Chapter 1

Conduction

CHAPTER HIGHLIGHTS

- Introduction to Heat Transfer
- Conduction
- Convection
- Radiation
- Heat Transfer by Conduction
- Thermal Conduction in Solid, Liquid and Gases
- Electrical Analogy
- Log Mean Area
- Critical Lagging Thickness
- Unsteady Heat Conduction
- Lumped Heat Analysis
- Temperature-time Response of Thermo Couple
- Fins and Heat Convection
- Efficiency
- Effectiveness

INTRODUCTION TO HEAT TRANSFER

Heat transfer is the science which deals with rates transfer of energy in the form of heat between two bodies which are at different temperatures. Heat is intimately related to Thermo dynamics. The energy possessed by a body can be classified as Kinetic energy, Potential energy and internal energy. When ever a body interact with another body or the surroundings its energy may undergo change. The interaction between the body and surroundings can be classified as work interaction and heat interaction.

Thermodynamic is the study of energy conversion and its effect on the system and surroundings. This includes all the principles which governs the conversion of the energy. The principles or laws of thermodynamics are zeroth, first, second and third law of thermodynamics.

The zeroth law of thermodynamics forms the basis for measurement of temperature. In thermodynamics, heat is defined as the energy transfer between a system and its surroundings. Hence Heat is a mode of energy transfer and it is energy in transit. Heat is stored in the form of internal energy in a body. Heat is not a property of the system but it is a mode of energy transfer from one body to other due to temperature difference. From Thermodynamic point of view heat transfer is not appropriate but it should be referred as energy transfer as heat.

The second law of thermodynamics rules out the possibility of spontaneous process reversing its own. It gives the direction of process and provides information regarding feasibility of a process. The transfer of energy from a body at high-temperature to a body at low-temperature is a spontaneous process. Energy transfer can not takes place spontaneously from a body at low temperature to a body at high-temperature. Spontaneous process proceeds till the system reach a state of equilibrium, all macroscopically observable properties remain constant. In thermodynamics, equilibrium refers to equilibrium with respect to mechanical thermal and chemical assets. However in the study of Heat transfer it is concerned with thermal equilibrium only. Heat transfer processes also cannot violate the laws of thermodynamics.

Heat can be transferred in three different modes, conduction, convection and radiation. All the modes of heat transfer require the existence of a 'temperature difference' between the points across which heat is transferred. In all modes, heat gets transferred from a high-temperature medium to a low-temperature medium.

Conduction

It is the mode of heat transfer from a point on a body to another point of the same body, or it is the heat transfer from one body to another body which is in contact with the first body.

Heat transfer by conduction can be from the more energetic particles of the substance to the adjacent less energetic ones–as a result of interaction between the particles.

In solids, particles of a substance vibrate from their mean position. When temperature increases the vibration becomes intense. This results in interactions between molecules, while vibrating about their mean positions.

Conduction is a mode of heat transfer without appreciable movements or displacement of molecules forming the substances. Heat transfer in solids occur by molecular interaction and also by free electrons.

Convection

It is the mode of heat transfer between a solid surface and the adjacent liquid or gas which is in motion. That is why convection is explained as involving the combined effect of conduction and fluid motion. The faster the fluids move, more is the convection heat transfer.

> In the absence of any bulk fluid motion, the heat transfer between a solid surface and the adjacent fluid is by pure conduction.

During convection heat transfer, there is actual movement of molecules and they mix or mingle in between. Convection can be in two ways:

1. Free natural convection
2. Forced convection

Convection is said to be free or natural if the motion of the fluid is caused by buoyancy forces that are induced by density difference due to the variation of temperature in the fluid layers.

The denser portion of fluid moves down because of greater force of gravity, as compared to the force of gravity on less dense portion.

Convection is said to be forced if the fluid is forced to flow over the surface by external means such as fans, pumps, blowers etc

> Heat transfer process that involves change of phase of a fluid is also considered to be convection.

It is because of the fluid motion induced during these processes. Vapour bubble rises during boiling and liquid droplets falls during condensation.

Radiation

Unlike conduction and convection, the transfer of heat by radiation does not require the presence of any intervening medium.

It is the heat energy emitted by matter in the form of electromagnetic waves as a result of the changes in the electronic configuration of atoms or molecules.

In fact, heat transfer by radiation is the fastest and it suffers no attenuation in vacuum.

Thermal radiation differs from other forms of electromagnetic radiations such as X rays, γ rays, microwaves, radio waves etc, as these are not related to temperature.

All bodies at a temperature above absolute zero emit thermal radiations.

Radiation is considered to be a surface phenomenon for solids.

HEAT TRANSFER BY CONDUCTION

The rate of heat flow through a homogeneous medium (solid) depends on the following.

1. Geometry of the medium
2. Thickness
3. Material of the medium
4. Temperature difference across the medium.

Considering these aspects, law of conduction is stated as follows.

> The rate of flow of heat through a single homogeneous solid is directly proportional to the area of the section at right angles to the heat flow and the change of temperature with respect to the length of the path of the heat flow.

The change of temperature with respect to the length of the path of heat flow of is known as temperature gradient $\frac{dT}{dx}$

$$\therefore \quad Q \propto A\frac{dT}{dx} \quad \text{or} \quad Q = KA\frac{dT}{dx}$$

or
$$Q = K\left(\frac{T_1 - T_2}{L}\right)A$$

Where 'K' a constant known as coefficient of thermal conductively. The equation is usually written as

$$Q = -KA\frac{dT}{dx}$$

The (–)ve sign is to take care of the fall of temperature along the direction of heat flow, i.e., temperature decreases as 'x' increases. 'Q' is the rate of heat flow and unit is Joules/s i.e., W.

$Q(\text{W}) = -KA\frac{dT}{dx}$. This is known as Fourier law of heat conduction.

$$Q(\text{W}) = -K\,A\,(\text{m}^2)\,\frac{dT}{dx}\ (\text{K/m})$$

$\therefore$ The unit of K is

$$\frac{\text{W}}{\text{m}^2\text{k/m}} = \frac{\text{W}}{\text{mk}}.$$

$$= \frac{\text{W}}{\text{metre} \times \text{kelvin}}.$$

Significance of Coefficient of Thermal Conductivity (K)

1. $\left(A\frac{dT}{dx}\right)$ remaining the same, rate of heat flow increases as K increases.
2. 'K' is the measure of the ability of a material to conduct heat. Thermal conductivity of iron is 120 times more than that of water.
3. Conduction of heat occurs faster in metals, slower in non-metals and very slow in insulators.
4. Gases with higher molecular weight have low thermal conductivity.
5. When the density of a gas is high conductivity is low.
6. Conductivity is directly proportional to the mean free path

Thermal Conduction in Solid, Liquid and Gases

In gas

- In gases the conduction takes place due to the collision of molecules. Because of collisions, vibrations occurs the conduction takes place.
- In gases, the molecules are far apart from each other and randomness effect is not so dominating as the effect of collision between the molecules. Therefore with the increase in temperature, thermal conductivity (k) of the gas increases.

In liquids

- In liquids also the conduction takes place due to the collision of molecules.
- In liquids, the randomness is dominating and by increasing the temperature, thermal conductivity (k) decreases.
- In case of pure water, k increases with temperature then it decreases.

$$K_{\text{liquids}} >>> K_{\text{gas}}\ [10^2 - 10^3 \text{ times}]$$

In solids

- Solid are tightly packed structure.
- Conduction takes place due to:
  1. Lattice Vibration (K_L). One lattice of molecules collides with each other.
  2. Movement of free electrons (K_c)

$$K_{\text{solids}} = K_L + K_e$$

- Every material has electrical resistivity (ρ_e). If ρ_e increases then resistance also increases and there is obstruction in electron movement and hence K_e decreases.

  For pure metals: ρ_e is very less and K_e is very high. In pure metals the conduction takes place mainly due to movement of free electrons (K_e).

  For alloys: ρ_e is less and K_e is high. In alloys both K_L and K_e are responsible for heat conduction.

  For non-metals: ρ_e is very high and K_e is very low. In non-metals, dominating factor is K_L for conduction.

- For solids, when temperature increases, the thermal conductivity decreases.

NOTE

For stainless steel, increment in temperature also increases thermal conductivity.

Thermal conductivity of some materials

Material	Thermal conductive watt/ mk
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury	8.54
Glass	0.78
Water	0.607
Human skin	0.37
Wood	0.17

(Continued)

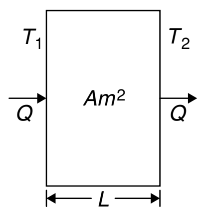
NOTE

Specific heat of a material is its ability to store thermal energy. Specific heat of water is 4.18 kJ/kg K and that of iron is 0.45 $kJ/kg\ K$. Water can store 10 times more thermal energy than iron can per unit mass.

Thermal Resistance

In the typical case of a slab having cross sectional area Am^2 perpendicular to the direction of heat flow, 'L' the thickness across which heat flow occurs and K, the coefficient of thermal conductivity, then Q

$$Q(\text{W/m}^2) = K\frac{(T_1 - T_2)}{L} \qquad (1)$$



Where T_1 is the higher temperature and T_2, the lower temperature. If Q is in W/m^2

$$Q = \frac{T_1 - T_2}{(L/K)} \qquad (2)$$

Where L/K is known as the thermal resistance of the material (if Q is in W/m^2).

The unit of thermal resistance

$$= \frac{m}{\frac{W}{mK}} = \frac{m^2k}{W}.$$

Electrical Analogy

Heat flow through a conductor is analogous to the current flow through a conductor. The current flowing along a conductor is given by the formula

$$I = \frac{E}{R} \qquad (1)$$

Where E is the voltage or potential difference between the ends of the conductor and 'R' the electrical resistance.

Consider heat flow through a conductor of cross sectional area Am^2, length L meters, the ends of the conductor maintained at temperatures

$$T_1 \text{ and } T_2 \,;\, T_1 > T_2$$

$$Q = KA\frac{(T_1 - T_2)}{L} \text{ W}$$

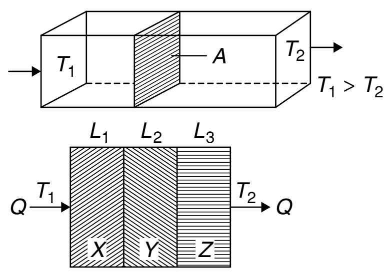
$$Q = \frac{T_1 - T_2}{L/KA} = \frac{T_1 - T_2}{R} \quad (2)$$

$T_1 - T_2$ is the temperature difference which is identical to the potential difference between the ends of the conductor 'E', and (L/KA) is the thermal resistance which is identical to 'R' the electrical resistance of the conductor according to equation (1).

Equations (1) and (2) are comparable

Electrical resistance $\propto \frac{L}{a}$, L = length, 'a' area of c/s of the conductor.

Thermal resistance $\propto \frac{L}{A}$, L = distance, A area of cross-section across the direction of heat flow, provided K is a constant.



Imagine the case of heat flow across three bodies X, Y and Z as shown in the figure, having thermal conductivities K_1, K_2, K_3 respectively.

As per equation (2) $Q = \dfrac{T_1 - T_2}{R_X + R_Y + R_Z}$

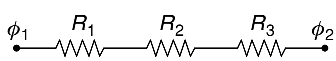
Where R_x = Thermal resistance of X,
R_y = Thermal resistance of Y and R_z = Thermal resistance of 'Z'

$$\therefore \qquad Q = \frac{T_1 - T_2}{\dfrac{L_1}{K_1 A} + \dfrac{L_2}{K_2 A} + \dfrac{L_3}{K_3 A}} \quad (3)$$

Where 'A' is the area of cross section, across which conduction takes place.

If we consider current flow

$$I = \frac{\phi_l - \phi_2}{R_1 + R_2 + R_2} \quad (4)$$



Where
$\phi_1 - \phi_2$ = potential difference.

(3) and (4) are identical. The unit of thermal resistance $= \dfrac{L}{K} = \dfrac{\text{m}}{\text{W/mK}} = \dfrac{\text{m}^2\text{k}}{\text{W}}$

The general equation of heat flow is given by

$$K\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + q' = \rho Cp \cdot \frac{dT}{dt} \quad (1)$$

This is 3-dimensional heat flow equation with heat generation. 'q'' denotes the internal heat generation/unit volume per unit time. 'ρ' is the density, and Cp is the specific heat.

Equation (1) can be written as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'}{K} = \frac{\rho Cp}{K}\frac{dT}{dt}$$

$$\text{That is, } \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'}{K} = \frac{1}{\alpha}\frac{dT}{dt} \quad (2)$$

where $\dfrac{K}{\rho Cp} = \alpha$ known as thermal diffusivity

Thermal diffusivity indicates the easiness with which heat dissipated. A higher value of 'α' indicates high rate of heat diffusion or dissipation.

Four Cases Arise from the General Heat Flow Equation

Case I: Steady state heat flow. Under steady state heat conduction, $\dfrac{dT}{dt} = 0$.

Equation (2) becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'}{K} = 0 \quad (3)$$

This is 3-dimensional steady state heat flow equation with internal heat generation.

Case II: Let $q' = 0$ i.e., there is no heat generation. Then equation (3) becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (4)$$

That is, this equation represents 3-dimenstional steady state heat flow equation without heat generation

Case III: Let us consider the case of 2-dimensional heat flow. Then $\dfrac{\partial^2 T}{\partial z^2}$ does not exist.

Consequently equation (4) becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (5)$$

This is 2-dimensional steady state heat flow equation without heat generates

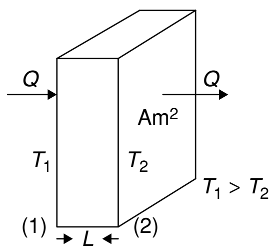
Case IV: Let us consider the heat flow in one direction only. Then $\frac{\partial^2 T}{\partial y^2}$ does not exist in equation (5).

It becomes $\frac{\partial^2 T}{\partial x^2} = 0$ (6)

This is one-dimensional steady state heat flow equation without heat generation

Heat flow across slab, hollow cylinder, spherical shell

(a) Slab



$$Q = -KA\frac{dT}{dx}$$

$$Qdx = -KAdT, \text{Integrating}$$

$$Q\int dx = -KA\int_1^2 dT$$

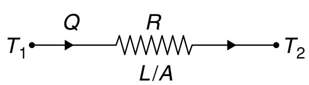
$$QL = -KA(T_2 - T_1)$$

$$Q = \frac{T_1 - T_2}{\left(\frac{L}{KA}\right)}$$

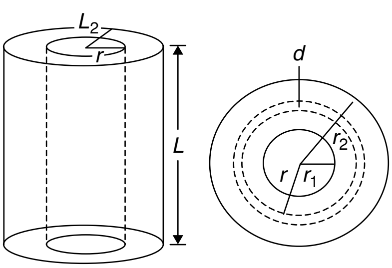
Thermal resistance in slab

$$Q = \frac{T_1 - T_2}{\frac{L}{kA}} \Rightarrow Q = \frac{(T_1 - T_2)}{\Sigma R}$$

$$\Sigma R = \text{L/K} = \text{Thermal resistance}$$



(b) Hollow cylinder



$$Q = -KA\frac{dT}{dr}$$

$$Qdr = -K \cdot 2\pi rL\, dT, \text{ Integrating}$$

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = -K2\pi L\int_{T_1}^{T_2} dT$$

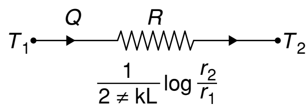
$$Q\log\frac{r_2}{r_1} = -2\pi KL[T_2 - T_1] = 2\pi KL[T_1 - T_2]$$

$$Q = \frac{2\pi KL(T_1 - T_2)}{\log r_2/r_1} = \frac{T_1 - T_2}{\frac{1}{2\pi KL}\log r_2/r_1}.$$

Thermal resistance in hollow cylinder

$$Q = \frac{(T_1 - T_2)}{\frac{1}{2\pi kL}\log\left(\frac{r_2}{r_1}\right)} \Rightarrow Q = \frac{(T_1 - T_2)}{\Sigma R}$$

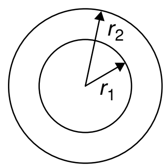
Where ΣR = Thermal resistance $= \frac{1}{2\pi kL}\log\left(\frac{r_2}{r_1}\right)$



(c) Spherical shell

$$Q = -KA\frac{dT}{dr}$$

$$Q = -K4\pi r^2 \cdot \frac{dT}{dr}$$



Where $4\pi r^2$ is the surface area of the shell at radius 'r'.

$$Qdr = -4\pi r^2 KdT, \text{ Integrating}$$

$$Q\int\frac{dr}{r^2} = -4\pi K\int_{T_1}^{T_2} dT = -4\pi K(T_2 - T_1)$$

$$Q\int_{r_1}^{r_2}\frac{1}{r^2}dr = 4\pi K[T_1 - T_2]$$

$$-Q\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = 4\pi K[T_1 - T_2]$$

$$\therefore \quad Q = \frac{4\pi K[T_1 - T_2]}{-\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} = \frac{4\pi K[T_1 - T_2]}{\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

$$= \frac{T_1 - T_2}{\frac{1}{4\pi K r_1 r_2}[r_2 - r_1]}$$

Thermal resistance of hollow sphere

$$Q = \frac{(T_1 - T_2)}{\frac{1}{4\pi k r_1 r_2}(r_2 - r_1)} \Rightarrow Q = \frac{(T_1 - T_2)}{\Sigma R}$$

Where ΣR = Thermal resistance

$$= \frac{1}{4\pi k r_1 r_2}(r_2 - r_1)$$

$$\frac{1}{4\pi k r_1 r_2}(r_2 - r_1)$$

NOTE

For a Slab $Q = \dfrac{T_1 - T_2}{\left(\dfrac{L}{KA}\right)}$

For a Cylinder $Q = \dfrac{T_1 - T_2}{\dfrac{1}{2\pi KL}\log\dfrac{r_2}{r_1}}$

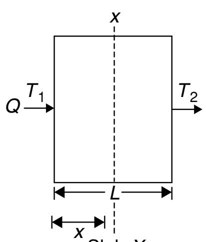
For a Spherical shell $Q = \dfrac{T_1 - T_2}{\dfrac{1}{4\pi r_1 r_2}[r_2 - r_1]}$

Temperature distribution

(a) In the case of a slab, the temperature distribution along the direction of heat flow is given by

$$T = T_1 + \left(\frac{T_2 - T_1}{L}\right) \times T_1 > T_2$$

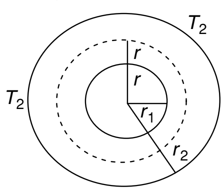
Where T is the temperature at section x-x, 'x' m away form the face with temperature T_1.



Slab X

(b) In the case of a hollow cylinder if the inside temperature is T_1 (at radius r_1) and outside temperature T_2 (at radius r_2) $T_1 > T_2$ then the temperature atany section at radius 'r'(in between r_1 and r_2) is given by $T = C_1 \log r + C_2$, C_1 and C_2 can be evaluated from the boundary conditions. At $r = r_1$,

$$T = T_1, \text{ at } r = r_2, T = T_2.$$

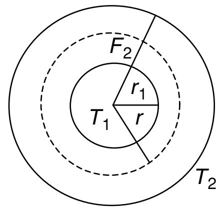


Hollow cylinder

(c) In the case of a spherical shell, Inside temperature being T_1 (at radius r_1) $T_1 > T_2$; the temperature at any radius 'r' (in between r_1 and r_2) is given by

$$T = \frac{-C_1}{r} + C_2,\ C_1 \text{ and } C_2$$

can be evaluated from the boundary conditions at $r = r_1$, $T = T_1$ at $r = r_2$, $T = T_2$



Spherrical shell

NOTES

After the evaluation of constants, we arrive at the following results.

1. In a cylindrical tube of inner radius r_1 and outer radius r_0 with temperature T_1 and T_0, respectively heat flowing from inside to outside, temperature at any point at radius 'r' is given by $T = T_1 - \dfrac{(T_1 - T_0)}{\log\dfrac{r_0}{r_1}}\log\dfrac{r}{r_0}$.
2. In a spherical shell of inside radius r_1 and outside radius r_0 with temperature T_1 and T_0 respectively, heat is flowing from inside to outside, the temperature at any point at radius 'r' is given by $T = T_1 - \dfrac{r_0}{r}\left(\dfrac{r - r_1}{r_0 - r_1}\right)(T_0 - T_1)$

When heat flows across a composite wall consisting of many layers of different materials, at steady state condition the total heat flow across each layer of the wall is numerically equal to the total heat flowing across the composite wall.

Log Mean Area

Hollow Cylinder

Problems of heat transfer involving cylinders can be converted to heat transfer problems across plane wall areas, in order to simplify the calculations. When heat flows across a cylinder is considered, the equivalent slab areas can be computed and the problem can be transferred to a case of heat flow through slab.

Let A_1 and A_2 be the inside and outside areas of a cylinder and L be its length. Let heat flows from inside to outside.

$$Q = \frac{2\pi KL(T_1 - T_2)}{\log\dfrac{r_2}{r_1}}$$

$Q = \dfrac{2\pi KL(T_1 - T_2)}{\log \dfrac{r_2}{r_1}}$ may be equal to $\dfrac{KA_e(T_1 - T_2)}{r_2 - r_1}$ where

Ae is the equivalent area of the slab across which heat flows $(r_2 - r_1)$ is the thickness of the slab.

$$\frac{2\pi KL(T_1 - T_2)}{\log \dfrac{r_2}{r_1}} = \frac{KA_e(T_1 - T_2)}{r_2 - r_1}$$

$$\therefore A_e = \frac{2\pi L(r_2 - r_1)}{\log \dfrac{r_2}{r_1}} = \frac{2\pi r_2 L - 2\pi r_1 L}{\log \dfrac{2\pi r_2 L}{2\pi r_1 L}}$$

$$A_e = \frac{A_2 - A_1}{\log \dfrac{A_2}{A_1}}$$

Where A_1 and A_2 are the inside and outside areas of the cylinder. The above expression for A_e is known as the log Mean Area. If $\dfrac{A_2}{A_1} < 2$ then the average area is taken as the area of the slab.

Hollow Sphere

Consider a hollow sphere of insider radius 'r_1' and outside radius r_2. Let T_1 and T_2 be the temperature inside and outside the sphere $T_1 >> T_2$

The heat transfer through the sphere

$$Q = \frac{4\pi r_1 r_2 K(T_1 - T_2)}{r_2 - r_1}$$

Let this quantity of heat is transferred through a slab of thickness '$r_2 - r_1$'. We have to find out the effective or equivalent area of the slab, $\perp^r$ to the direction of heat flow.

$Q = \dfrac{KAe(T_1 - T_2)}{r_2 - r_1}$ for the slab. It can be equated to the expression for heat flow across the spherical shell.

$$\therefore \frac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1} = \frac{KAe(T_1 - T_2)}{r_2 - r_1}$$

$$4\pi r_1 r_2 = A_e$$

$$A_e^2 = 16\pi^2 r_1^2 r_2^2$$

$$= 4\pi r_1^2 \times 4\pi r_2^2$$

$$= A_1 \times A_2$$

$$A_e = \sqrt{A_1 A_2}$$

Conduction and Convection

Consider the case of cooling a hot body blowing air over it. Heat transfer occurs in two different ways

1. Heat is transferred to the air layer (which is very thin) adjacent to the body by conduction.
2. This heat is then carried away from the surface air by air by convection.

The bulk motion of hot air from the surface gives room for cold air to occupy the space.

The rate of convection heat transfer is observed to be proportional to the temperature difference and the area of the surface exposed to heat transfer. Therefore heat transfer by convection can be written as $Q_c = hA(T_s - T_a)$ where T_s is the temperature of the hot body and T_a is the atmospheric temperature sensed away from the object. 'h' is known as the convection heat transfer coefficient or film coefficient.

Convection Heat Transfer Co-efficient

It is not a property of the fluid or solid undergoing convection heat transfer. Its value depends on all the variables influencing convection such as the surface geometry, nature of fluid motion, fluid properties etc.

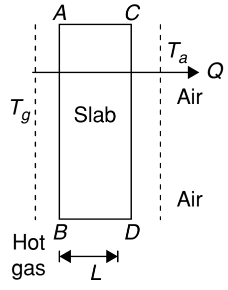
When, $Q = hA(T_1 - T_2)$

$$h = \frac{Q}{A(T_1 - T_2)}$$

Convection can be viewed as combined conduction and fluid motion. Conduction can be viewed as a special case of convection in the absence of fluid motion.

Imagine a hot gas at temperature T_g separated by a slab of thermal conductivity 'K' from atmospheric air at temperature 'T_a' $T_g >>> T_a$

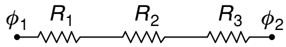
The hot gas is flowing along the face AB and air is flowing along the face CD.



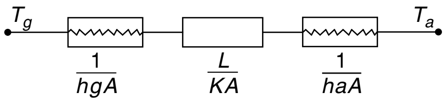
$\dfrac{1}{h_g A} + \dfrac{L}{KA} + \dfrac{1}{h_a A}$. Is the total thermal resistance, where hg and ha are convection heat transfer coefficient at faces AB and CD respectively.

$$\therefore Q = \frac{T_g - T_a}{\dfrac{1}{h_g A} + \dfrac{L}{KA} + \dfrac{1}{h_a A}} \quad \text{or} \quad \frac{A(T_g - T_a)}{\dfrac{1}{h_g} + \dfrac{L}{K} + \dfrac{1}{h_a}}$$

This case may be compared with the case of current flow along three resistors connected in series.



$\phi_1 - \phi_2$ = Potential Difference (Voltage)
R_1, R_2, R_3 are the resistances

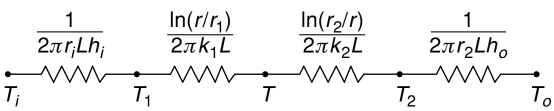


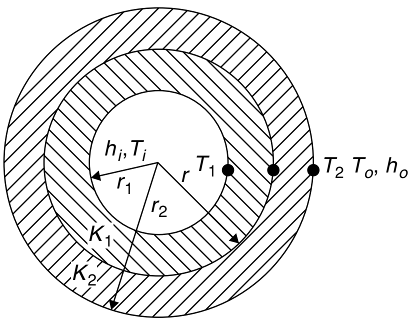
$$Q = \frac{T_g - T_a}{\frac{1}{h_g A} + \frac{l}{KA} + \frac{1}{h_a A}}$$

Heat Transfer Through Composite Cylinder

A composite cylinder consists of two or more coaxial cylinder as shown in the figure. The material thermal conductivities of the cylinders are K_1, K_2. Let the inner most temperature of composite cylinder is T_1 due to hot fluid flowing inside with temperature T_i and heat transfer coefficient of h_1 and outermost temperature is T_2 due to cold fluid flowing with temperature T_0 and heat transfer coefficient h_2.

If L is the length of the cylinder, the heat transfer rate Q from the hot to cold fluid is the same through each cylinder and are expressed as





$$Q = h_i 2\pi r_1 L(T_i - T_1); \; Q = \frac{2\pi k_1 L}{\ln\left(\frac{r}{r_1}\right)}(T_1 - T)$$

$$Q = \frac{2\pi k_2 L}{\ln\left(\frac{r_2}{r}\right)}(T - T_2); \; Q = h_o 2\pi r_2 L(T_2 - T_o)$$

$$(T_i - T_o) = (T_i - T_1) + (T_1 - T) + (T - T_2) + (T_2 - T_o)$$

$$\Rightarrow \quad (T_i - T_o) = Q\left[\frac{1}{h_i 2\pi r_1 L} + \frac{\ln\left(\frac{r}{r_1}\right)}{2\pi k_1 L} + \frac{\ln\left(\frac{r_2}{r}\right)}{2\pi k_2 L} + \frac{1}{h_o 2\pi r_2 L}\right]$$

$$\Rightarrow \quad (T_i - T_0) = Q[R_i + R_1 + R_2 + R_0$$

$$\text{or } Q = \frac{T_i - T_o}{R_i + R_1 + R_2 + R_o} = \frac{T_i - T_o}{\Sigma R}$$

$$Q = \frac{(T_i - T_o)}{\left[\frac{1}{h_i 2\pi r_1 L}\right] + \left[\frac{\ln r/r_1}{2\pi k_1 L}\right] + \left[\frac{\ln r_2/r}{2\pi k_2 L}\right] + \left[\frac{1}{h_o 2\pi r_2 L}\right]}$$

Overall heat transfer coefficient, U, can be expressed in this system.

$$U_i A_i = \frac{1}{\Sigma R} = \frac{1}{R_i + R_1 + R_2 + R_o}$$

$$\text{or } U_o A_o = \frac{1}{\Sigma R}$$

where $A_i = 2\pi r_1 L$ and $A_o = 2\pi r_2 L$

Heat Transfer through Composite Sphere

The total radial heat flow rate, Q through the composite sphere can be given as (refer pervious figure)

$$Q = h_i \cdot 4\pi r_1^2 [T_i - T_1]; \; Q = \frac{4\pi k_1 r_1 r(T_1 - T)}{(r - r_1)}$$

$$Q = \frac{4\pi k_2 r r_2}{(r_2 - r)}(T - T_2); \; Q = h_o \cdot 4\pi r_2^2 (T_2 - T_o)$$

$$\therefore T_i - T_o = \frac{Q}{4\pi}\left[\frac{1}{h_1 r_1^2} + \frac{(r - r_1)}{k_1 r_1 r} + \frac{(r_2 - r)}{k_2 r r_2} + \frac{1}{h_o \cdot r_2^2}\right]$$

$$\text{or } Q = \frac{4\pi(T_i - T_o)}{\left[\frac{1}{h_1 r_1^2} + \frac{(r - r_1)}{k_1 r_1 r} + \frac{(r_2 - r)}{k_2 r r_2} + \frac{1}{h_o \cdot r_2^2}\right]}$$

$$\text{or } Q = \frac{(T_i - T_o)}{R_i + R_1 + R_2 + R_o} = \frac{(T_i - T_o)}{\Sigma R}$$

The overall heat transfer coefficient for exterior surface will be given as

$$U_o \cdot A_o = \frac{1}{\Sigma R} \quad \Rightarrow \quad U_o \cdot 4\pi r_2^2 = \frac{1}{R_i + R_1 + R_2 + R_o}$$

Critical Lagging Thickness

We know that by adding insulation to a plane surface heat flow can be blocked. The heat transfer area remains the same, but the distance along which heat transfer taken place increases (L). Therefore the heat transfer rate decreases.

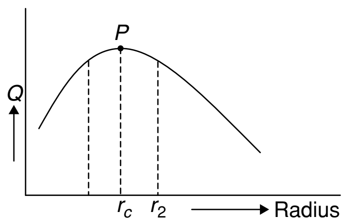
But this is not the case with spherical shells or cylindrical pipes. Addition of insulation increases the conduction resistance but will decrease the convection resistance. This is because the outside area of heat transfer increases.

Imagine a cylindrical pipe of outer radius r_1 and constant temperature T_1. Let this pipe be insulated with a material having thermal conductivity 'K'. Now the outer radius (including insulation) becomes r_2. Taking the atmospheric temperature as T_2 and convective heat transfer coefficient h, the rate of heat transfer.

$$Q = \frac{T_1 - T_2}{\frac{1}{2\pi KL}\log\frac{r_2}{r_1} + \frac{1}{h(2\pi r_2 L)}}$$

$$Q = \frac{T_1 - T_2}{\frac{1}{2\pi KL}\log\frac{r_2}{r_1} + \frac{1}{h\cdot 2\pi r_2 L}}$$

If we plot Q the rate of heat transfer against the out side radius of insulation we get a curve in the shape as shown.



The maximum heat transfer occurs at 'P'. At maximum $\frac{dQ}{dr_2} = 0$

Q will be maximum when $\frac{1}{2\pi KL}\log\frac{r_2}{r_1} + \frac{1}{2\pi r_2 L}$ is minimum.

$$f(r_2) = \frac{1}{2\pi KL}\log\frac{r_2}{r_1} + \frac{1}{2\pi h r_2 L} \text{ is minimum.}$$

$$f'(r_2) = 0$$

i.e.,
$$\frac{1}{2\pi KL} \times \frac{1 \times \frac{1}{r_1}}{\frac{r_2}{r_1}} + \frac{1}{2\pi hL}\left(\frac{1}{-r_2^2}\right) = 0$$

i.e.,
$$\frac{1}{K}\frac{1}{r_2} - \frac{1}{h}\left(\frac{-1}{r_2^2}\right) = 0$$

∴
$$\frac{1}{Kr_2} = \frac{1}{hr_2^2}$$

$$\boxed{r_2 = \frac{K}{h}}$$

This is the critical radius of insulation.

$$r_{cr} = \frac{K}{h}$$

When the radius of insulation is less than 'r_{cr}' heat transfer rate has an increasing trend. Values of radius of insulation up to r_{cr} the heat transfer will increase. For values of $r_2 > r_{cr}$ the heat transfer rate reduces.

The value of critical radius of insulation is large when 'K' is large and 'h' is small.

The lowest value of 'h' may be approximate to $5 \text{ W/m}^2\text{k}$. The highest value of thermal conductivity of insulating materials is of the order 0.05 W/mk.

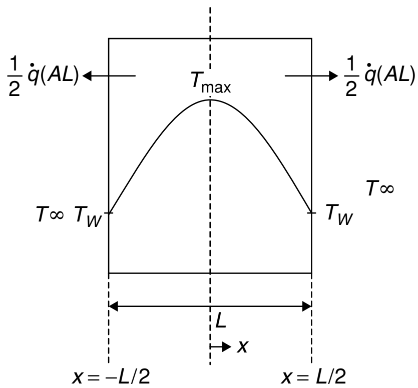
So the highest probable value of critical radius is

$$r_{cr} = \text{k/h} = \frac{0.05}{5} \text{ m}$$
$$= \frac{5\times10^{-2}}{5} = \frac{1}{100} \text{ m}$$
$$= 1 \text{ mm.}$$

So practically it is not a matter of concern. An insulation provided on a pipe shall have a thickness more than 1 mm.

Same way, we can evaluate the critical thickness of 'insulation' for a spherical shell. It is equal to '$2K/h$' Where 'K' is coefficient of thermal conductivity of the insulating materials and 'h' the convection heat transfer coefficient.

Plane Wall with Uniform Heat Generation



Let us consider a slab of thickness L, of uniform thermal conductivity K with internal heat generation of $\dot{q}$ (W/m³). The temperature of the two ends of the plate be same (T_W) because equal amount of heat will be lost due to connection on two sides.

At the centre, the temperature will be maximum because heat is generating at the centre.

$$\text{At } x = 0, \; \frac{dT}{dx} = 0$$

Energy Equation

$$k\frac{d^2T}{dx^2} + \dot{q} = 0 \text{ [For steady, } 1 - D \text{ heat transfer]}$$

or
$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \;\Rightarrow\; \frac{d}{dx}\left[\frac{dT}{dx}\right] = \frac{-\dot{q}}{k}$$

Integrating the above equation.

$$\frac{dT}{dx} = \left[\frac{-\dot{q}}{K}\right]x + C_1$$

Again integrating, $T = \left[\frac{-\dot{q}}{2k}\right]x^2 + C_1x + C_2$

Boundary Conditions

1. At $x = 0$, $\frac{dT}{dx} = 0 \Rightarrow C_1 = 0$
2. At $x = \pm L/2$, $T = T_W$

$$\therefore T_W = \frac{\dot{q}}{2k} \times \frac{L^2}{4} + C_2$$

$$\Rightarrow C_2 = T_W + \frac{\dot{q}}{2k}\left[\frac{L^2}{4}\right] \quad (1)$$

3. $-kA\left.\frac{dT}{dx}\right|_{x=+\frac{L}{2}} = hA[T_W - T_\infty]$

or $-kA\left.\frac{dT}{dx}\right|_{x=-L/2} = hA[T_\infty - T_w]$

$$\therefore \quad \frac{1}{2}\dot{q}AL = hA(T_W - T_\infty)$$

$$\Rightarrow T_w = T_\infty + \frac{\dot{q}AL}{2hA} = T_\infty + \frac{\dot{q}L}{2h}$$

From equation (1) we get

$$C_2 = T_\infty + \frac{\dot{q}L}{2h} + \frac{\dot{q}}{2K}\left[\frac{L^2}{4}\right]$$

$\therefore$ Temperature profile equation will be

$$T = \left[\frac{-\dot{q}}{2K}\right]x^2 + T_\infty + \frac{\dot{q}L}{2h} + \frac{\dot{q}}{2K}\left[\frac{L^2}{4}\right]$$

Unsteady Heat Conduction

During heat transfer if temperature varies with respect to time, such systems are known as unsteady state systems. Unsteady heat flow is very common. Oil cooling, air cooling etc carried out during heat treatment processes, cooling of castings etc are examples.

Newton's Law of Cooling states that the rate of cooling of a body by way of convection predominantly, is directly proportional to the mean temperature difference between the body and surroundings.

System with Negligible Internal Resistance – Lumped Heat Analysis

Let us assume a body which is cooling from initial temperature T_i being exposed to the atmospheric temperature T_∞. If the thermal conductivity of the body is infinity (or very high) then its internal resistance becomes zero and the surface resistance due to the convection is the only factor which is responsible for heat transfer. In these cases, there is no temperature change within the body with change in time. This process is known as Newtonian heating or cooling.

$$\therefore \ln(T - T_\infty) = \left[\frac{-hA_st}{\rho VC_p}\right] + \ln(T_i - T_\infty)$$

or

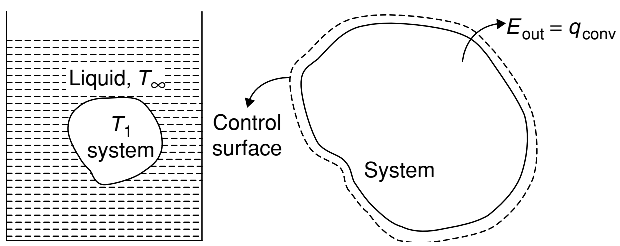
$$\ln\left[\frac{T - T_\infty}{T_i - T_\infty}\right] = -\frac{hA_st}{\rho VC_p}$$

$$\Rightarrow \frac{T - T_\infty}{T_i - T_\infty} = e^{-\left[\frac{hA_st}{\rho VC_p}\right]}$$

Where: C_p = Specific heat of the body, J/kg-K
h = Heat transfer coefficient, W/m^2-K
A_s = Surface area of body, m^2.
ρ = Density of the material, kg/m^3
V = Volume of the body, m^3.
T_∞ = Ambient temperature, °C.
T_1 = Initial temperature of body, °C.
T = Temperature of body at any time, °C.

When $R_{conduction} <<< R_{convection}$, then body can be treated like a lump.

Analysis of Quenching of Body



Energy input to the body in time dt = increase in internal energy of the body.

$$h.A_s.(T_\infty - T)dt = mC_P\,dt$$

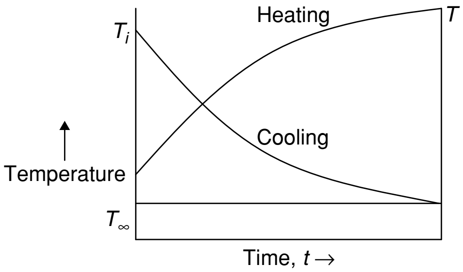
or

$$-\int \frac{hA_s}{mC_p}dt = \int \frac{dT}{(-T_\infty + T)}$$

$$\ln(T - T_\infty) = -\frac{hA_s}{C_p\rho V} \times t + C_1$$

Boundary conditions: At $t = 0$, $T = T_i$

$\therefore\ C_1 = \ln(T_i - T_\infty)$



The above expression gives the temperature distribution in the body for Newtonian heating or cooling and this indicates that the temperature will vary exponentially with time.

The expression can be arranged in dimensionless form as follows.

$$\frac{hA_sV}{\rho CV}t = \left(\frac{hV}{KA_s}\right)\left(\frac{A_s^2K}{\rho CV^2}t\right) = \left(\frac{hL_c}{K}\right)\left(\frac{\alpha\tau}{L_c^2}\right)$$

For flat plate $L_c = \dfrac{V}{A_s} = \dfrac{2Lbh}{2bh} = L$

Sphere $L_C = \dfrac{4/3\pi r^3}{4\pi r^2} = \dfrac{r}{3}$

Cylinder $L_C = \dfrac{\pi r^2 L}{2\pi rL} = \dfrac{r}{2}$

Cube $L_C = \dfrac{a^3}{6a^2} = \dfrac{a}{6}$

Here $\alpha =$ Thermal diffusivity $= \dfrac{K}{\rho C}$

$L_C =$ Characteristics length $= V/A_s$

Here $= \dfrac{hL_C}{K} = B_i$ Biot Number

$\dfrac{\alpha t}{L_C^2} = F_o$ Fourier number.

Biot number: This is the ratio of internal resistance (conduction) to external or surface resistance (convection). It provides a measure of the temperature drop in the solid relative to the temperature difference between the surface and the fluid.

When $B_i = \dfrac{hL_C}{K} \le 0.1$, then error associated with using the lumped capacitance method is small (5%).

Fourier number: This signifies the degree of penetration of heating or cooling effect through a solid.

$$\therefore\quad \frac{T - T_\infty}{T_i - T_\infty} = e^{-B_iF_o}$$

Temperature-time Response of Thermo Couple

Thermocouple works on the principle of unsteady state heat conduction with infinite thermal conductivity. This is temperature measuring device and should reach the temperature of the source, to which it is exposed as soon as possible. The time taken by the thermocouple to reach the source temperature is known as response time of thermocouple.

- The thermocouple reaches the source temperature as soon as the equation $e^{\left(\frac{hAt}{\rho C_P V}\right)}$ approaches to zero. This can be achieved by increasing the value of h or by decreasing the wire diameter. Therefore, a very thin wire is recommended for thermocouple for rapid response.
- The time required by a thermocouple to reach 63.2% of the value of initial temperature difference indicates the sensitivity of thermocouple and is known as time constant or response time.

$$\frac{T - T_\infty}{T_i - T_\infty} = 1 - 0.632 = 0.368 = e^{-1}$$

or $B_i F_o = 1$

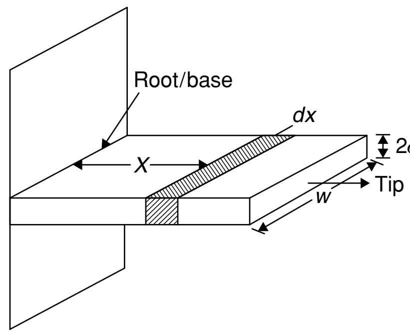
The response or thermal time constant is given by

$$t = \frac{\rho C_P V}{A_s h}$$

Fins and Heat Convection

The rate of heat transfer from a surface can be increased by adopting any of the following steps.

1. To increase the value of 'h'. This is possible by externally initiating fluid motion by fans, blowers, pumps etc.
2. To increase the temperature difference. Since the temperature of the surrounding and the hot surface remains almost constant, this method is also nearly impossible.
3. To increase the area of heat transfer. Provision of fins (Extended Surface) on hot bodies is a method popularly used for increasing heat transfer rate.



Difference between the temperature at any point on the fin and the atmospheric temperature $T - T_a$ is represented as $T - T_a = \theta$

At the root or base, let the temperature of the fin be T_0, then $T_o - T_a = \theta_0$.

The differential equation governing heat transfer from the fin is given by

$$\frac{d^2\theta}{dx^2} - \left(\frac{hP}{KA}\right)\theta = 0 \qquad (1)$$

Where h = convective heat transfer co-efficient

P is the perimeter of the cross-section of the fin $\perp r$ to the direction of heat flow

$P = 2W + 2(2\delta) = 2W + 4\delta$

K: Co-efficient of thermal conductivity of the material of fin.

A = the cross-sectional area of the fin.

Let $$\sqrt{\frac{hP}{KA}} = m.$$

Then the differential equation becomes $\frac{d^2\theta}{dx^2} - m^2\theta = 0$, This equation gives the distribution of temperature as a function of 'x' and 'm'.

$$\frac{d^2\theta}{dx^2} - m^2\theta.$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

Case I: Very long fin

For a long fin as $x \to 0$, $\theta \to 0$.

$\therefore 0 = C_1 e^{mx} \quad \therefore \quad C_1 = 0$

i.e., $\theta = C_2 e^{-mx}$ As $x \to 0$, $\theta \to \theta_0$

$\theta_0 = C_2 \times 1 \ \therefore\ C_2 = \theta_0$

i.e., $\theta = \theta_0 e^{-mx}$

$$\therefore \frac{\theta}{\theta_0} = e^{-mx} \quad or \quad \frac{T - T_a}{T_o - T_a} = e^{-m} \qquad (2)$$

If T_0 and T are the temperature values at two points at a distance of 'x' apart then equation (2) is valid.

After knowing the temperature distribution '$T - T_a$', the heat flow through the fin can be calculated either from the convective heat transfer from the entire surface of the fin body or from conduction heat transfer from the base/root of the fin.

In the first case the governing equation is

$$Q = \int_0^\infty hA(T - T_a)dx$$

In the second case, the governing equation is

$$Q = -KA\left(\frac{dT}{dx}\right)_{x=0}$$

Ultimately the total heat transfer becomes,

$$Q = \sqrt{hPKA} \cdot \theta_0 \quad \text{where } \theta_0 = T_0 - T_a \qquad (3)$$

Case II: Fin with insulated end (h-length of the fin)

When the tip of a fin is insulated, it appears that heat transfer from the fin may decrease. But this is not so. Since the area of the fin at the tip is negligible when compared to the lateral surface area, there is no appreciable loss of heat transfer.

The temperature variation along the fin length is given by

$$\theta = \theta_0 \cdot \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$\therefore \frac{\theta}{\theta_0} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$\frac{T - T_a}{T_0 - T_a} = \frac{\cosh[m(L-x)]}{\cosh(mL)} \qquad (4)$$

Total heat transfer is given by the formula

$$Q = \sqrt{hPKA}\,\theta_0 \tanh(mL) \qquad (5)$$

For long fins, when h is large tan $h\ m\ L \to 1$

$$\therefore \quad Q = \sqrt{hPKA} \cdot \theta_0 \text{ this is equivalent to equation} \qquad (6)$$

Case III: Short fin with convection off at the end

In this case, the amount of heat conducted to the end is convected off from the end is at $x = L$

$$\text{i.e., } -KA\frac{d\theta}{dx}\Big/ x = L = hA\theta / x = L$$

The temperature variation along the length of the fin is given by

$$\theta = \theta_0 \frac{\cosh(m[L-x]) + h/mk \ \sinh[m(L-x)]}{\cosh(mL) + h/mk \ \sinh(mL)} \text{ or}$$

$$\therefore \frac{\theta}{\theta_0} = \frac{T - T_a}{T_o - T_a}$$

$$\theta = \theta_0 \frac{\cosh[m(L-x)] + h/mk \ \sinh[m(L-x)]}{\cosh(mL) + h/mk \ \sinh(mL)}$$

When $h = 0$; we get the previous case of end insulated

The total heat flow

$$Q = \sqrt{hPKA} \cdot \theta_0 \cdot \frac{\tanh mL + h/mk}{1 + h/mk \tanh(mL)}$$

Points to remember

A. Temperature variation along the length of fin

1. Very long fin (L is large)

$$\frac{T - T_a}{T_o - T_a} = e^{-mx}; \ m = \sqrt{\frac{hp}{KA}}$$

2. Fin with insulated end

$$\frac{T - T_a}{T_0 - T_a} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

3. Short fin with convection off from end

$$\frac{T-T_a}{T_o-T_a} = \frac{\cosh[m(L-x)]+h/mk\ \sinh[m(L-x)]}{\cosh mL + h/mk\ \sinh mL}$$

B. Quantity of heat transfer

1. Very long fin

$$Q=\sqrt{hPKA}\cdot\theta_0.$$

2. Fin with insulated end

$$Q=\sqrt{hPKA}\cdot\theta_0\cdot\tanh(mL)$$

3. Short fin with convection off from end

$$Q=\sqrt{hPKA}\cdot\theta_0\cdot\frac{\tanh(mL)+h/mk}{1+h/mk\tanh mL}$$

Efficiency and Effectiveness of Fin

Fins are generally used for increasing the rate of heat transfer from hot surfaces. In IC Engine and small capacity compressors fins are used for increased heat removal. Fins also find application in heat exchangers to increase the rate of heat exchange.

Efficiency

The efficiency of a fin is defined as the ratio of the actual heat transferred by the fin to the maximum heat that could be transferred by the fin if the temperature of whole fin area is equal to the base temperature of fin, i.e., 'T_0', everywhere.

$$\eta_{fin}=\frac{Q_{fin}}{Q_{fin\ max\ possible}}$$

- For a very long fin (rectangular)

$$Q_{fin}=\sqrt{hPKA}\cdot\theta_0$$

If the whole surface of the fin were at the same temperature, then heat transfer $= h$ (Area) θ_0

$$= hP\times L\ \theta_0$$

$$\therefore\ \eta_{fin}=\frac{\sqrt{hPKA}\cdot\theta_0}{hPL\theta_0}=\frac{\sqrt{hPKA}}{hPL}=\sqrt{\frac{KA}{hPL^2}}$$

But we know that $m=\sqrt{\frac{hp}{KA}}$

$$\therefore\ \eta_{fin}=\sqrt{\frac{KA}{hPL^2}}=\frac{1}{mL}$$

- For a rectangular fin with insulated tip,

$$\eta_{fin}=\frac{Q_{fin}}{Q_{fin\ max}}$$

$$=\frac{\sqrt{hPKA}\cdot\theta_0\cdot\tanh mL}{hPL\theta_0}$$

$$=\sqrt{\frac{KA}{hPL^2}}\cdot\tanh mL=\frac{1}{mL}\tanh mL$$

Where $mL=\sqrt{\frac{hp}{KA}}L=\sqrt{\frac{h(2w+4\delta)}{K(w\times 2\delta)}}L$ if the fin is sufficiently wide such that $2w >>> 4\delta$

Then $mL=\sqrt{\frac{h2w}{Kw\times\delta}}L=\sqrt{\frac{2h}{K\delta}}L.$

Effectiveness

Effectiveness of the fin is defined as the amount of heat transferred with the fin to the amount of heat transferred without the fin in a specified time, and from a specified area.

If this ratio is less than '1' it means, addition of fin has reduced the heat transfer rate. That means the provision of fin for increasing the heat removal rate is not justified.

$\therefore$ Effectiveness 'ε' of the fin is

$$\varepsilon=\frac{\text{Heat lost from a surface, with fin}}{\text{Heat lost form the same surface without fins}}$$

ε should be > 1

$$\varepsilon=\frac{\sqrt{hPKA}\theta_0\dfrac{\tanh mL+h/mk}{1+h/mk\tanh mL}}{hA\theta_0}$$

$$=\sqrt{\frac{PK}{hA}}\left[\frac{\tanh mL+h/mk}{1+h/mk\ \tanh mL}\right]$$

Where $m=\sqrt{\frac{hP}{KA}}$

But, $$\frac{Km}{h}=\sqrt{\frac{k^2}{h^2}\cdot\frac{hP}{KA}}=\sqrt{\frac{KP}{hA}} \qquad (1)$$

$\varepsilon=\dfrac{Km}{h}\left[\dfrac{\tanh(mL)+h/mk}{1+h/mk\tanh(mL)}\right]$ or it can be written as

$$\frac{Km}{h}\left[\frac{\frac{mk}{h}\tanh(mL)+1}{\frac{mk}{h}+\tanh(mL)}\right]$$

For fin with insulated end,

$$\varepsilon = \frac{\sqrt{hPKA}\cdot\theta_0 \tanh(mL)}{hA\theta_0}$$

$$= \sqrt{\frac{PK}{hA}}\tanh(mL), \text{ applying (1)}$$

$$= \frac{Km}{h}\tanh(mL) \text{ `}\varepsilon\text{' increases when } h \text{ is less.}$$

That is, finning is justified when 'h' has low values.

When $m\,L$ is very large tanh $(mL) \to 1$.

Also, the expression for 'fin effectiveness' can be expressed in terms of Biot No.

$$\varepsilon = \frac{1}{\sqrt{B_i}}\left[\frac{1+\frac{1}{\sqrt{B_i}}\tanh\sqrt{\frac{h}{k\delta}}L}{\frac{1}{\sqrt{B_i}}+\tanh\sqrt{\frac{h}{k\delta}}L}\right]$$

Case I: When $B_i = 1;\ \varepsilon = 1,\ \varepsilon = \dfrac{1+\tanh\sqrt{\frac{h}{k\delta}}L}{1+\tanh\sqrt{\frac{h}{k\delta}}L} = 1$

Case II: When poor conducting material is used for fin and 'h' has higher values

$$B_i > 1, \varepsilon < 1$$

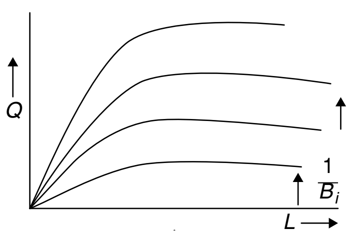
Here the fin will behave as an insulator

That is why we don't provide fins for steam condensers

Case III: When $B_i < 1$, $\varepsilon > 1$, this is the most desirable case. This is when the conductivity of the fin material is very high – for materials like Copper and Aluminium.

When length of the fin increases heat transfer increases.
When $\dfrac{1}{B_i}$ increases heat transfer increases.

The variation of heat transfer with Biot No is given in the following figure



Solved Examples

Example 1: Steady state one dimensional. Heat flow through slab, cylinder and sphere can be reduced to equations respectively

$$\frac{\partial t}{\partial x} = C,\ \frac{r\partial t}{\partial r} = C, \text{ and } \frac{r^2\partial t}{\partial r} = C$$

Solution: According to Fourier formula the general equation of heat transfer is

$K\left[\dfrac{d^2T}{dx^2}+\dfrac{\partial^2T}{dy^2}+\dfrac{\partial^2T}{dz^2}\right]+q' = \rho C_p\dfrac{dt}{dt}$, when there is no heat generation this becomes

$$\left[\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}+\frac{\partial^2T}{\partial_z{}^2}\right] = \frac{\rho C_p}{K}\cdot\frac{dt}{dt}$$

In the case of steady state heat flow $\dfrac{dt}{dt} = 0$

$$\therefore \quad \left[\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}+\frac{\partial^2T}{\partial z}\right] = 0$$

That is, $\dfrac{\partial^2T}{\partial x^2}+\dfrac{\partial^2T}{\partial y^2}+\dfrac{\partial^2T}{\partial z^2} = 0$ when it steady state one dimensional heat flow the equation

$$\frac{\partial^2T}{\partial x^2} = 0 \tag{1}$$

converting this to polar coordinates

$$\frac{\partial^2T}{dr^2}+\frac{1}{r}\frac{\partial T}{\partial r} = 0 \tag{2}$$

converting to spherical polar coordinates, it becomes

$$\frac{\partial^2T}{\partial r^2}+\frac{2}{\partial r}\frac{\partial T}{\partial r}0 \tag{3}$$

1. $\dfrac{\partial}{dx}\dfrac{(\partial T)}{\partial x} = 0,\ \dfrac{\partial T}{\partial x} = c$

2. $r\dfrac{\partial^2T}{\partial r^2}+\dfrac{\partial T}{\partial r} = 0 = \dfrac{\partial}{\partial r}\dfrac{(r\partial T)}{\partial r}$

   $r\cdot\dfrac{\partial T}{\partial r} = c$

3. $\dfrac{r^2\partial^2T}{\partial r^2}+2r\dfrac{\partial T}{\partial r} = 0$

   $\dfrac{\partial}{\partial r}\dfrac{(r^2\partial T)}{\partial r} = 0$

   $= r^2\dfrac{\partial T}{\partial r} = c$

Example 2: A composite wall of a furnace has 2 layers of equal thickness having thermal conductivities in the ratio of 3 : 2 what is the ratio of temperature drop across the two layers?

(A) 2 : 3 (B) 3 : 2
(C) 1 : 2 (D) $\log_e 2 : \log_e 3$

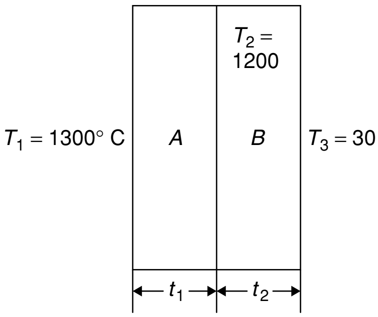
Solution:

$$\frac{\Delta T_1}{\left(\frac{\ell}{3}kA\right)} = \frac{\Delta T_2}{\left(\frac{\ell}{2}kA\right)}$$

$$\Delta T_1(3\ KA) = \Delta\ T_2\ (2\ kA)$$

$$\frac{\Delta T_1}{\Delta T_2} = \frac{2}{3}.$$

Example 3: A wall as given in fig is made up of two layers (A) and (B) The temperatures are as shown in the sketch. The ratio of thermal conductivities of two layers is given as $\frac{K_A}{K_B} = 3$ the ratio $t_1 : t_2$ is



(A) 0.256 (B) 0.38
(C) 0.31 (D) 0.22

Solution:

$$\frac{\frac{T_1 - T_2}{L_A}}{K_A} = \frac{\frac{T_2 - T_3}{L_B}}{K_B}$$

$$\frac{K_A(1300 - 1200)}{L_A} = \frac{(1200 - 30)K_B}{L_B}$$

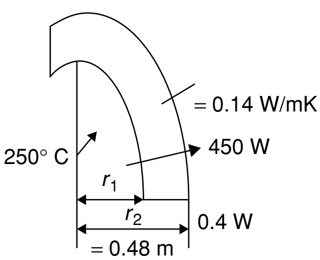
$$\frac{100K_A}{L_A} = \frac{1170K_B}{L_B}$$

$$\frac{L_A}{LB} = \frac{100K_A}{11170K_B}$$

$$= \frac{10}{117}\cdot\frac{K_A}{K_B} = \frac{10 \times 3}{117}$$

$$= \frac{30}{117} = \frac{10}{39} = 0.256.$$

Example 4:



A hot fluid contained in a spherical reactor vessel is at 250°C. The thermal resistance of the metallic wall is An insulation of 8 cm thick is provided on the vessel. The radius of vessel is 0.4 m The thermal conductivity of the insulation is 0.14 W/mK. The rate of heat loss is found to be 450 W. The temperature drop through the insulation and the outside surface temperature shall be

(A) 280.2, 110 (B) 143.4, 106.6
(C) 120.5, 106.6 (D) 110.2 , 110

Solution:

$$Q = \frac{T_1 - T_2}{\frac{1}{4\pi k}\left[\frac{1}{\frac{1}{r_1} - \frac{1}{r_2}}\right]}$$

Therefore $(T_1 - T_2)$

$$T_1 - T_2 = Q \times \left(\frac{1}{4\pi k}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\text{Temp drop } = \frac{450}{4\pi \times 0.14} \times \left[\frac{1}{0.4} - \frac{1}{0.48}\right]$$

$$= 106.6°\text{C}$$

$\therefore\ T_2 = 250\ 106.6 = 143.4°\text{C}$

Example 5: A steel plate of thermal conductivity 50 W/mK and thickness 10 cm passes a heat flux of 30 kW/m^2 by conduction if the temperature of the hot surface is 100°C then what is the temperature of the cooler side of the plate

(A) 40°C (B) 50°C
(C) 60°C (D) 70°C

Solution:

$$H(\text{kW/m}^2) = \frac{k(T_1 - T_2)}{t}$$

$$\text{That is, } = 30 \times 10^3 = \frac{50(100 - T_2)}{0.1}$$

$$= 40°\text{C}$$

Example 6: A current of 300A passes through a wire of diameter 2.5 mm and resistivity 70 μ ohm-cm. The length of the wire is 1.5 m. If the surface temperature of the wire is 180°C, the core temperature is ($K = 25$ W/mK)

(A) 221°C (B) 250°C
(C) 281°C (D) 238°C

Solution:

Heat generated is I^2R Watts

$$R = \rho\frac{\ell}{a} = \frac{70 \times 10^{-6} \times 150 \times 4}{\pi \times (0.25)^2}$$

$$= \frac{0.042}{\pi \times 0.0625} = 0.214\ \Omega$$

Volume of wire $V = \frac{\pi}{4} \times \left(\frac{2.5}{1000}\right)^2 \times 1.5$

$$= 7.35 \times 10^{-6} \text{ m}^3$$

Power $q = \frac{I^2 R}{V} = 300^2 \times \frac{0.214}{7.35} \times 10^{-6}$

$$= 2.62 \times 10^9 \text{ W/m}^3$$

$$T_{\max} = T_W + \frac{q^1 r_2^2}{4K}$$

$$= 180 + \frac{2.62 \times 10^9}{4 \times 25} \times \left[\frac{2.5}{2 \times 1000}\right]^2$$

$$= 180 + 41$$

$T_{\max} = 221°\text{C}$.

Example 7: An aluminum sphere of 7 kg mass initially at a temperature of 250°C is suddenly immersed in a fluid at 10°C. Determine the time required to cool the sphere to 90°C. Neglect the internal resistance and use the following properties of aluminum
($\rho = 2707$ kg/m^3, $C_p = 900$ J/kg-K and $K = 204$ W/mK, $h = 50$ W/m^{2}K)

(A) 25.36 (B) 24.48
(C) 30.8 (D) 41.5

Solution:

The temperature variation with respect to time is given by

$$\frac{T - T_a}{T_i - T_a} = e^{\frac{-hAt}{\rho V Q}} \quad \frac{A}{v} = \frac{4\pi R^2}{\frac{4}{3}\pi R^3} = \frac{3}{R}$$

The mass of the sphere is

$$m = \frac{4}{3}\pi R^3 \rho$$

$$7 = \frac{4}{3}\pi R^3 \times 2707$$

$$R = \frac{1}{11.74} \quad m = 8.52 \text{ cm}$$

$$\frac{h}{\rho_{c_p}} \times \frac{At}{v} = \frac{h}{\rho C_P} \times \frac{3t}{R} = \frac{50 \times 3 \times 11.74t}{2707 \times 900 \times 1} = \frac{1}{1383.5}$$

Substituting in equation

$$\frac{90 - 10}{250 - 10} = e^{-\frac{1}{1383.5}t}$$

$$\log_e \frac{80}{240} = -\frac{t}{1383.5}$$

$$\log \frac{240}{80} = \frac{t}{1383.5}$$

$t = 1520$ sec $= 25.33$ m.

Example 8: A thermo couple is to used to measure the temperature in a gas stream. The junction may be approximated as a sphere having thermal conductivity of 25W/m°C, $\rho = 8400$ kg/m^3 and $C_p = 0.4$/kg°C. The heat transfer coefficient between junction and gas is 560 W/m^2°C. Calculate the diameter of the junction if thermo couple should measure 95% of applied temperature difference in 3 seconds.

(A) 1 mm (B) 2 mm
(C) 3 mm (D) 5 mm

Solution:

$$\frac{T_g - T}{T_g - T_i} = e^{\left[\frac{-hA}{\rho C_p V}\right]t}$$

$$= e^{\left(\frac{-h}{sc_p} \cdot \frac{3}{R} t\right)} \text{ as } \frac{A}{V} = \frac{3}{R} \text{ for sphere}$$

$$0.05 = e^{\left[\frac{-560}{8400 \times 400} \times \frac{3 \times 3}{R}\right]} (t = 3)$$

$$= e^{\left[\frac{-1}{666.7R}\right]}$$

$$\log_e \left(\frac{1}{0.05}\right) = \frac{1}{667.7R}$$

$$\log_e 20 = \frac{1}{667.7R}$$

$$R = \frac{1}{667.7 \times 2.995} = \frac{1}{1998}$$

$$m = \frac{1000}{1998} = 0.5 \text{ mm}$$

$$D = 0.5 \times 2 = 1 \text{ mm}.$$

Example 9: A wall of 0.5 m thickness is to be constructed from a material having average thermal conductivity of 1.4 W/mK. The wall is to be insulated with a material having an average thermal conductivity of 0.35 W/mK. So that heat loss/m^2 will not exceed 1250 W. If inner and outer temperatures are 1200°C and 15°C respectively, determine the thickness of insulation required

(A) 20 cm (B) 15 cm
(C) 18 cm (D) 20 cm

Solution:

$$\frac{Q}{A} = 1250 \text{ W/m}^2$$

$$\frac{Q}{A} = \frac{(T_1 - T_2)}{\frac{L_1}{K_1} + \frac{L_2}{K_2}} = \frac{1185}{\frac{0.5}{1.4} + \frac{x}{0.35}}$$

$$1250 = \frac{1185}{\frac{5}{1.4} + \frac{x}{0.35}}$$

$$= \frac{5}{1.4} + \frac{x}{0.35} = \frac{1185}{1250} = 0.948$$

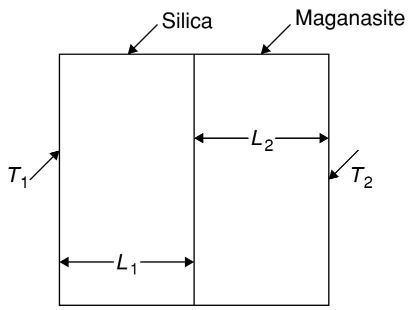
$$0.357 + \frac{x}{0.35} = 20 \text{ cm.}$$

Example 10: A furnace wall is made of inside silica brick (k = 1.858 W/m°C) and outside magnesite brick (5.8 W/m°C). The thickness of silica brick is 12 cm and that of magnesite brick is 20 cm. The surface of silica brick is 350°C and outside surface of magnesite brick is 150°C. Find the heat loss per m^2 of the furnace wall assuming contact resistance between two wall as 0.00258 m^2 °C/W/m^2. Contact area

(A) 3000 W (B) 1966 W
(C) 2800 W (D) 2500 W

Solution:

Heat flow through composite slab is



$$\text{given by} \left(\frac{T_1 - T_2}{\left(\frac{L_1}{K_1} + R_C + \frac{L_2}{K_2} \right)} \right)$$

where R_c is contact resistance between two faces.

$$\text{Substituting } \frac{(350 - 150)}{\frac{0.12}{1.858} + 0.00258 + \frac{0.20}{5.8}}$$

$$= \frac{200}{0.0648 + 0.00258 + 0.0345}$$

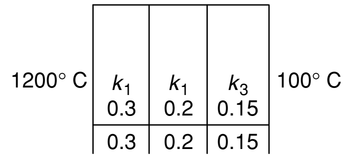
$$= \frac{200}{0.1017}$$

$$= 1966 \text{ W.}$$

Example 11: Composite wall has three layers of thicknesses 0.3, 0.2 and 0.15 meters and the thermal conductivities of these layers are 0.3, 0.2 and 15 W/m°K respectively. If the wall measures 10 × 5 m^2 and inner and outer surfaces of layers are at 1200°C and 100°C respectively, the flow rate of heat through the wall is

(A) 10.2 kW (B) 20.1 kW
(C) 18.33 kW (D) 13.1 kW

Solution:



$$A = 10 \times 5 = 50 \text{ m}^2$$

$$Q = \frac{A(t_1 - t_2)}{R}$$

$$R = \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3}$$

$$\frac{0.3}{0.3} + \frac{0.2}{0.2} + \frac{0.1}{0.15} = 1 + 1 + 1 = 3$$

$$Q = \frac{50}{3}(1100)\ w = 18.33 \text{ kW.}$$

Example 12: A hollow cylinder 5 cms ID and 10 cms OD has and inner surface temperature of 200°C and an outer surface temperature of 80°C. If the thermal conductivity of the cylinder material is 70 W/mK

(a) Determine the quantity of heat flow/m length of the cylinder
   (A) 91.4 kW/m (B) 76.1 kW/m
   (C) 83.4 kW/m (D) 99.1 kW/m
(b) The temperature at a point half way between inner and outer surfaces
   (A) 155.5°C (B) 182.8°C
   (C) 130.4°C (D) 170.°C

Solution:

(a) $$Q = \frac{T_1 - T_2}{\frac{1}{2\pi KL} \log \frac{r_2}{r_1}}$$

$$= \frac{120}{\frac{1}{2\pi \times 70 \times 1} \log \frac{5}{2.5}}$$

$$= \frac{62.8 \times 7 \times 120}{\log 2} = 76104 \text{ W/m}$$

$$= 76.1 \text{ kW/m.}$$

(b) $$T = T_1 - (T_1 - T_0) \frac{\log \frac{r}{r_1}}{\log \frac{r_0}{r_1}}$$

$$r = \frac{2.5 + 5}{2} = 3.75$$

$$200-(120)\frac{\log\left(\frac{3.75}{2.5}\right)}{\log\left(\frac{5}{2.5}\right)}$$

$$200-120\times\frac{0.4054}{0.693}$$

$= 200 – 120 \times 0.58$

$200 – 69.6 = 130.4°C.$

Example 13: A hollow sphere 10 cms ID and 30 cms OD of a material having thermal conductivity 60 W/mk holds liquid. The inner and outer temperatures 400°C and 100°C respectively. Determine (a) the heat flow arte through the sphere and (b) temperature at a point quarter away from the inner surface

(a) heat flow
 (A) 30.8 kW (B) 16.862 kW
 (C) 20.5 kW (D) 17.8 kW

(b) temperature
 (A) 319°C (B) 280°C
 (C) 325°C (D) 250.°C

Solution:

(a) $$Q=\frac{4\pi r_1 r_2 K(T_1-T_2)}{r_2-r_1}\begin{bmatrix} r_2=0.15\ m \\ r_1=0.05\ m\end{bmatrix}$$

$$=\frac{4\pi\times0.15\times.05\times60(300)}{0.15-0.05}\ \text{W}$$

$= 16.962$ kW.

(b) Quarter of the way means

$$r=\frac{30}{4}=75\text{ cms}=0.075\text{ m}$$

$$T=T_1-\frac{r_2}{r}\left(\frac{r-r_1}{r_2-r_1}\right)(T_2-T_1)$$

$$T=400-\frac{0.15}{0.075}\left(\frac{0.075-0.05}{0.15-0.05}\right)\times(100-400)$$

$= 400 – 150 = 250°C.$

Example 14: Calculate the rate of heat loss for a refractory brick wall of 15 m^2 area and thickness 40 cm. The temperature of the inner surface is 210°C and that of outer surface 20°C the coefficient of thermal conductivity of brick $K = 0.60$ W/mK

(a) Heat loss
 (A) 4279.29 (B) 4300
 (C) 3832 (D) 5315

(b) The temperature at an interior point of the wall 30cm distant from the inner wall is
 (A) 72.4°C (B) 50.°C
 (C) 67.5°C (D) 55.4°C

Solution:

(a) Heat loss $$=Q=\frac{(T_1-T_2)}{\frac{L}{KA}}$$

$$=\frac{(210-20)}{\frac{0.4}{0.6\times15}}$$

$$=\frac{190}{0.0444}=4275$$

(b) $$T=T_1-\frac{(T_1-T_2)}{L}x$$

$$=210-\frac{190}{0.4}\times0.3$$

$= 67.5°C.$

Example 15: In an experimental determination of thermal conductivity of a given material, a specimen of 2.5 cm diameter and 15 cm length is tested. This rod is maintained at 120°C at one end and 0°C at the other end and the cylindrical surface is completely insulated. The electrical measurement shows that the heat flow rate of 6 watts. Determine the thermal conductivity of the material

(A) 13.8 (B) 15.7
(C) 14.5 (D) 12.75

Solution:

The heat flow through the rod is given by $Q=\frac{KA_C(T_1-T_2)}{L}$

$$5=K\frac{(\pi)d^2}{4}\frac{(T_1-T_2)}{L}$$

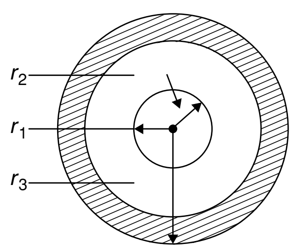
$$=K\times\frac{\pi}{4}\times\left(\frac{2.5}{100}\right)^2\times\frac{1}{.15}(120-0)$$

$$K=\frac{20\times10^4\times0.15}{\pi\times6.25\times120}=12.75.$$

Example 16: A stainless tube (K_B = 19 W/mK) of 4 cm ID and 10 cm OD insulated with 6 cm thick asbestos (K_a = 0.2 W/mK). It the temperature difference between the inner most and outer most surface is 600°C, the heat transfer rate per unit length is

(A) 0.94 W/m (B) 946 W/m
(C) 944.72 W/m (D) 9447.21 W/m

Solution:



Given $r_1 = 2$ cm, $r_2 = 5$ cm, $t = 6$ cm

$r_3 = r_2 + t$

$= 5 + 6 = 11$ cm

Equivalent thermal resistance

$$\frac{1}{2\pi K_1 \ell}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi K_2 \ell}\ln\left(\frac{r_3}{r_2}\right)$$

$$= \frac{1}{2\pi \times 19 \times 1}\ln\left(\frac{5}{2}\right) + \frac{1}{2\pi \times 0.2}\ln\left(\frac{11}{5}\right)$$

$= 0.634$ W

$\therefore$ Heat transfer rate per unit length

$$\frac{T_1 - T_2}{\Sigma R} = \frac{600}{0.634} = 946.37 \text{ W/m.}$$

Example 17: One dimensional unsteady state heat transfer equation for a sphere with heat generation at the rate of q can be written as

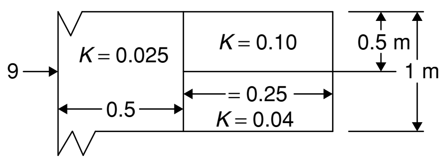
(A) $\frac{1\delta}{r\delta r}\left(r\frac{\delta T}{\delta r}\right) + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

(B) $\frac{1}{r_2}\frac{\partial}{dr}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

(C) $\frac{\partial^2 T}{\partial r^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

(D) $\frac{\partial^2}{\partial r^2}(rT) + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \cdot$ C

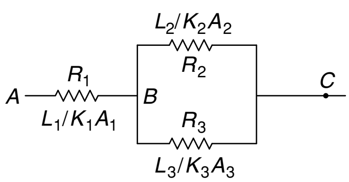
Example 18: Heat flows through a composite slab, as shown below. The depth of the slab is 1 m. The values of K are in W/mK. The over all thermal resistance in K/W is



(A) 15.8 K/W (B) 22 K/W
(C) 23.6 K/W (D) 19.5 K/W

Solution:

Resistance diagram



Now $R_1 = \frac{L_1}{K_1 A_1} = \frac{0.5}{0.025 \times 1 \times 1}$

$= 20$ K/W

$$R_2 = \frac{L_2}{K_2 A_2} = \frac{0.25}{0.1 \times 0.5 \times 1}$$

$= 5$ k/W

$$R_3 = \frac{L_3}{K_3 A_3} = \frac{0.25}{0.04 \times 0.5 \times 1}$$

$= 12.5$ K/W

Equivalent resistance between B and C

$$R_e = \frac{5 \times 12.5}{5 + 12.5} = 3.6 \text{ K/W}$$

Total resistance $= R_1 + R_e$

$= 20 + 3.6$

$= 23.6$ K/W.

NOTES

1. In the heat flow equation $Q = \frac{KA(t_1 - t_2)}{L}$ the term $\frac{L}{KA}$ is known as thermal resistance.
2. In the above equation $\frac{t_1 - t_2}{L}$ is known as temperature gradient.
3. Thermal diffusivity of a substance is proportional to thermal conductivity (k).
4. Unit of thermal diffusivity is m^2/hr.
5. Thermal diffusivities for gases are generally more than those of liquids.
6. Thermal diffusivity is expressed by the relation $\frac{k}{\rho C_p}$ where k = thermal conductivity, ρ = density. C_p = Specific heat at constant pressure.
7. The heat flow per unit length for steady conduction in a hollow cylinder of inside radius r_1 and outside radius r_2 is directly proportional to to $2\pi k(T_1 - T_2)$ and inversely proportional to $\log_e \frac{r_2}{r_1}$.
8. The heat flow across a hollow sphere of inner radius r_1 and outer radius r_2 is proportional to $\frac{r_1 r_2}{r_2 - r_1}$.
9. Thermal conductivity is a material property.
10. In good conductors lattice vibration does not contribute more for heat conduction.
11. Thermal conductivity of water increases with increase in temperature.
12. For the same amount of heat conduction through a slab, as thickness increases, the temperature gradient need not increase.
13. Fins for the same flow can be shorter if the thermal conductivity of the material increased.
14. For identical fins of different materials the tip to base temperature difference will be higher if the thermal conductivity is lower.

15. In a hollow cylinder, the temperature variation with radius is not linear.
16. The temperature gradient at the inner surface will be steeper compared to that at the outer surface in radial heat conduction in a hollow cylinder.
17. Fins are more useful with liquids than gases.
18. Fin effectiveness is generally greater than one.
19. Lumped capacity model can be used in the analysis of transient heat conduction if biot number is less than one.
20. Lumped parameter model can be used of the internal conduction resistance is low compared to the surface convection resistance.
21. To reduce the time constant of thermocouple its characteristic linear dimension (V/A)should be reduced
22. A solid of poor conductivity exposed for a short period to surface convection can be analyzed as semi infinite solid.
23. A slab will not cool faster compared to a long cylinder or sphere of the same characteristic dimensions when exposed to the same convection conditions.
24. Higher the value of biot number slower will be the cooling of a solid.
25. Convection coefficient is not a material property.
26. In a slab conducting heat the surface temperature are 200 and 100°C. The mid plane temperature will not be greater than 150°C.
27. In slab material variable conductivity with conductivity decreasing with temperature the surface temperature are 200°C and 100°C. The mid plane temperature will be higher than 150°C.
28. In hollow cylinder with radial conduction the mid plane temperature will be lower than the mean of surface temperatures
29. In a hollow sphere with radial conduction, the mid plane temperature will not be higher than the mean of surface temperature.
30. Any amount of additional insulation cannot reduce the heat flow through a hollow spherical insulation of the same material to half the original flow rate.
31. In case of small hollow cylinders or spheres, with outside convection, the thermal resistance may decrease by the addition of insulation.
32. Small electronic components may be kept cooler by encasing it in a glass like material.
33. Liquid metal flow can be approximated to slug flow.
34. Heat conduction in good conductors is largely by free electron movement.

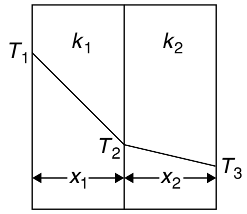
Exercises

Practice Problems I

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. A hollow cylinder has its inside maintained by an evaporating fluid at –40°C. The inside radius is 6 cm and wall thickness is 3 cm. The wall material has a thermal conductivity of 0.1 W/mK. The heat gain per m length due to evaporating liquid is 65.5 W/m length. Determine the outside temperature
   (A) 273 K (B) 288 K
   (C) 263 K (D) 320 K
2. A composite slab has two layers of different materials having internal conductivities k_1 and k_2. If each layer has the same thickness then, what is the equivalent thermal conductivity of the slab?
   (A) $\frac{k_1k_2}{k_1+k_2}$ (B) $\frac{k_1k_2}{2(k_1+k_2)}$
   (C) $\frac{2k_1}{(k_1+k_2)}$ (D) $\frac{2k_1k_2}{(k_1+k_2)}$
3. What is the heat lost per hour across a wall 4 m high, 10 m long and 115 mm thick, if the inside wall temperature is 30°C and outside ambient temperature is 10°C conductivity of brick wall is 1.15 W/mK, heat transfer coefficient for inside wall is 2.5 W/2 m^{2}K and that outside wall is 4 W/m^{2}K
   (A) 3635 kJ (D) 3750 kJ
   (C) 3830 kJ (D) 8310 kJ
4. A large concrete slab 1 m thick has one dimensional temperature distribution
   $T = 4 - 10x + 20x^2 + 10x^3$ where T is the temperature and x is the distance from one face towards other face of the wall. If the slab material has thermal diffusivity 2×10^{-3} m^2/hr. What is the rate of change of temperature at the other face of the wall
   (A) 0.1°C/h (B) 0.2°C/hr
   (C) 0.3°C/hr (D) 0.4°C/hr
5. Air at 20°C blows over a hot plate of 50 × 80 cm made out of carbon steel maintained at 220°C. The convective heat transfer coefficient is 25 W/m^{2}k what will be the heat loss from the plate?
   (A) 1500 W (B) 2500 W
   (C) 3000 W (D) 4000 W
6. A composite wall of a furnace has 3 layers of equal thickness having thermal conductivities in the ratio 1:2:4 where will be the temperature drop ratio a cross the three respective layers
   (A) 1:2:4 (B) 4:2:1
   (C) 1:1:1 (D) log4 : log2 : log1
7. The temperature drop through each layer of a two layer furnace wall is shown in figure. Assume that external temperature T_1 and T_3 are maintained constant and

$T_1 > T_3$. If the thickness of the layers X_1 is double of X_2 which one of the following statements is correct



(A) $k_1 > 2k_2$ where k is the thermal conductivity of the layer
(B) $k_1 < 2k_2$
(C) $k_1 = k_2$ but heat flow through maternal 1 is larger than through material 2
(D) $k_1 = k_2$ but flow through material 1 is less than that through material 2

Direction for questions 8, 9 and 10: A refrigerator has total inside surface area 3.06 m^2. The composite wall is made of two 3 mm mold sheets (K = 150 kJ/mhr K) with 6 mm thick glass wool sandwitched (K = 2 kJ/hr K)between them. The average values of convective heat transfer coefficients at the interior and exterior walls are 40 kJ/hrm^2 respectively.

8. The total resistance of the composite wall will be
(A) 12.42 K/kJhr (B) 15.7 K/kJhr
(C) 19.42 K/kJhr (D) .23 K/kJhr

9. The overall conductance is
(A) 63.70 kJ/Khr (B) 57.32 kJ/Khr
(C) 49.24 kJ/Khr (D) 42.7 kJ/Khr

10. For the air temperature inside the refrigerator 6.5°C and outside 25°C, the temperature on the outer surface of the metal sheet is
(A) 10.85°C (B) 12.34°C
(C) 17.31°C (D) 19.8°C

11. A composite wall of two layers of thicknesses Δx_1, Δx_2 and of thermal conductivities k_1 and k_2 is having, cross-sectional area A normal to the path of heat flow. If the wall surface temperature are T_1 and T_2 then rate of heat flow (Q) is equal to

(A) $\dfrac{A(T_1 - T_3)}{\dfrac{\Delta x_1}{k_2} + \dfrac{\Delta x_2}{k_1}}$ (B) $\dfrac{Ak_1k_2(T_1 - T_3)}{\Delta x_1 + \Delta x_2}$

(C) $\dfrac{(Ak_1 + Ak_2)(T_1 - T_3)}{\Delta x_1 + \Delta x_2}$ (D) $\dfrac{(T_1 - T_2)}{\dfrac{\Delta x_1}{Ak_1} + \dfrac{\Delta x_2}{Ak_2}}$

12. A furnace wall insulation is of fire clay brick with thermal conductivity

$k = 0.6925(1 + 9.747 \times 10^{-4})$ where T is °C and k is in W/mK. The wall is 30 cm thick. The inside surface is at 500°C while out side surface is 70°C.

The heat flow will be
(A) 1268.5 W/m^2 (B) 1312.5 W/m^2
(C) 1408.5 W/m^2 (D) 1250.8 W/m^2

13. The critical radius of insulation for asbestos with K = 0.17 W/m°C surrounding a pipe and exposed to a room air at 25°C with h = 4.0 W/m°C
(A) 4.25 cm (B) 5.8 cm
(C) 7 cm (D) 8.3 cm

Direction for questions 14 and 15: A composite wall of 3 layers of thickness 25 cm, 10 cm and 15 cm with thermal conductivities 1.7, K_B and 9.5 W/mK. The outside surface is exposed to 30°C with convection coefficient of 15 W/m^{2}K and inside is exposed to gases at 1200°C with a convection coefficient 30 W/m^{2}K and the inside surface is 1080°C.

14. The unknown thermal conductivity is
(A) 1.61 W/mK (B) 1.123 W/mK
(C) 1.352 W/mK (D) 1.245 W/mK

15. The overall heat transfer coefficient is
(A) 3.126 W/m^{2}K (B) 1.88 W/m^{2}K
(C) 2.38 W/m^{2}K (D) 3.03 W/m^{2}K

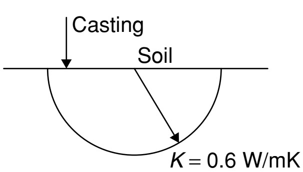
16. The over all heat transfer coefficient (U) for a composite wall of thickness t_1, t_2, t_3 and of corresponding thermal conductivities k_1, k_2, k_3 is given by equation is

(A) $\dfrac{1}{u} = \dfrac{k_1}{t_1} + \dfrac{k_2}{t_2} + \dfrac{k_3}{t_3}$ (B) $U = \dfrac{k_1}{t_1} + \dfrac{k_2}{t_2} + \dfrac{k_3}{t_3}$

(C) $\dfrac{1}{u} = \dfrac{t_1}{k_1} + \dfrac{t_2}{k_2} + \dfrac{t_3}{k_3}$ (D) $U = \dfrac{t_1}{k_1} + \dfrac{t_2}{k_2} + \dfrac{t_3}{k_3}$

17. A 800 kg vehicle traveling at 50 m/sec impacts a plunger attached to a piston cylinder arrangement. If all the energy of the vehicle is absorbed by the 20 kg of liquid contained in the cylinder what is the maximum temperature rise of the liquid (specific heat of the liquid is 4.0 kJ/kg°C)
(A) 55°C (B) 50°C
(C) 40°C (D) 12.5°C

18.



A casting in the form of a hemisphere of radius of 0.4 m is in a sand mould, on the ground with circular face parallel to the ground and as shown in figure. The surface temperature of the casting is 1000°C and the soil temperature is 50°C. The heat loss to soil is (K = 0.6 W/mK shape factor $2\pi r$)
(A) 158 (B) 1432
(C) 1632 (D) 1742

19. A furnace has inside dimensional of 1 m × 1.2 m × 1.5 m. The walls are .25 m thick the inside surface temperature is at 1000°C while outside surface is at 80°C. If the conductivity of the material is 0.45 W/mK. Determine the heat loss without taking into account the corner and edge effects

(A) 14.9 kW (B) 15.8 kW
(C) 12.3 kW (D) 20.3 kW

20. A composite wall is made of two layers of thickness δ_1 and δ_2 having thermal conductivity k and 2 k and equal surface area normal to the direction of heat flow. The outer surface of the composite wall are at 100°C and 200°C respectively the heat transfer takes place only by conduction and the required surface temperature at the junction is 150°C. The ratio of their thickness $\delta_1 : \delta_2$?

(A) 1 : 1 (B) 2 : 1
(C) 1 : 2 (D) 2 : 3

Practice Problems 2

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. The surface temperature of a furnace is 700°C. From the surface, 3 rods of equal length and cross section protrudes one made of steel, other made of copper and the third made of aluminum. The free ends of the rods are exposed. The atmospheric temperature is 27°C. For which of the rod tip temperature is highest

(A) Steel rod
(B) Copper rod
(C) Aluminium rod
(D) All the rods will have same tip temperature

2. If 'K' is the thermal conductivity τ is the density and CP the specific heat of a substance then thermal diffusivity is given by

(A) $\dfrac{K}{\tau_{CP}}$ (B) $\dfrac{C_P}{K\tau}$

(C) $\dfrac{\tau_{CP}}{K}$ (D) $\dfrac{K\tau}{C_P}$

3. Three metal walls of the same cross-sectional area with thermal conductivities in the ratio 1 : 2 : 4, heat transfer rate 13000kcal/hr. For the same thickness of the wall the temperature drop will be in the ratio

(A) 1 : 2 : 4 (B) 4 : 1 : 2

(C) $1:\frac{1}{2}:\frac{1}{4}$ (D) $\frac{1}{4}:\frac{1}{2}:1$

4. The rate of heat flow through common thick wall of material having thermal conductivity 40 W/mK for a temperature difference of 10°C will be

(A) 80 W/m^2 (B) 800 W/m^2
(C) 6666.66 W/m^2 (D) 80,000 W/m^2

5. The heat flux through a concrete slab 50 mm thick, whose inner surface is at 50°C and outer surface is at 20°C, is 50 W/m^2. The thermal conductivity of the concrete is

(A) 0.3 W/m°K (B) 0.53 W/m°K
(C) 0.08 W/m°K (D) 13 W/m°K

6. A very long rod 5 mm in diameter has one end at 100°C. The surface of the rod is exposed to ambient air at 30°C with convective heat transfer coefficient of 125 W/m°K. What is the heat loss from the rod if thermal conductivity is 398 W/m°K

(A) 1.03 W (B) 20 W
(C) 81.75 W (D) 40 W

7. The rate of heat flow through a composite wall of three layers of thickness 0.3 m, 0.2 m 0.15 m and of corresponding thermal conductivities in W/mK 1.2 0.3, 0.2, and 0.15 and heat is 1280 kJ/h°C. If the surface area normal to the direction flow of heat is 1 m^2 and inner surface temperature is 1000°C, then the interface temperature at the end of first layer will be

(A) 700°C (B) 680°C
(C) 500°C (D) 360°C

Direction for questions 8 and 9: A cold storage room has walls made of 0.23 m of brick on the outside, 0.08 m of plastic foam and finally 1.5 cm of wood on the inside. The out side and inside air temperatures are 22°C and –2°C respectively. If the inside and outside heat transfer coefficients are 30 and 12 W/m^2°C respectively and thermal conductivities of brick, form and wood are 0.92, 0.02, and 0.15 W/m°K respectively

8. The rate of heat removed by refrigeration if the total wall area is 90 m^2

(A) 444.5 W (B) 483.8 W
(C) 432.5 W (D) 493.6 W

9. The temperature of the inside surface of the brick is

(A) 45.5°C (B) 21.49°C
(C) 20.22°C (D) 18.45°C

10. At a given instant of time, temperature distribution with in an infinite homogonous body is given by a function

$T(x, y, z) = x^2 - 2y^2 + Z^2 - xy + 2yz.$

Determine the rate of change of temperature with respect to time at point (2, 2, 2). Assume constant properties and no internal heat generation

(A) 4 K/s (B) 0 K/s
(C) 10 K/s (D) 8 K/s

Direction for questions 11, 12, 13: A composite cylinder is made of 6 mm thick layers, each of two materials of thermal conductivities of 30 W/°mC and 5 W/°mC. The inside is exposed to fluid at 600°C with a convection coefficient of 40 W/m^2°C and the outside is exposed to at 35°C with a convection coefficient 25 W/m^{2}K. There is contract resistance of $1 \times 10^{+3}$ m^2°C/W between layers (inside diameter = 20 mm)

11. The heat loss for a length of 2 m is
(A) 1580 W (B) 1612.3 W
(C) 1428.1 W (D) 1318.1 W

12. The over all heat transfer coefficient (inside area)
(A) 22.7 (B) 13.8
(C) 14.7 (D) 51.8

13. Inter face temperature
(A) 29°C (B) 349°C
(C) 279°C (D) 253°C

14. The circular pipe of 0.2 m outside diameter is enclosed centrally in a square section insulation of 36 cm side. The thermal conductivity of the material is 8.5 W/mK. The inside surface is at 200°C. The outside is exposed to a convection at 30° with h = 35 W/m^{2}K. Heat flow per length of 5 m is
(A) 51281 W (B) 3849 W
(C) 46120 W (D) 26316 W

15. A hollow cylinder 5 cms ID and 10 cms OD has an inner surface temperature of 300°C and an outer surface temperature of 100°C. If the thermal conductivity of the cylinder material is 70 W/mK determine the quantity of heat flow/m length of the cylinder and the temperature at point half a between the inner and outer surfaces.
(A) 242°C (B) 235°C
(C) 208°C (D) 300°C

16. A hollow sphere 30 cm OD and 10cms ID of material having thermal conductivity 50W/mK is used as a container for a liquid chemical mixture. Its inner and outer surface temperature are 300°C and 50°C respectively. Determine the temperature at a point quarter the way between the miner and outside surfaces
(A) 175°C (B) 180.3°C
(C) 153.2°C (D) 140°C

17. A fire clay wall 20 cm thick has its two surfaces maintained at 1000°C and 200°C. The thermal conductivity of fire day varies with temperature as K = 0.813 + 0.000582. Determine the rate of heat flow
(A) 4.65 kw/m^2 (B) 5.33/kW
(C) 4.8/kW (D) 5.2/kW

18. A slab made of copper has its face 40 × 40 cm and thickness 5 cm. It is at a uniform temperature of 200°C and has its surface temperature suddenly cooled to 30°C. Find the time at which the slab temperature becomes 90° given

p = 9000 kg/m^3 C = 0.38 J/kgK

k = 370 W/mK and h = 90 W/m^{2}K
(A) 200 sec (B) 102 sec
(C) 99.42 sec (D) 301 sec

19. A metallic rod 15 mm diameter at 90°C is cooled to 30°C is 105 sec by placing it is air stream at 25°C. Calculate the heat transfer coefficient 'h' for air. For the rod mass is 1 kg, C_p = 350 J/kg surface area = 0.0004 m^2
(A) 213 W (B) 330 W
(C) 300 W (D) 402 W

20. Temperature distribution across a large size concrete slab 50 cm thick and an area of 5 m^2 from one side as measured by thermo couple approximate to the following relation

$T = 60 - 50x + 12x^2 + 20x^3 + 15x^4$

$x = 0, x = 0.5$. Find heat energy stored in the slab, given k = 1.2W/mK
(A) 117 W (B) 24 W
(C) 26 W (D) 29 W

21. A cylinder 2.5 cm radius and 1 m long is placed in an atmosphere at 45°C. The cylinder is provided with 10 nos of longitudinal straight fins of material with

k = 120 W/mK. The height of the fin is 1.27cms and thickness 0.76 mm. The convective heat transfer coefficient of air between cylinder and atmosphere is 17 W/mk°k. The cylinder surface temperature is 150°C. Calculate heat transfer rate per fin
(A) 30 W/fin (B) 24 W/fin
(C) 25 W/fin (D) 35 W/fin

22. A steam pipe 170/160 mm diameter is covered with two layers of insulation. The thickness of first layer is 30 mm and that of second layer is 50 mm. The thermal conductivities of the pipe and insulating layers are 40, 0.15, 0.07 W/mK respectively. The temperature of the inner surface of the steam pipe is 300°C and that of the outer surface of the insulation layer 50°C. The quantity of heat lost per m length of pipe is
(A) 875 kJ/h (B) 820 kJ/h
(C) 789 kJ/h (D) 748 kJ/h

23. The layer contact temperature of steam pipe with insulation
(A) 231°C, 300°C (B) 240°C, 350°C
(C) 244°C, 325°C (D) 258°C, 400°C

Direction for questions 24 and 25: An aluminum alloy fin (k = 200 W/mK) 3.5 mm thick and 2.5 mm long protrudes from a wall. The base is at 420°C and Ambient air temperature is 40°C. The heat transfer coefficient may be taken as 12 W/m^{2}K. Assume heat loss from the tip is negligible.

24. Find the heat loss
(A) 225 W (B) 210.5 W
(C) 150.5 W (D) 145.3 W

25. Fin efficiency is
(A) 99.30% (B) 88.34%
(C) 95.45% (D) 78.5%

26. A long rod 12 mm square section made of low carbon steel protrudes into air at 35°C from a furnace wall at 200°C. The convective heat transfer coefficient is estimated at 22 W/m^{2}K. The conductivity of the material is 51.9 W/mK. The location of the point from the wall where the temperature is at 60°C is

(A) 159 mm (B) 165 mm
(C) 173 mm (D) 148 mm

Direction for questions for 27, 28, 29: A rod of length 159 mm, cross-section 12 mm square of low carbon steel. The convective heat transfer coefficient is estimated 22 W/m^{2}K. It is protruded out from furnace wall at 200°C and to air at 35°C. The conductivity of material is 51.9 W/mK (Take as short fin).

27. The end temperature at 159 mm length is when heat convected at the end face is negligible is

(A) 83.72°C (B) 85.85°C
(C) 95.25°C (D) 90.85°C

28. Consider the fin to be 80 mm long and end face convection also exists. The end temperature will be

(A) 143.1°C (B) 280.2°C
(C) 135.8°C (D) 152.1°C

29. End temperature Considering the above fin as long fin (For 80 mm length)

(A) 133.23°C (B) 120°C
(C) 98.74°C (D) 145.8°C

30. The one dimensional steady state conduction equation and one dimensional transient steady state heat conduction equation both without heat generation respectively are

(A) $\dfrac{\partial 2T}{\partial x^2} = 0$ and $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial^2 T}{\partial r^2}$

(B) $\dfrac{\partial T}{\partial x} = 0$ and $\dfrac{\partial T}{\partial x} = \dfrac{1}{\alpha} = \dfrac{\partial T}{\partial r}$

(C) $\dfrac{\partial_2 T}{\partial x^2} = 0$ and $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial r}$

(D) $\dfrac{\partial^2 T}{\partial x^2} = 0$ and $\dfrac{\partial T}{\partial x} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial r}$

Previous Years' Questions

1. One dimensional unsteady state heat transfer equation for a sphere with heat generation at the rate of 'q' can be written as **[2004]**

(A) $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right) + \dfrac{q}{k} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$

(B) $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial T}{\partial r}\right) + \dfrac{q}{k} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$

(C) $\dfrac{\partial^2 T}{\partial r^2} + \dfrac{q}{k} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$

(D) $\dfrac{\partial^2}{\partial r^2}(rT) + \dfrac{q}{k} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$

2. A stainless steel tube (k_s = 19 W/mK) of 2 cm ID and 5 cm OD is insulated with 3 cm thick asbestos (k_a = 0.2 W/mK). If the temperature difference between the innermost and outermost surface is 600°C, the heat transfer rate per unit length is **[2004]**

(A) 0.94 W/m (B) 9.44 W/m
(C) 944.72 W/m (D) 9447.21 W/m

3. A spherical thermocouple junction of diameter 0.706 mm is to be used for the measurement of temperature of a gas stream. The convective heat transfer co-efficient on the bead surface is 400 W/mK. Thermophysical properties of thermocouple material are $k = 20$ W/m^{2}K, $C = 400$ J/kg K and $\rho = 8500$ kg/m^3. If the thermocouple initially at 30°C is placed in a hot stream of 300°C, the time taken by the bead to reach 298°C, is **[2004]**

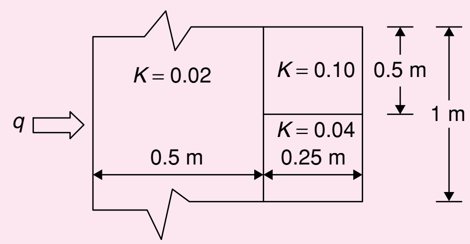
(A) 2.35 s (B) 4.9 s
(C) 14.7 s (D) 29.4 s

4. In a case of one dimensional heat conduction in a medium with constant properties, T is the temperature at position x, at time t. Then $\dfrac{\partial T}{\partial t}$ is proportional to **[2005]**

(A) $\dfrac{T}{x}$ (B) $\dfrac{\partial T}{\partial x}$

(C) $\dfrac{\partial^2 T}{\partial x \partial t}$ (D) $\dfrac{\partial^2 T}{\partial x^2}$

5. Heat flows through a composite slab, as shown below. The depth of the slab is 1 m. The K values are in W/mK. The overall thermal resistance in k/W is **[2005]**



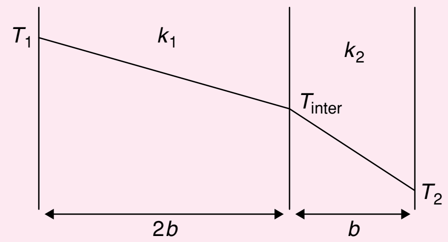
(A) 17.2 (B) 21.9
(C) 28.6 (D) 39.2

6. A small copper ball of 5 mm diameter at 500 K is dropped into an oil bath whose temperature is 300 K. The thermal conductivity of copper is 400 W/m.K, its density 9000 kg/m^3 and its specific heat 385 J/kg.K.

If the heat transfer coefficient is 250 W/m^2.K and lumped analysis is assumed to be valid, the rate of fall of the temperature of the ball at the beginning of cooling will be, in K/s, **[2005]**

(A) 8.7 (B) 13.9
(C) 17.3 (D) 27.7

7. In a composite slab, the temperature at the interface (T_{inter}) between two materials is equal to the average of the temperatures at the two ends. Assuming steady one-dimensional heat conduction, which of the following statements is true about the respective thermal conductivities? **[2006]**



(A) $2k_1 = k_2$ (B) $k_1 = k_2$
(C) $2k_1 = 3k_2$ (D) $k_1 = 2k_2$

8. With an increase in thickness of insulation around a circular pipe, heat loss to surrounding due to **[2006]**

(A) Convection increases, while that due to conduction decrease
(B) Convection decreases, while that due to conduction increases
(C) Convection and conduction decreases
(D) Convection and conduction increases

9. The average heat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observation of the change in plate temperature with time as it cools. Assume the plate temperature to be uniform at any instant of time and radiation heat exchange with the surroundings negligible. The ambient temperature is 25°C, the plate has a total surface area of 0.1 m^2 and a mass of 4 kg. The specific heat of the plate material is 2.5 kJ/kgK. The convective heat transfer coefficient in W/m^{2}K, at the instant when the plate temperature is 225°C and the change in plate temperature with time $\frac{dT}{dt}$ = –0.02 k/s, s **[2007]**

(A) 200 (B) 20
(C) 15 (D) 10

Direction for questions 10 and 11: Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 mW/m^3. The left and right faces are kept at constant temperature of 160°C and 120°C respectively. The plate has a constant thermal conductivity of 200 W/mK.

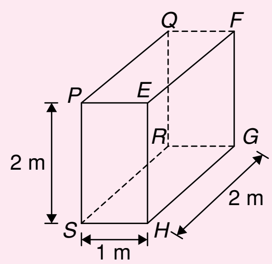
10. The location of maximum temperature within the plate from its left face is **[2007]**

(A) 15 mm (B) 10 mm
(C) 5 mm (D) 0 mm

11. The maximum temperature within the plate in °C is **[2007]**

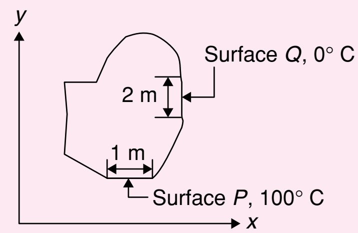
(A) 160 (B) 165
(C) 200 (D) 250

12. For the three-dimensional object shown in the figure below, five faces are insulated. The sixth face ($PQRS$), which is not insulated, interacts thermally with the ambient, with a convective heat transfer coefficient of 10 W/m^2.K. The ambient temperature is 30°C. Heat is uniformly generated inside the object at the rate of 100 W/m^3. Assuming the face $PQRS$ to be at uniform temperature, its steady state temperature is **[2008]**



(A) 10°C (B) 20°C
(C) 30°C (D) 40°C

13. Steady two-dimensional heat conduction takes place in the body shown in the figure below. The normal temperatures gradients over surfaces P and Q can be considered to be uniform. The temperature gradient $\frac{\partial T}{\partial x}$ at surface Q is equal to 10 K/m. Surfaces P and Q are maintained at constant temperature as shown in the figure, while the remaining part of the boundary is insulated. The body has a constant thermal conductivity of 0.1 W/m.K. The values of $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ at surface P are **[2008]**



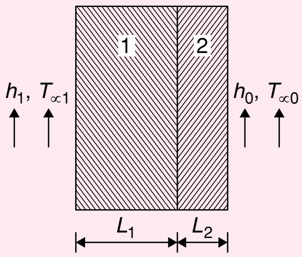
(A) $\frac{\partial T}{\partial x} = 20$ K/m, $\frac{\partial T}{\partial y} = 0$ K/m

(B) $\frac{\partial T}{\partial x} = 0$ K/m, $\frac{\partial T}{\partial y} = 10$ K/m

(C) $\dfrac{\partial T}{\partial x} = 10$ K/m, $\dfrac{\partial T}{\partial y} = 10$ K/m

(D) $\dfrac{\partial T}{\partial x} = 0$ K/m, $\dfrac{\partial T}{\partial y} = 20$ K/m

14. Consider steady-state heat conduction across the thickness in a plane composite wall (as shown in the figure) exposed to convection conditions on both sides.



Given:

$h_i = 20$ W/m^2 K; $h_0 = 50$ W/m^2 K; $T_{\infty,i} = 20$°C; $T_{\infty,o} = -2$°C; $k_1 = 20$ W/mK; $k_2 = 50$ W/mK; $L_1 = 0.30$ m and $L_2 = 0.15$ m.

Assuming negligible contact resistance between the wall surfaces, the interface temperature, T(in °C), of the two walls will be **[2009]**

(A) –0.50 (B) 2.75 (C) 3.75 (D) 4.50

15. A fin has 5 mm diameter and 100 mm length. The thermal conductivity of fin material is 400 Wm^{-1}K^{-1}. One end of the fin is maintained at 130°C and its remaining surface is exposed to ambient air at 30°C. If the convective heat transfer coefficient is 40 Wm^{-2}K^{-1}, the heat loss (in W) from the fin is **[2010]**

(A) 0.08 (B) 5.0 (C) 7.0 (D) 7.8

16. A pipe of 25 mm outer diameter carries steam. The heat transfer coefficient between the cylinder and surroundings is 5 W/m^{2}K.

It is proposed to reduce the heat loss from the pipe by adding insulation having a thermal conductivity of 0.05 W/mK. Which one of the following statements is TRUE? **[2011]**

(A) The outer radius of the pipe is equal to the critical radius.
(B) The outer radius of the pipe is less than the critical radius.
(C) Adding the insulation will reduce the heat loss.
(D) Adding the insulation will increase the heat loss.

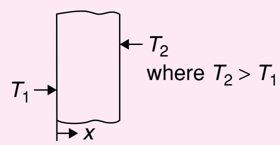
17. A spherical steel ball of 12 mm diameter is initially at 1000 K. It is slowly cooled in a surrounding of 300 K. The heat transfer coefficient between the steel ball and the surrounding is 5 W/m^{2}K. The thermal conductivity of steel is 20 W/mK. The temperature difference between the centre and the surface of the steel ball is **[2011]**

(A) Large because conduction resistance is far higher than the convective resistance.
(B) Large because conduction resistance is far less than the convective resistance.
(C) Small because conduction resistance is far higher than the convective resistance.
(D) Small because conduction resistance is far less than the convective resistance.

18. Consider one-dimensional steady state heat conduction along x-axis ($0 \le x \le L$), through a plane wall with the boundary surfaces ($x = 0$ and $x = L$) maintained at temperatures of 0°C and 100°C. Heat is generated uniformly throughout the wall. Choose the CORRECT statement. **[2013]**

(A) The direction of heat transfer will be from the surface at 100°C to the surface at 0°C.
(B) The maximum temperature inside the wall must be greater than 100°C.
(C) The temperature distribution is linear within the wall.
(D) The temperature distribution is symmetric about the mid-plane of the wall.

19. Consider one-dimensional steady state heat conduction, without heat generation, in a plane wall; with boundary conditions as shown in the figure below. The conductivity of the wall is given by $k = k_0 + bt$; where k_0 and b are positive constants, and T is temperature.



As x increases, the temperature gradient (d/dx) will **[2013]**

(A) Remain constant (B) Be zero
(C) Increase (D) Decrease

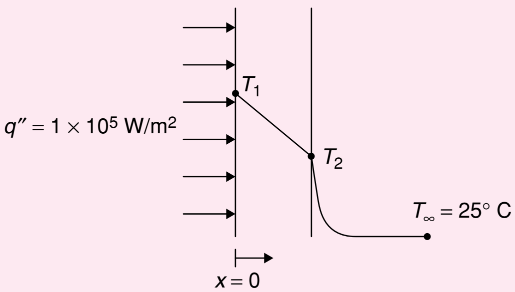
20. A steel ball of diameter 60 mm is initially in thermal equilibrium at 1030°C in a furnace. It is suddenly removed from the furnace and cooled in ambient air at 30°C, with convective heat transfer coefficient h = 20 W/m^{2}K. The thermo-physical properties of steel are: density ρ = 7800 kg/m^3, conductivity K = 40 W/mK and specific heat C = 600 J/kgK. The time required in seconds to cool the steel ball in air from 1030°C to 430°C is **[2013]**

(A) 519 (B) 931
(C) 1195 (D) 2144

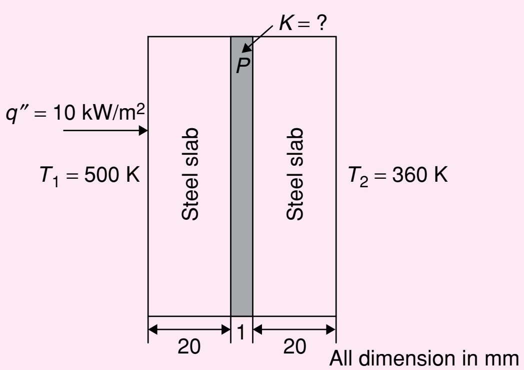
21. Biot number signifies the ratio of **[2014]**

(A) Convective resistance in the fluid to conductive resistance in the solid.
(B) Conductive resistance in the solid to convective resistance in the fluid.
(C) Inertia force to viscous force in the fluid.
(D) Buoyancy force to viscous force in the fluid.

22. Consider one dimensional steady state heat conduction across a wall (as shown in figure below) of thickness 30 mm and thermal conductivity 15 W/m. K. At $x = 0$, a constant heat flux, $q'' = 1 \times 10^5$ W/m^2 is applied. On the other side of the wall, heat is removed from the wall by convection with a fluid at 25°C and heat transfer coefficient of 250 W/m^2.K. The temperature (in °C), at $x = 0$ is _____ **[2014]**



23. A material P of thickness 1 mm is sandwiched between two steel slabs, as shown in the figure below. A heat flux 10 kW/m^2 is supplied to one of the steel slabs as shown. The boundary temperatures of the slabs are indicated in the figure. Assume thermal conductivity of this steel is 10 W/m.K. Considering one-dimensional steady state heat conduction for the configuration, the thermal conductivity (K, in W/m. K) of material P is _____ **[2014]**

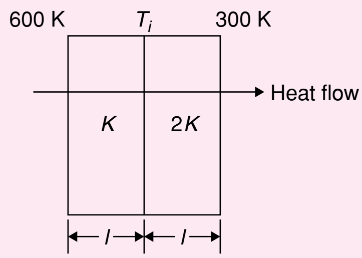


24. Consider a long cylindrical tube of inner and outer radii, r_i and r_o, respectively, length, L and thermal conductivity, K. Its inner and outer surfaces are maintained at T_i and T_o, respectively ($T_i > T_o$). Assuming one–dimensional steady state heat conduction in the radial direction, the thermal resistance in the wall of the tube is **[2014]**

(A) $\dfrac{1}{2\pi KL}\ln\left(\dfrac{r_i}{r_o}\right)$ (B) $\dfrac{L}{2\pi r_i K}$

(C) $\dfrac{1}{2\pi KL}\ln\left(\dfrac{r_o}{r_i}\right)$ (D) $\dfrac{1}{4\pi KL}\ln\left(\dfrac{r_o}{r_i}\right)$

25. Heat transfer through a composite wall is shown in figure. Both the sections of the wall have equal thickness (ℓ). The conductivity of one section is K and that of the other is $2K$. The left face of the wall is at 600 K and the right face is at 300 K.



The interface temperature T_i (in K) of the composite wall is _______ **[2014]**

26. As the temperature increases, the thermal conductivity of a gas **[2014]**

(A) Increases
(B) Decreases
(C) Remains constant
(D) Increases up to a certain temperature and then decreases

27. A plane wall has a thermal conductivity of 1.15 W/m. K. If the inner surface is at 1100°C and the outer surface is at 350°C, then the design thickness (in meter) of the wall to maintain a steady heat flux of 2500 W/m^2 should be ____ **[2014]**

28. A 10 mm diameter electrical conductor is covered by an insulation of 2 mm thickness. The conductivity of the insulation is 0.08 W/m-K and the convection coefficient at the insulation surface is 10 W/m^2-K. Addition of further insulation of the same material will: **[2015]**

(A) increase heat loss continuously.
(B) decrease heat loss continuously.
(C) increase heat loss to maximum and then decrease heat loss.
(D) decrease heat loss to maximum and then increase heat loss.

29. If a foam insulation is added to a 4 cm outer diameter pipe as shown in the figure, the critical radius of insulation (in cm) is _______. **[2015]**

30. A cylindrical uranium fuel rod of radius 5 mm in a nuclear reactor is generating heat at the rate of 4×10^7 W/m^3. The rod is cooled by a liquid (convective heat transfer coefficient 1000 W/m^2-K) at 25°C. At steady state, the surface temperature (in K) of the rod is: **[2015]**

(A) 308 (B) 398
(C) 418 (D) 448

31. A brick wall $\left(k = 0.9\dfrac{W}{m.K}\right)$ of thickness 0.18 m separates the warm air in a room from the cold ambient air. On a particular winter day, the outside air temperature is –5°C and the room needs to be maintained at 27°C. The heat transfer coefficient associated with outside air is $20\dfrac{W}{m^2K}$. Neglecting the convective resistance of the air inside the room, the heat loss, in $\left(\dfrac{W}{m^2}\right)$, is: **[2015]**

(A) 88 (B) 110
(C) 128 (D) 160

32. A plastic sleeve of outer radius r_0 = 1 mm covers a wire (radius r = 0.5 mm) carrying electric current. Thermal conductivity of the plastic is 0.15 W/m-K. The heat transfer coefficient on the outer surface of the sleeve exposed to air is 25 W/m²-K. Due to the addition of the plastic cover, the heat transfer from the wire to the ambient will: **[2016]**

(A) increase (B) remain the same
(C) decrease (D) be zero

33. A steel ball of 10 mm diameter at 1000 K is required to be cooled to 350 K by immersing it in a water environment at 300 K. The convective heat transfer coefficient is 1000 W/m²-K. Thermal conductivity of steel is 40 W/mK. The time constant for the cooling process τ is 16 s. The time required (in s) to reach the final temperature is ______. **[2016]**

34. A hollow cylinder has length L, inner radius r_1, outer radius r_2, and thermal conductivity k. The thermal resistance of the cylinder for radial conduction is: **[2016]**

(A) $\dfrac{\ln(r_2/r_1)}{2\pi kL}$ (B) $\dfrac{\ln(r_1/r_2)}{2\pi kL}$

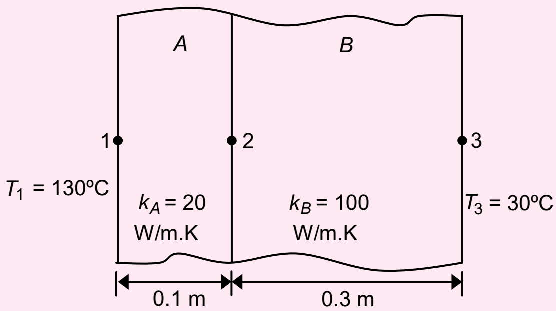
(C) $\dfrac{2\pi kL}{\ln(r_2/r_1)}$ (D) $\dfrac{2\pi kL}{\ln(r_1/r_2)}$

35. Two cylindrical shafts A and B at the same initial temperature are simultaneously placed in a furnace. The surfaces of the shafts remain at the furnace gas temperature at all times after they are introduced into the furnace. The temperature variation in the axial direction of the shafts can be assumed to be negligible. The data related to shafts A and B is given in the following table.

Quantity	Shaft A	Shaft B
Diameter (m)	0.4	0.1
Thermal conductivity (W/m-K)	40	20
Volumetric heat capacity (J/m³-K)	2×10^6	2×10^7

The temperature at the centerline of the shaft A reaches 400°C after two hours. The time required (in hours) for the centerline of the shaft B to attain the temperature of 400°C is ______. **[2016]**

36. Steady one-dimensional heat conduction takes place across the faces 1 and 3 of a composite slab consisting of slabs A and B in perfect contact as shown in the figure, where k_A, k_B denote the respective thermal conductivities. Using the data is given in the figure, the interface temperature T_2 (in °C) is ________. **[2016]**



37. A cylindrical steel rod, 0.01 m in diameter and 0.2 m in length is first heated to 750°C and then immersed in a water bath at 100°C. The heat transfer coefficient is 250 W/m²-K. The density, specific heat and thermal conductivity of steel are ρ = 7801 kg/m³, c = 473 J/kg-K, and k = 43 W/m-K, respectively. The time required for the rod to reach 300°C is ______ seconds. **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. C	**2.** D	**3.** C	**4.** B	**5.** D	**6.** B	**7.** B	**8.** B	**9.** A	**10.** C
11. D	**12.** A	**13.** A	**14.** A	**15.** D	**16.** C	**17.** D	**18.** B	**19.** A	**20.** C

Practice Problems 2

1. B	**2.** A	**3.** C	**4.** C	**5.** C	**6.** A	**7.** B	**8.** B	**9.** C	**10.** B
11. B	**12.** A	**13.** C	**14.** D	**15.** A	**16.** A	**17.** A	**18.** C	**19.** A	**20.** A
21. B	**22.** C	**23.** A	**24.** A	**25.** A	**26.** A	**27.** A	**28.** A	**29.** C	**30.** C

Previous Years' Questions

1. B **2.** C **3.** B **4.** D **5.** C **6.** C **7.** D **8.** A **9.** D **10.** C
11. B **12.** D **13.** D **14.** C **15.** B **16.** C **17.** D **18.** B **19.** D **20.** D
21. B **22.** 620 to 630 **23.** 0.09 to 0.11 **24.** C **25.** 399 to 401 **26.** A
27. 0.33 to 0.35 **28.** C **29.** 4.9 to 5.1 **30.** B **31.** C **32.** A **33.** 42 – 43
34. A **35.** 2.5 **36.** 67.5 **37.** 42–45