

# Chapter 10

## Wave Optics

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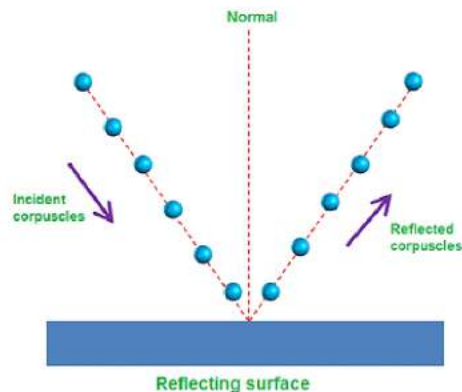
### Huygens Principle Interference of Light Waves & Young's Experiment

#### Wave Optics

Wave optics describes the connection between waves and rays of light. According to the wave theory of light, the light is a form of energy which travels through a medium in the form of transverse wave motion. The speed of light in a medium depends upon the nature of the medium.

#### Newton's Corpuscular Theory

- Light consists of very small invisible elastic particles which travel in vacuum with a speed of  $3 \times 10^8$  m/s.
- The theory could explain reflection and refraction.



#### Newton's Corpuscular Theory

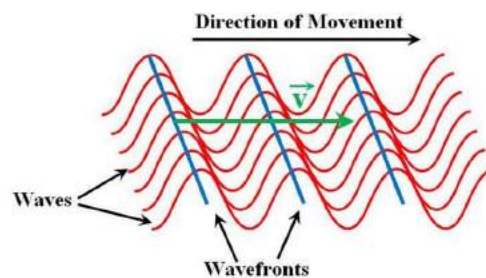
- The size of corpuscular of different colours of light are different.
- It could not explain interference, diffraction, polarization. photoelectric effect and Compton effect. The theory failed as it could not explain why light travels faster in a rarer medium than in a denser medium.

## Wavefront

A wavefront is defined as the continuous locus of all the particles of a medium, which are vibrating in the same phase.

These are three types:

- Spherical wavefront
- Cylindrical wavefront
- Plane wavefront

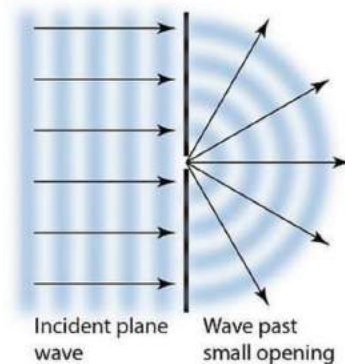


S = source of light.

AB = wavefront and SP SQ and SR are rays of light

## Huygen's Wave Theory

- Light travel in a medium in the form of wavefront.
- A wavefront is the locus of all the particles vibrating in same phase.
- All particles on a wavefront behaves as a secondary source of light, which emits secondary wavelets.



### Huygen's Principle

- The envelope of secondary wavelets represents the new position of a wavefront.
- When source of light is a point source the wavefront is spherical.
- Amplitude (A) is inversely proportional to distance (x) i.e.,  $A \propto 1 / x$ .  
 $\therefore \text{Intensity (I)} \propto (\text{Amplitude})^2$
- When Source of light is linear, the wavefront is cylindrical.
- Amplitude (A)  $\propto 1 / \sqrt{x}$   
 $\therefore \text{Intensity} \propto (\text{Amplitude})^2 \propto 1 / x$

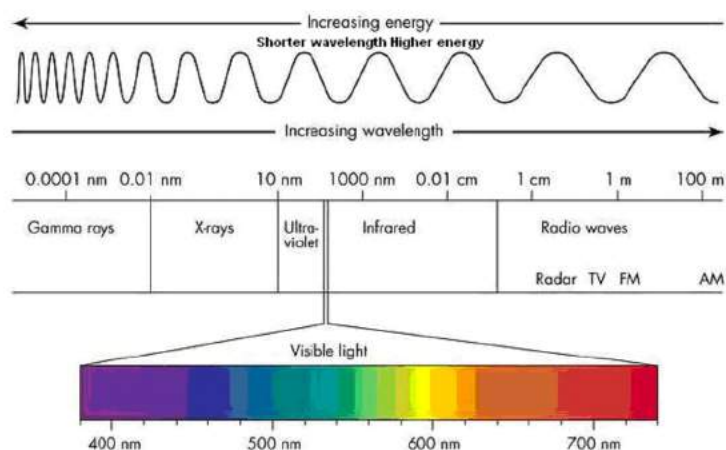
### Huygen's Principle

- Every point on given wavefront (called primary wavefront) acts as a fresh source of new disturbance called secondary wavelets.
- The secondary wavelets travels in all the directions with the speed of light in the medium.
- A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new (secondary) wave front of that instant.

### Maxwell's Electromagnetic Wave Theory

- Light waves are electromagnetic waves which do not require a material medium for their propagation.
- Due to transverse nature, light wave undergo polarisation.
- The velocity of electromagnetic wave in vacuum is  $c = 1 / \sqrt{\mu_0 \epsilon_0}$ .
- The velocity of electromagnetic waves in medium is less than that of light,  
 $v < c$   
 $v = 1 / \sqrt{\mu_0 \epsilon_0 \epsilon_r \mu_r} = c / \sqrt{\mu_r \epsilon_r}$
- The velocity of electromagnetic waves in a medium depend upon the electric and magnetic properties of the medium.  
where,  $\mu_0$  = absolute magnetic permeability and  
 $\epsilon_0$  = absolute electrical permittivity of free space.





### Electromagnetic Spectrum

- It failed to explain the phenomenon of photoelectric effect, Compton effect and Raman effect.

### Max Planck's Quantum Theory

- Light emits from a source in the form of packets of energy called quanta or photon.
- The energy of a photon is  $E = hv$ , where  $h$  is Planck's constant and  $v$  is the frequency of light.
- Quantum theory could explain photoelectric effect, Compton effect and Raman effect.
- Quantum theory failed to explain interference, diffraction and polarization of light.

### De - Broglie's Dual Theory

- Light waves have dual nature, wave nature according to Maxwell's electromagnetic wave theory and particle nature according to Max-Planck's quantum theory.
- Two natures of light are like the two faces of a coin. In any one phenomena only its one nature appears.
- Energy of photon  $= hv = hc / \lambda$   
 where,  $h$  = Planck's constant  $6.6 \times 10^{-34} \text{ J} \cdot \text{s}$   
 de-Broglie wave equation is  $\lambda = h / p = h / mv$   
 where  $h$  denotes Planck's constant.

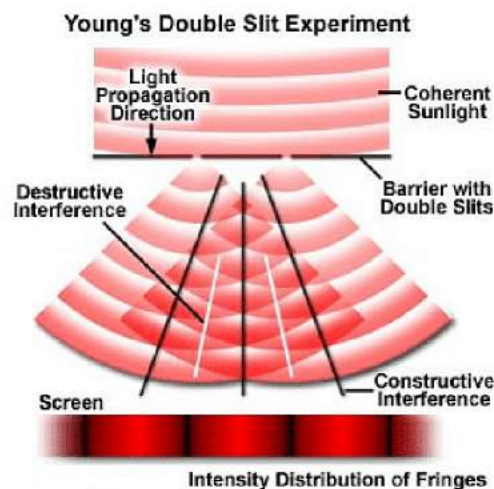
## Superposition of Waves

When two similar waves propagate in a medium simultaneously, then at any point the resultant displacement is equal to the vector sum of displacement produced by individual waves.

$$y = y_1 + y_2$$

## Interference of Light

- When two light waves of similar frequency having a zero or constant phase difference propagate in a medium simultaneously in the same direction, then due to their superposition maximum intensity is obtained at few points and minimum intensity at other few points.
- This phenomena of redistribution of energy due to superposition of waves is called interference of light waves.



- The interference taking place at points of maximum intensity is called **constructive interference**.
- The interference taking place at points of minimum intensity is **destructive interference**.

## Fringe Width

- The distance between the centers of two consecutive bright or dark fringes is called the fringe width.
- The angular fringe width is given by  $\theta = \lambda / d$ , where  $\lambda$  is the wavelength of light  $d$  is the distance between two coherent sources.

## Conditions for Constructive and Destructive Interference

### ➤ For Constructive Interference

- Phase difference,  $\phi = 2n\pi$
- Path difference,  $\Delta x = n\lambda$   
where,  $n = 0, 1, 2, 3, \dots$

### ➤ For Destructive Interference

- Phase difference,  $\phi = (2n - 1)\pi$
- Path difference,  $\Delta x = (2n - 1)\pi / 2$   
where,  $n = 1, 2, 3, \dots$

If two waves of exactly same frequency and of amplitude  $a$  and  $b$  interfere, then amplitude of resultant wave is given by

$$R = \sqrt{a^2 + b^2 + 2ab \cos \phi}$$

where  $\phi$  is the phase difference between two waves.

$$R_{\max} = (a + b)$$

$$R_{\min} = (a - b)$$

Intensity of wave

$$\therefore I = a^2 + b^2 + 2ab \cos \phi$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

where  $I_1$  and  $I_2$  are intensities of two waves.

$$\therefore I_1 / I_2 = a^2 / b^2 = \omega_1 / \omega_2$$

Where  $\omega_1$  and  $\omega_2$  are width of slits.

Energy remains conserved during interference.

Interference fringe width

$$\beta = D\lambda / d$$

where,  $D$  = distance of screen from slits,  $\lambda$  = wavelength of light and  $d$  = distance between two slits.

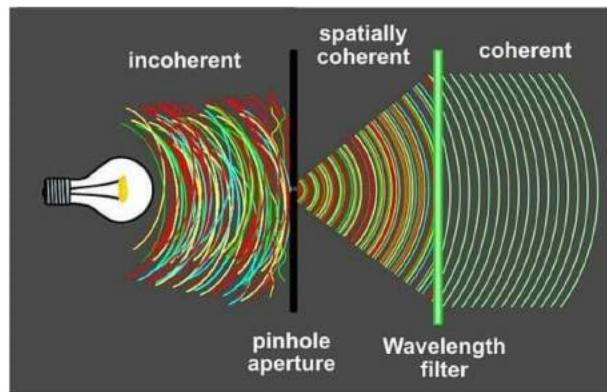
Distance of  $n$ th bright fringe from central fringe  $x_n = nD\lambda / d$

Distance of  $n$ th dark fringe from central fringe  $x'_n = (2n - 1) D\lambda / 2d$

## Coherent Sources of Light

The sources of light emitting light of the same wavelength, the same frequency having a zero or constant phase difference are called coherent sources of light.





### Coherent sources of Light

When a transparent sheet of refractive index  $\mu$  and of thickness  $t$  is introduced in one of the path of interfering waves, then fringe pattern shifts in that direction by a distance  $Y$

$$Y = D / d (\mu - 1) t = \beta / \lambda (\mu - 1) t$$

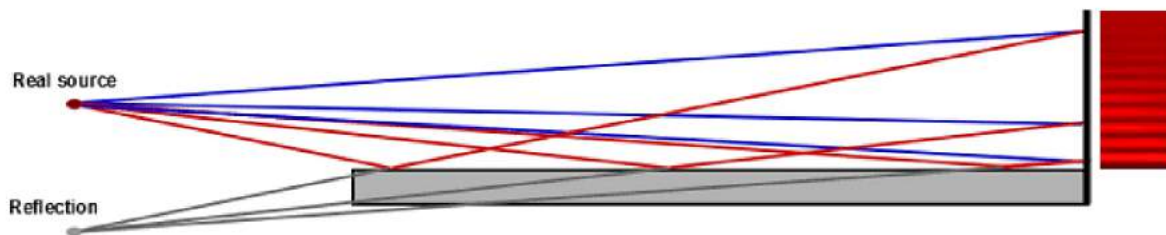
where,  $\beta$  = fringe width.

#### ➤ Fresnel's Biprism

- It is a combination of two prisms of very small refracting angles placed base to base. It is used to obtain two coherent sources from a single light source.

#### ➤ Llyod's Mirror

- The shape of interference fringes are usually hyperbolic.
- When screen is held at  $90^\circ$  to the line joining focii of the hyperbola, the fringes are circular.



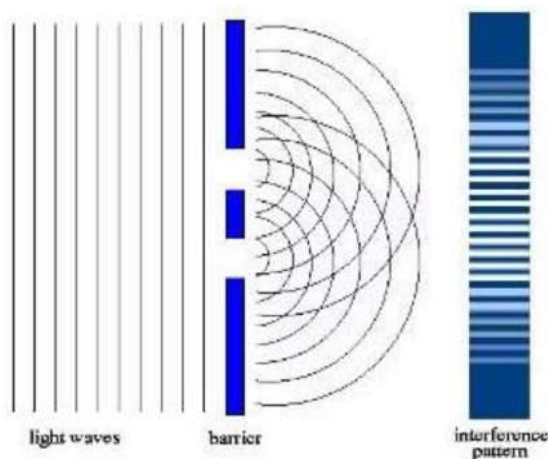
### Llyod's Mirror

- When distance of screen ( $D$ ) is very large compare to the distance between the slits ( $d$ ), the fringes are straight.

## Double Slit Experiment

**Conditions for Interference-:** Observable interference can take place if the following conditions are fulfilled:

- (a) The two sources should emit, continuously, waves of some wave-length or frequency. While deriving conditions for maxima and minima, we have taken 'l' for both the waves to be same.
- (b) The amplitudes of the two waves should be either or nearly equal. A good contrast between a maxima and minima can only be obtained if the amplitudes of two waves are equal or nearly equal.
- (c) The two sources should be narrow. A broader source can be supposed to be a combination of a number of narrow sources assembled side-by-side. Interference patterns due to these narrow sources may overlap each other.
- (d) The sources should be close to each other. The fringe width varies inversely as distance 'd' between the two sources. So, interference pattern will be more clear and distant if 'd' is small.
- (e) The two sources should be coherent one.

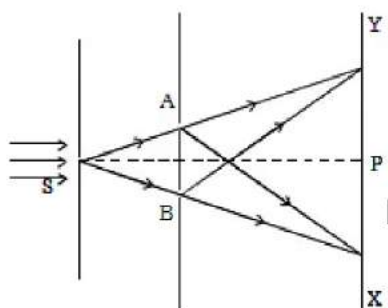


**Double Slit Experiment-:** The phenomenon of interference was first observed and demonstrated by Thomas Young in 1801. The experimental set up is shown in figure.

Light from a narrow slit S, illuminated by a monochromatic source, is allowed to fall on two narrow slits A and B placed very close to each other. The width of each slit is about 0.03 mm and they are about 0.3 mm apart. Since A and B are equidistant from S, light waves from S reach A and B in phase. So A and B act as coherent sources. According to Huygen's principle, wavelets from A and B spread out and overlapping takes place to the right side of AB. When a screen XY is placed at a distance of about 1 meter from the slits, equally spaced alternate bright and dark fringes appear on the screen. These are called interference fringes or bands. Using an eyepiece the fringes can be seen directly. At P on the screen, waves from A and B travel equal



distances and arrive in phase. These two waves constructively interfere and bright fringe is observed at P. This is called central bright fringe.



When one of the slits is covered, the fringes disappear and there is uniform illumination on the screen. This shows clearly that the bands are due to interference.

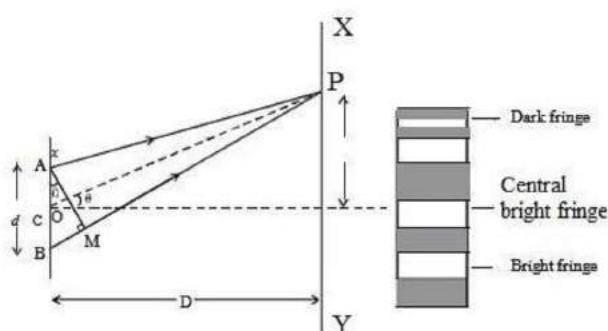
Let  $d$  be the distance between two coherent sources A and B of wavelength  $\lambda$ . A screen XY is placed parallel to AB at a distance  $D$  from the coherent sources. C is the midpoint of AB. O is a point on the screen equidistant from A and B. P is a point at a distance  $x$  from O, as shown in Fig 5.17. Waves from A and B meet at P in phase or out of phase depending upon the path difference between two waves

Draw AM perpendicular to BP

The path difference  $\delta = BP - AP$

$AP = MP$

$\delta = BP - AP = BP - MP = BM$



In right angled? ABM,  $BM = d \sin \theta$  If  $\theta$  is small,  
 $\sin \theta = \theta$

The path difference  $\delta = \theta \cdot d$

In right angled triangle COP,  $\tan \theta = OP/CO = x/D$

For small values of  $\theta$ ,  $\tan \theta = \theta$

Thus, the path difference  $\delta = xd/D$

Bright Fringes

By the principle of interference, condition for constructive interference is the path difference =  $n\lambda$

$$xd/D = n\lambda$$

Here,  $n = 0, 1, 2, \dots$  indicate the order of bright fringes

$$\text{So, } x = (D/d) n\lambda$$

This equation gives the distance of the  $n$ th bright fringe from the point O.

### Bright Fringes

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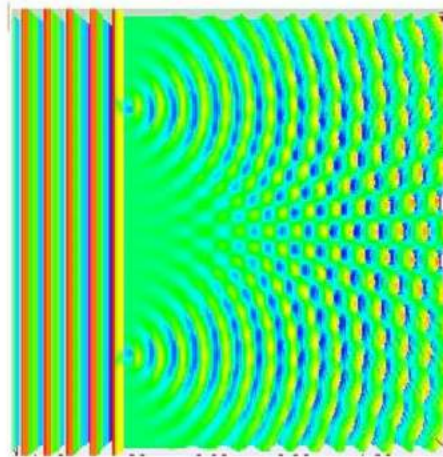
Here,  $n = 0, 1, 2, \dots$  indicate the order of bright fringes

$$\text{So, } x = (D/d) n\lambda$$

This equation gives the distance of the  $n$ th bright fringe from the point O.

### Dark Fringes

By the principle of interference, condition for destructive interference is the path difference =  $(2n-1) \lambda/2$



Here,  $n = 1, 2, 3, \dots$  indicate the order of the dark fringes.

$$\text{So, } x = (D/d) [(2n - 1) \lambda/2]$$

This equation gives the distance of the  $n$ th dark fringe from the point O. Thus, on the screen alternate dark and bright bands are seen on either side of the central bright band.

### Band Width ( $\beta$ )

The distance between any two consecutive bright or dark bands is called bandwidth.

The distance between  $(n+1)^{\text{th}}$  and  $n^{\text{th}}$  order consecutive bright fringes from O is given by,



$x_{n+1} - x_n = [(D/d) [(n+1) \lambda] - (D/d) [(n) \lambda]] = (D/d) \lambda$  Bandwidth,  $\beta = (D/d) \lambda$   
 Similarly, it can be proved that the distance between two consecutive dark bands is also equal to  $(D/d) \lambda$ . Since bright and dark fringes are of same width, they are equi-spaced on either side of central maximum.

### Condition for Obtaining Clear and Broad Interference Bands

The screen should be as far away from the source as possible.

The wavelength of light used must be larger.

The two coherent sources must be as close as possible.

The interference pattern in which the positions of maximum and minimum intensity of light remain fixed with time, is called sustained or permanent interference pattern. The conditions for the formation of sustained interference may be stated as :

- (a) The two sources should be coherent
- (b) Two sources should be very narrow
- (c) The sources should lie very close to each other to form distinct and broad fringes.

### Did You Know?

- The waves must be of the same type and must meet at a point.
- The waves must be both either unpolarised or have the same plane of polarisation.
- The dark and light regions are called interference fringes, the constructive and destructive interference of light waves.
- Interference fringes consisting of alternately bright and dark fringes (or bands) which are equally spaced are observed
- The distance  $D$  (from the double slits to the screen) is very much greater than  $d$ , typically  $\sim 1$  m.
- The fringe separation  $\Delta x$  is increased if distance to the screen  $D$  is increased.
- The fringe separation  $\Delta x$  is decreased if slit separation  $a$  is increased.
- The fringe separation  $\Delta x$  is increased as wavelength of light  $\lambda$  is increased.
- The wave characteristics of light cause the light to pass through the slits and interfere with each other, producing the light and dark areas on the wall behind the slits. The light that appears on the wall behind the slits is partially absorbed by the wall, a characteristic of a particle.
- Constructive interference occurs when waves interfere with each other crest-to-crest and the waves are exactly in phase with each other. Destructive interference occurs when waves interfere with each other crest-to-trough (peak-to-valley) and are exactly out of phase with each other.



- Each point on the wall has a different distance to each slit; a different number of wavelengths fit in those two paths. If the two path lengths differ by a half a wavelength, the waves will interfere destructively. If the path length differs by a whole wavelength the waves interfere constructively.
- Constructive interference – Occurs when waves interfere with each other crest to crest and the waves are exactly in phase with each other.
- Destructive interference – Occurs when waves interfere with each other crest to trough (peak to valley) and are exactly out of phase with each other.

### Problem (JEE Advanced):

In Young's double slit experiment, the intensities at two points  $P_1$  and  $P_2$  on the screen are respectively  $I_1$  and  $I_2$ . If  $P_1$  is located at the centre of a bright fringe and  $P_2$  is located at a distance equal to a quarter of fringe width from  $P_1$ , then find  $I_1/I_2$ .

#### Solution:

Here,  $y = \omega = \lambda D/4d$

$\Delta x = yd/D = \lambda/4$

$\phi = (2\pi/\lambda)\Delta x = \pi/2$

$\phi/2 = \pi/4$

Now,  $I_2 = I_1 \cos^2(\phi/2)$

Or,  $I_1/I_2 = 1/\cos^2(\phi/2) = 2$

From the above observation we conclude that, the ratio of  $I_1/I_2$  would be 2.

#### Problem:

A beam of light consisting of two wavelengths  $6500 \text{ \AA}$  and  $5200 \text{ \AA}$  is used to obtain interference fringes. The distance between the slits is  $2.0 \text{ mm}$  and the distance between the plane of the slits and the screen is  $120 \text{ cm}$ .

- Find the distance of the third bright fringe on the screen from the central maxima for the wavelength  $6500 \text{ \AA}$ .
- What is the least distance from the central maxima where the bright fringes due to both the wavelengths coincide?

#### Solution:

(i)  $y_3 = n \cdot D\lambda/d = 3 \times 1.2\text{m} \times 6500 \times 10^{-10}\text{m} / 2 \times 10^{-3}\text{m} = 0.12\text{cm}$

Let  $n$ th maxima of light with wavelength  $6500 \text{ \AA}$  coincides with that of  $m$ th maxima of  $5200 \text{ \AA}$ .

(ii)  $m \times 5200 \text{ \AA} \times D/d = n \times 6500 \text{ \AA} \times D/d \Rightarrow m/n = 6500/5200 = 5/4$

Least distance  $= y_4 = 4 \cdot D (5200 \text{ \AA})/d = 4 \times 6500 \times 10^{-10} \times 1.2 / 2 \times 10^{-3}\text{m} = 0.16\text{cm}$

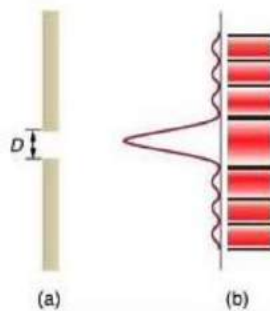
## Single Slit Experiment

### Learning Objectives:-

Discuss the single slit diffraction pattern.

Light passing through a single slit forms a diffraction pattern somewhat different from those formed by double slits or diffraction gratings. It shows a single slit diffraction pattern. Note that the central maximum is larger than those on either side, and that the intensity decreases rapidly on either side. In contrast, a diffraction grating produces evenly spaced lines that dim slowly on either side of center.

(a) Single slit diffraction pattern. Monochromatic light passing through a single slit has a central maximum and many smaller and dimmer maxima on either side. The central maximum is six times higher than shown. (b) The drawing shows the bright central maximum and dimmer and thinner maxima on either side.

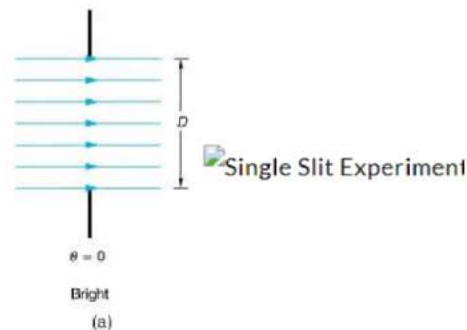


The analysis of single slit diffraction is illustrated in. Here we consider light coming from different parts of the *same* slit. According to Huygens's principle, every part of the wavefront in the slit emits wavelets. These are like rays that start out in phase and head in all directions. (Each ray is perpendicular to the wavefront of a wavelet.) Assuming the screen is very far away compared with the size of the slit, rays heading toward a common destination are nearly parallel. When they travel straight ahead, as in (a), they remain in phase, and a central maximum is obtained. However, when rays travel at an angle relative to the original direction of the beam, each travels a different distance to a common location, and they can arrive in or out of phase. In (b), the ray from the bottom travels a distance of one wavelength farther than the ray from the top. Thus a ray from the center travels a distance farther than the one on the left, arrives out of phase, and interferes destructively. A ray from slightly above the center and one from slightly above the bottom will also cancel one another. In fact, each ray from the slit will have another to interfere destructively, and a



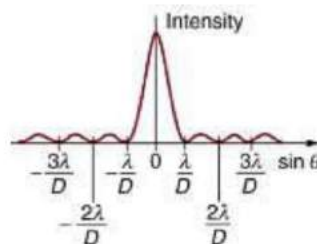
minimum in intensity will occur at this angle. There will be another minimum at the same angle to the right of the incident direction of the light.

Light passing through a single slit is diffracted in all directions and may interfere constructively or destructively, depending on the angle. The difference in path length for rays from either side of the slit is seen to be  $D \sin \theta$ .



At the larger angle shown in , the path lengths differ by  $3\lambda/2$  for rays from the top and bottom of the slit. One ray travels a distance different from the ray from the bottom and arrives in phase, interfering constructively. Two rays, each from slightly above those two, will also add constructively. Most rays from the slit will have another to interfere with constructively, and a maximum in intensity will occur at this angle. However, all rays do not interfere constructively for this situation, and so the maximum is not as intense as the central maximum. Finally, in the angle shown is large enough to produce a second minimum. As seen in the figure, the difference in path length for rays from either side of the slit is  $D \sin \theta$ , and we see that a destructive minimum is obtained when this distance is an integral multiple of the wavelength.

A graph of single slit diffraction intensity showing the central maximum to be wider and much more intense than those to the sides. In fact the central maximum is six times higher than shown here.



Thus, to obtain destructive interference for a single slit,  
 $D \sin \theta = m\lambda$ , for  $m = 1, -1, 2, -2, 3, \dots$  (destructive),



where  $D$  is the slit width,  $\lambda$  is the light's wavelength,  $\theta$  is the angle relative to the original direction of the light, and  $m$  is the order of the minimum. shows a graph of intensity for single slit interference, and it is apparent that the maxima on either side of the central maximum are much less intense and not as wide. This is consistent with the illustration in (b).

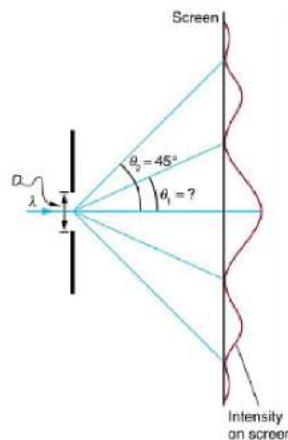
### Calculating Single Slit Diffraction

Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of  $\{45^\circ\}$  relative to the incident direction of the light.

(a) What is the width of the slit?

(b) At what angle is the first minimum produced?

A graph of the single slit diffraction pattern is analyzed in this example.



### Strategy

From the given information, and assuming the screen is far away from the slit, we can use the equation  $D \sin \theta = m \lambda$  first to find  $D$ , and again to find the angle for the first minimum  $\theta_1$ .

### Solution for (a)

We are given that  $\lambda = 550 \text{ nm}$ ,  $m = 2$  and  $\theta_2 = 45.0$ . Solving the equation  $D \sin \theta = m \lambda$  for  $D$  and substituting known values gives

$$\begin{aligned} D &= \frac{m\lambda}{\sin \theta_2} = \frac{2(550 \text{ nm})}{\sin 45.0} \\ &= \frac{1100 \text{ nm}}{0.707} \\ &= 1.56 \times 10^{-6} \text{ m} \end{aligned}$$

## Discussion

We see that the slit is narrow (it is only a few times greater than the wavelength of light). This is consistent with the fact that light must interact with an object comparable in size to its wavelength in order to exhibit significant wave effects such as this single slit diffraction pattern. We also see that the central maximum extends  $20.7^\circ$  on either side of the original beam, for a width of about  $41^\circ$ . The angle between the first and second minima is only about  $24^\circ$  ( $45.0^\circ - 20.7^\circ$ ). Thus the second maximum is only about half as wide as the central maximum.

## Section Summary

- A single slit produces an interference pattern characterized by a broad central maximum with narrower and dimmer maxima to the sides.
- There is destructive interference for a single slit when, where  $a$  is the slit width,  $\lambda$  is the light's wavelength,  $\theta$  is the angle relative to the original direction of the light, and  $m$  is the order of the minimum. Note that there is no  $m = 0$  minimum.

## Conceptual Questions

As the width of the slit producing a single-slit diffraction pattern is reduced, how will the diffraction pattern produced change?

## Problems & Exercises

(a) At what angle is the first minimum for 550-nm light falling on a single slit of width  $1.00 \text{ m}$ ? (b) Will there be a second minimum?

(a) Yes

(b) No

(a) Calculate the angle at which a  $2.00\text{-m}$ -wide slit produces its first minimum for 410-nm violet light. (b) Where is the first minimum for 700-nm red light?

(a) How wide is a single slit that produces its first minimum for 633-nm light at an angle of  $28.0^\circ$ ? (b) At what angle will the second minimum be?

(a)  $68.9^\circ$

(b)  $69.9^\circ$

(a) What is the width of a single slit that produces its first minimum at  $60.0^\circ$  for 600-nm light? (b) Find the wavelength of light that has its first minimum at  $62.0^\circ$ .

Find the wavelength of light that has its third minimum at an angle of  $48.6^\circ$  when it falls on a single slit of width 3.00 m.

750 nm

Calculate the wavelength of light that produces its first minimum at an angle of  $36.9^\circ$  when falling on a single slit of width 1.00 m.

(a) Sodium vapor light averaging 589 nm in wavelength falls on a single slit of width 7.50 m. At what angle does it produce its second minimum? (b) What is the highest-order minimum produced?

(a)  $9.04^\circ$

(b) 12

(a) Find the angle of the third diffraction minimum for 633-nm light falling on a slit of width 1.00 m. (b) What slit width would place this minimum at  $85.0^\circ$ ? Explicitly show how you follow the steps in Problem-Solving Strategies for Wave Optics

(a) Find the angle between the first minima for the two sodium vapor lines, which have wavelengths of 589.1 and 589.6 nm, when they fall upon a single slit of width 2.00 m. (b) What is the distance between these minima if the diffraction pattern falls on a screen 1.00 m from the slit? (c) Discuss the ease or difficulty of measuring such a distance.

(a) 0.0150

(b) 0.262 mm

(c) This distance is not easily measured by human eye, but under a microscope or magnifying glass it is quite easily measurable.

(a) What is the minimum width of a single slit (in multiples of  $\lambda$ ) that will produce a first minimum for a wavelength  $\lambda$ ? (b) What is its minimum width if it produces 50 minima? (c) 1000 minima?

(a) If a single slit produces a first minimum at  $30^\circ$ , at what angle is the second-order minimum? (b) What is the angle of the third-order minimum? (c) Is there a fourth-order minimum? (d) Use your answers to illustrate how the angular



width of the central maximum is about twice the angular width of the next maximum (which is the angle between the first and second minima).

(a) 30.1

(b) 31.1

(c) No

A double slit produces a diffraction pattern that is a combination of single and double slit interference. Find the ratio of the width of the slits to the separation between them, if the first minimum of the single slit pattern falls on the fifth maximum of the double slit pattern. (This will greatly reduce the intensity of the fifth maximum.)

### Integrated Concepts

A water break at the entrance to a harbor consists of a rock barrier with a 50.0-m-wide opening. Ocean waves of 20.0-m wavelength approach the opening straight on. At what angle to the incident direction are the boats inside the harbor most protected against wave action?

23.6 and 53.1

### Integrated Concepts

An aircraft maintenance technician walks past a tall hangar door that acts like a single slit for sound entering the hangar. Outside the door, on a line perpendicular to the opening in the door, a jet engine makes a 600-Hz sound. At what angle with the door will the technician observe the first minimum in sound intensity if the vertical opening is 0.800 m wide and the speed of sound is 340 m/s?

### Glossary

destructive interference for a single slit occurs when, where is the slit width, is the light's wavelength, is the angle relative to the original direction of the light, and is the order of the minimum

### Single Slit Interference & Diffraction Pattern

#### Diffraction

The bending of light waves around the corners of an obstacle or aperture is called diffraction of light.

The phenomenon of diffraction is divided mainly in the following two classes

- (a) Fresnel class
- (b) Fraunhofer class

S.No	Fresnel Class	Fraunhofer Class
1	The source is at a finite distance	The source is at infinite distance
2	No opticals are required.	Opticals are in the form of Collimating lens and focusing lens are required.
3	Fringes are not sharp and well defined.	Fringes are sharp and well defined.

### Fraunhofer Diffraction at a Single Slit

Linear Width of central maximum  $2D\lambda / a = 2f\lambda / a$

Angular width of central maximum  $= 2\lambda / a$

where,  $\lambda$  = wavelength of light,  $a$  = width of single slit,  $D$  = distance of screen from the slit and  $f$  = focal length of convex lens.

### For Secondary Minima

- (a) Path difference  $= n\lambda$
  - (b) Linear distance  $= nD\lambda / a = nf\lambda / a$
  - (c) Angular spread  $= n\lambda / a$
- where,  $n = 1, 2, 3$

### For Secondary Maxima

- (a) Path difference  $= (2n + 1) \lambda / 2$
- (b) Linear distance  $= (2n + 1) D\lambda / 2a = (2n + 1) f\lambda / 2a$
- (c) Angular spread  $= (2n + 1) \lambda / 2$

### Important Points

- A soap bubble or oil film on water appears coloured in white light due to interference of light reflected from upper and lower surfaces of soap bubble or oil film.
- In interference fringe pattern all bright and dark fringes are of same width,

- In diffraction fringe pattern central bright fringe is brightest and widest, and remaining secondary maxima are of gradually decreasing intensities.
- The difference between interference and diffraction is that the interference is the superposition between the wavelets coming from two coherent sources while the diffraction is the superposition between the wavelets coming from the single wavefront

## Polarization of Light

### Polarization

The phenomena of restructuring of electric vectors of light into a single direction is called **polarization**.

Ordinary light has electric vectors in all possible directions in a plane perpendicular to the direction of propagation of light.

When ordinary light is passed through a tourmaline, calcite or quartz crystal the transmitted light have electric vectors in a particular direction parallel to the axis of crystal. This light is plane polarized light.

[A plane containing the vibrations of polarized light is called plane of vibration.

A plane perpendicular to the plane of vibration is called **plane of polarization**.]

Polarization can take place only in transverse waves.

### Nicol Prism

A nicol prism is an optical device which is used for producing plane polarized light and analyzing light the same.

The nicol prism consists of two calcite crystal cut at  $68^\circ$  with its principal axis joined by a glue called Canada balsam.

### Law of Malus

When a beam of completely plane polarized light is incident on an analyzer, the intensity of transmitted light from analyzer is directly proportional to the square of the cosine of the angle between plane of transmission of analyzer and polarizer, i.e.,

$$I \propto \cos^2 \theta$$

When ordinary light is incident on a polarizer the intensity of transmitted light is half of the intensity of incident light.

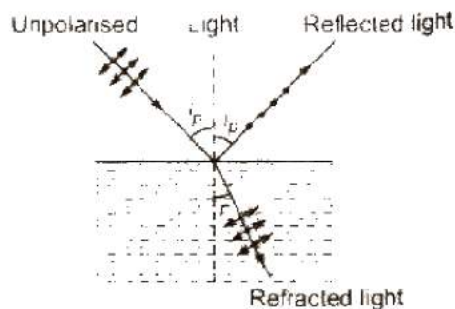
When a polarizer and analyzer are perpendicular to each other, then intensity of transmitted light from analyzer becomes 0.

### Brewster's Law



When unpolarized light is incident at an angle of polarization ( $i_p$ ) on the interface separating air from a medium of refractive index  $\mu$ , then reflected light becomes fully polarized, provided

$$\mu = \tan i_p$$



If angle of polarization is  $i_p$  and angle of refraction is  $\mu$  then

$$i_p + r = 90^\circ$$

$$\text{Refractive index } \mu = \tan i_p = 1 / \sin C$$

where,  $C$  = critical angle.

### Double Refraction

When unpolarized light is incident on a calcite or quartz crystal it splits up into two refracted rays. one of which follows laws of refraction, are called ordinary ray (O-ray) and others which do not follow laws of refraction, are called extraordinary ray (E-ray) This phenomena is called double refraction.

### Dichroism

Few double refracting crystals have a property of absorbing one of the two refracted rays and allowing the other to emerge out. This property of crystal is called dichroism.

### Polaroid

It is a polarizing film mounted between two glass plates. It is used to produce polarized light.

A polaroid is used to avoid glare of light in spectacles.

### Uses of Polaroid

- (i) Polaroids are used in sun glasses. They protect the eyes from glare.
- (ii) The polaroids are used in window panes of a train and especially of an aeroplane. They help to control the light entering through the window.

(iii) The pictures taken by a stereoscopic camera. When seen with the help of polarized spectacles, create three dimensional effect.

(iv) The windshield of an automobile is made of polaroid. Such a mind shield protects the eyes of the driver of the automobile from the dazzling light of the approaching vehicles.

### Resolving Power Experiment

#### Diffraction Limit

When a point object is imaged using a circular opening (or aperture) like a lens or the iris of our eye, the image formed is not a point but a diffraction pattern. This is true particularly when the size of the object is comparable to the wavelength of light.

Just as in single slit diffraction, a circular aperture produces a diffraction pattern of concentric rings that grow fainter as we move away from the center. These are known as Airy's discs.



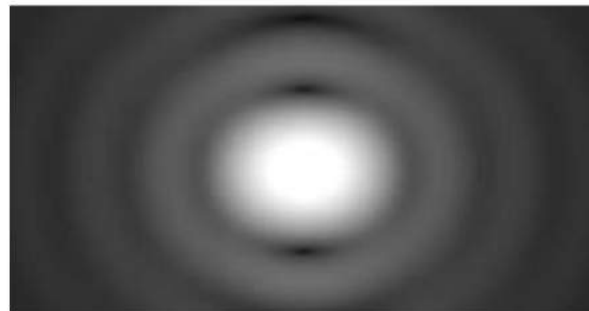
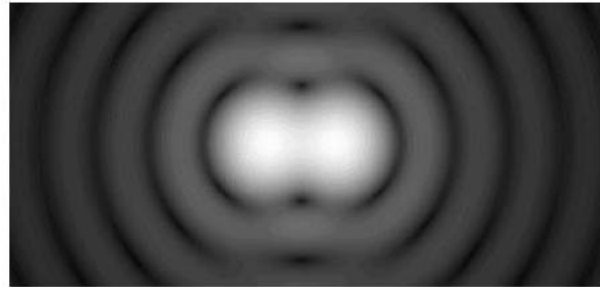
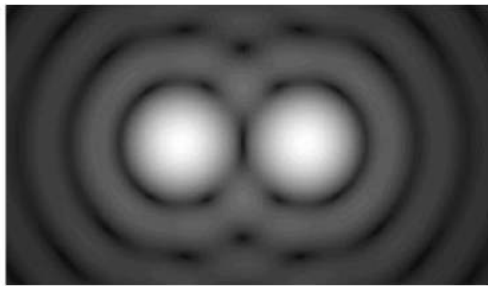
Because of this point sources close to one another can overlap and produce a blurred image. The half angle  $\theta$  subtended by the first minimum at the source is given by the relation

$$\sin\theta \approx 1.22 \lambda/d$$

To obtain a good image, point sources must be resolved i.e., the point sources must be imaged such that their images are sufficiently far apart that their diffraction patterns do not overlap. For this the minimum distance between images must be such that the central maximum of the first image lies on the first minimum of the second and vice versa. Such an image is said to be just resolved. This is the famous Rayleigh criterion.

The criterion is given by the above formula as:

$$\sin \theta_R \approx \theta_R \approx 1.22 \lambda/d$$



The first image is fully resolved, the second just resolved and third unresolved.

### **Resolving Power:**

It is defined as the inverse of the distance or angular separation between two objects which can be just resolved when viewed through the optical instrument.

### **Resolving Power of Telescope:**

In telescopes, very close objects such as binary stars or individual stars of galaxies subtend very small angles on the telescope. To resolve them we need very large apertures. We can use the Rayleigh's to determine the resolving power. The angular separation between two objects must be



$$\Delta \theta = 1.22 \frac{\lambda}{d}$$

$$\text{Resolving power} = \frac{1}{\Delta \theta} = \frac{d}{1.22 \lambda}$$

Thus higher the diameter  $d$ , better the resolution. The best astronomical optical telescopes have mirror diameters as large as 10m to achieve the best resolution. Also larger wavelengths reduce the resolving power and consequently radio and microwave telescopes need larger mirrors.

### Resolving Power of Microscope:

For microscopes, the resolving power is the inverse of the distance between two objects that can be just resolved. This is given by the famous Abbe's criterion given by Ernst Abbe in 1873 as

$$\Delta d = \frac{\lambda}{2 n \sin \theta}$$

$$\text{Resolving power} = \frac{1}{\Delta d} = \frac{2n \sin \theta}{\lambda}$$

Where  $n$  is the refractive index of the medium separating object and aperture. Note that to achieve high resolution  $n \sin \theta$  must be large. This is known as the Numerical aperture.

Thus, for good resolution:

1.  $\sin \theta$  must be large. To achieve this, the objective lens is kept as close to the specimen as possible.
2. A higher refractive index ( $n$ ) medium must be used. Oil immersion microscopes use oil to increase the refractive index. Typically for use in biology studies this is limited to 1.6 to match the refractive index of glass slides used. (This limits reflection from slides) Thus the numerical aperture is limited to just 1.4-1.6. Thus, optical microscopes (if you do the math) can only image to about 0.1 micron. This means that usually organelles, viruses and proteins cannot be imaged.
3. *Decreasing the wavelength by using X-rays and gamma rays.* While these techniques are used to study inorganic crystals, biological samples are usually damaged by x-rays and hence are not used.

The limit set by Abbe's criterion for optical microscopy cannot be avoided. However, using different fluorescence microscopy techniques the Abbe's limit

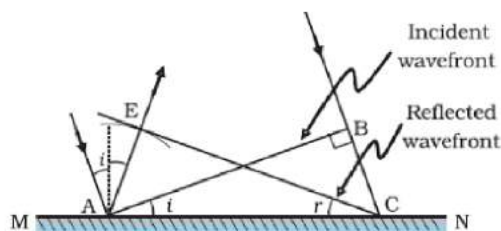
can be circumvented. Stefan Hell used a technique called Simulated Emission Depletion (STED) and the duo Eric Betzig and W.E. Moerner used super imposed images using green fluorescent proteins to bypass the resolution limit and obtain optical images in never before seen resolution. All three were awarded the 2014 Nobel Prize in Chemistry for their pioneering work.

### Refraction & Reflection of Plane Waves using Huygens Principle

As we know that when light falls on an object, it bends and move through the material, this is what refraction is. Also when the light bounces off the medium it is called a reflection. Let us know study reflection and refraction of waves by Huygen's principle.

#### Reflection using Huygens Principle

We can see a ray of light is incident on this surface and another ray which is parallel to this ray is also incident on this surface. Plane AB is incident at an angle ' $i$ ' on the reflecting surface MN. As these rays are incident from the surface, so we call it incident ray. If we draw a perpendicular from point 'A' to this ray of light, Point A, and point B will have a line joining them and this is called as wavefront and this wavefront is incident on the surface.



Reflection of a plane wave AB by the reflecting surface MN.  
AB and CE represent incident and reflected wavefronts.

These incident wave front is carrying two points, point A and point B, so we can say that from point B to point C light is travelling a distance. If  $v$  represents the speed of the wave in the medium and if ' $r$ ' represents the time taken by the wavefront from the point B to C then the distance

$$BC = vr$$

In order to construct the reflected wavefront we draw a sphere of radius  $vr$  from the point A. Let CE represent the tangent plane drawn from the point C to this sphere. So,

$$AE = BC = vr$$

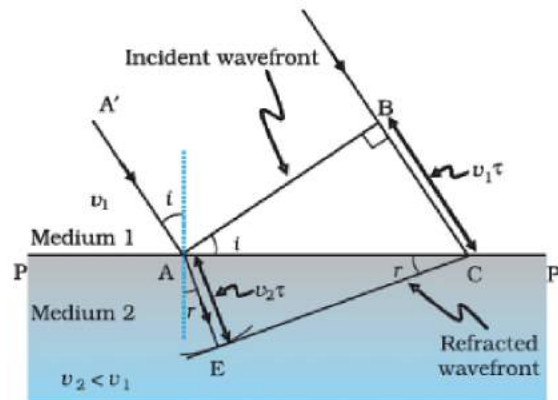
If we now consider the triangles EAC and BAC we will find that they are congruent and therefore, the angles ' $i$ ' and ' $r$ ' would be equal. This is the law of reflection

#### Refraction using Huygen's principle



We know that when a light travels from one transparent medium to another transparent medium its path changes. So the laws of refraction state that the angle of incidence is the angle between the incident ray and the normal and the angle of refraction is the angle between the refracted ray and the normal.

The incident ray, reflected ray and the normal, to the interface of any two given mediums all lie in the same plane. We also know that the ratio of the sine of the angle of incidence and sine of the angle of refraction is constant.



A plane wave AB is incident at an angle  $i$  on the surface PP' separating medium 1 and medium 2. The plane wave undergoes refraction and CE represents the refracted

wavefront. the figure corresponds to  $v_2 < v_1$  so that the refracted waves bends towards the normal.

We can see a ray of light is incident on this surface and another ray which is parallel to this ray is also incident on this surface. As these rays are incident from the surface, so we call it incident ray.

Let PP' represent the medium 1 and medium 2. The speed of the light in this medium is represented by  $v_1$  and  $v_2$ . If we draw a perpendicular from point 'A' to this ray of light, Point A, and point B will have a line joining them and this is called as wavefront and this wavefront is incident on the surface.

If  $r$  represents the time taken by the wavefront from the point B to C then the distance,

$$BC = v_1 r$$

So to determine the shape of the refracted wavefront, we draw a sphere of radius  $v_2 r$  from the point A in the second medium. Let CE represent a tangent plane drawn from the point C on to the sphere. Then,  $AE = v_2 r$ , and CE would represent the refracted wavefront. If we now consider the triangles ABC and AEC, we readily



obtain

$$\sin i = \frac{BC}{AC} = \frac{v_1 r}{AC}$$

$$\sin r = \frac{AE}{AC} = \frac{v_2 r}{AC}$$

where 'i' and 'r' are the angles of incidence and refraction, respectively. Substituting the values of  $v_1$  and  $v_2$  in terms of  $w$  we get the Snell's Law,

$$n_1 \sin i = n_2 \sin r$$

### **Coherent & Incoherent Addition of Waves**

You must have seen light coming out from the laser. Let us carry out a small activity. Take two needles and touch the needles on the surface of the water. Here if both the needles move with the same speed then they are said to be coherent. Let us learn more about coherent waves.

### **Coherent and Incoherent Addition of waves**

Suppose there is a surface of the water and you take a needle and touch the surface of the water. What will happen? Yes, ripples are formed. Now if you take two needles and you touch the surface of the water with the needles. What do you think will happen?

You will see a pattern. That pattern is the interference pattern. When you touch both the needles at the surface of the water at the same time, both the needles are in the same phase. Needle 1 will produce a wave. Also, needle 2 will produce its own ripples and they will intersect with waves of the first needle.

Now, if both the needles are moving with the same velocity, the wave formed here are coherent. If the velocity of a 1st needle and 2nd needle are not steady they won't intersect. This is because one is at a steady speed and other is at variable speed.

### **Coherent Waves**

If the potential difference between two waves is zero or is constant w.r.t time, then the two waves are said to be coherent.

### **Non-coherent Waves**



The waves are non-coherent if the potential difference between the two ways keeps on changing. Lightbulb, study lamp are the examples of the coherent waves. They emit waves at random potential difference.

### Explanation

Now let us consider there are two needles say  $S_1$  and  $S_2$  moving up and down on the surface of the water and are pointing at point P. So the path difference here is given as  $S_1P - S_2P$ . Now the displacement by two needles and  $S_1 S_2$  are:

$$y_1 = A \cos wt \dots\dots\dots (1)$$

$$y_2 = A \cos wt \dots\dots\dots (2)$$

So the resultant displacement at point P is,  $y = y_1 + y_2$ . When we substitute the value of  $y_1$  and  $y_2$  we write,

$$y = A \cos wt + A \cos wt$$

$$y = 2A \cos wt \dots\dots\dots (3)$$

Now, we know the intensity is proportional to the square of the amplitude waves.

$$I_0 \propto A^2$$

Where  $I_0$  is the initial intensity and  $A^2$  is the amplitude of the wave. From equation 3, we say that  $A = 2A$ . So,

$$I_0 \propto (2A)^2 \text{ or } I_0 \propto 4 A^2$$

$$I = 4 I_0$$

Now, if two needles that are  $S_1$  and  $S_2$  are in the same phase, the potential difference is,

$$S_1P - S_2P = n\lambda$$

Where  $n = 0, 1, 2, 3 \dots\dots\dots$  and  $\lambda =$  the wavelength of the wave. If the two needles

$S_1$  and  $S_2$  are vibrating at its destructive interference then, the potential difference is

$$S_1P - S_2P = (n + 1/2) \lambda$$

Now if the potential difference of the waves is  $\Phi$  then,

$$y_1 = a \cos wt$$

$$y_2 = a \cos wt$$

The individual intensity of each wave is  $I_0$ , we get,

$$y = y_1 + y_2$$

$$= \alpha \cos \omega t + \alpha \cos (\omega t + \Phi)$$

$$y = 2 \alpha \cos(\Phi/2) \cos (\omega t + \Phi/2)$$

Since, the intensity is  $I_0 \propto A^2$

$$I_0 \propto 4\alpha^2 \cos^2 (\Phi/2)$$

$$I = 4 I_0 \cos^2 (\Phi/2)$$

Well, the time-averaged value of  $\cos^2(\Phi_t/2)$  is  $1/2$ . So, the resultant intensity will be  $I = 2 I_0$  at all the points.