Work, Energy And Power

Work and Kinetic Energy - The Work Energy Theorem

- We know that according to the third equation of motion,

$$v^2 - u^2 = 2aS$$

Multiplying both sides by *m*/2, we obtain

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = maS$$
$$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = FS \qquad [\because F = ma]$$

$$\therefore \boxed{k_f - k_i = W} \qquad ...(i)$$

Where,

$\frac{1}{2}mv^2 = k_f$ = Final kinetic energy

$\frac{1}{2}mu^2 = k_i$ = Initial kinetic energy

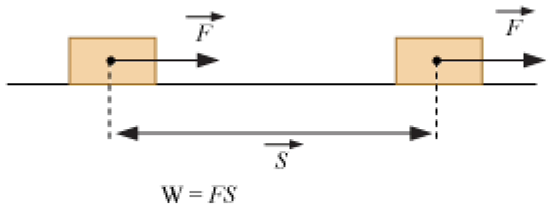
$W = FS$ = Work done

Equation (i) is a special case of work energy (*WE*) theorem. The change in kinetic energy of a particle is equal to the work done on it by the net force.

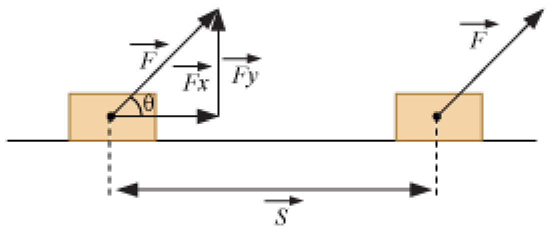
Work

- Work is said to be done when the point of application of the forces moves in the direction of the force.

- If a constant force $\vec{F}$ is applied on a body and the body has a displacement $\vec{S}$ in the direction of the force as shown in fig, then the work done on the body by the force is given by,



- When the displacement $\vec{S}$ is not in the direction of $\vec{F}$ as shown in figure,



In such a case, we find the work done by resolving $\vec{F}$ into two rectangular components.

(i) $\vec{F}_x$ in the direction of $\vec{S}$ (where $F_x = F \cos\theta$)

(ii) $\vec{F}_y$ perpendicular to $\vec{S}$ (where $F_y = F \sin\theta$)

The work is done along the component $\vec{F}_x$ only.

Thus,

$$W = F_x \times S$$
$$W = F\cos\theta \times S$$

$$W = FS\cos\theta$$

$$\therefore \overrightarrow{W} = \vec{F} \cdot \vec{S}$$

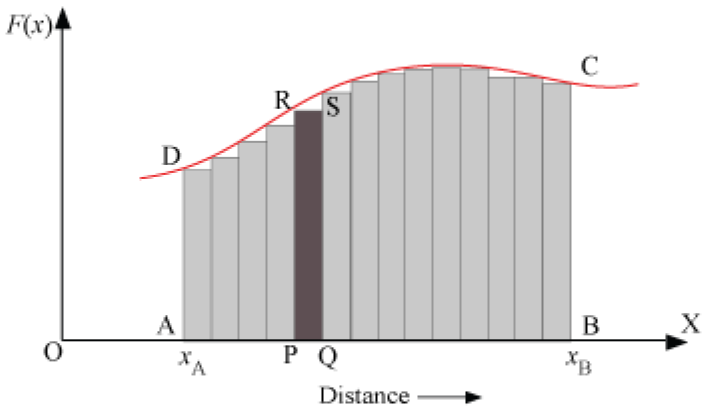
Kinetic Energy

- The kinetic energy of a mass *m* having velocity *v* is, $K = \frac{1}{2}mv^2$
- It is a scalar quantity.

Videos by teacher:

Work, Energy and Variable Force

Work Done by a Variable Force



- The above given figure shows a variable force.
- The entire path ABCD is broken into infinitesimally small displacements
- One such small displacement is from P to Q. Let PQ = *dx*
- Small amount of work done from P to Q is,

  $dW = F \times dx$

  Along the small displacement *dx*, force is constant in magnitude and direction.

  $dW = (PR) \times (PQ)$

  dW = Area of the strip PQRS
- Total work done in moving the body from A to B is,

  $$W = \sum dW = \sum F \times dx$$
- If the displacements are allowed to approach zero, then

  $$W = \lim_{dx \to 0} \sum F(dx)$$

  $$W = \int_{x_A}^{x_B} F(dx) = \int_{x_A}^{x_B} \text{Area of the strip PQRS} \qquad \text{(i)}$$

$\therefore W$ = Area of ABCDA

Work−Energy Theorem for a Variable Force

Suppose,

m = Mass of a body

u = Initial velocity of the body

v = Final velocity of the body

a = Acceleration

k_i = Initial kinetic energy of the body

k_f = Final kinetic energy of the body

The rate of change of kinetic energy is,

$$\frac{dk}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = m\frac{dv}{dt} \times v$$
$$= Fv \quad \left[\because F = m\frac{dv}{dt}\ (\text{Newton's second law})\right]$$

$$\therefore \frac{dk}{dt} = F\frac{dx}{dt}$$

$dk = Fdx$

On integrating from the initial position (x_A) to the final position (x_B), we have

$$\int_{k_i}^{k_f} dk = \int_{x_A}^{x_B} F dx$$

$$k_f - k_i = \int_{x_A}^{x_B} F dx = W$$ [From equation (i)]

$\therefore$Work done on the body = Increase in K.E. of the body

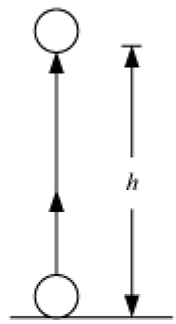
Video by teacher:

Potential Energy and Conservation of Mechanical Energy

- Potential energy is the energy possessed by the body by virtue of its position.

  Two important types of potential energy are

- Gravitational potential energy
- Elastic potential energy
- **Gravitational potential energy**: It is the energy possessed by the body by virtue of its position above the surface of the earth.



Let,

m = Mass of a body

g = Acceleration due to gravity

h = Height through which the body is raised.

The force applied to just overcome gravitational attraction is

$F = mg$

Work done = Force × Distance

OR

$W = (F) \times h = mgh$

This work gets stored as potential energy.

∴ Gravitational P.E. = $V(h) = mgh$

If *h* is taken as a variable, then

$$\frac{\mathrm{d}}{\mathrm{d}\,h} V\left(h\right) = -\frac{\mathrm{d}}{\mathrm{d}\,h}\left(mgh\right) = -mg = F$$

Where, *F* is the gravitational force on the body

The negative sign indicates that gravitational force is acting downwards.

Thus, $F = -\frac{\mathrm{d}}{\mathrm{d}h} V\left(h\right)$

Mathematically, the potential energy, *V* (*x*) is defined if the force *F* (*x*) can be written as

$$F(x) = -\frac{dv}{dx}$$

$$\boxed{\int_{x_1}^{x_2} F(x)dx = -\int_{V_1}^{V_2} dV = V_1 - V_2}$$

The above equation shows that the work done by a conservative force like gravity in taking the body from initial position, (x_1) to final position, (x_2) is equal to the difference between the initial and final P.E. of the body.

The Conservation of Mechanical Energy

- Total mechanical energy of a system is always conserved.

  Total mechanical energy = Potential energy (*V*) + Kinetic energy (*K*)

- Mechanical energy of a system is conserved if the forces doing work on it are conservative.

- Let us consider a body undergoing a small displacement, Δx under the action of a conservative force, *F*. According to work energy theorem,

  Change in K.E = Work done

  $\Delta k = F(x)\, \Delta x$ … (i)

  If the force is conservative, the P.E. function, $V(x)$ can be defined such that

  $-\Delta V = F\,(x)\, \Delta x$

$\Delta V = -F(x)\,\Delta x$ … (ii)

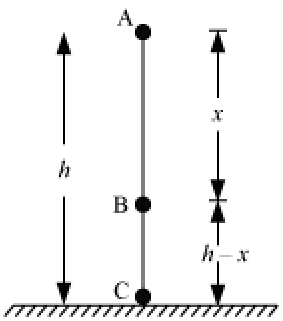
Adding equations (i) and (ii)

$\Delta k + \Delta V = 0$

$\Delta(k + V) = 0$, which means

$(k + V)$ = Constant.

- Example of the law of conservation of mechanical energy:



Let m be the mass of the body held at A, at a height, h above the ground.

At point A

The body is at rest at A.

K.E. of the body = 0

P.E. of the body = mgh

T.E. of the body = K.E + P.E. = 0 + mgh

$E_1 = mgh$ … (i)

At point C

Let the body be allowed to fall freely under gravity so as to strike the ground at C with a velocity, v.

From $v^2 - u^2 = 2as$

OR

$v^2 - 0 = 2\ (g)\ h$

OR

$v^2 = 2gh$ … (*ii*)

K.E. of the body = $\frac{1}{2} mv^2 = \frac{1}{2} m\ (2gh) = mgh$ …[From equation (ii)]

P.E. of the body = $mgh = mg\ (0) = 0$

Total energy of the body = K.E. + P.E.

$E_2 = mgh + 0 = mgh$ … (iii)

At point B

In free fall, suppose the body crosses point B with a velocity, v_1, where AB = x

$v^2 - u^2 = 2as$

$$v_1^2 - 0 = 2(g)x$$

OR

$$v_1^2 = 2gx$$

K.E. of the body $= \frac{1}{2} mv_1^2 = \frac{1}{2} m(2gx) = mgx$

Height of the body at B = CB = $(h - x)$

∴P.E. of the body at B = $mg\ (h - x)$

Total energy of the body at B = K.E. + P.E.

$E_3 = mgx + mg\ (h - x)$

$= mgx + mgh - mgx$

OR

$E_3 = mgh$ … (iv)

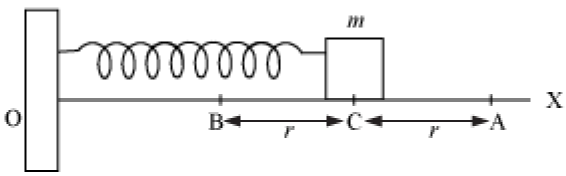
From equation (i), (iii) and (iv), we find that

$E_1 = E_2 = E_3 = mgh$

i.e., the total energy of the body during free fall remains constant at all positions

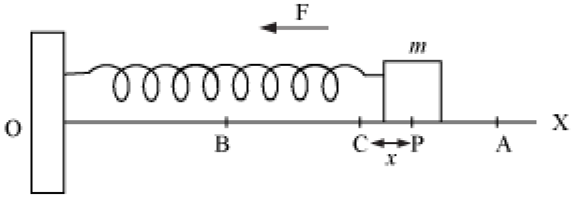
The Potential Energy of a Spring

Consider a light and perfectly elastic spring fixed at one end of a rigid support at point O and the other end attached to a block of mass 'm'.



When the block is pulled from its equilibrium position (C) to point A, the restoring force is set up in the spring due to elasticity.

The work done in stretching the spring from C to A is stored in the system in the form of potential energy of the spring.



Let us calculate the P.E stored in the spring, when the it is pulled from the mean position C up to a point P, such that CP = x

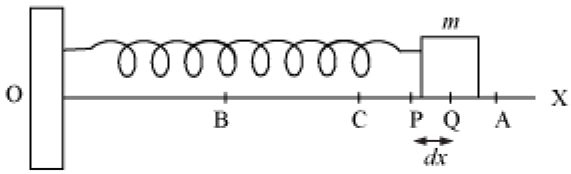
The restoring force set up in the string is given by,

$F \propto x$

$F = -kx$

Where, k is the constant of proportionality known as force constant or spring constant

Suppose that the block is further displaced through an infinitesimally small distance, PQ = dx



Small work done in increasing the length of the spring by *dx* is

$dW = Fdx = kxdx$ [Considering magnitude only]

The work done in increasing the length of the spring by an amount *x* can be calculated by integrating the above limits $x = 0$ to $x = x$ i.e,

$$W = \int_0^x kxdx = k\int_0^x xdx = k\left[\frac{x^2}{2}\right]_0^x$$

$$W = \frac{1}{2}kx^2$$

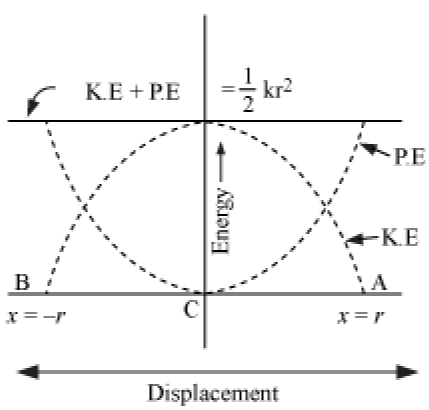
This work done is stored in the system as its potential energy at point P.

The potential energy of the system, when the block is pulled up to point A, can be obtained by setting $x = r$

∴P.E. of the system at point A $= \frac{1}{2}k(r)^2$

$$= \frac{1}{2}kr^2$$

If we plot the P.E and K.E against the displacement *x*, then the graph will be as depicted by the two dotted curves:



The Law of Conservation Of Energy and Power

Different Forms of Energy

- Internal energy − The sum of kinetic and potential energies of all the molecules constituting the body is called internal energy.
- Heat energy − A body possesses heat energy due to the disorderly motion of its molecules.
- Chemical energy − A body possesses chemical energy because of chemical bonding of its atoms.

  Exothermic reaction − A chemical reaction in which energy is released

  Endothermic reaction − A chemical reaction in which energy is absorbed

- Electrical energy − Work has to be done in order to move an electric charge from one point to another in an electric field. This work done appears as the electrical energy of the system.
- Nuclear energy − When a heavy nucleus (such as U − 235) breaks up into lighter nuclei on being bombarded by a slow neutron, a tremendous amount of energy is released. This energy is known as nuclear energy.

Principle of Conservation Of Energy

It states that energy can be neither created nor destroyed. It can only be converted from one form to another.

Power

The rate of doing work is called power. The average power is given by,

$$P_{av} = \frac{W}{t}$$

Where, *W* is work performed by the agent in time‘*t*’

Instantaneous power − Limiting value of the average power of an agent in a small time interval, when the time interval approaches zero.

If Δ*W* is work done in a small interval Δ*t*, then instantaneous power is defined as

$$P = \underset{\Delta t \to 0}{\mathrm{Lt}} \frac{\Delta W}{\Delta t} = \frac{\mathrm{d}W}{\mathrm{d}t}$$

Then,

$$\mathrm{d}P = \overrightarrow{F} \cdot \frac{\overrightarrow{\mathrm{d}s}}{\mathrm{d}t}$$

Again, $\frac{\overrightarrow{\mathrm{d}s}}{\mathrm{d}t} = \overrightarrow{v}$ the instantaneous velocity of the particle

Therefore,

$$\vec{P} = \vec{F} \cdot \vec{v}$$

If θ is angle between $\vec{F}$ and $\vec{v}$, then

$P = Fv \cos \theta$

If $\theta = 0°$, then

$P = Fv$

Unit of power − Watt

$$P = \frac{W}{t}$$

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ sec}} = 1 \text{Js}^{-1}$$

The bigger unit of power is kilowatt (kW).

1 kW = 10^3 W

Collisions

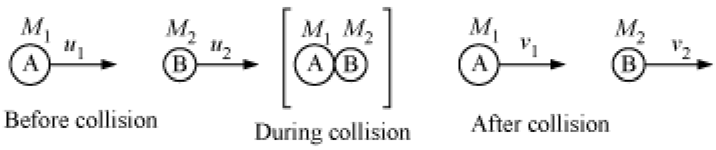
Collision between two particles is defined as mutual interaction of the particles for a short interval of time as a result of which the energy and momentum of the interacting particles change.

Types of Collision

- Elastic collision − Those collisions in which both momentum and kinetic energy of the system are conserved.
- Inelastic collision − Those collisions in which momentum of the system is conserved, but kinetic energy is not conserved.

Newton's Cradle:

Elastic Collision in One Dimension



Consider that two perfectly elastic bodies A and B of masses M_1 and M_2 moving with initial velocities u_1 and u_2 undergo head on collision and continue moving along the same straight line with final velocities v_1 and v_2.

As in an elastic collision, momentum is conserved.

$\therefore M_1u_1 + M_2u_2 = M_1v_1 + M_2v_2$ … (i)

Since kinetic energy is also conserved in an elastic collision, we obtain

$$\frac{1}{2}M_1u_1^2+\frac{1}{2}M_2u_2^2=\frac{1}{2}M_1v_1^2+\frac{1}{2}M_2v_2^2 \qquad ...(ii)$$

From equation (i), we obtain

$M_1 (u_1 − v_1) = M_2 (v_2 − u_2)$ … (iii)

From equation (ii), we obtain

$$M_1(u_1^2-v_1^2)=M_2(v_2^2-u_2^2) \qquad ...(iv)$$

Dividing equation (iv) by (iii), we obtain

$$\frac{u_1^2-v_1^2}{u_1-v_1}=\frac{v_2^2-u_2^2}{v_2-u_2}$$
$$u_1+v_1=v_2+u_2$$

$\therefore u_1 − u_2 = v_2 − v_1$ … (v)

From equation (v), it follows that in one-dimensional elastic collision, the relative velocity of approach ($u_1 - u_2$) before collision is equal to the relative velocity of separation ($v_2 - v_1$) after collision.

The ratio of relative velocity of separation after the collision to the relative velocity of the approach before the collision is known as coefficient of restitution or coefficient of resilience.

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

For perfectly elastic collision, $e = 1$

Calculation of velocities after collision:

Let us first find the velocity of body A after collision.

From equation (v), we have

$v_2 = u_1 - u_2 + v_1$

Substituting for v_2 in equation (i), we obtain

$$v_1 = \frac{(M_1 - M_2)u_1 + 2M_2u_2}{M_1 + M_2} \qquad \text{...(vi)}$$

Again, from equation (v), we have

$v_1 = v_2 - u_1 + u_2$

Substituting for v_1 in equation (i), we obtain

$$v_2 = \frac{(M_2 - M_1)u_2 + 2M_1u_1}{M_1 + M_2} \qquad \text{...(vii)}$$

Special Cases

- When the two bodies are of equal masses i.e.,

  $M_1 = M_2 = M$ (say)

  From equation (vi), we have

$$v_1 = \frac{(M-M)u_1 + 2Mu_2}{M+M} = \frac{0+2Mu_2}{2M}$$

$$\boxed{v_1 = u_2}$$

Also from equation (vii), we have

$$v_2 = \frac{(M-M)u_2 + 2Mu_1}{M+M} = \frac{0+2Mu_1}{2M}$$

$$\boxed{v_2 = u_1}$$

- When the target body (B) is at rest:

  In this case, $u_2 = 0$

  Substituting $u_2 = 0$ in equations (vi) and (vii), we obtain

$$v_1 = \frac{(M_1 - M_2)}{(M_1 + M_2)}u_1 \qquad \text{...(viii)}$$

$$v_2 = \frac{2M_1}{M_1 + M_2}u_1 \qquad \text{...(ix)}$$

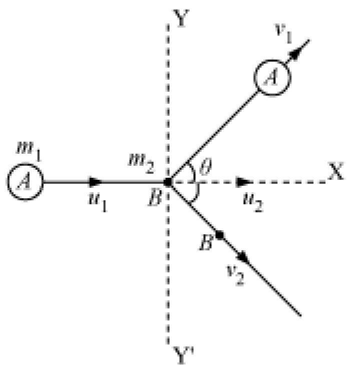
When $M_2 >> M_1$, in equation (viii) and (ix), M_1 can be neglected in comparison to M_2 i.e., $M_1 - M_2 \approx -M_2$ and $M_1 + M_2 \approx M_2$. Therefore, we have

$$v_1 = -\frac{M_2}{M_2}u_1 = -u_1 \text{ and } v_2 = \frac{2M_1}{M_2}u_1 \approx 0 (\because M_2 >> M_1)$$

Elastic Collision in Two Dimensions

Suppose m_1, m_2 are the masses of two bodies A and B moving initially along X-axis with velocities u_1 and u_2 respectively.

When $u_1 > u_2$, the two bodies collide. After collision, let body A move with a velocity v_1 at an angle θ with X-axis. Let body B move with a velocity v_2 at an angle Φ with X-axis.



As the collision is elastic, K.E. is conserved.

∴ Total K.E. after collision = Total K.E. before collision

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \qquad ...(i)$$

$$m_1v_1^2 + m_2v_2^2 = m_1u_1^2 + m_2u_2^2 \qquad ...(ii)$$

In elastic collision, linear momentum is also conserved (along X-axis).

∴ Total linear momentum after collision = Total linear momentum before collision

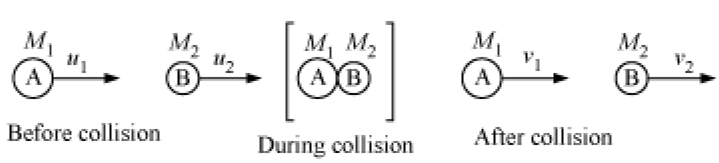
$$m_1v_1\cos\theta + m_2v_2\cos\phi = m_1u_1 + m_2u_2 \qquad ...(iii)$$

Along Y-axis, linear momentum before collision is zero (as both the bodies are moving along X-axis). After collision, total linear momentum along Y-axis is ($m_1v_1\sin\theta - m_2v_2\sin\Phi$).

$$\therefore \boxed{m_1v_1\sin\theta - m_2v_2\sin\phi = 0} \qquad ...(iv)$$

From equations (ii), (iii), and (iv), we have to calculate 4 variables v_1, v_2, θ, and Φ, which is not possible. We have to measure any one parameter experimentally.

Inelastic Collision in One Dimension



Consider that two bodies A and B of masses M_1 and M_2 moving with initial velocities u_1 and u_2 undergo head on collision and continue moving along the same straight line with final velocities v_1 and v_2.

As in an inelastic collision, momentum is conserved.

$\therefore M_1u_1 + M_2u_2 = M_1v_1 + M_2v_2$

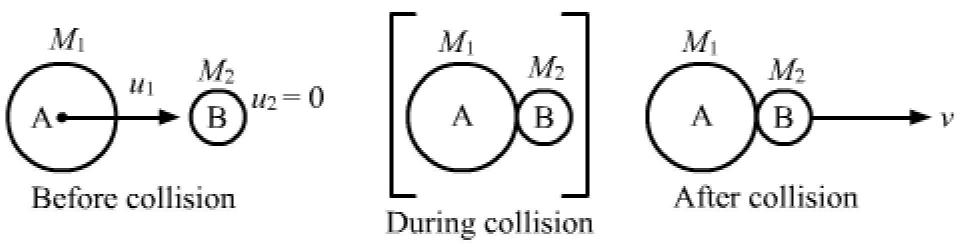
The kinetic energy of the system in inelastic collision is not conserved.

$$\frac{1}{2}M_1{u_1}^2 + \frac{1}{2}M_1{u_2}^2 \neq \frac{1}{2}M_1{v_1}^2 + \frac{1}{2}M_2{v_2}^2$$

Perfectly Inelastic Collision in One Dimension

Consider a perfectly inelastic collision between two bodies of masses M_1 and M_2. The mass M_2 was initially at rest ($u_2 = 0$) as shown in the figure. Here, the body of mass M_1 , moving with velocity u_1 collides with the body of mass M_2.

After the collision, the two bodies moves together with a common velocity v.



As in inelastic collision, the total linear momentum of the system remains constant. Therefore,

$$M_1u_1 + M_2u_2 = \left(M_1 + M_2\right)v$$

$$\Rightarrow M_1u_1 + 0 = \left(M_1 + M_2\right)v$$

$$\Rightarrow v = \frac{M_1u_1}{(M_1+M_2)}$$

As we know that, $\left(\frac{M_1}{M_1+M_2}\right) < 1$

Therefore, $v < u_1$.

Total kinetic energy before the collision is,

$$E_i = \frac{1}{2}M_1{u_1}^2$$

Total kinetic energy after the collision is,

$$E_f = \frac{1}{2}\left(M_1 + M_2\right)v^2$$

$$\Rightarrow E_f = \frac{1}{2}\left(M_1 + M_2\right)\left(\frac{M_1u_1}{M_1+M_2}\right)^2$$

$$\Rightarrow E_f = \frac{{M_1}^2{u_1}^2}{2(M_1+M_2)}$$

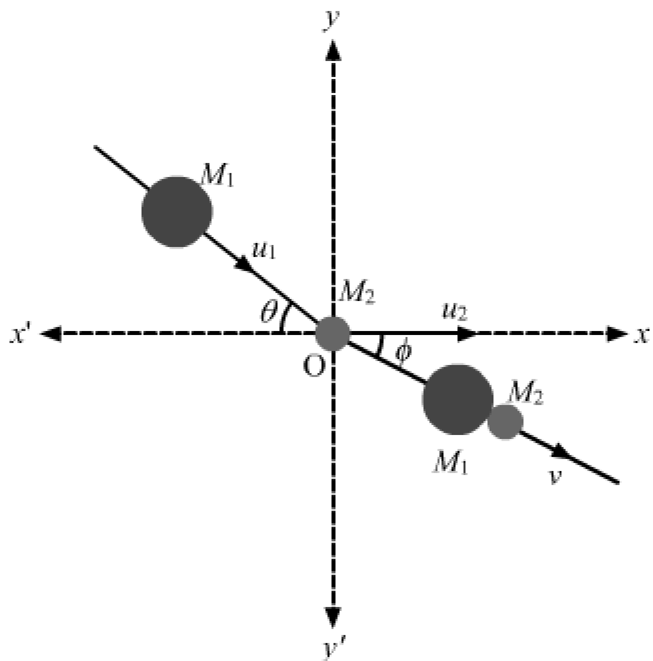
Loss in kinetic energy can be calculated by subtracting final kinetic energy from the initial kinetic energy,

$$E_{loss} = E_i - E_f = \frac{1}{2}M_1{u_1}^2 - \frac{{M_1}^2{u_1}^2}{2(M_1+M_2)}$$

$$\Rightarrow E_{loss} = \frac{M_1M_2{u_1}^2}{2(M_1+M_2)}$$

Perfectly Inelastic Collision in Two Dimensions

Consider a body of mass M_1 moving with velocity u_1 collides with another mass M_2 moving with velocity u_2 in perfectly inelastic way as shown in the figure.



Initially mass M_1 is moving with velocity u_1 at an angle θθ with x-axis and the mass M_2 is moving with velocity u_2 along the x-axis.

After the collision at point O, the two masses stick to each and start moving together with a new velocity v.

As in inelastic collision, the total linear momentum of the system remains constant. Applying conservation of momentum along x-axis, we get

$$M_1 u_1 \cos\theta + M_2 u_2 = (M_1 + M_2) v \cos\phi$$

Applying conservation of momentum along y-axis , we get

$$M_1 u_1 \sin\theta + 0 = (M_1 + M_2) v \sin\phi$$

v and ϕ can be calculated using these equations.