CBSE Test Paper 04

Chapter 15 Waves

1. A travelling wave, at a rigid boundary or a closed end, **1**
   a. is reflected with a phase difference of 30^{o}
   b. is reflected with same phase
   c. is reflected with a phase difference of 70^{o}
   d. is reflected with a phase reversal
2. A tuning fork produces 4 beats/sec. with 50 cm and 40 cm of a stretched wire, of a sonometer. The frequency of fork is **1**
   a. 90 Hz
   b. 36 Hz
   c. 110 Hz
   d. 50 Hz
3. When a wave is reflected from a rigid surface, it undergoes a phase change given by **1**
   a. 3π
   b. 2π
   c. 0
   d. π
4. An observer is watching two vehicles of same velocity (4 m/s). The former is approaching towards the observer while the latter is receding. If the frequency of the siren of the vehicle is 240 Hz and velocity of sound in air is 320 m/s , then the beats produced is **1**
   a. 12
   b. zero
   c. 6
   d. 3
5. A certain string fixed at both ends will resonate to several frequencies, the lowest of which is 200 Hz. What are the next three higher frequencies to which it resonates? **1**
   a. 50, 150, 300 Hz
   b. 200, 250, 300 Hz
   c. 400 , 600 , 800 Hz

d. 100, 200, 300 Hz

6. A steel wire has a length of 12 m and a mass of 2.10kg. What will be the speed of a transverse wave on this wire when a tension of 2.06×10^4 N is applied? **1**
7. Why do tuning forks have two prongs? **1**
8. What is the nature of the thermal change in air, when a sound wave propagates through it? **1**
9. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C (is equal to 343 ms^{-1})? **2**
10. Differentiate between closed pipe and open pipe at both ends of same length for frequency of fundamental note and harmonics. **2**
11. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will this source be in resonance with the pipe if both the ends are open? **2**
12. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s-1, and that of P wave is 8.0kms^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur? **3**
13. What do you mean by interference of waves? What do you mean by constructive and destructive interference? State their conditions. **3**
14. The length of a sonometer wire between two fixed ends is 110cm. Where the two bridges should be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio of 1:2:3? **3**
15. The transverse displacement of a string (clamped at its both ends) is given by $y(x,t) = 0.06 \sin \frac{2}{3} x \cos(120\pi t)$ Where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.
    Answer the following: **5**
    i. Does the function represent a travelling wave or a stationary wave?
    ii. Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?
    iii. Determine the tension in the string.

Answer

1. d. is reflected with a phase reversal

   Explanation: In reflection of this kind there must be node at the support as the string is fixed there. The reflected and incident pulses must have opposite signs, so as to cancel each other at that point. Thus in case of a travelling wave, the reflection at rigid boundary will take place with a phase reversal or with a phase difference of 180^0

2. b. 36 Hz

   Explanation: beat frequency is given by

   $f_{beat} = f_1 - f_2$

   $4 = f_1 - f_2 \rightarrow (1)$

   also frequency $f\alpha\ \frac{1}{L}$

   $$\frac{f_1}{f_2} = \frac{L_2}{L_1}$$

   $$\frac{f_1}{f_2} = \frac{40}{50} \rightarrow (2)$$

   on solving equation 1 and 2

   $f_1 = 16$ Hz

   $f_2 = -20$ Hz

   $|f_1 - f_2| = 36$ Hz

3. d. π

   Explanation: When a wave travel from rarer medium to denser medium and undego reflection at interface. Its phase changes by π.

4. c. 6

   Explanation: Using formula of doppler's shift for stationary observer and source moving away from observer

   $$f_1 = \frac{v}{v+u} \times f$$

   $$f_1 = \frac{320}{320+4} \times 240$$

f_1 = 237.03

Now for source moving towards observer

$$f_2 = \frac{v}{v-u} \times f$$
$$f_2 = \frac{320}{320-4} \times 240$$

f_2 = 243.03

Beat frequency $f_{beat} = f_2 - f_1$

f_{beat} = 243.03 - 237.03

f_{beat} = 6

5. c. 400 , 600 , 800 Hz

Explanation: $f_n = \frac{nv}{2L}$

given f_1 = 200 Hz

$f_2 = 2f_1$ = 400 Hz

f3 = $3f_1$ = 600 Hz

f4 = $4f_1$ = 800 Hz

6. l = 12m, M(Total mass) = 2.10 kg

$$m = \frac{M}{l} = \frac{2.1}{12} \quad T = 2.06 \times 10^4$$
$$\therefore v = \sqrt{\frac{T}{m}} = \sqrt{\frac{2.06 \times 10^4 \times 12}{2.10}} = \sqrt{\frac{1236 \times 10^4}{105}} v$$
$$= \sqrt{11.77} \times 10^2 = 3.43 \times 10^2$$
$$v = 343.0\ m/\text{s}.$$

7. The two prongs of a tuning fork set each other is resonant vibrations and help to maintain the vibrations for a longer time.

8. When the sound wave travel through air adiabatic changes take place in the medium.

9. Given speed of sound in air, v = 343 ms^{-1}

Length of wire, I = 12.0 m, total mass of wire M = 2.10 kg

masss per unit length $m = \frac{M}{l} = \frac{2.10}{12.0} = 0.175 kgm^{-1}$

Now

$v = \sqrt{\frac{T}{m}} \Rightarrow T = v^2 m$ = $(343)^2 \times 0.175$ = 20,588.6N = 2.06×10^4N

10. i. In a pipe open at both ends, the frequency of fundamental note produced is twice as that produced by a closed pipe of same length.
    ii. An open pipe produces all the harmonics, while in a closed pipe, the even harmonics are absent.

11. Length of pipe = L = 20cm = 0.2m

    Frequency of nth node = υ_n = 430 Hz

    Velocity of sound = v = 340m|s

    Now, υ_n of closed pipe is : $\rightarrow$

    $\upsilon_n = \frac{(2n-1)v}{4L}$

    $430 = \frac{(2n-1)\times 340}{4\times 0.2}$

    $2n - 1 = \frac{430\times 4\times 0.2}{340}$

    2n - 1 = 1.02

    2n = 1.02 + 1

    2n = 2.02

    n = 1.01

    Hence, it will be the first normal mode of vibration, In a pipe, open at both ends we, have

    $\upsilon_n = \frac{n\times v}{2L} = \frac{n\times 340}{2\times 0.2} = 430$

    So, $430 = \frac{n\times 340}{2\times 0.2}$

    $n = \frac{430\times 2\times 0.2}{340}$

    n = 0.5

    As n has to be an integer, open organ pipe cannot be in resonance with the source.

12. Let v_S and v_P be the velocities of S(Secondary wave) and P(Primary Wave) waves respectively.

    Let L be the distance between the epicentre and the seismograph.

    We have:

    L = $v_S t_S$ ....(i)

    L = $v_P t_P$ ....(ii)

    Where,

    t_S and t_P are the respective times taken by the S and P waves to reach the seismograph from the epicentre

It is given that:

v_P = 8 km/s

v_S = 4 km/s

From equations (i) and (ii), we have:

$v_S t_S = v_P t_P$

$4 \times t_S = 8 \times t_P$

$t_S = 2t_P$ ....(iii)

It is also given that:

$t_S - t_P$ = 4 min = 240 s

$2t_P - t_P = 240$

t_P = 240 sec

so time taken by P wave to reach from epicentre to seismograph is 240 sec and we know that $t_S = 2 * t_P$

$t_s = 2 \times 240$ = 480 sec

From equation (ii), we get:

$L = 8 \times 240$

= 1920 km

From above calculation it was found that distance of earthquake from epicentre is 1920 km.

13. **Interference** of waves is the phenomenon of superposition of two waves having same frequency, constant phase travelling in the same direction due to which amplitude and intensity becomes maximum at some points and minimum at some points.
Constructive interference occurs when waves superimpose in the same phase i.e., the crest of one wave (in transverse waves) coincides with crest of another wave and vice-versa. As a result, the resultant amplitude and hence intensity of the resultant wave becomes maximum. 'Thus, for constructive interference, the phase difference between the superposing waves is given by $\Delta\phi$ = 0 or $2n\pi$, where n is an integer i.e., n = 1, 2, 3...
Destructive interference occurs when waves superimpose in mutually opposite phase i.e., in a superposition of two transverse waves crest of one wave exactly coincides with trough of another wave. As a result, the resultant amplitude and hence

intensity of the resultant wave becomes minimum. For destructive interference, the phase difference is given by $\Delta\phi$ = (2n - 1)π , where n =1, 2, 3...

14. Let l_1, l_2 and l_3 be the length of the three parts of the wire and f_1, f_2 and f_3 be their respective frequencies.

Since T and m are fixed quantities, and 2 is constant

$\Rightarrow f = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$\Rightarrow f = \alpha \frac{1}{l}$

or r_1 = constant

So, f_1 l_1 = Constant $\rightarrow$ (1)

f_2 l_2 = Constant $\rightarrow$ (2)

f_3 l_3 = Constant $\rightarrow$ (3)

Equating equation 1), 2) & 3)

$f_1l_1 = f_2l_2 = f_3l_3$

Now, $l2 = \frac{f_1}{f_2} l|11$

$\Rightarrow l_2 = \frac{1}{2} l_1 \rightarrow (4) \left(\frac{f_1}{f_2} = \frac{1}{2}\right)$ Given

Also, $l_3 = \frac{f_1}{f_3} l_1$

$\Rightarrow l_3 = \frac{1}{3} l_1 \left(\frac{f_1}{f_3} = \frac{1}{3}(\text{given})\right)$

Now, Given Total length = 110cm

i.e l_1 +l_2+l_3= 110cm

$l_1 + \frac{1}{2} l_1 + \frac{1}{3} l_1 = 110$

$\Rightarrow \frac{11 l_1}{6} = 110$

$\Rightarrow l_1 = \frac{110 \times 6}{11} = 60cm$

i. e

$\Rightarrow$. l_1 = 60cm

$\Rightarrow l_2$ = 30cm

Now,

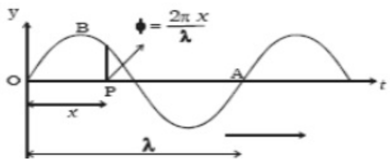
$\Rightarrow l_3 = \frac{l_1}{3}$

$\Rightarrow l_3 = \frac{60}{3}$

$\Rightarrow l_3$ = 20 cm

So length of three parts are 60cm, 30cm, 20cm.

15. A simple harmonic progressive wave is a wave which continuously advances in a given direction without change of form and the particles of the medium perform simple harmonic motion about their mean position with the same amplitude and period, when the waves pass over them.



consider a particle of the medium situated at point P at a distance x from origin, this particle also performs SHM with the same amplitude and period. Hence, the displacement of the particle at P at the instant t is given by -

$y = a\sin(\omega t - \phi)$

here ϕ is a phase constant which describes particle's position at t = 0 if at t=0 ,y=0 then ϕ = 0

The general equation representing a stationary wave is given by the displacement function:

y (x, t) = 2a sin kx cosωt

This equation is similar to the given equation:

$y(x,t) = 0.06\sin\left(\frac{2}{3}x\right)\cos(120\pi t)$

Hence, the given function represents a stationary wave.

i. A wave travelling along the positive x-direction is given as:

$y_1 = a\sin(\omega t - kx)$

The wave travelling along the negative x-direction is given as:

$y_2 = a\sin(\omega t - kx)$

The transverse displacement of the string is given as:

$y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$

2a = 0.06, a = 0.03

$\omega = 120\pi$

$\mathrm{k} = \frac{2\pi}{3}$

So we get two waves as

$y_1(x,t) = 0.03\sin(120\pi t - 2\pi x/3)$

$y_2(x,t) = 0.03\sin(120\pi t - 2\pi x/3)$
$\omega = 120\pi$
Frequency of each wave, $f = \frac{\omega}{2\pi}$ = 60 Hz
Wavelength of each wave, $\lambda = \frac{2\pi}{k}$ = 3 m
Speed of each wave, $v = \lambda . f = 180\text{m/s}$

ii. The velocity of a transverse wave travelling in a string is given by the relation:
$v = \sqrt{\frac{T}{\lambda}}$ .....(i)
Where,
Velocity of the transverse wave, v = 180 m/s
Mass of the string, m = 3.0×10^{-2} kg
Length of the string, l = 1.5 m
Mass per unit length of the string, $\lambda = \frac{m}{l}$
$= \frac{3.0}{1.5} \times 10^{-2}$
$= 2 \times 10^{-2} kgm^{-1}$
Tension in the string = T
From equation (i), tension can be obtained as:
$T = v^2\mu$
$= (180)^2 \times 2 \times 10^{-2}$= 648 N
Note that here we have taken speed of wave v = 180 m/sec, velocity of the component wave, not of the standing wave
[velocity of standing wave is always zero as it doesn’t propagate]