Chapter-14

Factorisation of Algebraic Expressions

Z7T2I3

You have already learnt the factorisation of natural numbers. For example

$105 = 1\times105 = 3\times35 = 5\times21 = 7\times15 = 3\times5\times7$

Here, 105 is expressed as the product of different numbers in 5 different ways. In this way if a number is expressed as product of two or more numbers then the later numbers are called *factors* of the *first number*. Therefore, 1, 3, 5, 7, 15, 21, 35 and 105 all are factors of 105. In this case observe that each of 3, 5, 7 are prime factors. That means none of them can be expressed as multiple of other smaller numbers except 1.

That means 3, 5 and 7 as factors of 105, are *irreducible*.

On the other hand, we can say $105 = 3\times5\times7$ is the *prime factor form* of 105.

Prime factorisation of numbers is essential for various mathematical discussions.

We know that algebraic expressions are formed by addition or subtraction of different terms. Again these terms are formed by product of different numbers, constants and algebraic symbols or variables. For example, $10x^2y+6y^2+12y$ is an algebraic expression where $10x^2y$, $6y^2$ and $12y$ are three terms of the expression. That means it is a polynomial (here Trinomial). Can this expression be expressed as the product of two or more algebraic expressions? That means, is it possible to factorise the expression $10x^2y+6y^2+12y$? If possible, what are the factors? To find the answer to this question, let us discuss first how the factors of algebraic expression are formed.

14.1 Factors of Algebraic Expressions

Consider the first term of the expression discussed above i.e. $10x^2y$. In how many different ways the term can be expressed as the product of different factors?

$10x^2y = 10 \times x^2y$

Therefore, 10 and x^2y are two factors of $10x^2y$.

Here, $10 = 2\times5$ and $x^2y = x^2 \times y$

Therefore, $10x^2y = 10 \times x^2y = 2 \times 5 \times x^2 \times y$

i.e. Factors of $10x^2y$ are 2, 5, x^2 and y

Again, $x^2 = x \times x$

Therefore, $10x^2y = 2\times5\times x\times x\times y$

i.e. factors of $10x^2y$ are 2, 5, x, x and y

Now as factors of $10x^2y$, 2 and 5 are *irreducible* factors of the term.

Similarly, as factors of $10x^2y$, x and y are also *irreducible*. i.e. x or y cannot be expressed as the product of any other terms.

Therefore, $10x^2y = 2\times5\times x\times x\times y$ is the written form of product of irreducible factors.

Observe (i) Instead of $10x^2y = 10 \times x^2y$ we could have written $10x^2y = 10x\times xy$ or, $2\times5x^2\times y$ or $2x\times5xy$ etc. Then we could have got $10x$, xy, 2, $5x^2$, y, $2x$, $5xy$ etc. as factors of $10x^2y$. But among these factors $10x$, $5x^2$, $2x$, $5xy$ etc. are reducible. But reducing these factors, we would have the irreducible factors 2, 5, x, x and y of $10x^2y$.

(ii) We can express $10x^2y = 1\times10x^2y$. But 1 is multiple of any of the terms. Therefore, it is not required to mention that 1 as factor of $10x^2y$.

Similarly, if we observe the second term of the given expression i.e. $6y^2$, then we can see the written form of the product of $6y^2$ in irreducible factor as

$6y^2 = 6\times y^2 = 2\times3\times y\times y$

The third terms $12y$ expressed as product of irreducible factors is —

$12y = 12\times y = 4\times3\times y = 2\times2\times3\times y$

How will we be benefitted by expressing the three terms of the given expression as the product of irreducible factors?

We have seen that all the three terms have factors 2 and y.

i.e. $2\times y = 2y$ is *common factor* of all the three terms.

For example,

$10x^2y = 2\times5\times x\times x\times y$

$= (2\times y)\times(5\times x\times x)$

$= 2y\times5x^2$

$6y^2 = 2\times3\times y\times y$

$= (2\times y)\times(3\times y)$

$= 2y\times3y$

$12y = 2\times2\times3\times y$

$= (2\times y)\times(2\times3)$

$= 2y\times6$

Therefore, we can arrange the given expression as

$10x^2y+6y^2+ 12y = 2y \times 5x^2+ 2y \times 3y + 2y \times 6$

$2y$ is the *common factor* of all the three terms of the right hand side of the expression. On the other hand $5x^2$, $3y$ and 6 are not common factors of the terms of the expression.

Now let us examine.

Add the uncommon factors and multiply it by the common factors.

i.e.

$2y \times (5x^2 + 3y + 6)$

Using the distributive law of multiplication over addition we get,

$2y \times (5x^2+3y +6) = 2y \times 5x^2 + 2y \times 3y + 2y \times 6$

$= 2 \times 5x^2 \times y + 2\times 3\times y\times y+ 2\times 6 \times y$

$= 10x^2 y + 6y^2+ 12y$

Isn't the expression in the right hand side of above is the same as the expression discussed in the begining?

Therefore it is seen that the given expression is product of two expressions $2y$ and $5x^2+3y+6$.

i.e. $10x^2 y + 6y^2+ 12y = 2y\times(5x^2+3y+6)$

$= 2\times y\times(5x^2+3y+6)$

Here, 2, y and $5x^2+3y + 6$ are the irreducible factors of the given expression.

In the previous lesson, we have learnt how to get new expression by multiplying two or more expressions. Next we discuss how one expression can be expressed as the product of two or more irreducible factors.

The process of expressing an algebraic expression as the product of two or more irreducible factors is known as *factorisation*.

14.2 Highest Common Factor (HCF)

To introduce the concept, let us take two terms $3xy$ and $2x^2y$.

First we factorise each of the terms.

$3xy = 3 \times x \times y$

$2x^2y = 2 \times x \times x \times y$ [Irreducible factor form of the two terms]

Now, factors of $3xy$ are 1, 3, x, y, $3x$, $3y$, xy, $3xy$.

Factors of $2x^2y$ are 1, 2, x, x^2, y, $2x$, $2y$, $2x^2$, xy, x^2y, $2xy$, $2x^2y$.

Therefore, factors common to $3xy$ and $2x^2y$ are 1, x, y, xy.

In this case, xy is the greatest common factor amongst all the factors of $3xy$ and $2x^2y$. We say HCF of $3xy$ and $2x^2y$ is xy.

One more example : Let us take $12ab^2$ and $14ab^3$

$12ab^2 = 2 \times 2 \times 3 \times a \times b \times b$

$14ab^3 = 2 \times 7 \times a \times b \times b \times b$

$\therefore$ HCF of $12ab^2$ and $14ab^3$ is $2 \times a \times b \times b = 2ab^2$

Thus, to get the HCF of the two terms, the factors which are common to them are multiplied together.

Similarly, let us see the common factors of pq and abc.

$pq = p \times q$

$abc = a \times b \times c$

pq and abc have no common factor. But we know that 1 is factor of each term.

Therefore, 1 is a common factor of pq and abc, 1 being the only common factor it is the HCF of the given expressions.

Try yourself Find the highest common factor (HCF) of the following expressions –

(i) a^2 and a^3 (ii) p^2q and pq^2 (iii) $3a^3b^2$ and $6a^2b^3$

(iv) $24l^2mn$ and $36l^2m^3p$ (v) $18x^2yz^2$, $24x^3y^3z$ and $20x^2y$ (v) a and b

Let us see one technique

(i) HCF of x and x^2 is x

(ii) HCF of a^4 and a^3 is a^3

(iii) HCF of p^6 and p^2 is p^2

What have you learnt?

The HCF of two powers is the same variable is the one having the lowest power.

14.3 Factorisation of Algebraic Expressions

Let us discuss some methods of factorisation of algebraic expressions.

14.3.1 Method of common factors

In this method, the common factors are separated from the terms of the given expressions. and using distributive law $a \times b + a \times c = a \times (b + c)$ the factorisation is completed.

Example 1 : Factorise $3x + 6$

Solution : $3x = 3 \times x$

$6 = 3 \times 2$

HCF of $3x$ and 6 is 3

$\therefore \quad 3x + 6 = 3 \times x + 3 \times 2$

$= 3 \times (x + 2)$ [using $a \times b + a \times c = a \times (b+c)$]

i.e. $3x + 6 = 3(x + 2)$

Example 2 : $3xy + 3x$

Solution : $3xy + 3x$

$= 3x \times y + 3x \times 1$

$= 3x(y+1)$

Example 3 : Factorise $12a^2b + 3ab^2$

Solution : $12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

$3ab^2 = 3 \times a \times b \times b$

HCF of $12a^2b$ and $3ab^2$ is $3 \times a \times b = 3ab$

$\therefore \quad 12a^2b + 3ab^2$

$= 3ab \times 4a + 3ab \times b$

$= 3ab(4a+b)$

Example 4 : Factorise $12a^3b^2 - 15a^2b^3$

Solution : $12a^3b^2 = 2 \times 2 \times 3 \times a \times a \times a \times b \times b$

$15a^2b^3 = 3 \times 5 \times a \times a \times b \times b \times b$

HCF of $12a^3b^2$ and $15a^2b^3 = 3 \times a \times a \times b \times b$

$= 3a^2b^2$

Therefore,

$12a^3b^2 - 15a^2b^3$

$= 3a^2b^2 \times 4a - 3a^2b^2 \times 5b$

$= 3a^2b^2 \times (4a - 5b)$

In short,

$12a^3b^2 - 15a^2b^3$

$= 3 \times 2 \times 2\, a^3b^2 - 3 \times 5 \times a^2 b^3$

$= 3a^2b^2(4a - 5b)$

Example 5 : Factorise $10a^2b^3 - 12a^3b^2 + 18ab^2$

Solution : $10a^2b^3 - 12a^3b^2 + 18ab^2$

$= 2ab^2(5ab - 6a^2 + 9)$

Observe

(i) HCF of 10, 12 and 18 is 2

(ii) HCF of a^2, a^3 and a is a

(iii) HCF of b^3 and b^2 is b^2

Therefore, HCF of $10a^2b^3$, $12a^3b^2$ and $18ab^2$ is $2ab^2$

Example 6 : Factorise $3pq^2 + 15pq + 7p^2q$

Solution : $3pq^2 + 15pq + 7p^2q$

$= pq \times 3q + pq \times 15 + pq \times 7p$ [Note that pq is HCF of $3pq^2$, $15pq$ and $7p^2q$]

$= pq \times (3q + 15 + 7p)$

$= pq(3q + 15 + 7p)$

Try yourself

Factorise :

(i) $2xy + 2y$ (ii) $10x^2y^3 - 15xy^3$

14.3.2 Factorisation of Algebraic Expressions by proper grouping of terms :

Sometimes factorisation of algebraic expression is done by rearranging its terms in proper groups. Observe the following example.

Example 7 : Factorise $ab + ay + xb + xy$

You might notice that a is the common factor of the first two terms of the expressions ab and ay.

On the other hand, x is the common factor of xb and xy. But all the terms have no common factor except 1. How shall we proceed then?

First let us factorise the terms having common factors.

$ab + ay = a \times b + a \times y$

$= a \times (b + y)$

$= a(b + y)$

Similarly, $xb + xy = x \times b + x \times y$

$= x \times (b + y)$

$= x(b+y)$

Now, $ab + ay + xb + xy$

$= a(b + y) + x(b + y)$

$= (b + y)(a + x)$ [$\because$ $(b + y)$ is common factor, we can use distributive law]

Observe that $ab + ay + xb + xy = ab + xb + ay + xy$ [Rearrangement of the terms]

$= b(a + x) + y(a + x)$

$= (a + x)(b + y)$

Example 8 : Factorise $3xy + 2y + 3x + 2$

Observe that in this expression the first term and second term have no other common factor except y. Similarly, $3x$ and 2 also have no other common factor except 1.

Solution : $3xy + 2y + 3x + 2 = y(3x + 2) + 1\,(3x + 2) = (3x + 2)\,(y + 1)$

or, $3xy + 2y + 3x + 2$

$= 3xy + 3x + 2y + 2$

$= 3x\,(y + 1) + 2(y + 1)$

$= (y + 1)\,(3x + 2)$

Example 9 : Factorise $15xy - 6x + 5y - 2$

Solution : $15xy - 6x + 5y - 2$

$= 3 \times x \times 5 \times y - 3 \times x \times 2 + 5 \times y - 2$

$= 3x \times (5y - 2) + 1 \times (5y - 2) \times 1$

$= (5y - 2)\,(3x + 1)$

or,

$15xy - 6x + 5y - 2$

$= 15xy + 5y - 6x - 2$

$= 5y \times 3x + 5y \times 1 + (-2) \times 3x + (-2)$

$= 5y\,(3x + 1) + (-2)\,(3x + 1)$

$= (3x + 1)\,[5y + (-2)]$

$= (3x + 1)\,(5y - 2)$

Try yourself

Factorise the following expressions :

(i) $x^2 + xy + 4x + 4y$ (ii) $5xy + 5y + 2x + 2$

(iii) $px + qx - py - qy$ (iv) $x - 3 + 3yz - xyz$

14.3.3 Factorisation using identities

Already you are familiar with the following identities

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

We can also factorise using these identities. To do this we have to identify the right hand side of the exact identity given above that fits into the given expression. Then the corresponding left hand side of the identity given the factorisation of the given expression.

Example 10 : Factorise $25x^2 - 30x + 9$

Solution : $25x^2 - 30x + 9$
$= (5x)^2 - 2 \times (5x) \times 3 + 3^2$
$= (5x - 3)^2$
$= (5x - 3)(5x - 3)$

Observe that the given expression is of the form $a^2 - 2ab + b^2$ where $a = 5x$ and $b = 3$ and $2ab = 2 \times 5x \times 3$

Example 11 : Factorise $9a^2 - 81b^2$

Solution : $9a^2 - 81b^2$
$= (3a)^2 - (9b)^2$
$= (3a + 9b)(3a - 9b)$
$= 3(a + 3b)\ 3(a - 3b)$
$= 9\ (a + 3b)\ (a - 3b)$

Aliter,
$9a^2 - 81b^2$
$= 9\ (a^2 - 9b^2)$
$= 9\ \{a^2 - (3b)^2\}$
$= 9\ (a + 3b)\ (a - 3b)$

Example 12 : Factorise $16a^4 - 81$

Solution : $16a^4 - 81$
$= (4a^2)^2 - 9^2$
$= (4a^2 + 9)(4a^2 - 9)$
$= (4a^2 + 9)\ \{(2a)^2 - 3^2\}$
$= (4a^2 + 9)(2a + 3)(2a - 3)$

Example 13 : Factorise $a^2 - b^2 + 2bc - c^2$

Solution : $a^2 - b^2 + 2bc - c^2$
$= a^2 - (b^2 - 2bc + c^2)$
$= a^2 - (b - c)^2$
$= \{a + (b - c)\}\{a - (b - c)\}$
$= (a + b - c)(a - b + c)$

Try yourself

Factorise the following expressions :

(i) $a^2 + 12a + 36$ (ii) $p^4 + 8p^2 + 16$ (iii) $m^2 + 144 - 24m$
(iv) $16x^2 + 49 - 56x$ (v) $x^2 - 25$ (vi) $x^8 - m^8$

14.3.4 Factorisation of expressions of the type $x^2 + px + q$

Look at the expressions $x^2 + 3x + 2$, $x^2 + 9x + 14$, $x^2 - 5x + 6$, $x^2 - 6x - 8$ etc.

These expressions are of the type $x^2 + px + q$ where coefficient of x is p and constant term is q. We cannot express this epression in the form $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$ or $a^2 - b^2$. Let us see how to factorise this type of expressions.

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$ where a and b constants. Here, the sum of a and b as well as product of a and b are constants.

Let $\quad a + b = p$ and $ab = q$

$\therefore \quad x^2 + (a + b)x + ab = x^2 + px + q$

Conversely, to factorise $x^2 + px + q$ we need two numbers such that their sum and product are respectively equal to the coefficient of x and the constant term of the expression.

Let us try to learn using some examples.

Example 14 : Factorise $x^2 + 7x + 12$

Solution : In above expression $p = 7$ and $q = 12$

We are to find two numbers a and b such that their sum is 7 and sum is 12

i.e. $a \times b = 12$, $a + b = 7$

Now let us look at the following manipulations.

$1 \times 12 = 12, \quad 1 + 12 = 13$
$2 \times 6 = 12, \quad 2 + 6 = 8$
$3 \times 4 = 12, \quad 3 + 4 = 7$

Select 3 and 4 for a and b respectively since the sum of 3 and 4 is equal to 7 which is coefficient of x and the product of 3 and 4 is equal to the constant term 12.

No other factors except 3 and 4 satisfy the second relation.

Therefore, $\quad x^2 + 7x + 12$
$= x^2 + (3 + 4)x + 3 \times 4$
$= x^2 + 3x + 4x + 3 \times 4$
$= x(x + 3) + 4(x + 3)$
$= (x + 3)(x + 4)$

$\therefore$ Factorisation of $x^2 + 7x + 12$ is $(x + 3)(x + 4)$

Example 15 : Factorise $x^2 - 5x + 6$

Solution : Here, $p = -5$ and $q = 6$

We are to find two numbers such that their product is 6 and sum is – 5.

Let us see factors of 6

$1 \times 6 = 6 \qquad 1 + 6 = 7$
$2 \times 3 = 6 \qquad 2 + 3 = 5$

But here, we shall not get negative numbers as a sum.

So, let us proceed in another way.

$(-1) \times (-6) = 6 \quad (-1) + (-6) = -7$

$(-2) \times (-3) = 6 \quad (-2) + (-3) = -5$

It is seen that adding –2 and –3 we get – 5.

Therefore $x^2 - 5x + 6$

$= x^2 - 2x - 3x + 6$

$= x(x - 2) - 3(x - 2)$

$= (x - 2)(x - 3)$

$\therefore$ Factorisation of $x^2 - 5x + 6$ is $(x - 2)(x - 3)$

Example 16 : Factorise $x^2 + 8x - 20$

Solution : In this expression $p = 8$, $q = -20$

We are to find two numbers a and b such that their sum is 8 and their product is – 20. It is to be noted that the product of two numbers is negative, one of a and b must be negative.

Let us examine the various options in this case –

a	b	$a \times b$	$a + b$
– 1	20	– 20	19
1	– 20	– 20	– 19
2	– 10	– 20	– 8
– 2	10	– 20	8
4	– 5	– 20	– 1
– 4	5	– 20	1

From the above table we get two numbers –2 and 10 for which the sum is $(-2) + 10 = 8$ and the product is

$2 \times (-10) = -20$

Therefore, $x^2 + 8x - 20$

$= x^2 - 2x + 10x - 20$

$= x(x - 2) + 10(x - 2)$

$= (x - 2)(x + 10)$

Example 17 : Factorise $x^2 - 5x - 24$

Solution : Find a and b such that

$a + b = -5$ and $ab = -24$

Observe the adjoining table

a	b	$a \times b$	$a + b$
24	– 1	– 24	23
– 24	1	– 24	– 23
2	– 12	– 24	– 10
– 2	12	– 24	10
3	– 8	– 24	– 5
– 3	8	– 24	5
– 6	4	– 24	– 2
6	– 4	– 24	2

We have, $x^2 - 5x - 24$

$= x^2 + 3x - 8x - 24$

$= x(x + 3) - 8(x + 3)$

$= (x + 3)(x - 8)$

Example 18 : Factorise $x^2 + 3x - 108$

Solution : Let us practise without the help of such table.

$x^2 + 3x - 108$

$= x^2 + (12 - 9)x - 108$

$= x^2 + 12x - 9x - 108$

$= x(x + 12) - 9(x + 12)$

$= (x + 12)(x - 9)$

Direction for the teachers : First help the students learn by preparing the table on factors. After a number of practice they can try orally.

Try yourself

Factorise the following expressions.

(i) $x^2 + 7x + 10$ (ii) $x^2 - 7x + 10$ (iii) $x^2 - 7x - 10$

(iv) $x^2 + 11x + 24$ (v) $x^2 - 15x + 36$ (vi) $x^2 - 20x - 64$

14.3.5 Factorisation of expressions of the type $mx^2 + px + q$

You have learnt about the factorisation of expressions of the form $x^2 + px + q$ where coefficient of x^2 is 1. Now we will see the factorisation of such algebraic expressions where coefficient of x^2 is other than 1.

Consider one example $15x^2 + 11x + 2$

To factorise this expression like the expression $x^2 + px + q$ we consider two numbers a and b such that $a + b = 11$ and $ab = 15 \times 2 = 30$

By observation we get $a = 6$ and $b = 5$.

This means, $15x^2 + 11x + 2 = 15x^2 + 6x + 5x + 2$
$= 3x(5x + 2) + (5x + 2)$
$= (5x + 2)(3x + 1)$

Observe the following product :

$$(ax + b)(cx + d)$$
$$= ax(cx + d) + b(cx + d)$$
$$= acx^2 + adx + bcx + bd$$
$$= acx^2 + (ad + bc)x + bd$$
$$= mx^2 + px + q \quad \text{where } m = ac,\ p = ad + bc,\ q = bd$$

Observe $m \times q = ac \times bd = ad \times bc$

$\therefore$ Product of m and q is equal to the product of ad and bc, i.e. coefficient of x is sum of ad and bc.

Thus to factorise $mx^2 + px + q$ we need two numbers whose product is equal to mq (i.e product of coefficient of x^2 and constant term) and algebraic sum of the two numbers is equal to coefficient of x.

Let us take few examples –

Example 19 : $2x^2 + 9x + 9$
$= 2x^2 + (3 + 6)x + 9$
$= 2x^2 + 3x + 6x + 9$
$= x(2x + 3) + 3(2x + 3)$
$= (2x + 3)(x + 3)$

Product of coefficient of x^2 and constant term is $2 \times 9 = 18$

a	b	ab	$a + b$
2	9	18	11
3	**6**	**18**	**9**
1	18	18	19

Example 20 : $4m^2 + 25m - 21$

Solution : $4m^2 + 25m - 21$
$= 4m^2 + 28m - 3m - 21$
$= 4m(m + 7) - 3(m + 7)$
$= (m + 7)(4m - 3)$

Product of coefficient of m^2 and constant term is $4 \times (-21) = -84$

Since the product is negative, one of a and b must be negative.

Example 21 : $6x^2 - 20x - 16$

Solution : $6x^2 - 20x - 16$

$= 6x^2 - 24x + 4x - 16$

$= 6x(x - 4) + 4(x - 4)$

$= (x - 4)(6x + 4)$

$= 2(x - 4)(3x + 2)$

Aliter : $6x^2 - 20x - 16$

$= 2[3x^2 - 10x - 8]$

$= 2[3x^2 - 12x + 2x - 8]$

$= 2[3x(x - 4) + 2(x - 4)]$

$= 2[(x - 4)(3x + 2)]$

$= 2(x - 4)(3x + 2)$

Example 22 : $56y - 3 - 221y^2$

Solution : $56y - 3 - 221y^2$

$= - 221y^2 + 56y - 3$ [Arranging in the form $mx^2 + px + q$]

$= -(221y^2 - 56y + 3)$

$= -[221y^2 - (39 + 17)y + 3]$

$= -[221y^2 - 39y - 17y + 3]$

$= -[13y(17y - 3) - 1(17y - 3)]$

$= -(17y - 3)(13y - 1)$

Try yourself

Factorise the following expressions :

(i) $6x^2 + 5x + 1$ (ii) $2x^2 + 6x + 4$ (iii) $3a^2 + 2a - 8$

(iv) $4b^2 - 2b - 6$

Exercuse 14.1

1. Factorise the following expressions

   (i) $3x^2y + 5xy$ (ii) $10x^2y - 5xy^2$ (iii) $7a^2bc - 21ab^2c + 14abc$

2. Factorise

   (i) $a^2 + ab + 6a + 6b$ (ii) $a^2 + bc + ab + ac$ (iii) $1 + x + x^2 + x^3$

   (iv) $ab + a + b + 1$ (v) $4ax + 3ay - 4bx - 3by$

3. Factorise

   (i) $x^2 - 36$ (ii) $9x^2 + 30x + 25$ (iii) $16a^2 - 88a + 121$

   (iv) $11x^2 - 44$ (v) $x^4 - 81$ (vi) $4 - x^2 - y^2 + 2xy$

   (vii) $x^8 - y^8$ (viii) $a^3 - ab^2 - a^2b + b^3$

4. Factorise

   (i) $16 + 8x + x^2$ (ii) $15 - 2x - x^2$ (iii) $x^2 + 8x - 20$

   (iv) $x^2 + 2x - 3$ (v) $a^2 - 4a - 12$ (vi) $x^2 - 21x + 104$

   (vii) $2x^2 + 18x + 40$ (viii) $l^2 - 13l + 42$ (ix) $-a^2 - a + 20$

5. Factorise the following expressions

(i) $3x^2 + 8x + 4$ (ii) $2m^2 + 7m + 3$ (iii) $2p^2 + p - 28$

(iv) $9a^2 + 21a - 8$ (v) $4y^2 + 25y - 21$ (vi) $3m^6 - 6m^4n - 45m^2n^2$

(vii) $1 - x - 6x^2$ (viii) $6a^2 + 7ab - 3b^2$

6. Fill in the blanks (by observation)

(i) $9x^2 + 15x + 4 = (3x + ...) (.... + 1)$

(ii) $12y^2 - 17y + 6 = (.... - 2) (4y - ....)$

(iii) $6m^2 - m - 15 = (3m ....) (2m ....)$

14.4 Division of Algebraic Expressions

So far you have learnt how to add, subtract and multiply algebraic expressions. In this part. we will discuss how to divide an algebraic expression by another expression.

We know that division is the reverse operation of multiplication.

For example, $7 \times 8 = 56$

Therefore, $56 \div 7 = 8$ and $56 \div 8 = 7$

Similarly, in case of algebraic expressions also we can proceed in the same way.

(i) $4x \times 3x = 12x^2$ Therefore, $12x^2 \div 4x = 3x$ and $12x^2 \div 3x = 4x$

(ii) $6x^2 + 7x + 2 = (3x + 2)(2x + 1)$

Therefore, $(6x^2 + 7x + 2) \div (3x + 2) = 2x + 1$

and $(6x^2 + 7x + 2) \div (2x + 1) = 3x + 2$ etc.

(iii) $3x + 2 = 1 \times (3x + 2)$,

Therefore, $(3x + 2) \div (3x + 2) = 1$ and $(3x + 2) \div 1 = (3x + 2)$

Let us discuss it in details.

14.4.1 Division of a monomial by another monomial

Example : (i) $4x^3 \div 2x$

Solution : $4x^3 \div 2x$

$= \frac{4x^3}{2x}$ [Using the law $\frac{a^m}{a^n} = a^{m-n}$]

$= 2.x^{3-1}$

$= 2x^2$

(ii) $-30x^4 \div 5x^2$

Solution : $\frac{-30x^4}{5x^2}$

$= \frac{-6 \times 5 \times x^4}{5 \times x^2} = -6x^{4-2}$

$= -6x^2$

(iii) $34x^3y^3z^3 \div 51xy^2z^3$

Solution : $\dfrac{34x^3y^3z^3}{51xy^2z^3}$

$= \dfrac{2\times17\times x^3y^3z^3}{3\times17\times xy^2z^3}$

$= \dfrac{2}{3}x^{3-1}y^{3-2}z^{3-3}$

$= \dfrac{2}{3}x^2y^1z^0$

$= \dfrac{2}{3}x^2y \qquad [\because z^0 = 1]$

(iv) $39p^2q^3r \div 26p^4qr^2$

Solution : $\dfrac{39p^2q^3r}{26p^4qr^2}$

$= \dfrac{13\times3\times p^2q^3r}{13\times2\times p^4qr^2}$

$= \dfrac{3}{2}p^{2-4}q^{3-1}r^{1-2} \qquad = \dfrac{3}{2}p^{-2}q^2r^{-1}$

$= \dfrac{3q^2}{2p^2r}$

Try yourself

(i) $48y^3 \div 12y$ (ii) $-35a^3 \div 5a$ (iii) $19x^2y^3 \div 7xz$

(iv) $28p^4 \div 56p$ (v) $12a^8b^8 \div (-6a^6b^4)$

14.4.2 Division of a polynomial by a monomial

Let us try to understand with the help of some examples.

Example : (i) $(12x^2 - 6x) \div 3x$

Solution : $12x^2 - 6x = 6x(2x-1)$ (using distributive law)

$\therefore\ (12x^2 - 6x) \div 3x = \dfrac{6x(2x-1)}{3x}$

$= 2(2x-1)$

or,

$(12x^2 - 6x) \div 3x$

$= (12x^2 - 6x) \times \frac{1}{3x}$

$= 12x^2 \times \frac{1}{3x} - 6x \times \frac{1}{3x}$ (using distributive law)

$= \frac{12x^2}{3x} - \frac{6x}{3x}$

$= 4x - 2 \quad = 2(2x - 1)$

(ii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 2x^2y^2z^2$

Solution : $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3)$

$= 8x^2y^2z^2 (x + y + z)$

$\therefore\ 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 2x^2y^2z^2$

$= \frac{8x^2y^2z^2(x+y+z)}{2x^2y^2z^2}$

$= 4(x + y + z)$

or,

$8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 2x^2y^2z^2$

$= 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \times \frac{1}{2x^2y^2z^2}$

$= \frac{8x^3y^2z^2}{2x^2y^2z^2} + \frac{8x^2y^3z^2}{2x^2y^2z^2} + \frac{8x^2y^2z^3}{2x^2y^2z^2}$ (using distributive law)

$= 4x + 4y + 4z$

$= 4(x + y + z)$

14.4.3 Division of a polynomial by a polynomial

Example 1 : $(5x^2 + 15x) \div (x + 3)$

Solution : First we factorise $5x^2 + 15x$

$5x^2 + 15x$

$= 5x(x + 3)$

Now,

$(5x^2 + 15x) \div (x + 3)$

$= \frac{5x(x+3)}{(x+3)} \quad = 5x$

Example 2 : Divide $4m^2 + 4m - 15$ by $(2m - 3)$

Solution : Factorising $4m^2 + 4m - 15$

we get $4m^2 + 4m - 15$

$= 4m^2 + (10 - 6)m - 15$

$= 4m^2 + 10m - 6m - 15$

$= 2m \times 2m + 5 \times 2m - 3 \times 2m - 3 \times 5$

$= 2m \times (2m + 5) - 3\,(2m + 5)$

$= (2m + 5)\,(2m - 3)$

$$\therefore \quad \frac{4m^2 + 4m - 15}{2m - 3}$$

$$= \frac{(2m + 5)(2m - 3)}{(2m - 3)}$$

$= 2m + 5$

Example 3 : $16x^3y\,(x^8 - y^8) \div 4xy^2(x + y)$

Solution : $(x^8 - y^8) = (x^4)^2 - (y^4)^2$

$= (x^4 + y^4)\,(x^4 - y^4)$

$= (x^4 + y^4)\,[(x^2)^2 - (y^2)^2]$

$= (x^4 + y^4)\,(x^2 + y^2)\,(x^2 - y^2)$

$= (x^4 + y^4)\,(x^2 + y^2)\,(x + y)\,(x - y)$

$\therefore \quad 16x^3y\,(x^8 - y^8) \div 4xy^2(x + y)$

$$= \frac{16x^3y(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)}{4xy^2(x + y)}$$

$$= \frac{4x^2(x^4 + y^4)(x^2 + y^2)(x - y)}{y}$$

Let us find out error (if any)

Three students who were given a problem by their teacher in a classroom had done it in different ways as given below.

Parijat : $\frac{x+7}{7} = x + 1$

Tomal : $\frac{x+7}{7} = x$

Maloti : $\frac{x+7}{7} = \frac{x}{7} + 1$

Who did the division correctly? Who did it incorrectly? Discuss.

Let us take some more examples :

Example 1 : $3x^2 + 10x + 8$

Solution : First find a and b such that $a + b = 10$ and $ab = 24$

$3x^2 + 10x + 8$
$= 3x^2 + 6x + 4x + 8$
$= 3x(x + 2) + 4(x + 2)$
$= (x + 2)(3x + 4)$

Example 2 : $3x^2 - 10x + 8$
$= 3x^2 - 6x - 4x + 8$
$= 3x(x - 2) - 4(x - 2)$
$= (x - 2)(3x - 4)$

Example 3 : $3x^2 + 10x - 8$
$= 3x^2 + 12x - 2x - 8$
$= 3x(x + 4) - 2(x + 4)$
$= (x + 4)(3x - 2)$

Example 4 : $3x^2 - 10x - 8$
$= 3x^2 - 12x + 2x - 8$
$= 3x(x - 4) + 2(x - 4)$
$= (x - 4)(3x + 2)$

Exercise 14.2

1. Divide the following

   (i) $x^5 \div x^2$ (ii) $6p^3 \div 3p$ (iii) $36m^3n^2 \div (-4mn^3)$
   (iv) $96p^2q^2r^4 \div 72pqr$ (v) $-12a^8b^7 \div 17a^4b^9$

2. Divide each polynomial by the monomials in the following :

   (i) $(5y^3 - 3y^2) \div y^2$
   (ii) $(5a^8 - 4a^6 + 3a^4) \div 2a^4$
   (iii) $(5p^2q^3r^4 - 10p^2q^2r^2 + 15p^3q^3r^4) \div 5p^2q^2r^2$
   (iv) $(ax^3 + bx^2 - cx) \div ax$
   (v) $(m^3n^6 - m^6n^3) \div m^3n^3$

3. Divide the following

   (i) $(9x - 21) \div (3x - 7)$
   (ii) $10m(8m + 12) \div (4m + 6)$
   (iii) $7p^2q^2(22p - 6) \div pq(121p - 33)$

(iv) $1729xyz(3x + 12)(4y - 24) \div 19(x + 4)(y - 6)$

4. Divide

(i) $(x^2 - 25) \div (x + 5)$
(ii) $(4a^2 + 8a + 4) \div (a + 1)^2$
(iii) $(9p^2 - 18p + 9) \div (p - 1)$
(iv) $26pqr(p + q)(q + r)(r + p) \div 52pq(q + r)(r + p)$
(v) $(x^4 - 81) \div (3 - x)$
(vi) $(x^2 + 10x + 21) \div (x + 3)$
(vii) $(m^2 + 6m - 27) \div (m - 3)$
(viii) $(4y^2 + 25y - 21) \div (y + 7)$
(ix) $(4u^2 + 25u + 21) \div (u + 1)$
(x) $52y^3(50y^2 - 98) \div 26y^2(5y + 7)$

5. Find out the incorrect statements from the following and correct them.

(i) $\frac{9x^2}{9x^2} = 0$

(ii) $\frac{4x^2 + 1}{4x^2} = 1 + 1 = 2$

(iii) $\frac{3x + 2}{3x} = \frac{1}{2}$

(iv) $\frac{7x + 5}{5} = 7x$

(v) $\frac{4x^2 + 8x + 4}{4} = x^2 + 8x + 4$

What have we learnt

1. One algebraic expression can be expressed as the product of its factors.
2. An irreducible factor is a factor which cann't be expressed as product of other factors.
3. There are three steps to factorise algebraic expressions–

(i) First write down each term of the expression as the product of irreducible factors.

(ii) Separate the common factors of the terms of the expression.

(iii) Use distributive law to collect the uncommon factors.

4. To factorise $x^2 + px + q$ we need two numbers whose sum is equal to coefficient of x and their product is equal to the constant term.

5. To factorise $mx^2 + px + q$ we need two numbers whose product is equal to mq (i.e. product of coefficient of x^2 and constant term) and sum equal to p (the coefficient of x.)

6. To divide a polynomial by another polynomial, both the dividend and divisor are expressed in terms of factors. Then the numerator and denominator are divided by the common factor (s).

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