}{2}$$

Q. 14. Solve for x: (1 – i) x + (1 + i) y = 1 – 3i.

Answer : We have, (1 – i) x + (1 + i) y = 1 – 3i

⇒ x-ix+y+iy = 1-3i

⇒ (x+y)+i(-x+y) = 1-3i

On equating the real and imaginary coefficients we get,

⇒ x+y = 1 (i) and –x+y = -3 (ii)

From (i) we get

x = 1-y

Substituting the value of x in (ii), we get

-(1-y)+y=-3

⇒ -1+y+y = -3

⇒ 2y = -3+1

$\Rightarrow$ y = -1

$\Rightarrow$ x=1-y = 1-(-1)=2

Hence, x=2 and y = -1

Q. 15. Solve for x: $x^2 - 5ix - 6 = 0$.

Answer : We have, $x^2 - 5ix - 6 = 0$

Here, $b^2-4ac = (-5i)^2-4\times1\times-6$

$= 25i^2+24 = -25+24 = -1$

Therefore, the solutions are given by $x = \frac{-(-5i)\pm\sqrt{-1}}{2\times1}$

$$x = \frac{5i \pm i}{2 \times 1}$$

$$x = \frac{5i \pm i}{2}$$

Hence, x= 3i and x = 2i

Q. 16. Find the conjugate of $\frac{1}{(3+4i)}$.

Answer : Let $z = \frac{1}{3+4i}$

$$= \frac{1}{3+4i} \times \frac{3-4i}{3-4i} = \frac{3-4i}{9+16}$$

$$= \frac{3}{25} - \frac{4}{25}i$$

$$\Rightarrow \ \bar{z} = \frac{3}{25} + \frac{4}{25}i$$

Q. 17. If z = (1 – i), find z^{-1}.

Answer : We have, z = (1 – i)

$\Rightarrow \bar{z} = 1 + i$

$\Rightarrow |z|^2 = (1)^2 + (-1)^2 = 2$

∴ The multiplicative inverse of (1 – i),

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{1+i}{2}$$

$$z^{-1} = \frac{1}{2} + \frac{1}{2}i$$

Q. 18. If z = $\left(\sqrt{5} + 3i\right)$, find z^{-1}.

Answer : We have, $z = (\sqrt{5} + 3\,i)$

$\Rightarrow \bar{z} = (\sqrt{5} - 3\,i)$

$\Rightarrow |z|^2 = \left(\sqrt{5}\right)^2 + (3)^2$

$= 5 + 9 = 14$

∴ The multiplicative inverse of $(\sqrt{5} + 3\,i)$,

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3\,i}{14}$$

$$z^{-1} = \frac{\sqrt{5}}{14} + \frac{3}{14}i$$

Q. 19. Prove that arg (z) + arg $\left(\bar{z}\right)$ = 0

Answer : Let z = r(cosθ + i sinθ)

⇒ arg(z) = θ

Now, $\overline{z} = r(\cos\theta - i\sin\theta) = r(\cos(-\theta) + i\sin(-\theta))$

$\Rightarrow \arg(\overline{z}) = -\theta$

Thus, arg (z) + $\arg(\overline{z})$ = θ - θ = 0

Hence proved.

Q. 20. If |z| = 6 and arg (z) = $\frac{3\pi}{4}$, find z.

Answer : We have, |z| = 6 and arg (z) = $\frac{3\pi}{4}$

Let z = r(cosθ + i sinθ)

We know that, |z| = r = 6

And arg (z) = θ = $\frac{3\pi}{4}$

Thus, z = r(cosθ + i sinθ) = $6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

Q. 21. Find the principal argument of (–2i).

Answer : Let, z = -2i

Let 0 = rcosθ and -2 = rsinθ

By squaring and adding, we get

$(0)^2 + (-2)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$\Rightarrow 0+4 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 4 = r^2$

$\Rightarrow r = 2$

∴ cosθ= 0 and sinθ=-1

Since, θ lies in fourth quadrant, we have

$$\theta = -\frac{\pi}{2}$$

Since, $\theta \in (-\pi, \pi]$ it is principal argument.

Q. 22. Write the principal argument of $(1 + i\sqrt{3})^2$.

Answer : Let, $z = \left(1 + i\sqrt{3}\right)^2$

$$= (1)^2 + \left(i\sqrt{3}\right)^2 + 2\sqrt{3}i$$

$$= 1 - 1 + 2\sqrt{3}i$$

$$z = 0 + 2\sqrt{3}i$$

Let $0 = r\cos\theta$ and $2\sqrt{3} = r\sin\theta$

By squaring and adding, we get

$(0)^2 + (2\sqrt{3})^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$\Rightarrow 0+(2\sqrt{3})^2 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow (2\sqrt{3})^2 = r^2$

$\Rightarrow r = 2\sqrt{3}$

$\therefore \cos\theta = 0$ and $\sin\theta = 1$

Since, θ lies in first quadrant, we have

$$\theta = \frac{\pi}{2}$$

Since, $\theta \in (-\pi, \pi]$ it is principal argument.

Q. 23. Write –9 in polar form.

Answer : We have, $z = -9$

Let -9 = rcosθ and 0 = rsinθ

By squaring and adding, we get

$(-9)^2 + (0)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$\Rightarrow 81 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 81 = r^2$

$\Rightarrow r = 9$

$\therefore \cos\theta = -1 \text{ and } \sin\theta = 0$

$\Rightarrow \theta = \pi$

Thus, the required polar form is 9(cos π+i sin π)

Q. 24. Write 2i in polar form.

Answer : Let, z = 2i

Let 0 = rcosθ and 2 = rsinθ

By squaring and adding, we get

$(0)^2 + (2)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$\Rightarrow 0+4 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 4 = r^2$

$\Rightarrow r = 2$

∴ cosθ= 0 and sinθ=1

Since, θ lies in first quadrant, we have

$\theta = \frac{\pi}{2}$

Thus, the required polar form is $2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

Q. 25. Write –3i in polar form.

Answer : Let, $z = -3i$

Let $0 = r\cos\theta$ and $-3 = r\sin\theta$

By squaring and adding, we get

$(0)^2 + (-3)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$\Rightarrow 0+9 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 9 = r^2$

$\Rightarrow r = 3$

$\therefore \cos\theta = 0$ and $\sin\theta = -1$

Since, θ lies in fourth quadrant, we have

$$\theta = \frac{3\pi}{2}$$

Thus, the required polar form is $3\left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right)$

Q. 26. Write z = (1 – i) in polar form.

Answer : We have, $z = (1 - i)$

Let $1 = r\cos\theta$ and $-1 = r\sin\theta$

By squaring and adding, we get

$(1)^2 + (-1)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$\Rightarrow 1+1 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 2 = r^2$

$\Rightarrow r = \sqrt{2}$

$$\therefore \cos\theta = \frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{-1}{\sqrt{2}}$$

Since, θ lies in fourth quadrant, we have

$\theta = -\frac{\pi}{4}$

Thus, the required polar form is $\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

Q. 27. Write z = (–1 + i$\sqrt{3}$) in polar form.

Answer : We have, z = (–1 + i$\sqrt{3}$)

Let -1 = rcosθ and √3 = rsinθ

By squaring and adding, we get

$(-1)^2 + (\sqrt{3})^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$\Rightarrow 1+3 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 4 = r^2$

$\Rightarrow r = 2$

$\therefore \cos\theta = \frac{-1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$

Since, θ lies in second quadrant, we have

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Thus, the required polar form is $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

Q. 28. If |z| = 2 and arg (z) = $\frac{\pi}{4}$, find z.

Answer : We have, |z| = 2 and arg (z) = $\frac{\pi}{4}$,

Let z = r(cosθ + i sinθ)

We know that, $|z| = r = 2$

And $\arg(z) = \theta = \frac{\pi}{4}$

Thus, $z = r(\cos\theta + i \sin\theta) = 2\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$