14. Introduction to graphs

Exercise 14.1

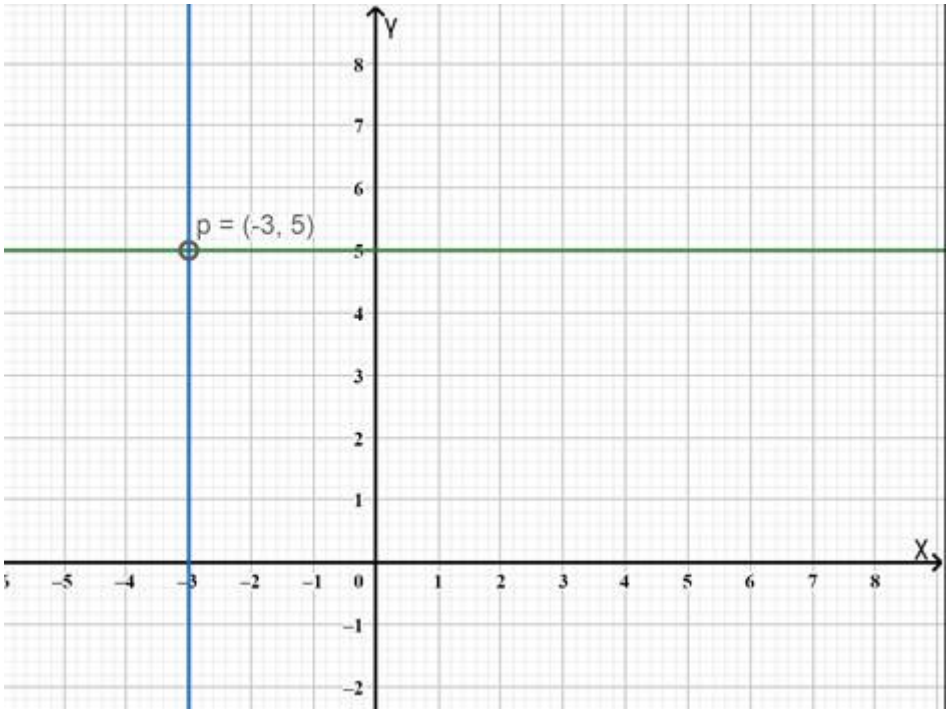
1 A. Question

Fix up your own coordinate system on a graph paper and locate the following points on the sheet:

P (−3,5)

Answer

P (−3,5)



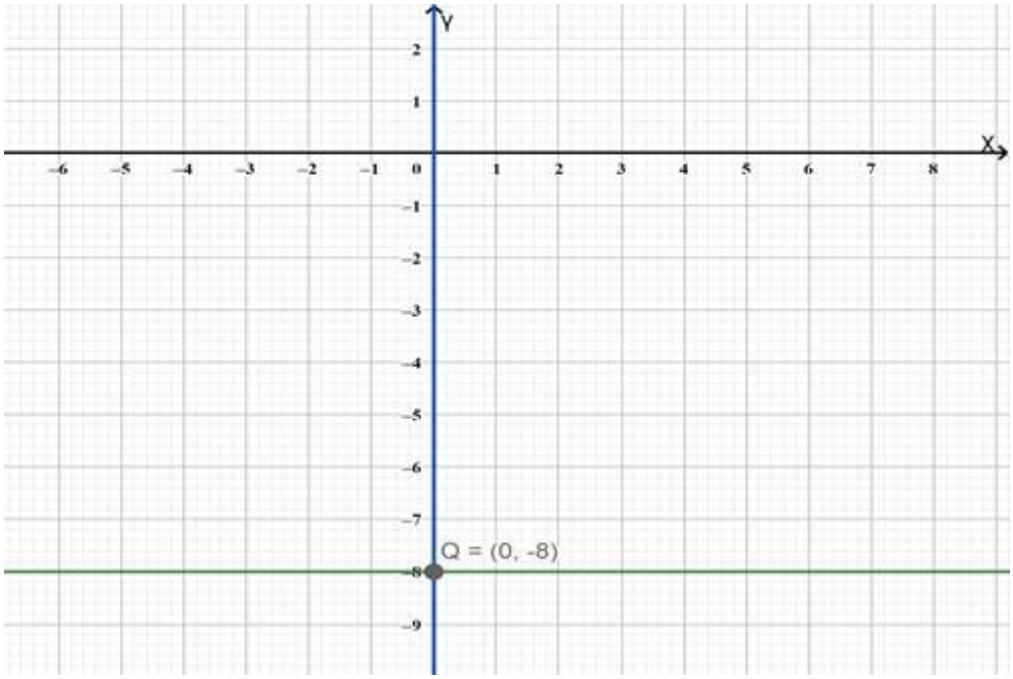
1 B. Question

Fix up your own coordinate system on a graph paper and locate the following points on the sheet:

Q (0,−8)

Answer

Q (0,−8)



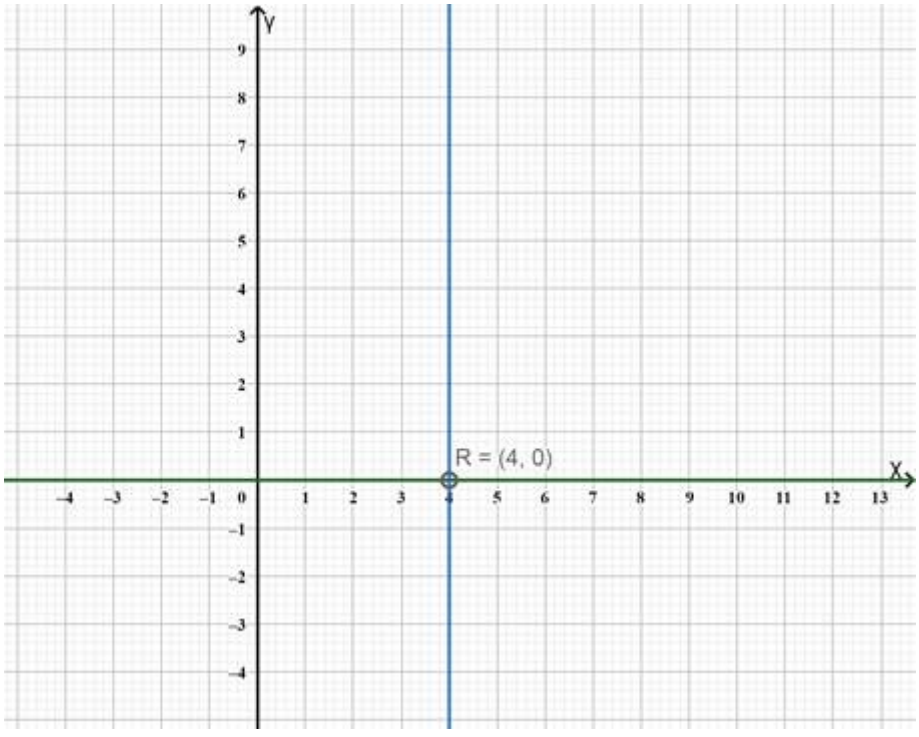
1 C. Question

Fix up your own coordinate system on a graph paper and locate the following points on the sheet:

R (4,0)

Answer

R (4,0)



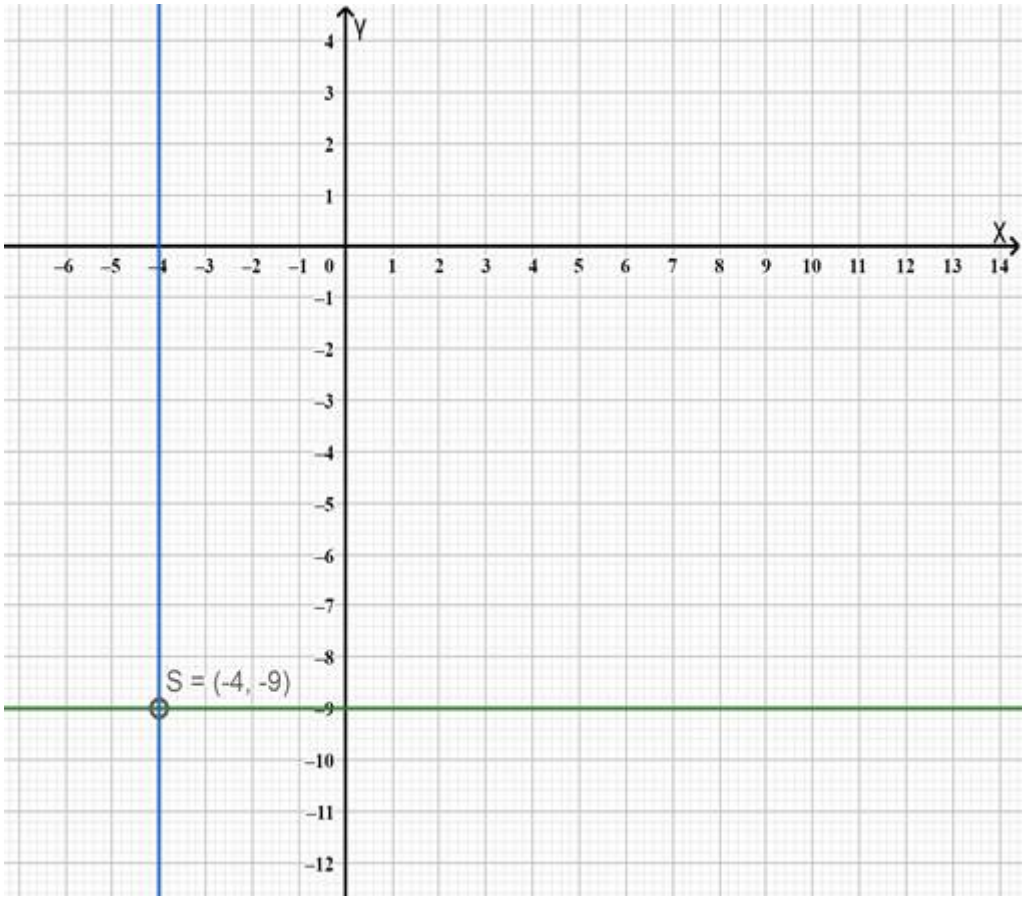
1 D. Question

Fix up your own coordinate system on a graph paper and locate the following points on the sheet:

S (−4,−9).

Answer

S (−4,−9)



2. Question

Suppose you are given a coordinate system. Determine the quadrant in which the following points lie:

(i) A (4,5) (ii) B (−4,−5) (iii) C (4,−5).

Answer

A. Since both X coordinate and Y coordinate are Positive

⇒ (+,+) which is in the first coordinate

B. Since both X coordinate and Y coordinate are Negative

⇒ (-, -) which is in the third coordinate

C. Since both X coordinate is POSITIVE and Y coordinate is Negative

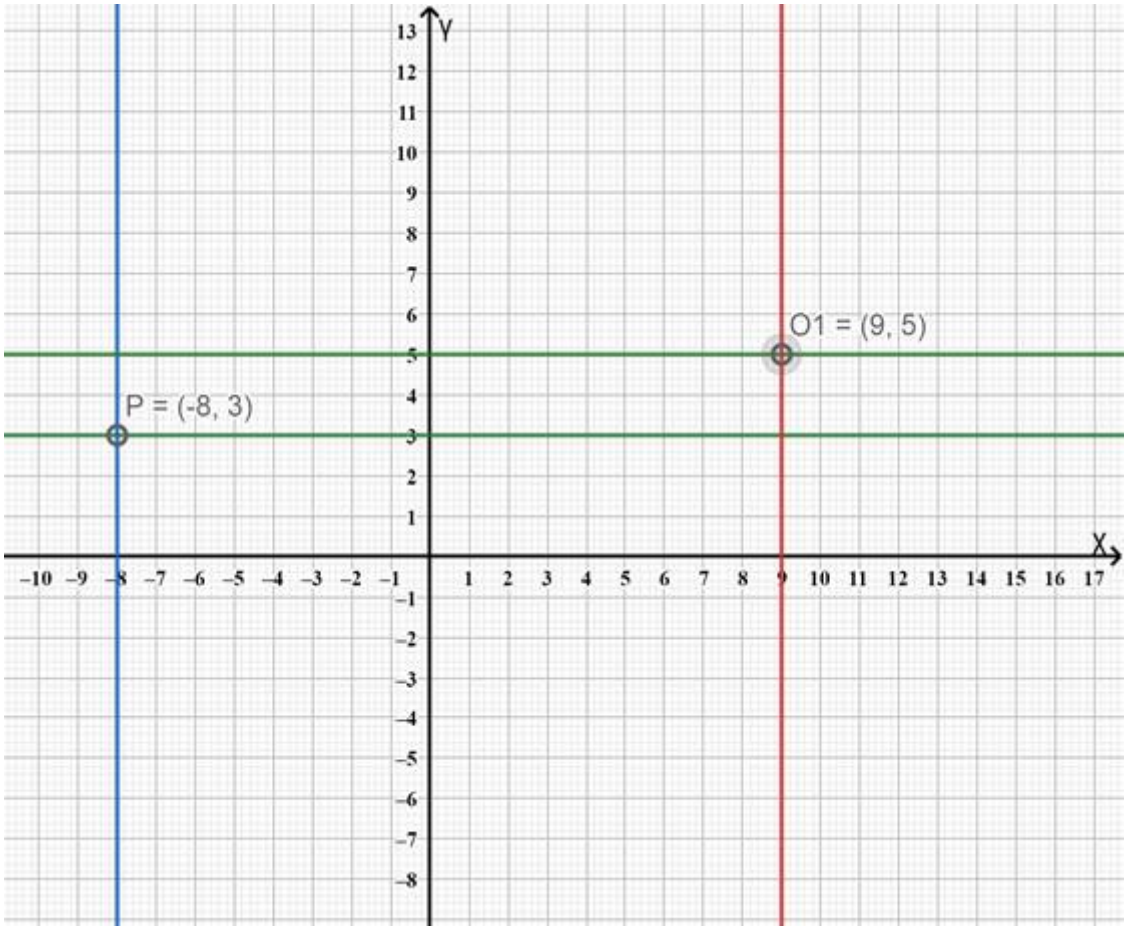
⇒ (+, -) which is in fourth coordinate

3. Question

Suppose P is a point with coordinates (−8,3) with respect to a coordinate system X'OX ↔ Y'OY. Let $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ be another system with X'OX

$\| X_1'O_1X_1$ and suppose O_1 has coordinates (9,5) with respect to X ’OX ↔ Y ’OY. What are the coordinates of P in the system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$?

Answer



In the figure, we have two points marked, P = (-8, 3) and O_1 = (9, 5)

According to question O_1 is the origin for a different coordinate system, $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$.

For coordinates of P in the new system, we have to find the distance between $O_1$and point P, **measured FROM P.**

By measuring distance on the graph above, we get

X_1= -17 and Y_1= -2

⇒ So, the coordinates of Point P in new system is (-17, -2)

OR,

Given, x= -8, y= 3

and a= 9, b=5 (Coordinates of O_1)

As we know, x= a + x’

-8 = 9+ x’

x’ = -8-9

x’ = -17

similarly,

y= b + y_1

3 = 5+ y_1

y_1 = 3-5

y_1 = -2

New coordinate, (x_1, y_1) are (-17,-2)

4. Question

Suppose P has coordinates (10,2) in a coordinate system X ’OX–Y ’OY and (–3,–6) in another coordinate system X_1’O_1X_1↔ Y_1’O_1Y_1? with X ’OX ‖ X_1’O_1X_1. Determine the coordinates of O with respect to the system X_1’O_1X_1↔ Y_1’O_1Y_1.

Answer

Given coordinate of P(x, y) = (10,2)

Coordinates of P with respect = (-3,-6)

toX_1’O_1X_1↔ Y_1’O_1Y_1(x’, y’)

to find, a & b

As we know, x = a + x’

10 = a - 3

a = 10+3

a = 13

similarly,

y = b + y’

2 = b - 6

b = 2+6

b = 8

So, Coordinates of O_1 = (13, 8)

⇒ Coordinates of O With respect to O_1 = (-13, -8)

Exercise 14.2

1 A. Question

Draw the graphs of the following straight-lines:

y = 3 - x

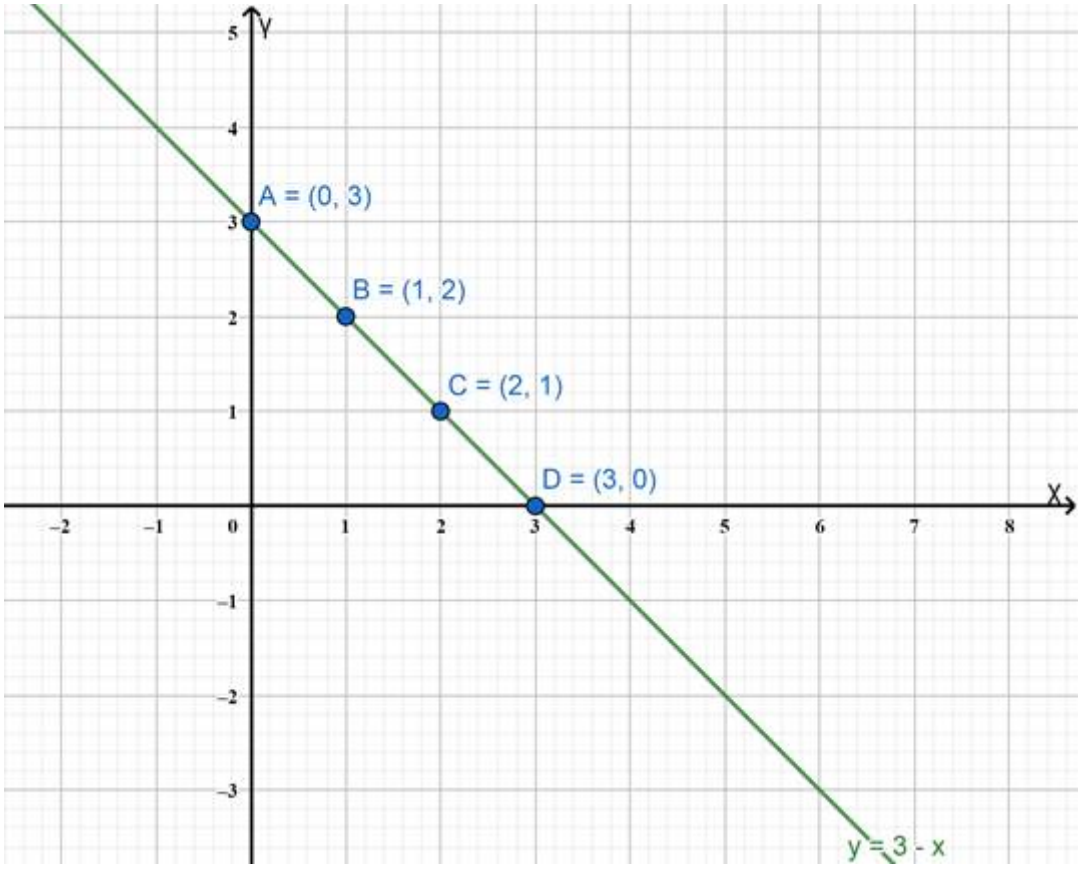
Answer

Y = 3 - x

For different values of X and Y

x	0	1	2	3
y	3	2	1	0

Plotting above values in graph, we get



1 B. Question

Draw the graphs of the following straight-lines:

y = x - 3

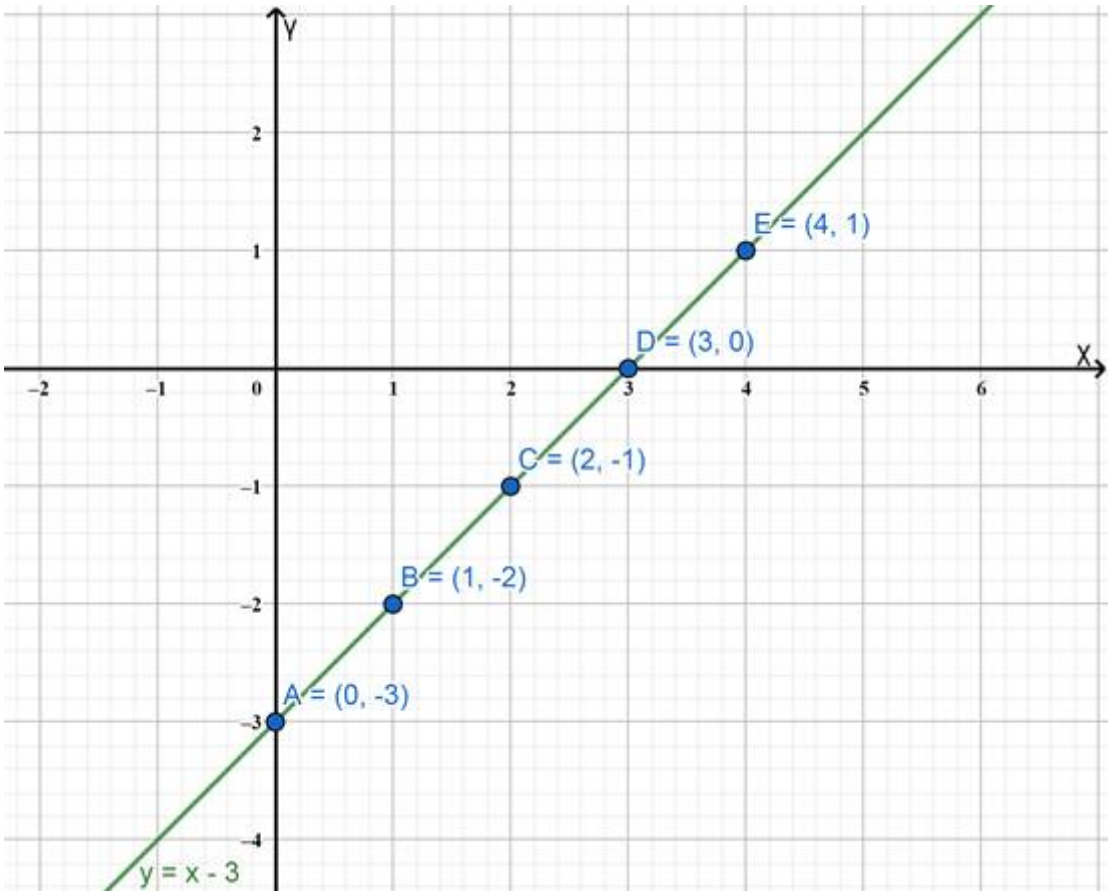
Answer

y = x - 3

For different values of X and Y

x	0	1	2	3	4
y	-3	-2	-1	0	1

Plotting above values in graph, we get



1 C. Question

Draw the graphs of the following straight-lines:

y = 3x - 2

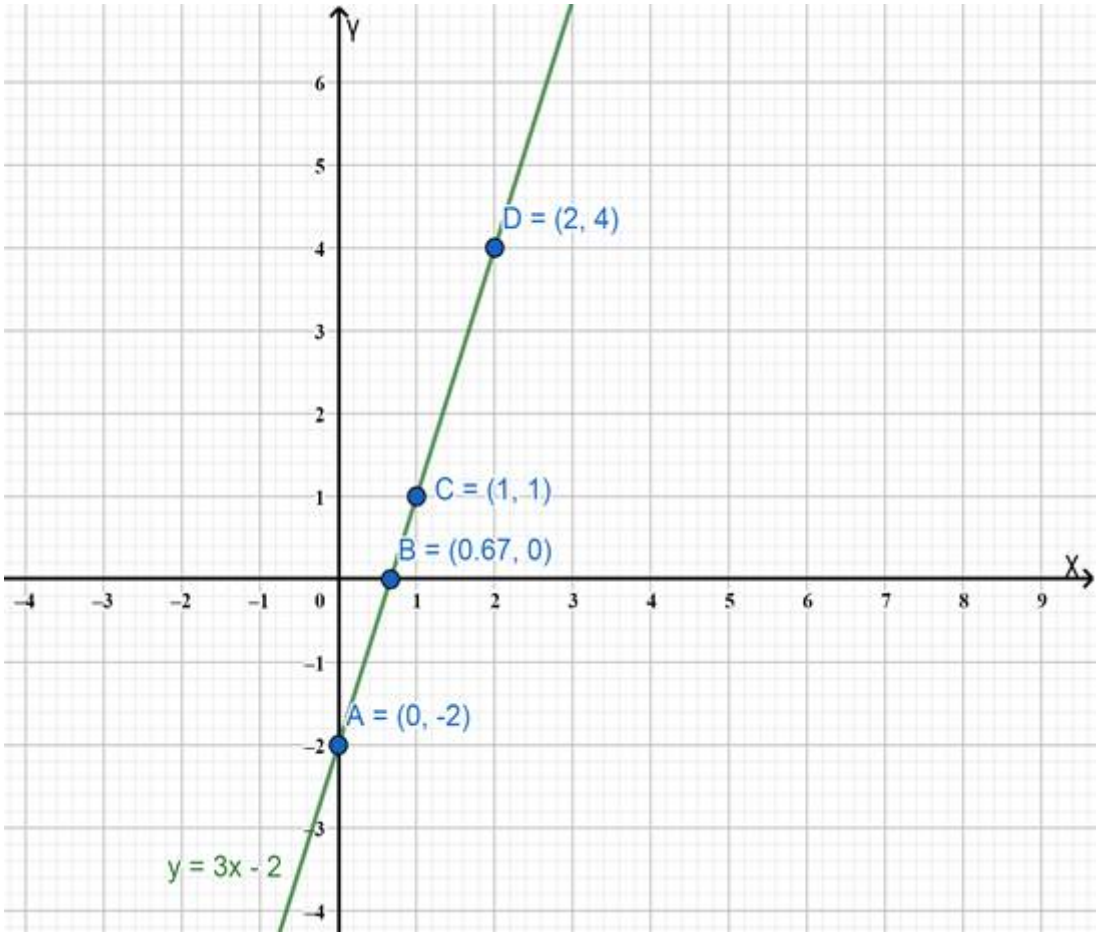
Answer

y = 3x - 2

For different values of X and Y

x	0	$\frac{2}{3}$	1	2
y	-2	0	1	4

Plotting above values in the graph, we get



1 D. Question

Draw the graphs of the following straight-lines:

y = 5 - 3x

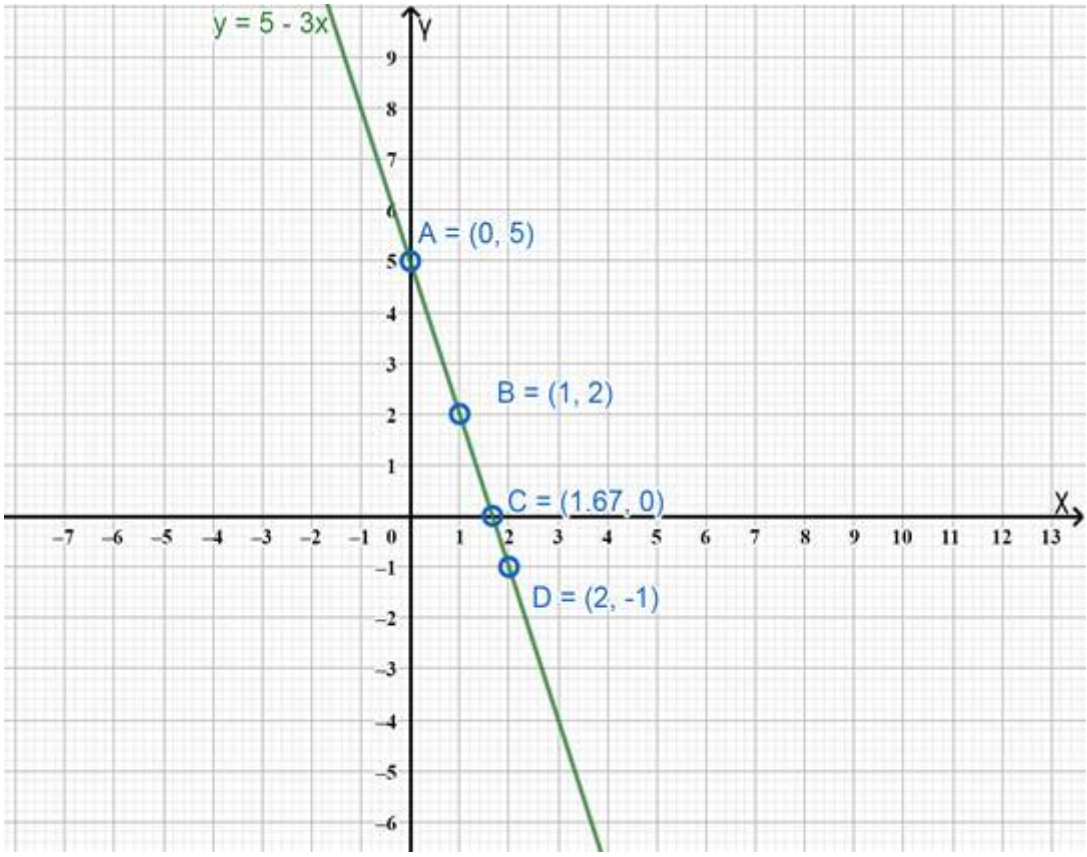
Answer

y = 5 - 3x

For different values of X and Y

x	0	1	$\frac{5}{3}$	2
y	5	2	0	-1

Plotting above values in the graph, we get



1 E. Question

Draw the graphs of the following straight-lines:

4y = −x + 3

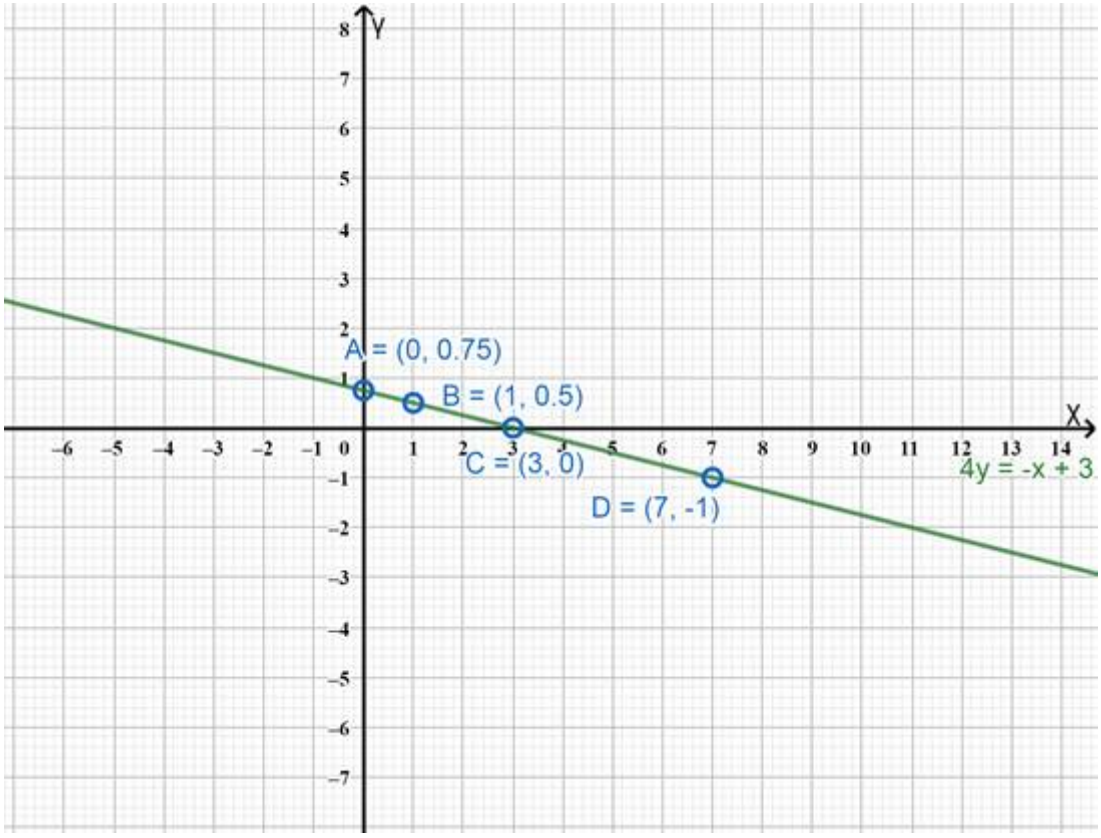
Answer

4y = −x + 3

For different values of X and Y

X	0	1	3	7
Y	$\frac{3}{4}$	$\frac{1}{2}$	0	-1

Plotting above values in graph, we get



1 F. Question

Draw the graphs of the following straight-lines:

3y = 4x + 1

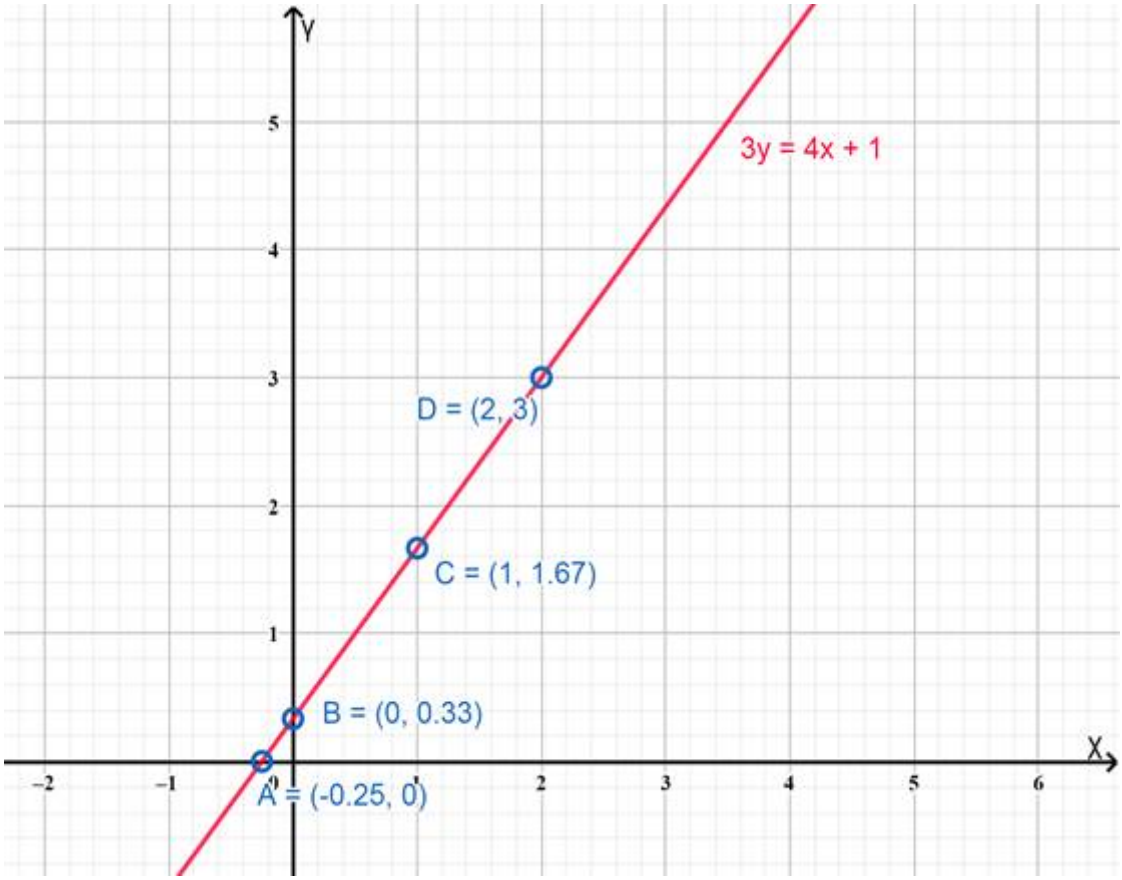
Answer

3y = 4x + 1

For different values of X and Y

X	$\frac{-1}{4}$	0	1	2
Y	0	$\frac{1}{3}$	$\frac{5}{3}$	3

Plotting above values in graph, we get



1 G. Question

Draw the graphs of the following straight-lines:

x = 4

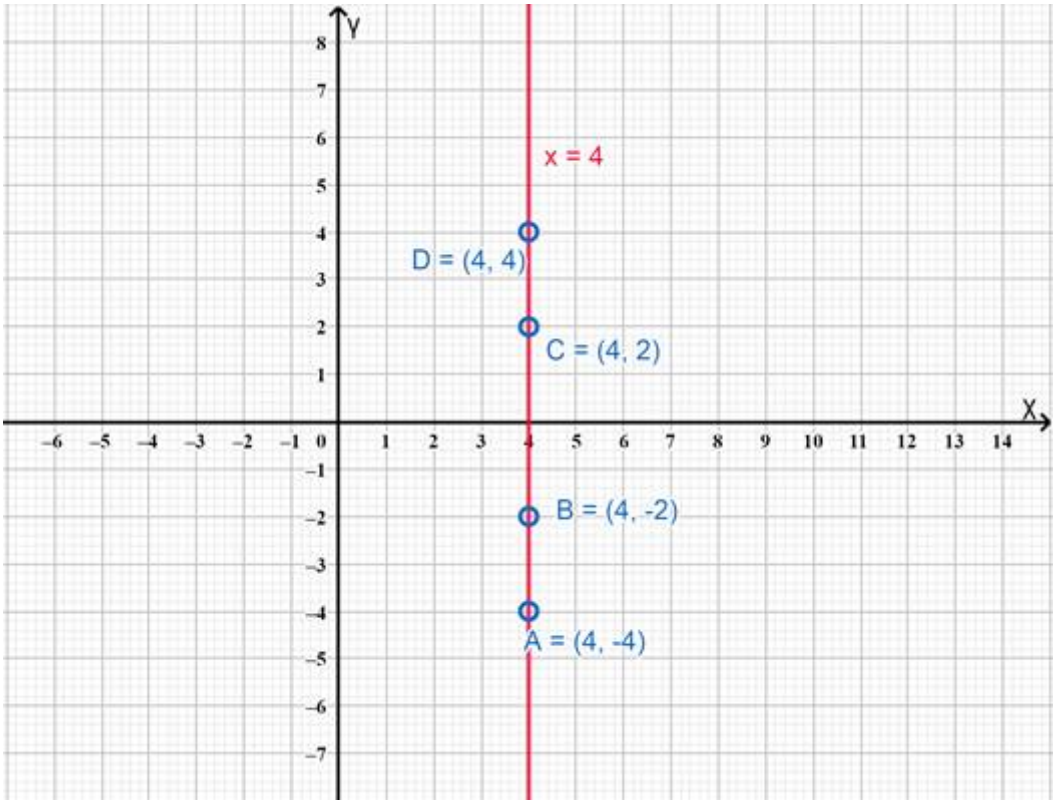
Answer

x = 4

For different values of X and Y

x	4	4	4	4
y	-4	-2	2	5

Plotting above values in graph, we get



General equation for line is ay = bx +c

Here the line is, **0**y=**1**x **-4**

Since 'y' is multiplied by zero every time, the value of 'y' does not affect the value of x or the equation.

⇒ For every value of y, the value of x will be constant, i.e. 4.

1 H. Question

Draw the graphs of the following straight-lines:

3y = 1.

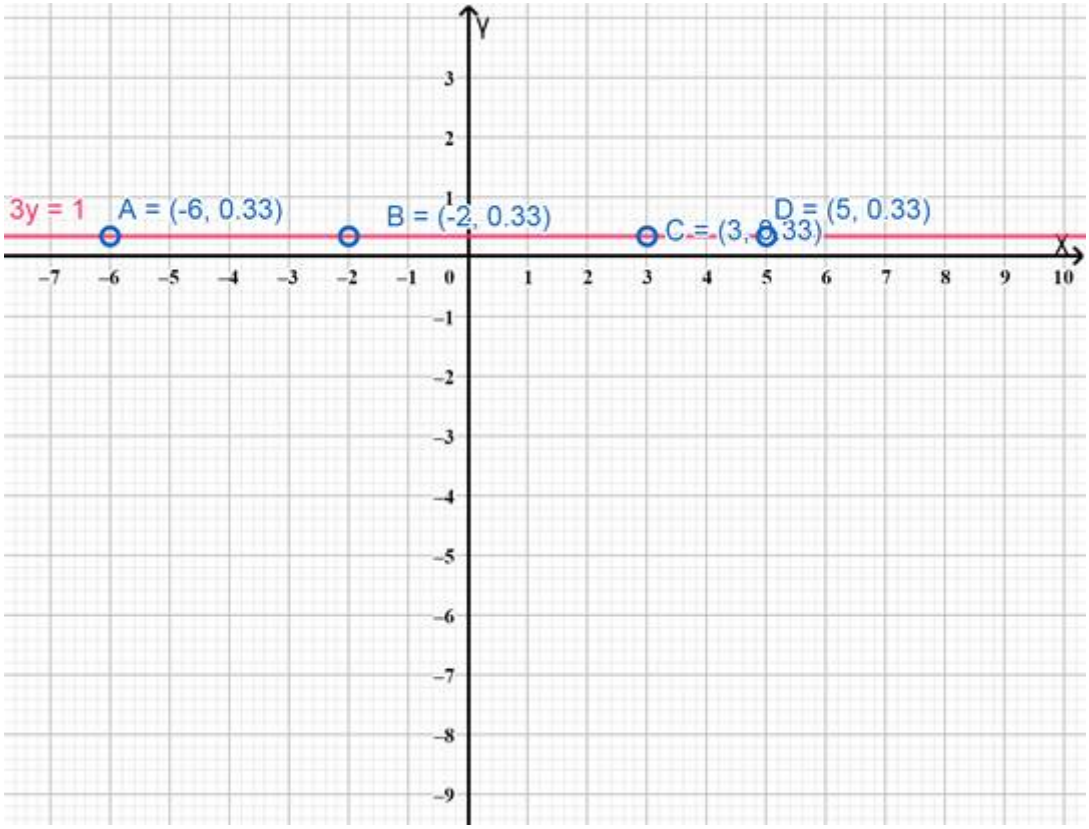
Answer

3y =1.

For different values of X and Y

x	-6	-2	3	5
y	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Plotting above values in the graph, we get



General equation for line is ay = bx +c

Here the line is, **3**y=**0**x+ **1**

Since 'x' is multiplied by zero every time, the value of 'x' does not affect the value of y or the equation.

⇒ For every value of x, the value of y will be constant, i.e. $\frac{1}{3}$

2. Question

Draw the graph of y/2 = y + 1/x + 2

Answer

The given equation is $\frac{y}{2} = \frac{y+1}{x+2}$

Simplifying the equation, $y(x + 2) = 2(y + 1)$

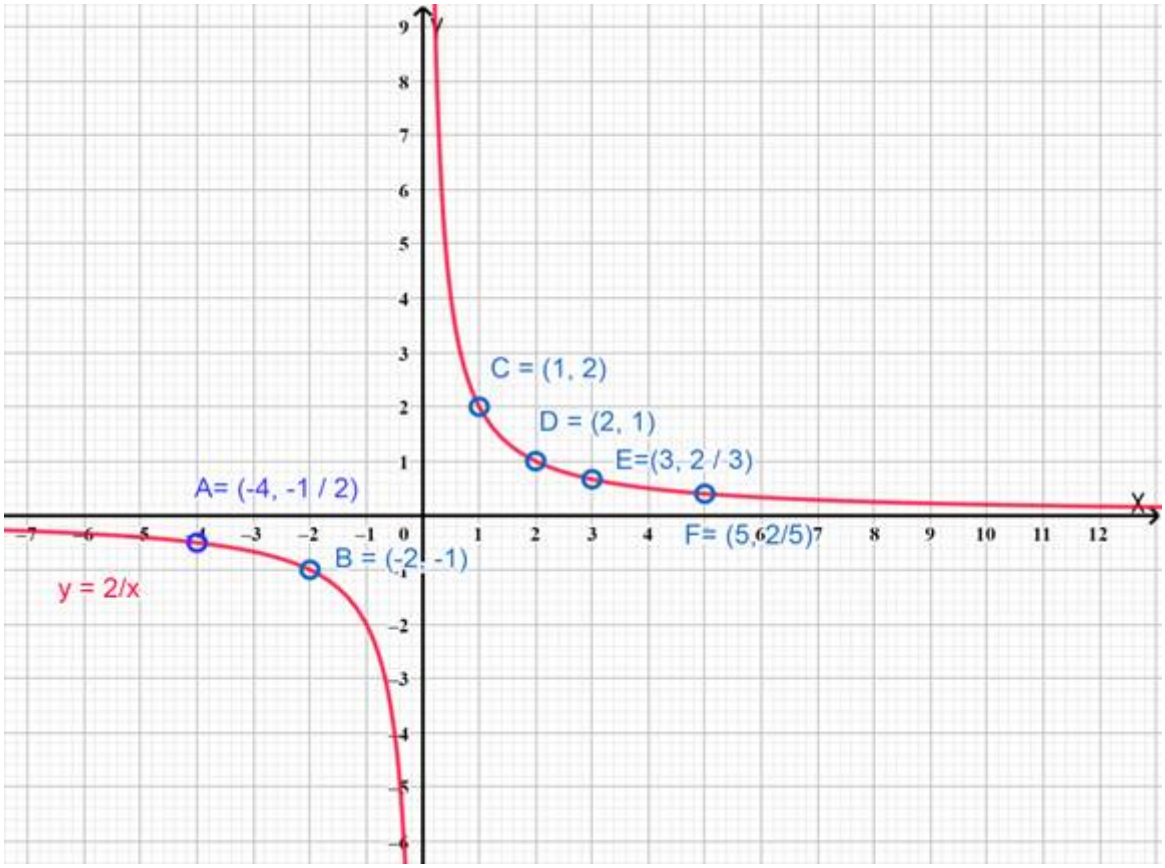
$\Rightarrow xy + 2y = 2y + 2$

$\Rightarrow xy = 2$

For different values of X and Y

x	-4	-2	1	2	3	5	0	∞
y	$\frac{-1}{2}$	-1	2	1	$\frac{2}{3}$	$\frac{2}{5}$	∞	0

Plotting above values in the graph, we get

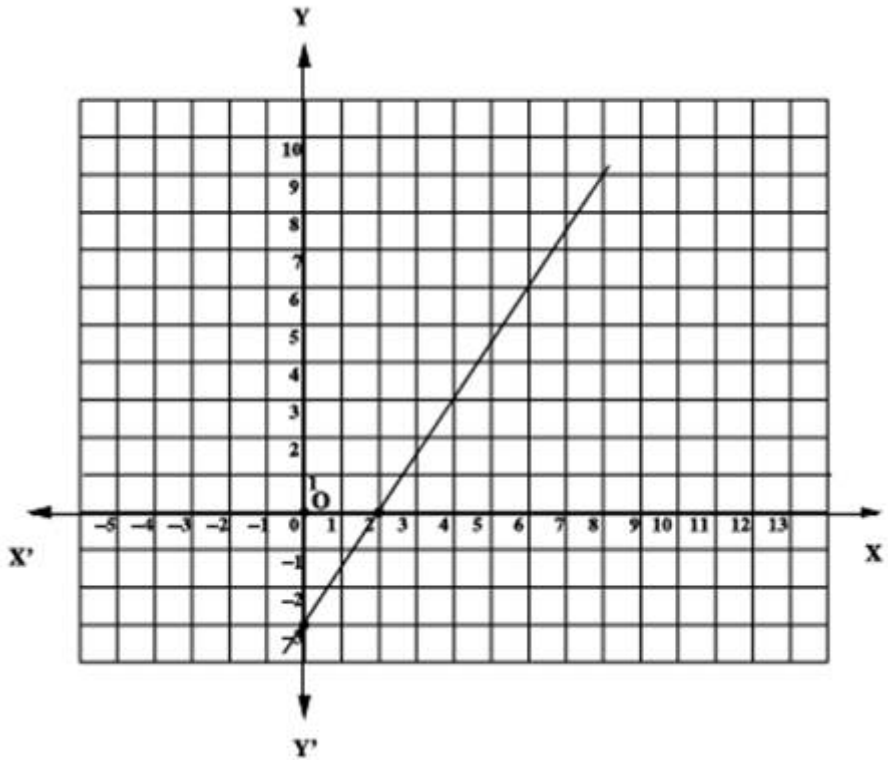


Do note that point G (0, ∞) and H (∞, 0) does not lie on the graph. As the Graph is Dis-Continuous, that means graph breaks at point G and H.

The above-mentioned graph is called HYPERBOLA, it has a general equation xy = c.

3 A. Question

Determine the equation of the line in each of the following graphs:



Answer

The general equation of the line is y = mx + c, where m and c are constants.

From the graph, it is visible that the line passes through the point (2, 0) and (0,-3).

Substituting both the values in general equation, y = mx + c, we get

0 = m(2) + c ..(i)

-3 = m(0) + c ..(ii)

From equation (ii), we get c = -3,

Putting c = -3 in equation (i),

2m + (-3) = 0

$$\Rightarrow m = \frac{3}{2}$$

Putting both m and c in general equation y=mx+c,

$$\Rightarrow y = \frac{3}{2}x - 3$$

$$\Rightarrow y + 3 = \frac{3}{2}x$$

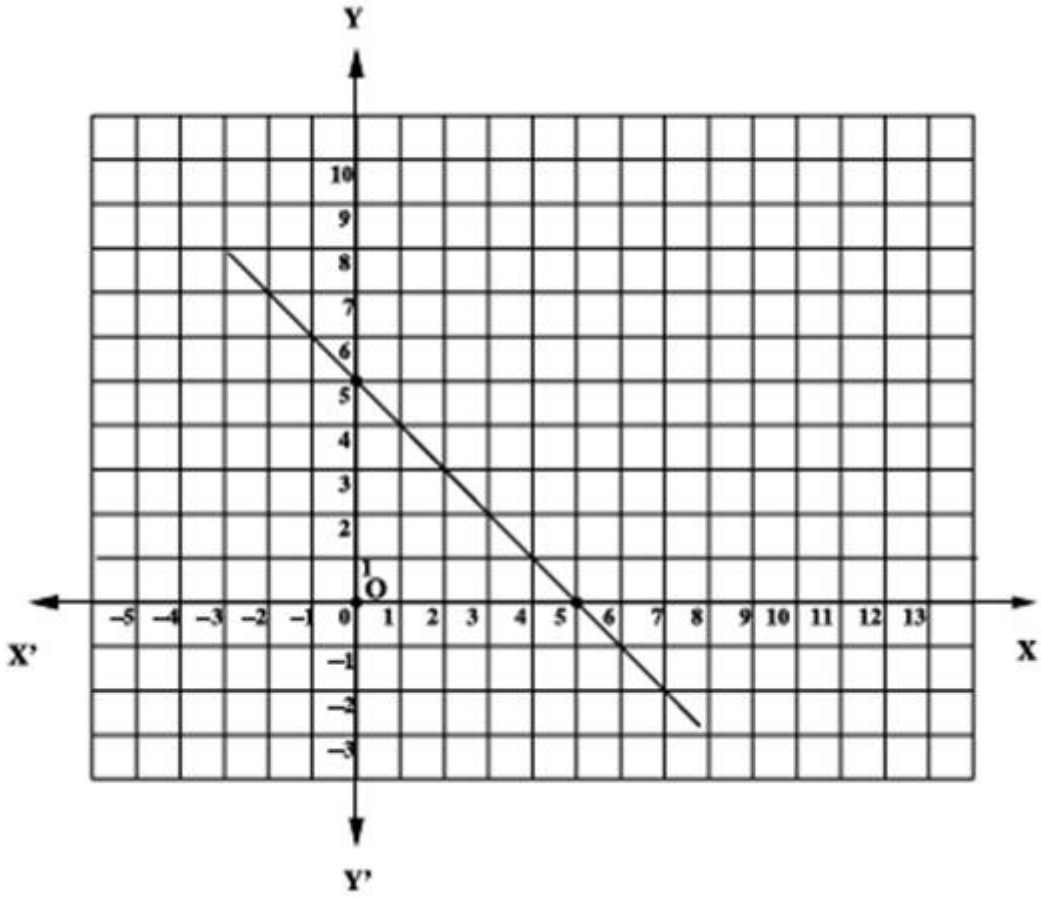
$$\Rightarrow 2y + 6 = 3x$$

$$\Rightarrow 3x - 2y = 6$$

So equation of line is 3x-2y = 6

3 B. Question

Determine the equation of the line in each of the following graphs:



Answer

The general equation of the line is y=mx+c, where m and c are constants.

From the graph, it is visible that the line passes through the point (5, 0) and (0,5).

Substituting both the values in general equation, y=mx + c, we get

0 = m(5) + c ..(i)

5 = m(0) + c ..(ii)

From equation (ii), we get c = 5,

Putting c = 5 in equation (i),

5m + (5) = 0

$\Rightarrow m = -1$

Putting both m and c in general equation y = mx + c,

$\Rightarrow y = -x + 5$

$\Rightarrow y + x = 5$

$\Rightarrow x + y = 5$

So equation of line is x + y = 5

4. Question

A boat is moving in a river, downstream, whose stream has speed 8 km per hour. The speed of the motor of the boat is 22 km per hour. Draw the graph of the distance covered by the boat versus an hour.

Answer

Given, Speed of boat = 22 km/h

Speed of stream = 8 km/h

Movement of boat downstream means that the boat is moving with the flow of the river and thus the total speed of boat will be the sum of the speed of boat and speed of the river. Thus,

Net speed = 22 + 8

= 30 km/h

$$\text{We know that, Speed} = \frac{\text{Distance}}{\text{Time}}$$

⇒ Distance = Speed × time

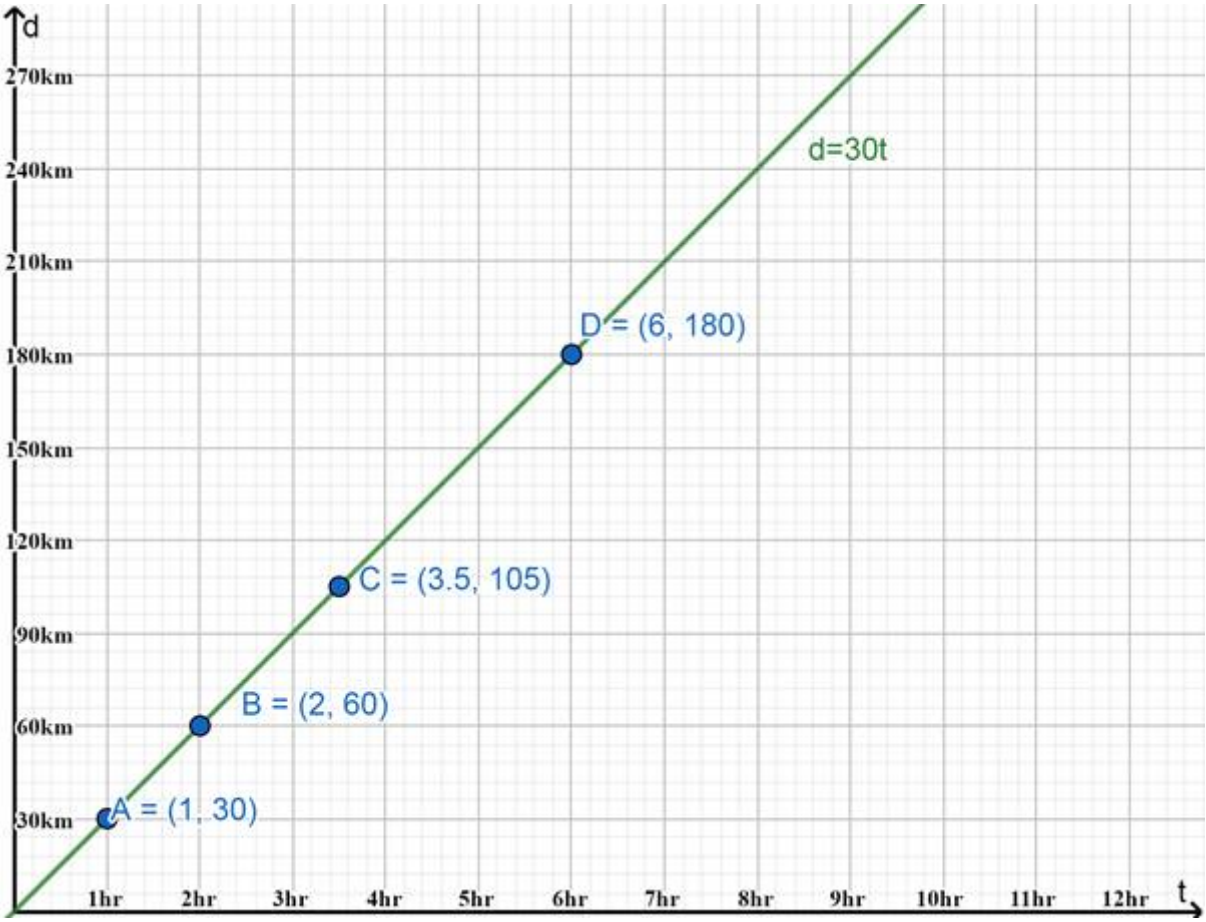
Let distance be ‘d’ and time be ‘t’

⇒ d = (30)t

For different values of t and d

t	1	2	3.5	6
d	30	60	105	180

Plotting above values in the graph, we get



5. Question

Find the point of intersection of the straight lines 3y+4x=7 and 4y + 3x = 7, by drawing their graphs and looking for the point where they meet.

Answer

For equation 3y + 4x = 7

For different values of x and y

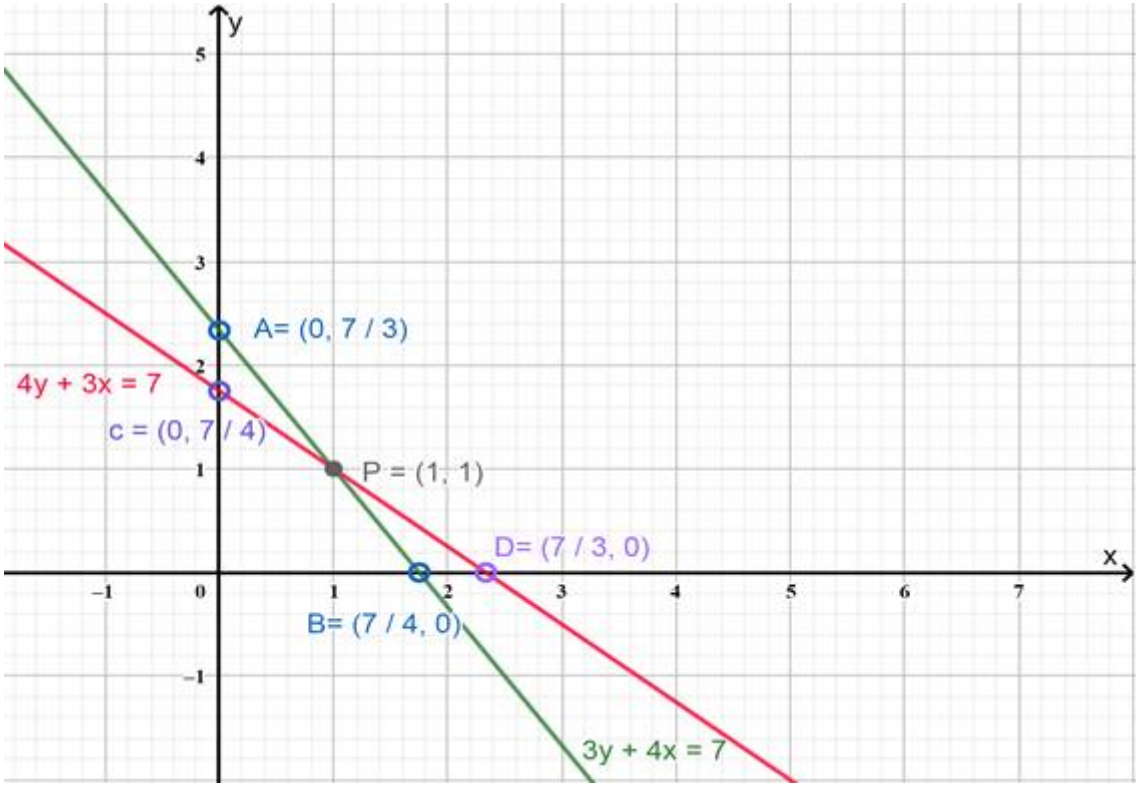
x	0	$\frac{7}{4}$
y	$\frac{7}{3}$	0

For equation 4y + 3x = 7

For different values of x and y

x	0	$\frac{7}{3}$
y	$\frac{7}{4}$	0

Plotting for both equations in same graph,



From graph it is visible that intersection point is P (1,1).

→ **NOTE:** You can verify intersection point by substituting it in both equations, and it should satisfy both equations.

Additional Problems 14

1 A. Question

The point (4,0) lie on the line _____

A. y− x = 0

B. y =0

C. x = 0

D. y+ x =0

Answer

Only when we put the point in equation y=0, it satisfies.

1 B. Question

The point (−5,4) lie in _____

A. the first quadrant

B. the second quadrant

C. the third quadrant

D. the fourth quadrant

Answer

Since here the x coordinate is negative and y coordinate is positive, it implies that the point lies in the second quadrant.

1 C. Question

If a straight-line pass through (0,0) and (1,5), then its equation is _____

A. y = x

B. y =5x

C. 5y = x

D. y = x +5

Answer

Since, Line passes through origin(0,0), this means equation will be of form: y=mx.

And,

$$m = \frac{y2 - y1}{x2 - x1}$$

$$\Rightarrow m = \frac{5 - 0}{1 - 0}$$

⇒ m = 5

⇒ Equation: y = 5x

1 D. Question

If a point P has coordinates (3,4) in a coordinate system $X'OX \leftrightarrow Y'OY$, and if O has coordinates (4,3) in another system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ with $X'OX \parallel X'O_1X_1$, then the coordinates of P in the new system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ is _____

A. (3,4)

B. (1,−1)

C. (7,7)

D. (−1,1)

Answer

If a point P has coordinates (x1,y1) in a coordinate system $X'OX \leftrightarrow Y'OY$, and if O has coordinates (a,b) in another system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ with $X'OX \parallel$

$X'O_1X_1$, then the coordinates of P say (x, y) in the new system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ will be x = x1 + a and y = y1 + a

⇒ x = 3 + 4 and y = 4 + 3

⇒ x = 7 and y = 7

1 E. Question

The coordinates of a point P in a system X ′OX ↔ Y ′OY are (5,8). The coordinates of the same point in the system Y ′OY ↔ XOX′ are _____

A. (−8,5)

B. (8,5)

C. (8,−5)

D. (−8,−5)

Answer

If the coordinates of a point P in a system X ′OX ↔ Y ′OY are (x,y). The coordinates of the same point in the system Y ′OY ↔ XOX′ will be (y,-x)

⇒ co-ordinates will be (8, -5).

1 F. Question

The signs of the coordinates of a point in the third quadrant are _____

A. (+,−)

B. (−,+)

C. (+,+)

D. (−,−)

Answer

In the third quadrant both x and y coordinates are negative.

1 G. Question

If a person moves either 1 unit in the direction of positive x-axis or 1 unit in the direction of positive y-axis per step, then the number of steps he requires to reach (10,12) starting from the origin (0,0) is _____

A. 10

B. 12

C. 22

D. 120

Answer

Firstly, to reach (0,10) it will take 10 steps and afterward 12 steps will be taken to reach (10,12).

1 H. Question

The y-coordinate of the point of intersection of the line y =3x + 4 with x =3 is _____

A. 4

B. 7

C. 10

D. 13

Answer

Putting x=3 in y=3x+4,

we get,

$y=3 \times 3 + 4$

$\Rightarrow y = 13$

1 I. Question

The equation of the line which passes through (0,0) and (1,1) is _____

A. $y = x$

B. $y = -x$

C. $y = 1$

D. $x = 1$

Answer

Only when we put the points in equation y=x, it satisfies.

2. Question

Find the quadrant in which the following points lie:

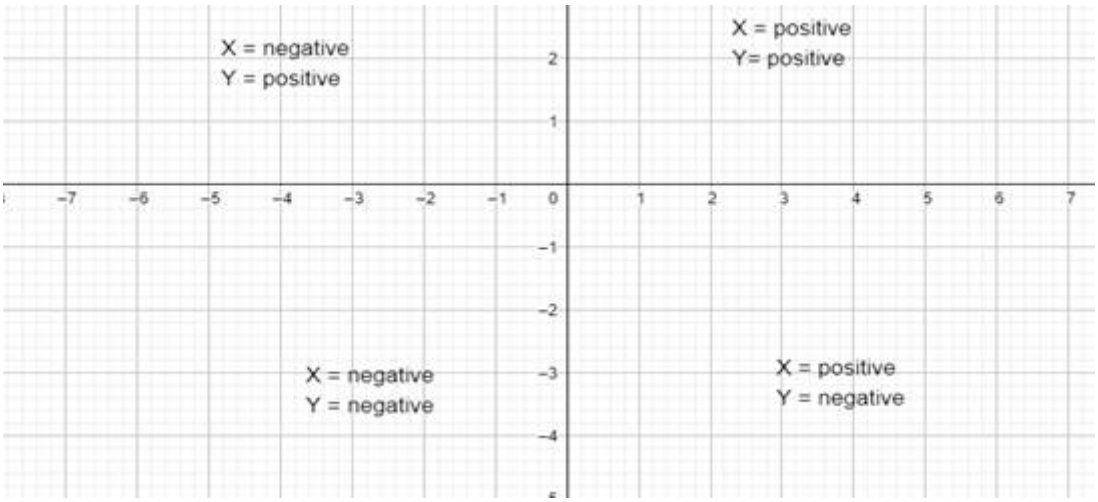
(i) (5, 10);

(ii) (−8, 9);

(iii) (−800, −3000);

(iv) (8, −100).

Answer



(i) Since, here both x and y coordinate is positive implies that the point (5, 10) lies in the first quadrant.

(ii) Since, here x coordinate is negative and y coordinate is positive implies that the point (-8, 9) lies in the second quadrant.

(iii) Since, here both x and y coordinate are negative implies that the point (-800, -3000) lies in the third quadrant.

(iv) Since, here x coordinate is positive and y coordinate isneaitive implies that the point (8, -100) lies in first quadrant.

3. Question

Match the following:

(A) On the x-axis	(i) x coordinate is negative
(B) In the second quadrant	(ii) cuts the y-axis at (0,4)
(C) The line y = 3x + 4	(iii) coordinates of a point are of the form (a,0).

Answer

(A)→ (iii)

⇒ On the x-axis, coordinates of a point are of the form (a,0).

And, (B)→ (i)

⇒ In the second quadrant, x coordinate is negative.

And, (C)→ (ii)

⇒ The line y = 3x + 4, cuts the y-axis at (0,4), as it satisfies the equation too.

4. Question

Fill in the blanks:

(a) The y-coordinate of a point on the x-axis is _____.

b) The x-coordinate is called as _____.

(c) The x-axis and Y-axis intersect at _____.

(d) If a point (x,y) ≠ (0,0) is in the third quadrant, then x + y has _____sign.

(e) If a point (x,y) lies above horizontal axis, then y is always _____.

(f) The point of intersection of x = y and x = −y is _____.

(g) The line y =4x +5 intersects y-axis at the point _____.

Answer

(a) The y-coordinate of a point on the x-axis is **zero.**

Explanation:

On x – axis, the values of x keep on increasing and value of y remains zero as there is no movement in vertical direction.

(b) The x-coordinate is called as **abscissa**.

Explanation: The x coordinate of a coordinate system is called abscissa and y coordinate is called ordinate.

(c) The x-axis and Y-axis intersect at **origin (0,0).**

At origin values of x and y coordinates are 0 and thus they start increasing or decreasing in value from origin. Thus, they intersect at origin.

(d) If a point (x,y) ≠ (0,0) is in the third quadrant, then x + y has **–(negative)** sign.

Explanation: In the third quadrant, x and y both are negative, thus x – y = -x + (-y) = - (x + y)

And so its negative.

(e) If a point (x,y) lies above horizontal axis, then y is always **+(positive).**

(As, in first and second quadrant y is positive.)

(f) The point of intersection of x = y and x = −y is **origin(0,0)**.

(As, after solving both the equations simultaneously by using substitution method, we get- x=0 and y=0).

(g) The line y =4x +5 intersects y-axis at the point **(0, 5)**.

(Since, the point is at y-axis implies that x=0, and after substituting x=0 in equation, we get y=5.

5. Question

True or false?

(a) The equation of the x-axis is x =0.

(b) The line x =4 is parallel to Y-axis.

(c) The line y =8 is perpendicular to x-axis.

(d) The lines x = y and x = −y are perpendicular to each other.

(e) The lines x =9 and y =9 are perpendicular to each other.

(f) The graph of $y = x^2$ is a straight line.

(g) The line y =3x +4 does not intersect x-axis.

(h) In a rectangular coordinate system, the coordinate axes are chosen such that they form a pair of perpendicular lines.

Answer

(a) False

The equation of the x-axis is y =0.

(b) True

The y coordinate is 0 and x is constant, so value of y will remain same for this line, so it will be parallel to y axis.

(c) False

Since, x coordinate is 0 and y is constant implies that the line y=8 is parallel to x-axis.

(d) True

Since the multiplication of slope of both the equation is -1, implies that the lines are perpendicular to each other.

(e) True

Since, the one is parallel to horizontal axis and other one is parallel to the vertical axis implies that the both the lines are perpendicular as the axes are perpendicular too.

(f) False

Since, y will always be positive implies that the graph will be a V- shaped graph lying in first and second quadrant.

(g) False

Since, for y=0, $x = -\frac{4}{3}$ and this the point where line intersect x-axis.

(h) True

Since, property of rectangle states that all the angles of a rectangle are 90°.

6. Question

Determine the equation of the line which passes through the points (0,–8) and (7,0).

Answer

Let $(x_1, y_1) = (0, -8)$ and $(x_2, y_2)=(7, 0)$

Now the equation of the line passing through (x_1, y_1) and (x_2, y_2) is given by-

$$y - y1 = \frac{y2 - y1}{x2 - x1}(x - x1)$$

$$\Rightarrow\ y - (-8) = \frac{0 - (-8)}{7 - 0}(x - 0)$$

$$\Rightarrow\ y + 8 = \frac{8}{7}x$$

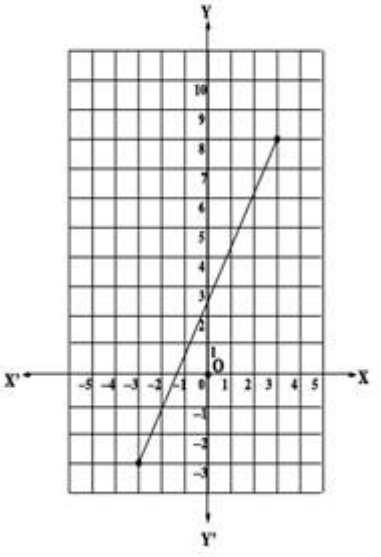
⇒ 7y+56=8x

⇒ 7y=8x-56

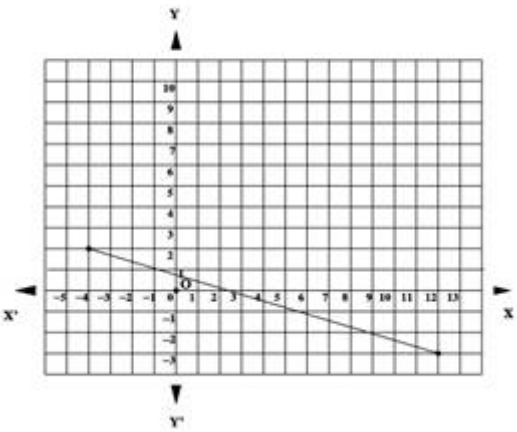
7. Question

Determine the equation of the line in each of the following graphs:

(i)



(ii)



Answer

(i) We can see in the graph that the graph passes through the points (3, 8) and (-3, -3)

Now, let (x1, y1) = (3, 8) and (x2, y2) = (-3,-3)

Now the equation of the line passing through (x1, y1) and (x2, y2) is given by-

$$y - y1 = \frac{y2 - y1}{x2 - x1}(x - x1)$$

$$\Rightarrow\ y - (8) = \frac{-3 - (8)}{-3 - 3}(x - 3)$$

$$\Rightarrow\ y - 8 = \frac{-11}{-6}(x - 3)$$

⇒ 6y-48 = 11x-33

⇒ 6y = 11x + 15

(ii) We can see in the graph that the graph passes through the points (-4, 2) and (12, -3)

Now, let (x1, y1)=(-4, 2) and (x2, y2)=(12,-3)

Now the equation of the line passing through (x1,y1) and (x2,y2) is given by-

$$y - y1 = \frac{y2 - y1}{x2 - x1}(x - x1)$$

$$\Rightarrow\ y - (2) = \frac{-3 - (2)}{12 - (-4)}(x - (-4))$$

$$\Rightarrow\ y - 2 = \frac{-5}{16}(x + 4)$$

⇒ 16y – 32 = - 5x - 20

⇒ 16y = - 5x + 12

8. Question

A point P has coordinates (7,10) in a coordinate system X′OX ↔ Y′OY. Suppose it has coordinates (10,7) in another coordinate system $X'_1O_1X_1$↔ $Y'_1O_1Y_1$ with X′OX || $X'_1O_1X_1$. Find the coordinates of O_1 in the system X′OX ↔ Y′OY.

Answer

(If a point P has coordinates (x1,y1) in a coordinate system X ′OX ↔ Y′OY, and if O has coordinates (a,b) in another system $X_1'O_1X_1$↔ $Y_1'O_1Y_1$ with X ′OX ||

$X'O_1X_1$, then the coordinates of P(say x,y) in the new system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ will be x = x1 + a and y = y1 + a)

Here, x = 7, y = 10, x1 = 10, and y1 = 7

⇒ a = x - x1

⇒ a = 7 - 10

⇒ a = -3

And, b = y - y1

⇒ b = 10 -7

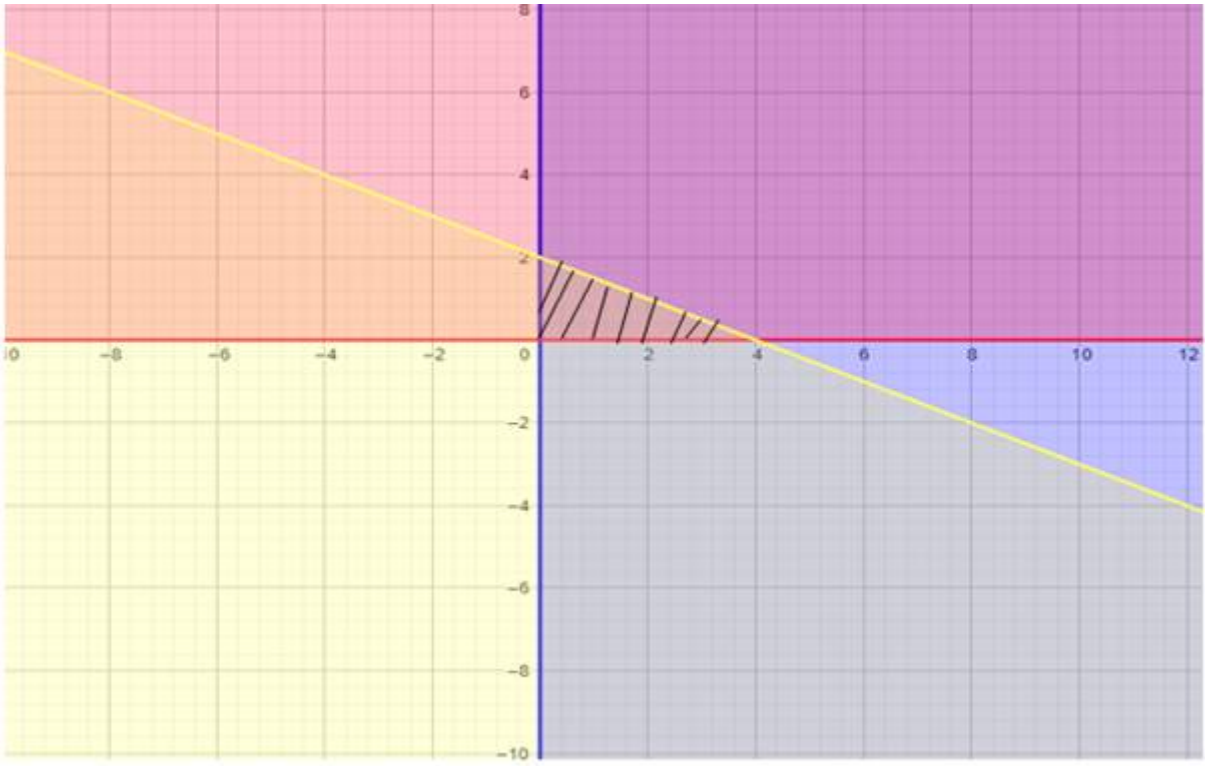
⇒ b = 3

⇒ the coordinates of O_1 in the system X′OX ↔ Y′OY are **(-3, 3).**

9. Question

Sketch the region {(x, y): x ≥ 0, y ≥ 0, x + 2y ≤ 4} in a coordinate system set up by you.

Answer

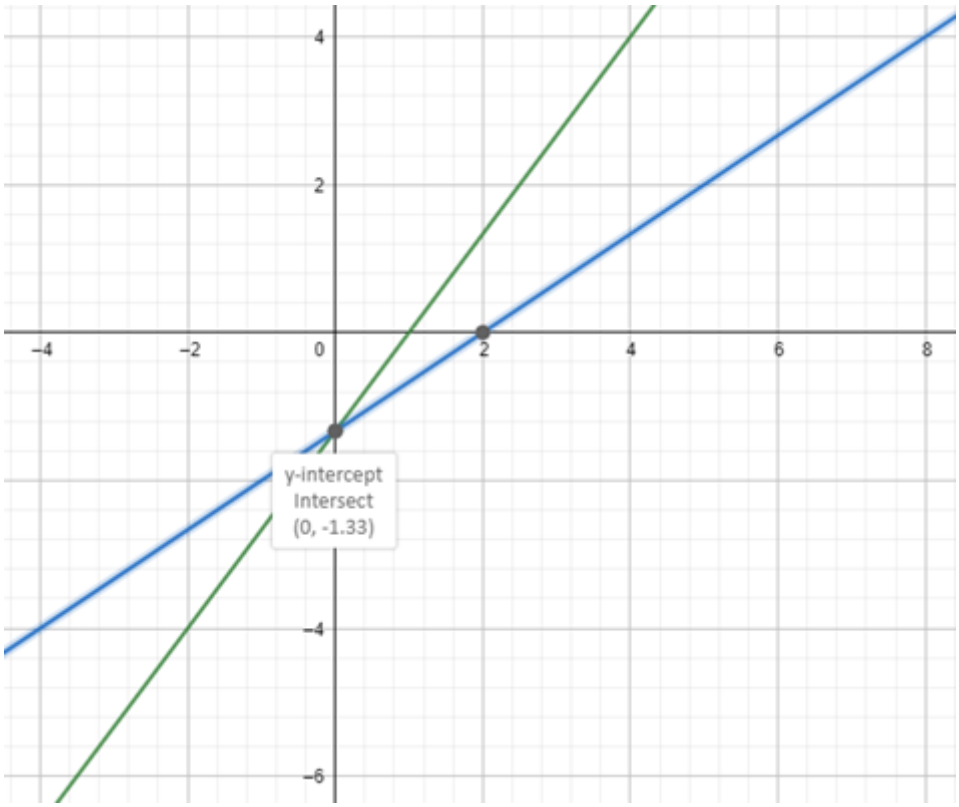


In the graph the black- lined shaded region is the desired region. (whose left side is covered by the y-axis, downside by the x-axis and remaining by line x+2y=4).

10. Question

Draw the graphs of lines 3y =4x − 4 and 2x = 3y+4 and determine the point at which these lines meet.

Answer



In the above graph, the green line represents the equation 3y=4x-4 and the blue line represents equation 2x=3y+4 and we can see that they are intersecting at (0,-1.33) or $(0, -\frac{4}{3})$

Also, when we'll solve this two equation simultaneously we'll get-

x=0 and $y = -\frac{4}{3}$.

11. Question

If a ⋆ b = ab + a + b, draw the graph of y =3 ⋆ x +1 ⋆ 2.

Answer

Given, a*b=ab+a+b…….(1)

Now, y=3 * x + 1 * 2

⇒ y = 3x + 3 + x + 2 + 1 + 2 (Using (1))

⇒ y = (3x + x) + (3 + 2 + 1 + 2)

⇒ y = 4x + 8