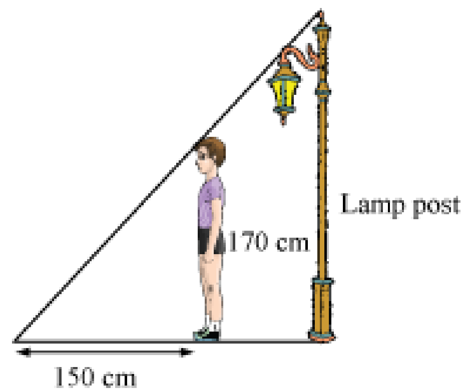
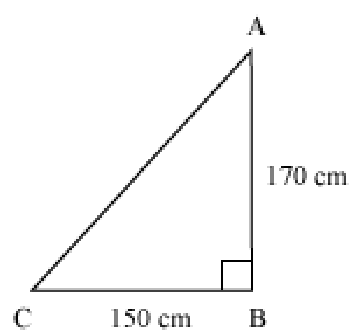
Trigonometry

Trigonometric Ratios

Suppose a boy is standing in front of a lamp post at a certain distance. The height of the boy is 170 cm and the length of his shadow is 150 cm.



You can see from the above figure that the boy and his shadow form a right-angled triangle as shown in the figure below.



The ratio of the height of the boy to his shadow is 170:150 i.e., 17:15.

Is this ratio related to either of the angles of ΔABC?

We can also conclude the following:

$\cos A = \frac{1}{\sec A}$, $\tan A = \frac{1}{\cot A}$

Also, note that

tan A = $\frac{\sin A}{\cos A}$ **and cot A =** $\frac{\cos A}{\sin A}$

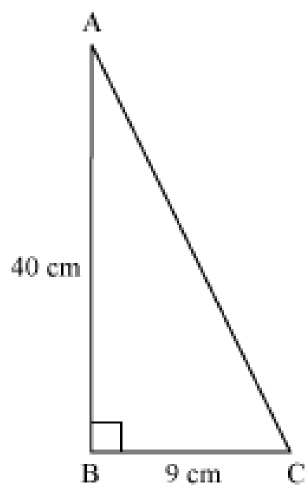
Let us now solve some more examples based on trigonometric ratios.

Example 1:

In a triangle ABC, right-angled at B, side AB = 40 cm and BC = 9 cm. Find the value of sin A, cos A, and tan A.

Solution:

It is given that AB = 40 cm and BC = 9 cm



Using Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (40)^2 + (9)^2$

$(AC)^2 = 1600 + 81$

$(AC)^2 = 1681$

$(AC)^2 = (41)^2$

AC = 41 cm

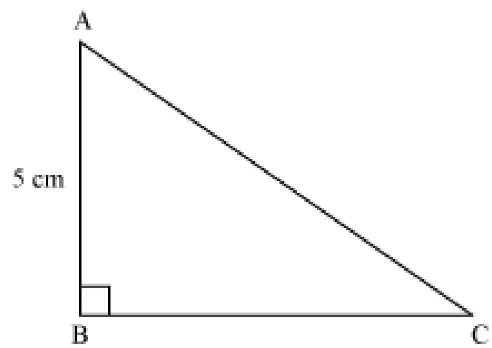
Now, sin A $= \frac{BC}{AC} = \frac{9}{41}$

cos A $= \frac{AB}{AC} = \frac{40}{41}$

tan A $= \frac{BC}{AB} = \frac{9}{40}$

Example 2:

From the given figure, find the values of cosec C and cot C, if AC = BC + 1.



Solution:

Now, it is given that AB = 5 cm and

AC = BC + 1 … (1)

By Pythagoras theorem, we obtain

$(AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (AC)^2 - (BC)^2 = (AB)^2$

$\Rightarrow (BC + 1)^2 - (BC)^2 = (5)^2$ [Using (1)]

$\Rightarrow (BC)^2 + 1 + 2BC - (BC)^2 = 25$

$\Rightarrow$ 2BC = 25 – 1

$\Rightarrow$ 2BC = 24

$\Rightarrow$ BC = 12 cm

$\therefore$ AC = 12 + 1 = 13 cm

Thus, cosec C = $\frac{AC}{AB}$

$= \frac{13}{5}$

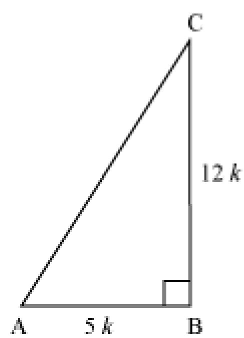
And, cot C = $\frac{BC}{AB}$

$= \frac{12}{5}$

Example 3:

In a right-angled triangle ABC, which is right-angled at B, tan A = 12/5. Find the value of cos A and sec A.

Solution:



It is given that tan A $= \frac{12}{5}$

We know that tan A $= \frac{BC}{AB}$

$\Rightarrow \frac{BC}{AB} = \frac{12}{5}$

Let BC = 12*k* and AB = 5*k*

Using Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$= (5k)^2 + (12k)^2$

$= 25k^2 + 144k^2$

$(AC)^2 = 169k^2$

$AC = 13k$

Now, cos A $= \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$

$= \frac{5k}{13k}$

$= \frac{5}{13}$

Sec A $= \frac{1}{\cos A}$

$= \frac{1}{\frac{5}{13}}$

$= \frac{13}{5}$

Use Trigonometric Ratios In Solving Problems

If we know the value of a trigonometric ratio, then we can find the values of other trigonometric ratios and the value of any expression involving these trigonometric ratios.

Let us take an example in the video to understand the method.

Let us now solve more problems.

Example1:

If cot A $= \frac{1}{3}$, then find the value of $\frac{\tan^2 A - 1}{\tan^2 A + 1}$.

Solution:

It is given that cot A $= \frac{1}{3}$.

We know that

tan A $= \frac{1}{\cot A}$

$= \frac{1}{\frac{1}{3}}$

tan A = 3

Then, $\frac{\tan^2 A - 1}{\tan^2 A + 1} = \frac{(3)^2 - 1}{(3)^2 + 1}$

$= \frac{9-1}{9+1}$

$= \frac{8}{10}$

$= \frac{4}{5}$

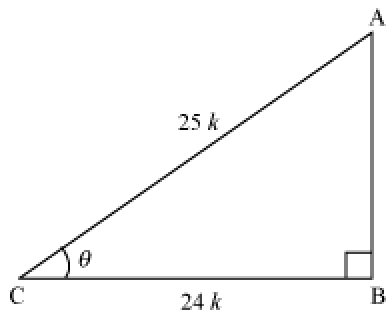
Example 2:

Find the value of ($\sin^2 \theta + \cos^2 \theta$), if sec θ $= \frac{25}{24}$.

Solution:

We have sec θ $= \frac{25}{24}$ … (i)

We know that sec θ $= \frac{\text{Hypotenuse}}{\text{Side adjacent to } \theta}$



Let ABC be a triangle in which ∠C = θ, therefore we have

$\sec\theta = \frac{AC}{BC}$ ...(ii)

From equations (i) and (ii), we have

$\frac{AC}{BC} = \frac{25}{24}$

Let us take AC = 25k and BC = 24k.

Using Pythagoras theorem, we have,

$(AC)^2 = (AB)^2 + (BC)^2$

$(25k)^2 = (AB)^2 + (24k)^2$

$625k^2 = (AB)^2 + 576k^2$

$AB^2 = (625 - 576)k^2$

$AB^2 = 49k^2$

AB = 7k

$\therefore \sin\theta = \frac{AB}{AC}$

$= \frac{7k}{25k}$

$= \frac{7}{25}$

$\cos\theta = \frac{BC}{AC}$

$= \frac{24k}{25k}$

$= \frac{24}{25}$

Now, $\sin^2\theta + \cos^2\theta = \left(\frac{7}{25}\right)^2 + \left(\frac{24}{25}\right)^2$

$= \frac{49}{625} + \frac{576}{625}$

$= \frac{625}{625}$

= 1

$\therefore \sin^2\theta + \cos^2\theta = 1$

Trigonometric Ratios Of Some Specific Angles

Let us now solve some examples based on the trigonometric ratios of some specific angles.

Example 1:

Find the value of cos 2A if A = 30°.

Solution:

It is given that

A = 30°

$\therefore \cos 2A = \cos(2 \times 30°)$

= cos 60°

$= \frac{1}{2}$

Example 2:

Prove that sin 2A = 2 sin A cos A, for A = 45°.

Solution:

L.H.S = sin 2 A = sin (2 × 45°)

= sin 90 °

= 1

R.H.S. = 2 sin A cos A = 2 × sin 45° × cos 45°

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

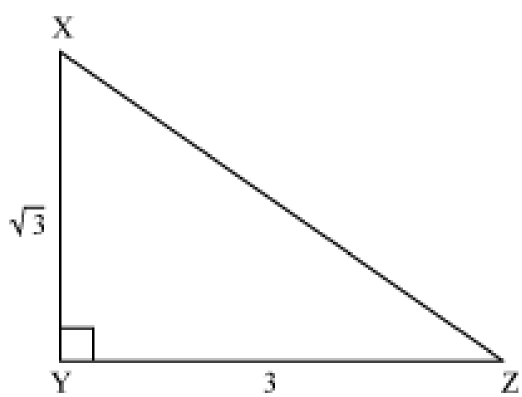
= 1

L.H.S = R.H.S.

∴ sin 2A = 2 sin A cos A

Example 3:

From the given figure XYZ, find ∠YZX and ∠ZXY.



Solution:

It is given that XY = $\sqrt{3}$ and YZ = 3

Now, $\frac{XY}{YZ} = \tan Z$

$\Rightarrow \tan Z = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\therefore \angle YZX = 30^\circ$

By angle sum property in ΔXYZ, we obtain

$\angle ZXY = 180° - 30° - 90°$

$\therefore \angle ZXY = 60^\circ$

Example 4:

If sin (A + B) = 1 and tan (A −B) = 0, where $0^\circ < A + B \leq 90^\circ$, then find A and B.

Solution:

sin (A + B) = 1,

We know that sin 90° = 1

$\Rightarrow A + B = 90^\circ$ ... (1)

tan (A −B) = 0

And, we know that tan 0° = 0

$\Rightarrow A - B = 0^\circ$ ... (2)

On adding (1) and (2), we obtain

$2A = 90^\circ$

$\Rightarrow A = 45^\circ$

On putting this value of A in (1), we obtain

$B = 45^\circ$

Thus, the value of both A and B is 45°.

Example 5:

Find the value of $\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B + \sin A \sin B}$, if A = 60° and B = 30°.

Solution:

$$\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B + \sin A \sin B} = \frac{\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ}{\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}}$$

$$= \frac{\frac{3}{4} + \frac{1}{4}}{\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}}$$

$$= \frac{\frac{4}{4}}{\frac{2\sqrt{3}}{4}}$$

$$= \frac{2}{\sqrt{3}}$$

Example 6:

Find the value of $\sin^2 60^\circ + 2\cos^2 30^\circ - \tan^2 45^\circ + \sec^2 30^\circ$

Solution:

$\sin^2 60^\circ + 2\cos^2 30^\circ - \tan^2 45^\circ + \sec^2 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2\left(\frac{\sqrt{3}}{2}\right)^2 - \left(1\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= \frac{3}{4} + 2 \times \frac{3}{4} - 1 + \frac{4}{3}$$

$$= \frac{31}{12}$$

Example 7:

If sin x = sin 60° cos 30° + cos 60° sin 30° then find the value of x.

Solution:

We have

sin x = sin 60° cos 30° + cos 60° sin 30°

$$\Rightarrow \sin\ x = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$\Rightarrow \sin\ x = \frac{3}{4} + \frac{1}{4}$$

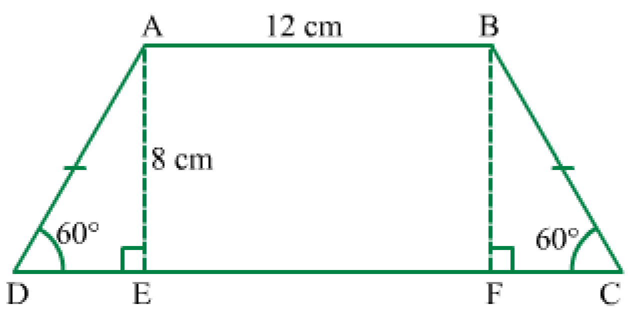
⇒ sin x = 1

⇒ sin x = sin 90°

⇒ x = 90°

Example 8:

ABCD is a trapezium such that AB || CD and AD = BC.



Find the area of the trapezium.

Solution:

In right-angled ΔAED, we have

$\tan 60° = \frac{AE}{ED}$

$\Rightarrow ED = \frac{AE}{\tan 60°}$

$\Rightarrow ED = \frac{8}{\sqrt{3}}$ cm

In ΔAED and ΔBFC, we have

AD = BC

∠AED = ∠BEC = 90° and

AE = BF   (Perpendiculars drawn between two parallel lines)

∴ ΔAED ≅ ΔBFC

By CPCT, we have

ED = FC = 83√83cm

ABFE is a rectangle, so we have

AB = FE = 12 cm

Now, DC = ED + FE + FC = $\frac{8}{\sqrt{3}}$ cm + 12 cm + $\frac{8}{\sqrt{3}}$ cm = $\left(12 + \frac{16}{\sqrt{3}}\right)$ cm

Area of trapezium ABCD = (AB + DC) × AE

$$= \left\{\frac{1}{2} \times \left(12 + 12 + \frac{16}{\sqrt{3}}\right) \times 8\right\} \text{cm}^2$$

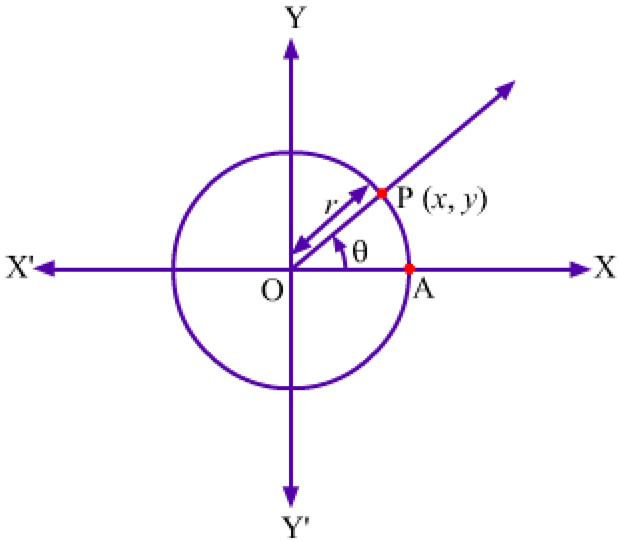
$$= \left\{\left(24 + \frac{16}{\sqrt{3}}\right) \times 4\right\} \text{cm}^2$$

$$= \left(96 + \frac{64}{\sqrt{3}}\right) \text{cm}^2$$

Trigonometric Identities

Now, let us prove these identities.

Let us take a standard circle with radius r such that it intersects the X-axis at point A. Also, let the initial arm OA is rotated in anti-clockwise direction by an angle ?



In the figure, the terminal arm intersects the circle at point P (x, y) where $x, y \neq 0$ and OP = r.

By the definition of trigonometric ratios, we have

$$\sin\theta = \frac{y}{r}, \qquad \operatorname{cosec}\theta = \frac{r}{y}$$
$$\cos\theta = \frac{x}{r}, \qquad \sec\theta = \frac{r}{x}$$
$$\tan\theta = \frac{y}{x}, \qquad \cot\theta = \frac{x}{y}$$

Now, OP is a distance between origin O (0, 0) and point P (x, y) which can be obtained by distance formula as follows:

$$OP = \sqrt{(x-0)^2 + (y-0)^2}$$
$$\Rightarrow r = \sqrt{x^2 + y^2}$$
$$\Rightarrow r^2 = x^2 + y^2 \quad \ldots(i)$$

(1) On dividing both sides of the equation (i) by r^2, we get

$$\frac{r^2}{r^2} = \frac{x^2}{r^2} + \frac{y^2}{r^2}$$

$$\Rightarrow 1 = \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2$$

$$\Rightarrow 1 = \cos^2\theta + \sin^2\theta$$

$$\Rightarrow \sin^2\theta + \cos^2\theta = 1$$

From this identity, we get two results

I. $\sin^2\theta = 1 - \cos^2\theta$

II. $\cos^2\theta = 1 - \sin^2\theta$

(2) On dividing both sides of the equation (i) by x^2 ($x \neq 0$), we get

$$\frac{r^2}{x^2} = \frac{x^2}{x^2} + \frac{y^2}{x^2}$$

$$\Rightarrow \left(\frac{r}{x}\right)^2 = 1 + \left(\frac{y}{x}\right)^2$$

$$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$$

$$\Rightarrow 1 + \tan^2\theta = \sec^2\theta$$

From this identity, we get two results

I. $\tan^2\theta = \sec^2\theta - 1$

II. $\sec^2\theta - \tan^2\theta = 1$

(3) On dividing both sides of the equation (i) by y^2 ($y \neq 0$), we get

$$\frac{r^2}{y^2} = \frac{x^2}{y^2} + \frac{y^2}{y^2}$$

$$\Rightarrow \left(\frac{r}{y}\right)^2 = \left(\frac{x}{y}\right)^2 + 1$$

$$\Rightarrow \operatorname{cosec}^2\theta = \cot^2\theta + 1$$

$$\Rightarrow 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

From this identity, we get two results

I. $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

II. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Corollary:

(i) When *x* = 0 then we have

$\sin\theta = \frac{y}{r}$, $\operatorname{cosec}\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r} = \frac{0}{r} = 0$, $\sec\theta = \frac{r}{x} = \frac{r}{0}$(Undefined)

$\tan\theta = \frac{y}{x} = \frac{y}{0}$(Undefined), $\cot\theta = \frac{x}{y} = \frac{0}{y} = 0$

In this case, the identities $\sin^2\theta + \cos^2\theta = 1$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ exist but the identity $1 + \tan^2\theta = \sec^2\theta$ does not exist.

(ii) When *y* = 0 then we have

$\sin\theta = \frac{y}{r} = \frac{0}{r} = 0$, $\operatorname{cosec}\theta = \frac{r}{y} = \frac{r}{0}$(Undefined)

$\cos\theta = \frac{x}{r}$, $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x} = \frac{0}{x} = 0$, $\cot\theta = \frac{x}{y} = \frac{x}{0}$(Undefined)

In this case, the identities $\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$ exist but the identity $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ does not exist.

Now, we know the basic trigonometric identities, let us see the following video to know how to use these identities.

Let us now solve some more problems using trigonometric identities.

Example 1:

Find the value of the expression ($\sec^2 27°$ – tan27°.cot63°).

Solution:

$\sec^2 27°$ – tan27°.cot63° = $\sec^2 27°$ – tan27°.cot (90° – 27°)

[27° and 63° are complementary angles]

= $\sec^2 27°$– tan27°.tan27° [cot (90° – ?) = tan ?]

= $\sec^2 27° - \tan^2 27°$

= $1 + \tan^2 27° - \tan^2 27°$ [Using the identity $1 + \tan^2 \theta = \sec^2 \theta$]

= 1

Thus, the value of the given expression is 1.

Example 2:

Write all the trigonometric ratios in terms of sin A.

Solution:

Using the identity

$\sin^2 A + \cos^2 A = 1$,

we can write, $\cos^2 A = 1 - \sin^2 A$

Taking square root on both sides,

$$\cos A = \sqrt{1-\sin^2 A} \qquad ...(i)$$

Now, $\tan A = \dfrac{\sin A}{\cos A}$

$$= \frac{\sin A}{\sqrt{1-\sin^2 A}}$$ [Using (i)]

$\Rightarrow \tan A = \dfrac{\sin A}{\sqrt{1-\sin^2 A}}$ ...(ii)

$\cot A = \dfrac{1}{\tan A}$

$\Rightarrow \cot A = \dfrac{\sqrt{1-\sin^2 A}}{\sin A}$ ...(iii)

$\sec A = \dfrac{1}{\cos A}$

$\Rightarrow \sec A = \dfrac{1}{\sqrt{1-\sin^2 A}}$ ...(iv) [Using (i)]

and, $\operatorname{cosec} A = \dfrac{1}{\sin A}$ ...(v)

The trigonometric ratios in terms of sin A are given by (i), (ii), (iii), (iv), and (v).

Example 3:

Simplify the following expression.

[(1 + cot A – cosec A)(1 + tan A + sec A)]

Solution:

(1 + cot A – cosec A)(1 + tan A + sec A)

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{(\sin A + \cos A)^2 - 1^2}{\sin A.\cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2.\sin A.\cos A - 1}{\sin A.\cos A}$$

$$= \frac{1 + 2.\sin A.\cos A - 1}{\sin A.\cos A} = \frac{2.\sin A.\cos A}{\sin A.\cos A} = 2$$

Thus, the value of the given expression is 2.

Solving Real World Problems Involving Heights and Distances

Tushar is looking at the top of a tree. The height of Tushar is 1.6 m. The distance between the foot of the tree and Tushar is 12 m. The line joining his eye and the top of the tree makes an angle of 30° with the horizontal line. Tushar wants to know the height of the tree.

Can we help him?

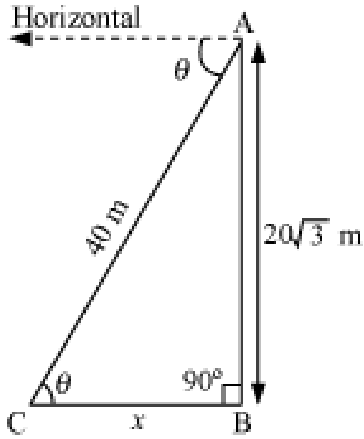
In this way, we can find the unknown distance or height using the concept of trigonometric ratios. Let us discuss some examples based on problems related to heights and distances.

Example 1:

A boy is looking at a ball on the ground from the top of a building which is $20\sqrt{3}$ m high. If the distance between his eye and the ball is 40 m, then what will be the angle of depression and the distance between the foot of the building and the ball?

Solution:

The diagram of this situation can be drawn as follows



Let the angle of depression be θ and the distance between the foot of the building and the ball be x m.

It is given that,

AC = line of sight = 40 m

AB = height of the building = $20\sqrt{3}$ m

The angle of depression = ∠ACB = θ

Now, in right-angled triangle ABC, we have

$$\sin\theta = \frac{\text{Perpndicular}}{\text{Hypotenuse}} = \frac{\text{AB}}{\text{AC}}$$
$$\Rightarrow \sin\theta = \frac{20\sqrt{3}}{40} = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \theta = 60°$$

Thus, the angle of depression is 60°.

Again, in ΔABC, we have

$$\tan\theta = \frac{\text{Perpndicular}}{\text{Base}} = \frac{\text{AB}}{\text{BC}}$$
$$\Rightarrow \tan 60° = \frac{20\sqrt{3}}{x}$$
$$\Rightarrow x = \frac{20\sqrt{3}}{\tan 60°} = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m}$$

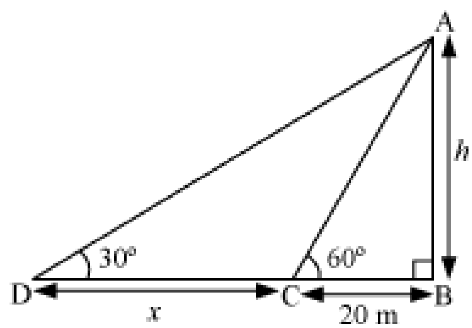
Thus, the distance between the foot of the building and the ball is 20 m.

Example 2:

A man is standing at a distance of 20 m at a point C in front of a lamp post AB. The angle of elevation at point C is 60º. Find the distance covered by the man if he walks away from the lamp post so that the angle of elevation becomes 30°.

Solution:

The figure of this situation can be drawn as follows.



Let the man walk *x* m and reach at point D. Let the height of the lamp post AB be *h*.

Now, in right-angled triangle ABC, we have

$$\tan 60° = \frac{AB}{BC} = \frac{h}{20}$$
$$\Rightarrow h = 20 \times \tan 60° = 20 \times \sqrt{3} = 20\sqrt{3} \text{ m}$$

Thus, the height of the lamp post is $20\sqrt{3}$ m.

In ΔABD, we have

$$\tan 30° = \frac{AB}{BD} = \frac{h}{x+20}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{20 \times \sqrt{3}}{x+20}$$
$$\Rightarrow x + 20 = 20 \times \sqrt{3} \times \sqrt{3} = 60 \text{ m}$$
$$\Rightarrow x = 60 - 20$$
$$\Rightarrow x = 40 \text{ m}$$

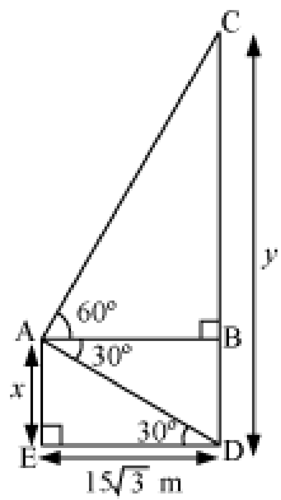
Thus, the required distance is 40 m.

Example 3:

Two buildings are at a distance of $15\sqrt{3}$ m from each other. A boy standing at the top of the smaller building is looking at the top and bottom of the other building. The angle of elevation and depression are 60° and 30° respectively. What will be the heights of the two buildings?

Solution:

The given situation can be represented geometrically by the following figure.



Let the height of smaller building be *x* m and larger building be *y* m.

Here, the angle of elevation, i.e., ∠CAB = 60° and the angle of depression, i.e., ∠BAD = 30°.

∠BAD and ∠ADE are alternate interior angles.

∴ ∠ADE = ∠BAD

= 30°

From the figure, we have

BD = AE = *x*

∴ BC = CD − BD

= *y* − *x*

Now, in right-angled ΔAED, we have

$$\tan\ 30^\circ = \frac{AE}{DE}$$
$$\Rightarrow \tan 30^\circ = \frac{x}{15\sqrt{3}}$$
$$\Rightarrow x = 15\sqrt{3} \times \tan 30^\circ$$
$$= 15\sqrt{3} \times \frac{1}{\sqrt{3}}$$
$$\Rightarrow x = 15 \text{ m} \qquad ...(1)$$

Now, in right angled ΔABC, we have

$$\tan 60^\circ = \frac{BC}{AB}$$
$$\Rightarrow \tan 60^\circ = \frac{y - x}{15\sqrt{3}}$$
$$\Rightarrow y - x = 15\sqrt{3} \times \tan 60^\circ$$
$$= \left(15\sqrt{3} \times \sqrt{3}\right) \text{ m}$$
$$\Rightarrow y - x = 45 \text{ m}$$

Using equation (1), we have

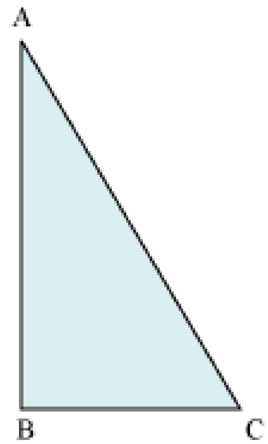
$y - 15 = 45$

$\Rightarrow y = 45 + 15 = 60$ m

Thus, the heights of the two buildings are 15 m and 60 m respectively.

Trigonometric Ratios Of Complementary Angles

Consider the following figure.



Here, a right-angled triangle ABC has been shown. In this triangle, suppose that the value of sin C is $\frac{12}{13}$.

Can we find the value of cos A?

Let us look at the given video to answer this question.

We can use these relations for simplifying the given expression.

For example: Let us express **sec 55°− cosec 89°** in terms of trigonometric ratios of angles between 0° and 45°.

Now, how can we do so? Let us see.

Since 55° and 35° are complementary angles and also 89° and 1° are complementary angles, we can write 55° as (90° − 35°) and 89° as (90° − 1°).

Therefore,

sec 55° − cosec 89° = sec (90° − 35°) − cosec (90° − 1°)

= cosec 35° − sec 1°

[∵ sec (90° – A) = cosec A and cosec (90° – A) = sec A]

∴ sec 55° − cosec 89°= cosec 35° − sec 1°

Let us now solve some more examples involving trigonometric ratios of complementary angles.

Example 1:

Find the value of sin 53° – cos 37°.

Solution:

We know that 53° and 37° are complementary angles as

53° + 37° = 90°

∴ We can write 37° as (90° – 53°).

∴ sin 53° – cos 37° = sin 53° – cos (90° – 53°)

= sin 53° – sin 53°

[∵ cos (90° – A) = sin A]

= 0

Thus, the value of (sin 53° – cos 37°) is 0.

Example 2:

Evaluate $\frac{\textbf{cosec } 27^\circ}{\textbf{sec } 63^\circ}$

Solution:

Here, 27° and 63° are complementary angles as 27° + 63° = 90°

∴ We can write 27° = 90° – 63°

Now,

$$\frac{\operatorname{cosec} 27^\circ}{\sec 63^\circ} = \frac{\operatorname{cosec}(90^\circ - 63^\circ)}{\sec 63^\circ}$$

$$= \frac{\sec 63^\circ}{\sec 63^\circ} \qquad [\because \operatorname{cosec}(90^\circ - A) = \sec A]$$

$$= 1$$

Example 3:

Prove that tan 2A = cot 3A, when A = 18°.

Solution:

When A = 18°,

L.H.S = tan 2A = tan (2 × 18°)

= tan 36°

R.H.S = cot 3A = cot (3 × 18)

= cot 54°

54° and 36° are complementary angles.

∴ We can write 54° as 90° – 36°.

Therefore, cot 3A = cot 54°

= cot (90° – 36°)

= tan 36° [∵ cot (90° – A) = tan A]

∴ L.H.S = R.H.S = tan 36°

∴ tan 2A = cot 3A

Example 4:

If sin A = cos A, then prove that A = 45°.

Solution:

It is given that sin A = cos A

⇒ sin A = sin (90° – A)

[∵ sin (90° – A) = cos A]

⇒ A = 90° – A

⇒ 2A = 90°

$$\Rightarrow A = \frac{90°}{2}$$
$$\Rightarrow A = 45°$$

Hence, proved

Example 5:

If P, Q, and R are interior angles of a triangle PQR, which is right-angled at Q, then show that

$$\cot\left(\frac{P+R}{2}\right) = \tan\frac{Q}{2}$$

Solution:

Now, P, Q, and R are the interior angles of the triangle PQR. Therefore, their sum should be 180°.

∴ P + R = 180 – Q

Now, consider the L.H.S. = $\cot\left(\frac{P+R}{2}\right)$

= $\cot\left(\frac{180° - Q}{2}\right)$

= $\cot\left(90° - \frac{Q}{2}\right)$

= $\tan\frac{Q}{2}$ [∵ cot (90° – A) = tan A]

= R.H.S.

∴ $\cot\left(\frac{P+R}{2}\right) = \tan\frac{Q}{2}$

Hence, proved

Example 6:

Prove that

$\tan 1° \times \tan 2° \times \tan 3° \ldots \tan 87° \times \tan 88° \times \tan 89° = 1$

Solution:

Here, 1° and 89° are complementary angles as $1° + 89° = 90°$

Therefore, we can write $89° = 90° – 1°$

Similarly, $88° = 90° – 2°$

$87° = 90° – 3°$

$46° = 90° – 44°$ and so on

Now, the L.H.S is

$\tan 1° \times \tan 2° \times \tan 3° \ldots \tan 44° \times \tan 45° \times \tan 46° \ldots \tan 87° \times \tan 88° \times \tan 89°$

$= \tan 1° \times \tan 2° \times \tan 3° \ldots \tan 44° \times \tan 45° \times \tan (90° – 44°) \ldots \tan (90° – 3°) \tan (90° – 2°) \tan (90° – 1°)$

$= \tan 1° \times \tan 2° \times \tan 3° \ldots \tan 44° \times \tan 45° \times \cot 44° \ldots \cot 3° \times \cot 2° \times \cot 1°$

$[\because \tan (90° – A) = \cot A]$

$$= \tan 1° \times \tan 2° \times \tan 3° \ldots \tan 44° \times \tan 45° \times \frac{1}{\tan 44°} \ldots \frac{1}{\tan 3°} \times \frac{1}{\tan 2°} \times \frac{1}{\tan 1°}$$

$$\left[\because \cot A = \frac{1}{\tan A}\right]$$

$= \tan 45°$

$= 1$

= R.H.S

$\therefore \tan 1° \times \tan 2° \times \tan 3° \ldots \tan 87° \times \tan 88° \times \tan 89° = 1$

Hence, proved