
CBSE Sample Paper-05 (Solved)
Mathematics
Class – XII

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- d) Use of calculators is not permitted.

Section A

1. If a matrix has 12 elements, what are the possible orders it can have?
2. Prove that $A - A'$ is skew symmetric.
3. Find the values of x, y, z s.t $\begin{bmatrix} x-y & 0 & 0 \\ z & 6 & 0 \\ 0 & 0 & 2y \end{bmatrix}$ is a scalar matrix?
4. Is R defined on the set $A = \{1, 2, 3, \dots, 14, 15\}$ defined as $R = \{(x, y) : 3x - y = 0\}$ reflexive?
5. Find the angle between the vectors $\vec{a} = 4i + 4j$ and $\vec{b} = 4i - 2j$.
6. Evaluate $\sin^{-1}\left(\frac{1}{6}\right) + \cos^{-1}\left(\frac{1}{6}\right)$?

Section B

7. If $A = \begin{bmatrix} 0 & -\tan \alpha / 2 \\ \tan \alpha / 2 & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that
$$I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$
 8. Find the equations of the tangent and the normal to the curve $x = \cos t, y = \sin t$ at $t = \frac{\pi}{4}$.
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9. Show that the relation R in the set \mathbb{Z} of integers given by $R=\{(a,b):7 \text{ divides } a-b\}$ is an equivalence relation.

10. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots}}}$ prove that $(2y-1) \frac{dy}{dx} = \frac{1}{x}$.

11. Solve: $3 \sin^{-1} \frac{2x}{1+x^2} - 4 \cos^{-1} \frac{1-x^2}{1+x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$

12. Find a unit vector perpendicular to each of the vectors $(\vec{a}-\vec{b})$ and $(\vec{a}+\vec{b})$ where $\vec{a} = i + j + k$ and $\vec{b} = i + 2j + 3k$.

13. If A and B are independent events such that $P(A \cup B) = 0.6$, $P(A) = 0.2$. Find $P(B)$

14. The relation between the total cost y and the total output x is given by

$y = \frac{3x(x+7)}{x+5} + 5$. Prove that the marginal cost continuously falls as output increases.

15. Solve $\frac{dy}{dx} + \frac{2y}{3} = \frac{x}{\sqrt{y}}$

16. For any two vectors \vec{a} and \vec{b} , prove that $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.

17. Integrate $\int \frac{e^x}{e^{2x} - 4} dx$. Write ant points for promoting national integration.

18. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2i + 2j - 3k) = 7$, $\vec{r} \cdot (2i + 5j + 3k) = 9$ and the point $(2, 1, 3)$.

19. Find the equation of the plane passing through the line

$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point $(0, 7, -7)$. Show that the line $x = \frac{7-y}{3} = \frac{z+7}{2}$ lies on the plane.

Section C

20. Prove that the volume of the largest cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ of the volume of the sphere.

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21. Solve the following system of equations using matrix method

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

22. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per kg of vitamin A and 2 unit per kg of vitamin C. Food 1 costs Rs.50 per kg and Food 2 costs Rs.70 per kg. Using linear programming, find the minimum cost of such a mixture.
23. Draw a rough sketch of the region $\{(x, y) : y^2 < 4x, 4x^2 + 4y^2 \leq 9\}$ and find the area enclosed.
24. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assume that a student who guesses the answer would answer correctly with probability $\frac{1}{4}$. What is the probability that a student knows the answer, given that he has answered it correctly.

25. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

26. Integrate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log(\sin x) dx$.

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ANSWERS

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Section A

1. Solution:

$(1,12),(2,6),(3,4),(4,3),(6,2),(12,1)$

2. Solution:

$$(A - A')' = A' - (A')' = A' - A = -(A - A')$$

Thus, $A - A'$ is skew symmetric.

3. Solution:

A scalar matrix is a diagonal matrix with equal diagonal elements.

$$\therefore z = 0, 2y = 6, x - y = 6$$

$$\Rightarrow x = 9, y = 3, z = 0$$

4. Solution:

NO.

$$\text{Let } 1 \in A, 3(1) - 1 = 2 \neq 0$$

Thus, $(1,1) \notin R$

5. Solution:

$$\vec{a} = 4i + 4j \text{ and } \vec{b} = 4i - 2j$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{16 - 8}{(4\sqrt{2}) 2\sqrt{5}} = \frac{8}{8\sqrt{10}} \Rightarrow \theta = \cos^{-1} \left(\frac{8}{8\sqrt{10}} \right)$$

6. Solution:

$$\frac{\pi}{2}$$

Section B

7. Solution:

$$\begin{aligned}
 I - A &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan \alpha / 2 \\ \tan \alpha / 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan \alpha / 2 \\ -\tan \alpha / 2 & 1 \end{bmatrix} \\
 (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} &= \begin{bmatrix} 1 & \tan \alpha / 2 \\ -\tan \alpha / 2 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\
 &= \begin{bmatrix} \cos \alpha + (\tan \alpha / 2) \sin \alpha & -\sin \alpha + (\tan \alpha / 2) \cos \alpha \\ \sin \alpha - (\tan \alpha / 2) \cos \alpha & \cos \alpha + (\tan \alpha / 2) \sin \alpha \end{bmatrix} \\
 &= \begin{bmatrix} (2 \cos^2 \alpha / 2 - 1) + (\tan \alpha / 2)(2 \sin \alpha / 2 \cos \alpha / 2) & -(2 \sin \alpha / 2 \cos \alpha / 2) + (\tan \alpha / 2)(2 \cos^2 \alpha / 2 - 1) \\ (2 \sin \alpha / 2 \cos \alpha / 2) - (\tan \alpha / 2)(2 \cos^2 \alpha / 2 - 1) & (2 \cos^2 \alpha / 2 - 1) + (\tan \alpha / 2)(2 \sin \alpha / 2 \cos \alpha / 2) \end{bmatrix} \\
 &= \begin{bmatrix} 1 & -\tan \alpha / 2 \\ \tan \alpha / 2 & 1 \end{bmatrix} = I + A
 \end{aligned}$$

8. Solution:

$$\text{When } t = \frac{\pi}{4}, x = \frac{1}{\sqrt{2}}, y = \frac{1}{\sqrt{2}}$$

$$\therefore \text{ point of contact } \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

$$y = \sin t \Rightarrow \frac{dy}{dt} = \cos t, x = \cos t \Rightarrow \frac{dx}{dt} = -\sin t$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t} = -\cot t$$

$$\therefore \text{ slope at } t = \frac{\pi}{4} = -\cot \frac{\pi}{4} = -1$$

$$\therefore \text{ equation of tan gent } = (y - \frac{1}{\sqrt{2}}) = -1(x - \frac{1}{\sqrt{2}}) \Rightarrow x + y - \sqrt{2} = 0$$

$$\text{Slope of normal} = m, m(-1) = -1 \Rightarrow m = 1$$

$$\therefore \text{ equation of normal } = (y - \frac{1}{\sqrt{2}}) = 1(x - \frac{1}{\sqrt{2}}) \Rightarrow x - y = 0$$

9. Solution:

Reflexive:

Let $a \in \mathbb{Z}$

$a - a = 0$, 7 divides 0.

$$\therefore (a, a) \in R \forall a \in \mathbb{Z}$$

Hence, R is reflexive.

Symmetric:

$$\begin{aligned} \text{Suppose } (a, b) \in R &\Rightarrow 7 \mid a - b \Rightarrow a - b = 7k \text{ f.s. } k \in \mathbb{Z} \\ &\Rightarrow b - a = -7k = 7k' \text{ f.s. } k' \in \mathbb{Z} \\ \therefore (b, a) &\in \mathbb{Z} \end{aligned}$$

Transitive:

$$\begin{aligned} \text{Suppose } (a, b), (b, c) &\in R \\ &\Rightarrow b - a = 7m \text{ f.s. } m \in \mathbb{Z}, c - b = 7n \text{ f.s. } n \in \mathbb{Z} \\ \therefore c - a &= c - b + b - a = 7(n - m), \text{ where } n - m \in \mathbb{Z} \\ \therefore (a, c) &\in R \end{aligned}$$

Hence the above relation is an equivalence relation.

10. Solution:

$$\begin{aligned} y &= \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots}}} \\ &\Rightarrow y = \sqrt{\log x + y} \\ &\Rightarrow y^2 = \log x + y \end{aligned}$$

Differentiating both sides w.r.t x, we get

$$2y \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx} \quad (2y - 1) \frac{dy}{dx} = \frac{1}{x}$$

11. Solution:

$$3 \sin^{-1} \left(\frac{2x}{1+x^2} \right) - 4 \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) + 2 \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \frac{\pi}{3}$$

$$3(2 \tan^{-1} x) - 4(2 \tan^{-1} x) + 2(2 \tan^{-1} x) = \frac{\pi}{3}$$

$$2 \tan^{-1} x = \frac{\pi}{3}$$

$$\tan^{-1} x = \frac{\pi}{6}$$

$$x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

12. Solution:

$$\vec{a} = i + j + k, \vec{b} = i + 2j + 3k$$

$$\therefore \vec{a} + \vec{b} = 2i + 3j + 4k, \vec{a} - \vec{b} = -j - 2k$$

Vector \perp to $\vec{a} + \vec{b}, \vec{a} - \vec{b}$ is $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2i + 4j - 2k = \vec{c}$$

$$\therefore |\vec{c}| = 2\sqrt{6}$$

$$\text{Unit vector} = \frac{-2i + 4j - 2k}{2\sqrt{6}}$$

13. Solution:

Since A and B are independent events $P(A \cap B) = P(A)P(B)$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0$$

$$\Rightarrow 0.6 = 0.2 + P(B) - .2P(B)$$

$$\Rightarrow P(B) = \frac{.4}{.8} = \frac{1}{2}$$

14. Solution:

$$\text{Cost} = y = \frac{3x(x+7)}{x+5} + 5 = \frac{3x^2 + 21}{x+5} + 5$$

$$\therefore MC = \frac{dy}{dx} = \frac{(x+5)(6x) - (3x^2 + 21) \cdot 1}{(x+5)^2} = \frac{3x^2 + 30x + 105}{(x+5)^2} = 3 + \frac{30}{(x+5)^2}$$

$$\frac{d}{dx}(MC) = \frac{-60}{(x+5)^2} < 0$$

Thus, marginal cost is a decreasing function of output(x).

15. Solution:

$$\frac{dy}{dx} + \frac{2y}{3} = \frac{x}{\sqrt{y}}$$

$$\sqrt{y} \frac{dy}{dx} + \frac{2y^{3/2}}{3} = x$$

$$\text{Let } z = y^{3/2} \quad \frac{dz}{dx} = \frac{3}{2} \sqrt{y} \frac{dy}{dx}$$

$$\frac{2}{3} \frac{dz}{dx} - \frac{2}{3} z = x$$

$$\frac{dz}{dx} - z = \frac{3}{2} x$$

$$\therefore P=1, Q=\frac{3}{2}x$$

$$\therefore I.F = e^{\int P dx} = e^x$$

Solution is :

$$ze^x = \int \frac{3}{2} xe^x dx$$

$$y^{3/2} e^x = \frac{3}{2} x e^x - e^x + c$$

$$y^{3/2} = \frac{3}{2} (x - 1) + C e^{-x}$$

16. Solution:

$$\begin{aligned} |\vec{a} + \vec{b}|^2 &= (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) \\ &= \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} \\ &= |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 \leq |\vec{a}|^2 + 2|\vec{a} \cdot \vec{b}| + |\vec{b}|^2 \leq |\vec{a}|^2 + 2|\vec{a}||\vec{b}| + |\vec{b}|^2 \\ &= (|\vec{a}| + |\vec{b}|)^2 \\ \therefore |\vec{a} + \vec{b}| &\leq |\vec{a}| + |\vec{b}| \end{aligned}$$

17. Solution:

$$I = \int \frac{e^x}{e^{2x} - 4} dx$$

$$\text{Let } e^x = t \quad e^x dx = dt$$

$$I = \int \frac{1}{t^2 - 4} dt = \int \frac{1}{(t-2)(t+2)} dt$$

$$= \frac{1}{4} \left(\frac{1}{t-2} - \frac{1}{t+2} \right) dt$$

$$= \frac{1}{4} (\log|t-2| - \log|t+2|) + c$$

$$= \frac{1}{4} \log \left| \frac{t-2}{t+2} \right| + c$$

$$= \frac{1}{4} \log \left| \frac{e^x - 2}{e^x + 2} \right| + c$$

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1. We should not judge people by their religion.
 2. Women should be empowered by educating them.

18 Solution:

$$\vec{n}_1 = 2i + 2j - 3k, d_1 = 7$$

$$\vec{n}_2 = 2i + 5j + 3k, d_2 = 9$$

Equation of plane :

$$\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$$

$$\vec{r} \cdot (2i + 2j - 3k + \lambda(2i + 5j + 3k)) = 7 + 9\lambda$$

$$\text{Let } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\therefore x(2 + 2\lambda) + y(2 + 5\lambda) + z(-3 + 3\lambda) = 7 + 9\lambda$$

$$\text{Putting } (x, y, z) = (2, 1, 3) \text{ we get } \lambda = \frac{10}{9}$$

$$\text{Substituting the value of } \lambda \text{ we get, } \vec{r} \cdot (38i + 68j + 3k) = 153$$

19. Solution:

Equation of any plane containing the given line is:

$$A(x+1) + B(y-3) + C(z+2) = 0, \text{ where } -3A + 2B + C = 0$$

Since, plane passes through (0, 7, -7) we have,

$$A(1) + B(4) + C(-5) = 0, \text{ i.e } A + 4B - 5C = 0$$

Solving the above equations for A, B, C we have

$$\frac{A}{-10-4} = \frac{B}{1-15} = \frac{C}{-12-2} \Rightarrow \frac{A}{1} = \frac{B}{1} = \frac{C}{1} = k$$

Thus, substituting the value of A, B, C we get the equation of the plane as $x + y + z = 0$

Now, the line $x = \frac{7-y}{3} = \frac{z+7}{2}$ or $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ lies on the plane if it is parallel to the plane and any point on it lies on the plane.

Since, $1(1) + (-3)(1) + 2(1) = 0 \Rightarrow$ line is parallel to the plane.

Again as (0, 7, -7) lies on the plane, we conclude the line lies on the plane.

Section C

20. Solution:

Consider a sphere of radius a with center O s.t. $OD=x$ and $DC=r$.

Let h be the height of the cone.

Then, $h=OA+x=a+x$

Also, for $\triangle ODC$, $x^2 + r^2 = a^2$

Let V be the volume of the cone.

$$\therefore V = \frac{1}{3} \pi r^2 h$$

$$\Rightarrow V(x) = \frac{1}{3} \pi (a^2 - x^2)(a + x)$$

$$\therefore V'(x) = \frac{1}{3} \pi [(a^2 - x^2) + (a + x)(-2x)] = \frac{\pi}{3} (a + x)(a - 3x)$$

$$\text{Also, } V''(x) = \frac{\pi}{3} [(a + x)(-3x) + (a - 3x)1]$$

$$V'(x) = 0 \Rightarrow x = -a, \frac{a}{3}$$

Neglecting $x = -a$,

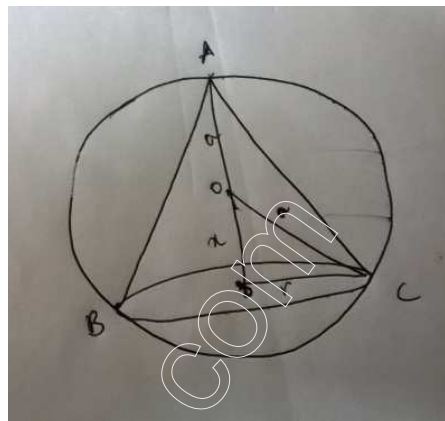
$$V''\left(\frac{a}{3}\right) = \frac{-4\pi a}{3} < 0$$

\therefore volume is maximum when $x = a/3$.

$$\therefore h = a + \frac{a}{3} = \frac{4a}{3}$$

$$r^2 = a^2 - \frac{a^2}{9} = \frac{8a^2}{9}$$

$$\therefore \text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left(\frac{8a^2}{9} \right) \left(\frac{4a}{3} \right) = \frac{8}{27} \left(\frac{4}{3} \pi a^3 \right)$$



21. Solution:

$$\text{Let } \frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w$$

\therefore the system of equations becomes,

$$2u + 3v + 10w = 4$$

$$4u - 6v + 5w = 1$$

$$6u + 9v - 20w = 2$$

$$\text{Let } A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$|A| = 1200 \neq 0, A^{-1} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}, U = A^{-1}b = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$$\therefore u = 1/2 \Rightarrow x = 2, v = 1/3 \Rightarrow y = 3, w = 1/5 \Rightarrow z = 5$$

22. Solution:

Suppose the mixture contains x kg of food 1 and y kgs of food 2.

Then, Cost $Z = 50x + 70y$

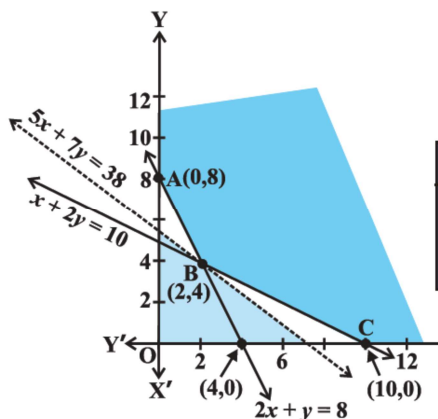
The mathematical formulation of the problem is as follows:

$$\text{Min } Z = 50x + 70y$$

$$2x + y \geq 8 (\text{requirement of VitA})$$

$$\text{s.t } x + 2y \geq 10 (\text{requirement of VitC})$$

$$x \geq 0, y \geq 0$$



We graph the above inequalities. The feasible region as shown in the figure is unbounded. The corner points are A,B and C. The co-ordinates of the corner points are (0,8), (2,4),(10,0).

Corner Point	$Z=50x + 70y$
(0,8)	560
(2,4)	<u>380</u>
(10,0)	500

Thus cost is minimized by mixing 2 units of food 1 and 4 units of food 2 and minimum cost is 380.

23. Solution:

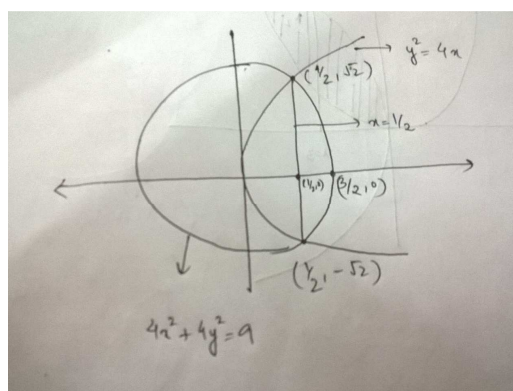
The point of intersection of the curves $y^2=4x$, $4x^2+4y^2=9$:

$$4x^2 + 4(4x) = 9 \quad 4x^2 - 16x - 9 = 0 \quad (2x - 1)(2x + 9) = 0$$

$$x = 1/2, -9/2$$

but $x = -9/2$ is not a possible solution ($\because y^2 = -18$, not possible)

$$\therefore x = 1/2 \quad y = \sqrt{2}$$



The shaded area is the required area.

$$Area = 2 \left[\int_0^{1/2} 2\sqrt{x} dx + \int_{1/2}^{3/2} \sqrt{\left(\frac{3}{2}\right)^2 - x^2} dx \right]$$

$$= 2 \left[\frac{2x^{3/2}}{3/2} \Big|_0^{1/2} + \left(\frac{x}{2} \sqrt{\left(\frac{9}{4}\right) - x^2} + \frac{9}{8} \sin^{-1} \left(\frac{x}{3/2} \right) \right) \Big|_{1/2}^{3/2} \right] = \frac{9}{8} \pi - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) + \frac{5}{6\sqrt{2}}$$

24. Solution:

Let E be the event that the student answered correctly.

Let E_1 denote the event that the student knows the answer. Let E_2 denote the event that the student guesses the answer .

$$P(E_1/E)=?$$

$$P(E_1)=3/4, P(E_2)=1/4$$

$$P(E/E_1)=1, P(E/E_2)=1/4$$

Thus, by Baye's Theorem

$$\begin{aligned} P(E_1/E) &= \frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2)} \\ &= \frac{(3/4)(1)}{(3/4)1 + (1/4)(1/4)} = \frac{12}{13} \end{aligned}$$

25. Solution:

$$\text{Let } y = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right), z = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

Put in g $x = \tan \theta$,

$$\begin{aligned} y &= \tan^{-1} \left(\frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right) = \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right) = \tan^{-1} \left(\frac{1-\cos \theta}{\sin \theta} \right) \\ &= \tan^{-1} \left(\frac{2 \sin^2 \theta / 2}{2 \sin \theta / 2 \cos \theta / 2} \right) = \tan^{-1} (\tan \theta / 2) = \theta / 2 \end{aligned}$$

$$z = \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = \sin^{-1} (\sin 2\theta) = 2\theta$$

$$\therefore \theta = z / 2$$

$$\therefore y = \frac{1}{2} \left(\frac{z}{2} \right) = \frac{z}{4}$$

$$\frac{dy}{dz} = \frac{1}{4}$$

26. Solution :

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log(\sin x) dx.$$

$$= \log(\sin x) \cdot \frac{\sin 2x}{2} \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\sin x} \cos x \cdot \frac{\sin 2x}{2} dx$$

$$= 0 - \log\left(\frac{1}{\sqrt{2}}\right) \frac{1}{2} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\cancel{\sin x}} \cos x \cdot \frac{\cancel{2} \cancel{\sin x} \cos x}{\cancel{2}} dx$$

$$= -\frac{1}{2} \log\left(\frac{1}{\sqrt{2}}\right) - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 + \cos 2x}{2} dx = -\frac{1}{2} \log\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{8} + \frac{1}{4}$$
