Sample Question Paper - 5

Mathematics (041)

Class- XII, Session: 2021-22

TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Evaluate: $\int \cos^3 x \sin 2x\, dx$. **[2]**

OR

Evaluate: $\int (x + 1) \log x\, dx$

2. Solve differential equation: $\frac{dy}{dx} - y \cot x = cosec\ x$ **[2]**

3. Find the value of a for which the vector $\vec{r} = \left(a^2 - 4\right) \hat{i} + 2\hat{j} - \left(a^2 - 9\right) \hat{k}$ make acute angles with the coordinate axes. **[2]**

4. Find the equation of a plane passing through the point P(6, 5, 9) and parallel to the plane determined by the points A(3, -1, 2), B(5, 2, 4) and (-1, -1, 6). Also, find the distance of this plane from the point A. **[2]**

5. If A and B are two independent events such that P ($\bar{A} \cap B$) = $\frac{2}{15}$ and P ($A \cap \bar{B}$) = $\frac{1}{6}$, then find P(B). **[2]**

6. If two events A and B are such that P ($\overline{A}$) = 0.3, P (B) = 0.4 and P (A ∩ $\overline{B}$) = 0.5, find P ($\frac{B}{\bar{A} \cap \bar{B}}$) **[2]**

Section B

7. Evaluate the integral: $\int \frac{1}{x\sqrt{1+x^n}} dx$ **[3]**

8. Verify that $y^2 = 4a(x + a)$ is a solution of the differential equation $y\left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$. **[3]**

OR

Find one-parameter families of solution curves of the differential equation: $x\frac{dy}{dx} - y = (x + 1) e^{-x}$

9. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, find the unit vector in the direction of $6\vec{b}$. **[3]**

10. Find the vector and Cartesian equations of the plane passing through the point (3, -1, 2) and parallel to the lines $\overrightarrow{r} = (-\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + \hat{k}) + \mu(-5\hat{i} + 4\hat{j})$. **[3]**

OR

Prove that the line through A (0, -1, -1) and B (4, 5,1) intersects the line through C (3,9,4) and D (-4, 4, 4).

Section C

11. Prove that: $\int_{0}^{\pi/2} x \cot x dx = \frac{\pi}{4}(\log 2)$. **[4]**

12. Find the area of circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$. **[4]**

OR

Find the area between the curves $y = x$ and $y = x^2$

13. Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. **[4]**

CASE-BASED/DATA-BASED

14. In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class. **[4]**

Based on the above information, answer the following questions.

i. The probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics?

ii. The probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics?

Solution

MATHEMATICS BASIC 041

Class 12 - Mathematics

Section A

1. Here,
$I = \int$ sin 2 x cos ^{3} x dx
$\Rightarrow \int$ 2 sin x cos x cos ^{3}x dx
$\Rightarrow \int$ 2 sin x cos ^{4} x dx
Now put cos x = t
$\Rightarrow$-sin x dx = dt
$\Rightarrow -2\int t^4 dt$
$\Rightarrow -2 \times \frac{t^5}{5} + c$
Re-substituting the value of t = cos x we get,
$\Rightarrow \frac{-2\cos^5 x}{5} + c$

OR

Let I = $\int$(x + 1) log x dx, then we have
I = log x $\int$(x + 1) dx - $\int \left(\frac{1}{x}\right)(x+1)dx\Big) dx$
= $\left(\frac{x^2}{2} + x\right)\log x - \int \frac{1}{x}\left(\frac{x^2}{2} + x\right) dx$
= $\left(\frac{x^2}{2} + x\right)$log x - $\frac{1}{2}\int x dx - \int dx$
= $\left(\frac{x^2}{2} + x\right)$log x - $\frac{1}{2} \times \frac{x^2}{2}$- x + C
I = $\left(\frac{x^2}{2} + x\right)$log x - $\left(\frac{x^2}{4} + x\right)$+ C

2. Given that$\frac{dy}{dx} - y\cot x = \cos ecx$
It is linear differential equation.
Comparing it with $\frac{dy}{dx}$ + py = Q
P = -cot x, Q = cosec x
I.F. = $e^{\int Pdx}$
= $e^{-\int \cot x dx}$
= $e^{-|log|\sin x|}$
= cosec x
Solution of the given equation is given by,
y (I.F.) $= \int Q \times (1.F.)dx + c$
$y \cos ecx = \int \cos ecx \times \cos ecx dx + c$
y cosec x = $\int \csc^2 x dx + c$
y cosec x = -cot x + c

3. We know that, For vector $\vec{r}$ to be inclined with acute angles with the coordinate axes, we must have,
$\vec{r}\cdot\hat{i} > 0, \vec{r}\cdot\hat{j} > 0$ and $\vec{r}.\hat{k} > 0$
$\Rightarrow \vec{r}\cdot\hat{i} > 0$ and $\vec{r}\cdot\hat{k} > 0$ $[\because \vec{r}.\hat{j} = 2 > 0]$
$\Rightarrow$(a^2 - 4) > 0 and - (a^2 - 9)>0$\left[\because \vec{r}.\hat{i} = a^2 - 4 \text{ and } \vec{r}\cdot\hat{k} = -\left(a^2 - 9\right)\right]$
$\Rightarrow$ (a - 2)(a + 2) > 0 and (a + 3)(a - 3) < 0
$\Rightarrow$ a < -2 or, a > 2 and -3 < a < 3
$\Rightarrow a \in (-3, -2) \cup (2, 3)$

4. We have a vector $\vec{n}$ normal to the plane determined by the points A (3, -1 , 2), B(5, 2, 4) and C(-1, -1, 6) is given by $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$

$\therefore \vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 12\hat{i} - 16\hat{j} + 12\hat{k}$

Clearly, $\vec{n} = 12\hat{i} - 16\hat{j} + 12\hat{k}$ is also normal to the plane passing through P(6,5, 9) and parallel to the plane determined by point A, B and C. So, its equation is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$ and $\vec{a} = 6\hat{i} + 5\hat{j} + 9\hat{k}$

or, $\vec{r} \cdot (12\hat{i} - 16\hat{j} + 12\hat{k}) = (12\hat{i} - 16\hat{j} + 12\hat{k}) \cdot (6\hat{i} + 5\hat{j} + 9\hat{k})$

or, $\vec{r} \cdot (12\hat{i} - 16\hat{j} + 12\hat{k}) = 72 - 80 + 108$

or, $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) = 25$

The cartesian equation of this plane is 3x - 4y + 3z = 25

Hence The required distance d of this plane from point A(3, -1, 2) is given by

$d = \left|\frac{3\times3-4\times-1+3\times2-25}{\sqrt{9+16+9}}\right| = \frac{6}{\sqrt{34}}$

5. Let: P(A) = x, P(B) = y

$P(\bar{A} \cap B) = \frac{2}{15}$

$\Rightarrow P(\bar{A}) \times P(B) = \frac{2}{15}$

$\Rightarrow (1 - x)y = \frac{2}{15}$ ...(i)

$P(A \cap \bar{B}) = \frac{1}{6}$

$\Rightarrow P(A) \times P(B) = \frac{1}{6}$

$\Rightarrow$ (1 - y)x = $\frac{1}{6}$ ...(ii)

subtracting (i) from (ii), we get,

x - y = $\frac{1}{30}$

x = $y + \frac{1}{30}$

putting the value of x in (ii), we have,

$\left(y + \frac{1}{30}\right)(1 - y) = \frac{1}{6}$

$\Rightarrow$ $30y^2$ - 29y + 4 = 0

$\Rightarrow$ y = $\frac{1}{6}, \frac{4}{5}$

6. According to Baye's Theorem

$P(\frac{B}{\bar{A}\cap\bar{B}}) = \frac{P(B\cap(\bar{A}\cap\bar{B}))}{P(\bar{A}\cap\bar{B})}$

$= \frac{P(B\cap\overline{(A\cup B)})}{P(\bar{A}\cap\bar{B})}$

$= \frac{P(\bar{B}\cap\overline{(A\cup B)})}{P(\bar{A}\cap B)}$

$= \frac{P(B\cup(A\cup B))}{P(\bar{A}\cap B)}$

Now $\bar{B} \cup B = \bar{U} = \phi$

So $P(\bar{B} \cup (A \cup B)) = \phi$

Therefore $P(\frac{B}{\bar{A}\cap\bar{B}}) = 0$

Section B

7. Let the given integral be,

$I = \int \frac{dx}{x\sqrt{1+x^n}}$

$= \int \frac{x^{n-1}dx}{x^{n-1}x^1\sqrt{1+x^n}}$

$= \int \frac{x^{n-1}dx}{x^n\sqrt{1+x^n}}$

Putting x^n = t

$\Rightarrow$ n x^{n-1}dx = dt

$\Rightarrow x^{n-1}$dx $= \frac{dt}{n}$

$\therefore I = \frac{1}{n}\int \frac{dt}{t\sqrt{1+t}}$

let 1 + t = p^2

$\Rightarrow$ dp = 2p dp

$\therefore I = \frac{1}{n} \int \frac{2\text{pdp}}{(p^2-1)p}$

$= \frac{2}{n} \int \frac{dp}{p^2-1^2}$

$= \frac{2}{n} \times \frac{1}{2} \log \left| \frac{p-1}{p+1} \right| + C$

$= \frac{1}{n} \log \left| \frac{\sqrt{1+t}-1}{\sqrt{1+t}+1} \right| + C$

$= \frac{1}{n} \log \left| \frac{\sqrt{1+x^n}-1}{\sqrt{1+x^n}+1} \right| + C$

8. The given functional relation is,

$y^2 = 4a(x + a)$

Differentiating above equation with respect to x

$2y\frac{dy}{dx} = 4a$

Substituting above results in

$y\left(1-\left(\frac{dy}{dx}\right)^2\right) = 2x\frac{dy}{dx}$,we get,

$y\left(1-\left(\frac{dy}{dx}\right)^2\right) = 4\frac{ax}{y}$

=> y - $\frac{4a^2}{y} = 4\frac{ax}{y}$

$\Rightarrow \frac{y^2-4a(a+x)}{y} = 0$

$\Rightarrow \frac{4a(a+x)-4a(a+x)}{y} = 0$

$\Rightarrow \ 0 = 0$,which is true.

$\therefore$ $y^2 = 4a(x + a)$ is the solution of $y\left(1-\left(\frac{dy}{dx}\right)^2\right) = 2x\frac{dy}{dx}$

OR

The given differential equation is,

$x\frac{dy}{dx} - y = (x + 1)\, e^{-x}$

$\frac{dy}{dx} - \frac{y}{x} = \left(\frac{x+1}{x}\right) e^{-x}$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx} + Py = Q$

$P = -\frac{1}{x}$, $Q = \left(\frac{x+1}{x}\right) e^{-x}$

$\text{I.F.} = e^{\int pdx}$

$= e^{-\int \frac{1}{x} dx}$

$= e^{-\log|x|}$

$= e^{\text{tag}\left(\frac{1}{x}\right)}$

$= \frac{1}{x}$, x > 0

Solution of the equation is given by,

$y \times (\text{I.F.}) = \int Q \times (\text{I.F.})\, dx + c$

$y \times \left(\frac{1}{x}\right) = \int \left(\frac{x+1}{x}\right) e^{-x} \times \left(\frac{1}{x}\right) dx + c$

$\frac{y}{x} = \int \left(\frac{1}{x} + \frac{1}{x^2}\right) e^{-x}\, dx + c$

Let -x = t

-dx = dt

$y\left(-\frac{1}{x}\right) = \int \left(-\frac{1}{t} + \frac{1}{t^2}\right) e^t\, dt + c$

$y\left(-\frac{1}{x}\right) = -\frac{1}{t}\, e^t + c$

[Since $\int \{f(x) + f'(x)\}\, e^x\, dx = f(x)\, e^x + c$]

$-\frac{y}{x} = \frac{1}{x}\, e^{-x} + c$

y = -(e^{-x} + cx)

y = -e^x + c_1x ,where $c_1 = -c$

9. We have,

$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

We need to find the unit vector in the direction of $6\vec{b}$.

First, let us calculate $6\vec{b}$.

As we have,

$\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

Multiply it by 6 on both sides.

$\Rightarrow 6\vec{b} = 6(2\hat{i} + \hat{j} - 2\hat{k})$

For finding unit vector, we have the formula:

$6\hat{b} = \frac{6\vec{b}}{|6\vec{b}|}$

Now we know the value of $6\vec{b}$, so just substitute the value in the above equation.

$\Rightarrow 6\hat{b} = \frac{12\hat{i}+6\hat{j}-12\hat{k}}{|12\hat{i}+6\hat{j}-12\hat{k}|}$

Here, $|12\hat{i} + 6\hat{j} - 12\hat{k}| = \sqrt{12^2 + 6^2 + (-12)^2}$

$\Rightarrow 6\hat{b} = \frac{12\hat{i}+6\hat{j}-12\hat{k}}{\sqrt{144+36+144}}$

$\Rightarrow 6\hat{b} = \frac{12\hat{i}+6\hat{j}-12\hat{k}}{\sqrt{324}}$

$\Rightarrow 6\hat{b} = \frac{12\hat{i}+6\hat{j}-12\hat{k}}{18}$

Let us simplify.

$\Rightarrow \widehat{6b} = \frac{6(2\hat{i}+\hat{j}-2\hat{k})}{18}$

$\Rightarrow \widehat{6b} = \frac{2\hat{i}+\hat{j}-2\hat{k}}{3}$

Thus, unit vector in the direction of $6\vec{b}$ is $\frac{2\hat{i}+\hat{j}-2\hat{k}}{3}$.

10. We know that $(\vec{r} - \vec{a})(\vec{b} \times \vec{c}) = 0$

Here $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$

$\vec{b} = 2\hat{i} - 5\hat{j} - \hat{k}$ and $\vec{c} = -5\hat{i} + 4\hat{j}$

Now, $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{vmatrix} = \begin{vmatrix} -5 & -1 \\ 4 & 0 \end{vmatrix}\hat{i} - \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix}\hat{j} + \begin{vmatrix} -2 & -5 \\ -5 & 4 \end{vmatrix}\hat{k} = 4\hat{i} + 5\hat{j} - 17\hat{k}$

Therefore the required equation is $(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$

$\Rightarrow [(x-3)\hat{i} + (y+1)\hat{j} + (z-2)\hat{k}] \cdot (4\hat{i} + 5\hat{j} - 17\hat{k}) = 0$

$\Rightarrow$ 4(x - 3) + 5(y + 1) - 17(z - 2) = 0

$\Rightarrow$ 4x - 12 + 5y + 5 - 17z + 34 = 0

$\Rightarrow$ 4x + 5y - 17z + 27 = 0

This is the Cartesian of plane

The required vector equation of the plane is $\vec{r} \cdot (4\hat{i} + 5\hat{j} - 17\hat{k}) + 27 = 0$

OR

The equation of line through $A(0,-1,-1)$ and $B(4,5,1)$ is

$\frac{x-0}{4-0} = \frac{y+1}{5+1} = \frac{z+1}{1+1}$

i.e. $\frac{x}{4} = \frac{y+1}{6} = \frac{z+1}{2}$.....(i)

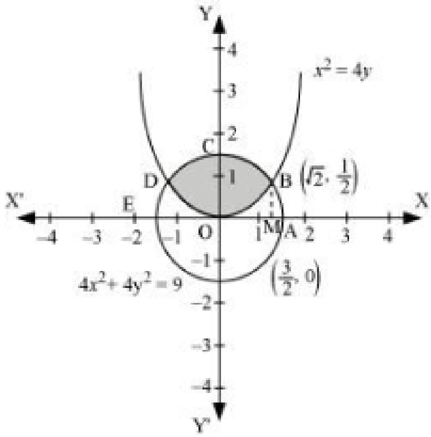
Equation of line through $C(3,9,4)$ and $D(-4,4,4)$ is

$\frac{x-3}{-4-3} = \frac{y-9}{4-9} = \frac{z-4}{0}$

i.e., $\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0}$ ....(ii)
We know that, the lines
$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and
$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ will intersect,
if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$
:. The given lines will intersect, if
$\begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}=0$
Now,
$\begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}=\begin{vmatrix} 3 & 10 & 5 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}$
$=3(0+10-10(0+14)+5(-20+42)$
$=30-140+110=0$
Hence, the given lines intersect.

Section C

11. To solve this we Use integration by parts that is,
$\int I\times II\,dx=I\int II\,dx-\int\frac{d}{dx}I\left(\int II\,dx\right)dx$
$y=x\int\cot x\,dx-\int\frac{d}{dx}x\left(\int\cot x\,dx\right)dx$
$y=(x\log\sin x)_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\log\sin x dx$
Let, $I=\int_0^{\frac{\pi}{2}}\log\sin x dx$ .... (i)
Use King theorem of definite integral
$\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$
$I=\int_0^{\frac{\pi}{2}}\log\sin\left(\frac{\pi}{2}-x\right)dx$
$I=\int_0^{\frac{\pi}{2}}\log\cos x dx$ ..... (ii)
Adding eq. (i) and (ii) we get,
$2I=\int_0^{\frac{\pi}{2}}\log\sin x dx+\int_0^{\frac{\pi}{2}}\log\cos x dx$
$2I=\int_0^{\frac{\pi}{2}}\log\frac{2\sin x\cos x}{2}dx$
$2I=\int_0^{\frac{\pi}{2}}\log\sin 2x-\log 2dx$
Let, 2x = t
⇒ 2dx =dt
At x = 0, t = 0
At $x=\frac{\pi}{2},t=\pi$
$2I=\frac{1}{2}\int_0^{\pi}\log\sin t dt-\frac{\pi}{2}\log 2$
$2I=\frac{2}{2}\int_0^{\frac{\pi}{2}}\log\sin x dx-\frac{\pi}{2}\log 2$
$2I=I-\frac{\pi}{2}\log 2$
$I=\int_0^{\frac{\pi}{2}}\log\sin x dx=-\frac{\pi}{2}\log 2$
$y=(x\log\sin x)_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\log\sin x dx$
$y=\frac{\pi}{2}\log\sin\frac{\pi}{2}-\left(-\frac{\pi}{2}\log 2\right)$
$y=\frac{\pi}{2}\log 2$
Hence proved..

12. 

Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the point of intersection as B$\left(\sqrt{2}, \frac{1}{2}\right)$ and D$\left(-\sqrt{2}, \frac{1}{2}\right)$

It can be observed that the required area is symmetrical about y-axis.

$\therefore$ Area OBCDO = 2 $\times$ Area OBCO

We draw BM perpendicular to OA.

Therefore, the coordinates of M are ($\sqrt{2}$, 0)

Therefore, Area OBCO = Area OMBCO - Area OMBO

$= \int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}} dx - \int_0^{\sqrt{2}} \frac{x^2}{4} dx$

$= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9-4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx$

$= \frac{1}{4} \left[x\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$

$= \frac{1}{4}\left[\sqrt{2}\sqrt{9-8} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{1}{12}(\sqrt{2})^3$

$= \frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$

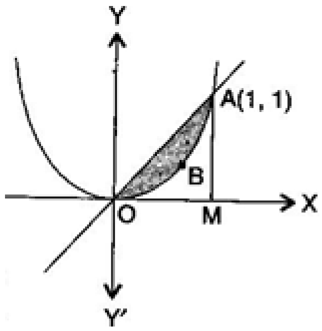
$= \frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$

$= \frac{1}{2}\left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right)$

Therefore, the required area OBCDO = $2 \times \frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]$ = $\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]$ sq. units.

OR

Equation of one curve (straight line) is y = x .....(i)



Equation of second curve (parabola) is $y = x^2$ ... (ii)

Solving eq. (i) and (ii), we get x = 0 or x = 1 and y = 0 or y = 1

$\therefore$ Points of intersection of line (i) and parabola (ii) are O (0, 0) and A (1, 1).

Now Area of triangle OAM

= Area bounded by line (i) and x - axis

$= \left|\int_0^1 y dx\right| = \left|\int_0^1 x dx\right| = \left(\frac{x^2}{2}\right)_0^1$

$= \frac{1}{2} - 0 = \frac{1}{2}$ sq units

Also Area OBAM = Area bounded by parabola (ii) and x - axis

$= \left|\int_0^1 y dx\right| = \left|\int_0^1 x^2 dx\right| = \left(\frac{x^3}{3}\right)_0^1$

$= \frac{1}{3} - 0 = \frac{1}{3}$ sq. units

$\therefore$ Required area OBA between line (i) and parabola (ii)

= Area of triangle OAM – Area of OBAM

$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$ sq. units

13. Suppose the required line is parallel to vector $\vec{b}$

Which is given by $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

We know that the position vector of the point (1, 2, 3) is given by

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$ ...(i)

The equation of the given planes are

$\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ ...(ii)

and $\vec{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$ ...(iii)

The line in Equation (i) and plane in Eq. (ii) are parallel.

Therefore, the normal to the plane of Eq. (ii) is perpendicular to the given line

$\therefore (\hat{i} - \hat{j} + 2\hat{k}) \cdot \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$

$\Rightarrow (b_1 - b_2 + 2b_3) = 0$ ....(iv)

Similarly, from Eqs. (i) and (iii), we get

$(3\hat{i} + \hat{j} + \hat{k}).(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$

$\Rightarrow (3b_1 + b_2 + b_3) = 0$......(v)

On solving Equations . (iv) and (v) by cross-multiplication, we get

$\frac{b_1}{(-1)\times1-1\times2} = \frac{b_2}{2\times3-1\times1} = \frac{b_3}{1\times1-3(-1)}$

$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$

Therefore, the direction ratios of $\vec{b}$ are (-3, 5, 4).

$\vec{b} = -3\hat{i} + 5\hat{j} + 4\hat{k}\left[\because \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right]$

On substituting the value of b in Equation (i), we get

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$

which is the equation of the required line.

CASE-BASED/DATA-BASED

14. Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore \quad P(E) = \frac{50}{100} = \frac{1}{2}, P(M) = \frac{35}{100} = \frac{7}{20}$ and $P(E \cap M) = \frac{25}{100} = \frac{1}{4}$

i. Required probability = P(E|M)

$= \frac{P(E\cap M)}{P(M)} = \frac{\frac{1}{4}}{\frac{7}{20}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$

ii. Required probability = P(M|E)

$= \frac{P(M\cap E)}{P(E)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$