Chapter 10. Quadratic And Exponential Functions

Ex. 10.3

Answer 1CU.

Use algebra titles

x	1	1
x	1	1
x^2	x	x

Hence the area of the square is x^2+4x+4.

Answer 1GCI.

Consider the function $y=a(x-h)^2+k$

Use the rule

"The standard form of the parabola is with vertex (h,k) is given by

$y-k=a(x-h)^2$ or $y=a(x-h)^2+k$

Therefore the vertex of the function $y=a(x-h)^2+k$ is

Answer 1PQ.

Consider the equation $y=x^2-x-6$

Step1: Write the equation of the axis of symmetry. The given equation is $y=x^2-x-6$

Use the rule "The equation of the axis of symmetry for the graph $y=ax^2+bx+c$ where $a\neq 0$ is $x=\frac{-b}{2a}$"

Now compare the equation $y=x^2-x-6$ with $y=ax^2+bx+c$.

We have $a=1, b=-1 \text{ and } c=-6$

$$x=\frac{-b}{2a} \quad \text{(Equation for the axis of symmetry of a parabola)}$$

$$x=-\frac{(-1)}{2\cdot(1)} \quad \text{(Replace } a \text{ by 1 and } b \text{ by } -1\text{)}$$

$$x=\frac{1}{2} \quad \text{(Simplification)}$$

Hence, the equation of the axis of symmetry is $\boxed{x=\frac{1}{2}}$

Step2: Find the coordinates of the vertex. Since the equation of the axis of symmetry is $x = \frac{1}{2}$ and the vertex lies on the axis, the *x* – Coordinate for the vertex is $x = \frac{1}{2}$

$$y = x^2 - x - 6 \qquad \text{(Original equation)}$$

$$y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6 \qquad \left(\text{Replace } x \text{ by } \frac{1}{2}\right)$$

$$y = \frac{1}{4} - \frac{1}{2} - 6$$

$$y = \frac{1 - 1 \cdot 2 - 6 \cdot 4}{4} \qquad \left(\begin{array}{l}\text{Keep as a common denominator} \\ \text{and combine the numerator}\end{array}\right)$$

$$y = \frac{1 - 2 - 24}{4}$$

$$y = \frac{1 - 26}{4}$$

$$y = \frac{-25}{4}$$

Step3: Identify the maximum or minimum

The equation is $y = x^2 - x - 6$

Use the rule "The equation of the parabola is $y = ax^2 + bx + c$. Suppose the coefficient of *x*2 term is positive, the parabola opens upwards and the vertex is a minimum point. Suppose the coefficient of *x*2 term is negative, the parabola open downward and the vertex is maximum points."

Since the coefficient of the *x*2 term i8s positive the parabola open upwards and vertex is a minimum point.

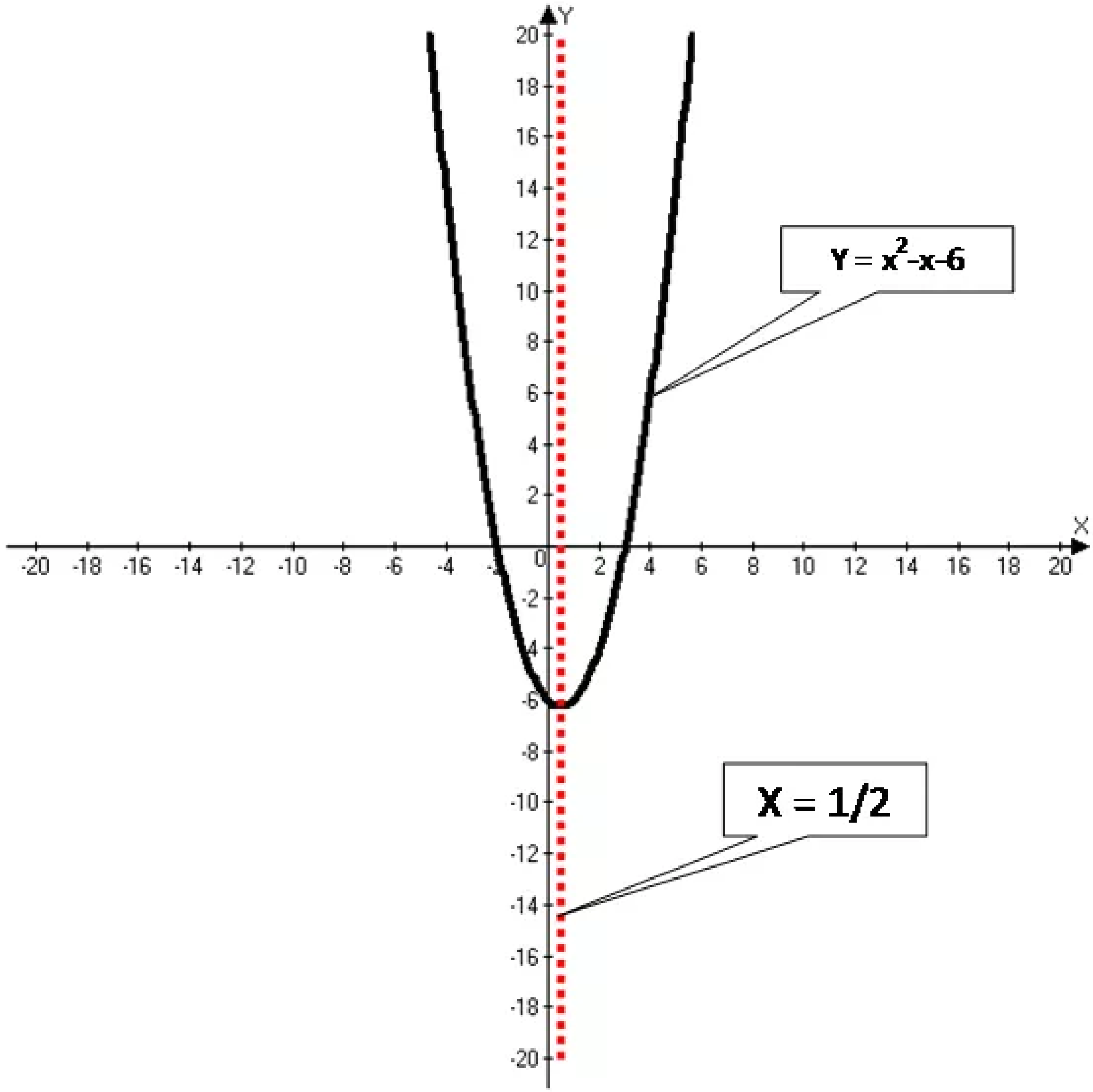
Hence, the minimum points of the parabola is $\left(\frac{1}{2}, \frac{-25}{4}\right)$

Step4: Graph the function $y = x^2 - x - 6$

Now, we consider the table for $y = x^2 - x - 6$. We can substitute the different values of *x* is $y = x^2 - x - 6$, we get the *y* – values. Graph these ordered pairs and connect them, we get the smooth curve.

x	$x^2 - x - 6$	y	(x, y)
$\frac{-1}{4}$	$\left(\frac{-1}{4}\right)^2 - \left(\frac{-1}{4}\right) - 6 = \frac{-91}{16}$	$\frac{-91}{16}$	$\left(\frac{-1}{4}, \frac{-91}{16}\right)$
$\frac{-1}{2}$	$\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right) - 6 = \frac{-21}{4}$	$\frac{-21}{4}$	$\left(\frac{-1}{2}, \frac{-21}{4}\right)$
0	$(0)^2 - (0) - 6 = -6$	-6	−6
$\frac{1}{2}$	$\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6 = \frac{-25}{4}$	$\frac{-25}{4}$	$\left(\frac{1}{2}, \frac{-25}{4}\right)$
$\frac{1}{4}$	$\left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right) - 6 = \frac{-27}{16}$	$\frac{-27}{16}$	$\left(\frac{1}{4}, \frac{-24}{16}\right)$

Add these all ordered pairs; we get a parabola open upward.



Answer 2CU.

Consider the trinomial $x^2 - 5x - 7 = 0$

Claim Solve the equation $x^2 - 5x - 7 = 0$

Step 1 : Isolated x^2 and *x* terms

$$x^2 - 5x - 7 = 0 \qquad \text{[Original equation]}$$

$$x^2 - 5x - 7 + 7 = 0 + 7 \qquad \text{[Add 7 to each side]}$$

$$x^2 - 5x = 7$$

Step 2: Complete the square and solve the equation $x^2 - 5x = 7$

$$x^2 - 5x = 7$$

$$x^2 - 1 \cdot 5 \cdot x = 7$$

$$x^2 - \frac{2}{2} \cdot 5x = 7$$

$$x^2 - 2 \cdot x \cdot \frac{5}{2} = 7$$

$$x^2 - 2 \cdot x \cdot \frac{5}{2} + \left(\frac{5}{2}\right)^2 = 7 + \left(\frac{5}{2}\right)^2 \qquad \left[\text{Add } \left(\frac{5}{2}\right)^2 \text{ to each side}\right]$$

$$\left(x - \frac{5}{2}\right)^2 = 7 + \frac{25}{4} \qquad \left[\text{Factor } x^2 - 2 \cdot x \cdot \frac{5}{2} + \left(\frac{5}{2}\right)^2\right]$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{28 + 25}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{53}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = 13.25$$

$$\left(x - \frac{5}{2}\right)^2 = \pm\sqrt{13.25} \quad \text{or} \quad x = \frac{5}{2} - \sqrt{13.25}$$

$$\approx 2.5 + 3.6 \quad \text{or} \quad \approx 2.5 - 3.6 \qquad \left[\text{Since } \sqrt{13.25} \approx 3.6\right]$$

$$\approx 6.1 \quad \text{or} \quad \approx -1.1$$

Therefore, $x \approx 6.1$ or $x \approx -1.1$

Hence, the solution set is $\boxed{\{-1.1, 6.1\}}$

Answer 2GCI.

Consider the equation $x^2 - 2x - 3$

Claim: Write the equation $x^2 - 2x - 3$ vertex form

To rewrite the equation $x^2 - 2x - 3$ in the standard form of parabola with vertex (h, k) is

$$y = a(x-h)^2 + k$$

$$\begin{aligned} y &= x^2 - 2x - 3 \qquad \text{(original equation)} \\ &= (x^2 - 2x - 1) - 3 - 1 \\ &= (x-1)^2 - 4 \end{aligned}$$

Therefore $y = x^2 - 2x - 3$ vertex form is $\boxed{y = (x-1)^2 - 4}$

Answer 2PQ.

Consider the equation $y = 2x^2 + 3$

Step1: Write the equation of the axis of symmetry. The given equation is $y = 2x^2 + 3$

Use the rule "The equation of the axis of symmetry for the graph $y = ax^2 + bx + c$ where $a \neq 0$ is $x = \frac{-b}{2a}$"

Now compare the equation $y = 2x^2 + 3$ with $y = ax^2 + bx + c$.

We have $a = 2, b = 0$ and $c = 3$

$$\begin{aligned} x &= \frac{-b}{2a} && \text{(Equation for the axis of symmetry of a parabola)} \\ x &= -\frac{(0)}{2\cdot(2)} && \text{(Replace } a \text{ by 2 and } b \text{ by 0)} \\ x &= 0 && \text{(Simplification)} \end{aligned}$$

Hence, the equation of the axis of symmetry is $\boxed{x = 0}$

Step2: Find the coordinates of the vertex. Since the equation of the axis of symmetry is $x = 0$ and the vertex lies on the axis, the x – Coordinate for the vertex is $x = 0$

$$\begin{aligned} y &= 2x^2 + 3 && \text{(Original equation)} \\ y &= 2(0)^2 + 3 && \text{(Replace } x \text{ by 0)} \\ y &= 0 + 3 \\ y &= 3 \end{aligned}$$

Step3: Identify the maximum or minimum

The equation is $y = 2x^2 + 3$

Use the rule "The equation of the parabola is $y = ax^2 + bx + c$. Suppose the coefficient of *x*2 term is positive, the parabola opens upwards and the vertex is a minimum point. Suppose the coefficient of *x*2 term is negative, the parabola open downward and the vertex is maximum points."

Since the coefficient of the *x*2 term is positive the parabola open upwards and vertex is a minimum point.

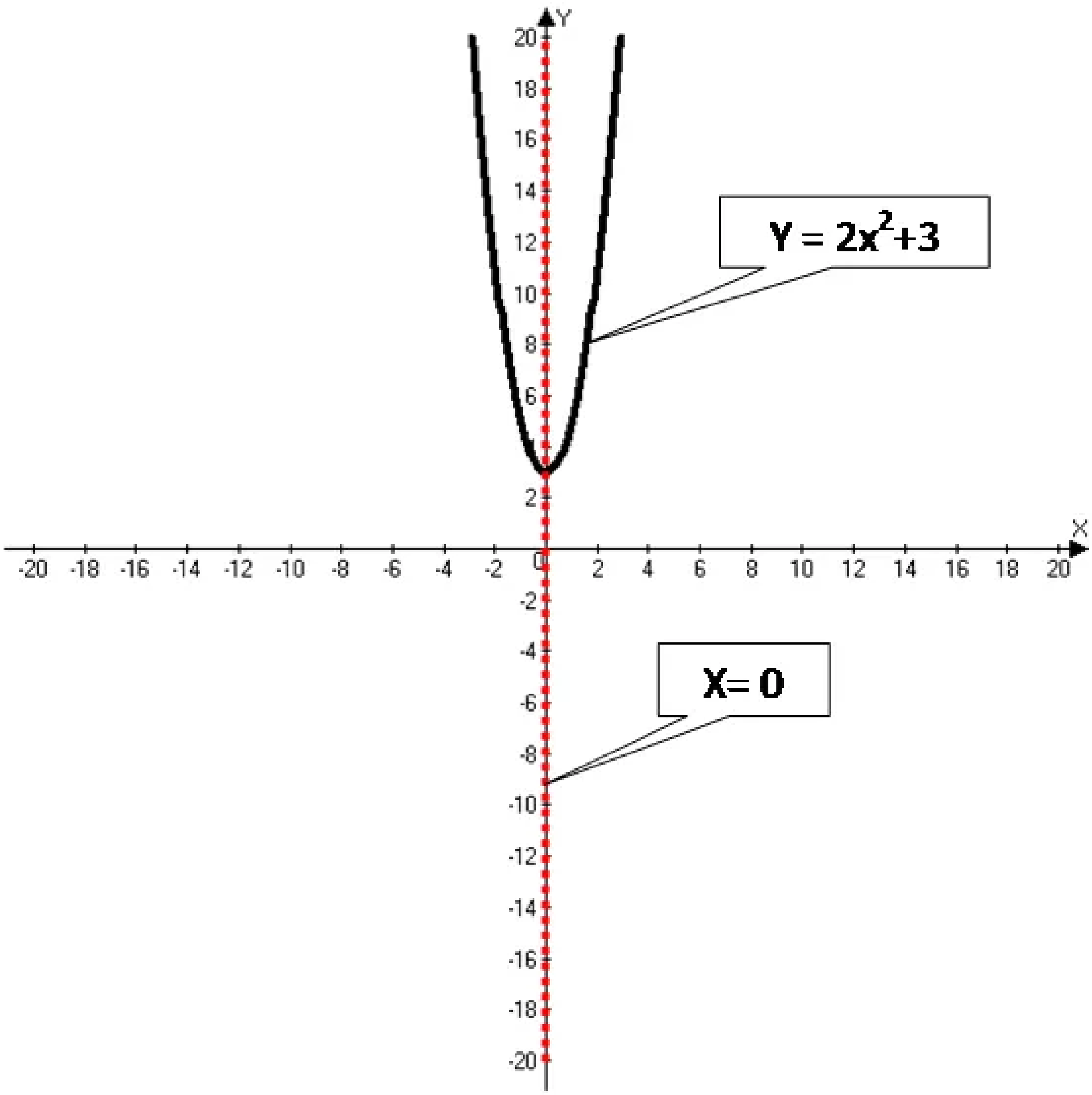
Hence, the minimum points of the parabola is $(0,3)$

Step4: Graph the function $y = 2x^2 + 3$

Now, we consider the table for $y = 2x^2 + 3$. We can substitute the different values of *x* is $y = 2x^2 + 3$, we get the *y* – values. Graph these ordered pairs and connect them, we get the smooth curve.

x	$2x^2 + 3$	y	(x, y)
−2	$2(-2)^2 + 3 = 11$	11	$(-2, 11)$
−1	$2(-1)^2 + 3 = 5$	5	$(-1, 5)$
0	$2(0)^2 + 3 = 3$	3	$(0, 3)$
1	$2(1)^2 + 3 = 5$	5	$(1, 5)$
2	$2(2)^2 + 3 = 11$	11	$(2, 11)$

Add these all ordered pairs, we get a parabola open upward.



Answer 3CU.

Consider the equation $5x^2+12x=15$

Step-1:- Solve the equation $5x^2+12x=15$

first divided each side by x^2 co-efficient

$$5x^2+12x=15 \qquad \text{[Original equation]}$$

$$\frac{5x^2+12x}{5}=\frac{15}{5}$$

$$x^2+\frac{12}{5}x=3$$

Therefore, the first step need to solve the equation $5x^2+12x=15$ by completing square is divided each side by 5

Answer 3GCI.

Consider the equation x^2+2x-7

<u>Claim</u>: Write the equation x^2+2x-7 in vertex form

To rewrite the equation x^2+2x-7 in the standard form of parabola with vertex (h,k) is

$$y=a(x-h)^2+k$$

$$x^2+2x-7 \qquad \text{(original equation)}$$

$$=x^2+2x-7$$

$$=x^2+2x\cdot1+1^2-1^2-7$$

$$=(x+1)^2-1-7$$

$$=(x+1)^2-8$$

Therefore x^2+2x-7 vertex form is $\boxed{y=(x+1)^2-8}$

Answer 3PQ.

Consider the equation $y=-3x^2-6x+5$

Step1: Write the equation of the axis of symmetry. The given equation is $y=-3x^2-6x+5$

Use the rule "The equation of the axis of symmetry for the graph $y=ax^2+bx+c$ where $a\neq 0$ is $x=\frac{-b}{2a}$"

Now compare the equation $y=2x^2+3$ with $y=ax^2+bx+c$.

We have $a=-3, b=-6 \text{ and } c=5$

$$x=\frac{-b}{2a} \quad \text{(Equation for the axis of symmetry of a parabola)}$$

$$x=-\frac{(-6)}{2\cdot(-3)} \quad (\text{Replace } a \text{ by } -3 \text{ and } b \text{ by } -6)$$

$$x=\frac{6}{-2\cdot 3} \quad \text{(Simplification)}$$

$$x=\frac{2\cdot 3}{-2\cdot 3} \quad \text{(Cancellation of the numerator and the denominator)}$$

$$x=-1$$

Hence, the equation of the axis of symmetry is $\boxed{x=-1}$

Step2: Find the coordinates of the vertex. Since the equation of the axis of symmetry is $x=-1$ and the vertex lies on the axis, the x – Coordinate for the vertex is $x=-1$

$$y=-3x^2-6x+5 \quad \text{(Original equation)}$$

$$y=-3(-1)^2-6(-1)+5 \quad (\text{Replace } x \text{ by } -1)$$

$$y=-3(1)+6+5$$

$$y=-3+11$$

$$y=8$$

Step3: Identify the maximum or minimum

The equation is $y = -3x^2 - 6x + 5$

Use the rule "The equation of the parabola is $y = ax^2 + bx + c$. Suppose the coefficient of *x*2 term is positive, the parabola opens upwards and the vertex is a minimum point. Suppose the coefficient of *x*2 term is negative, the parabola open downward and the vertex is maximum points."

Since the coefficient of the *x*2 term is positive the parabola open downwards and vertex is a maximum point.

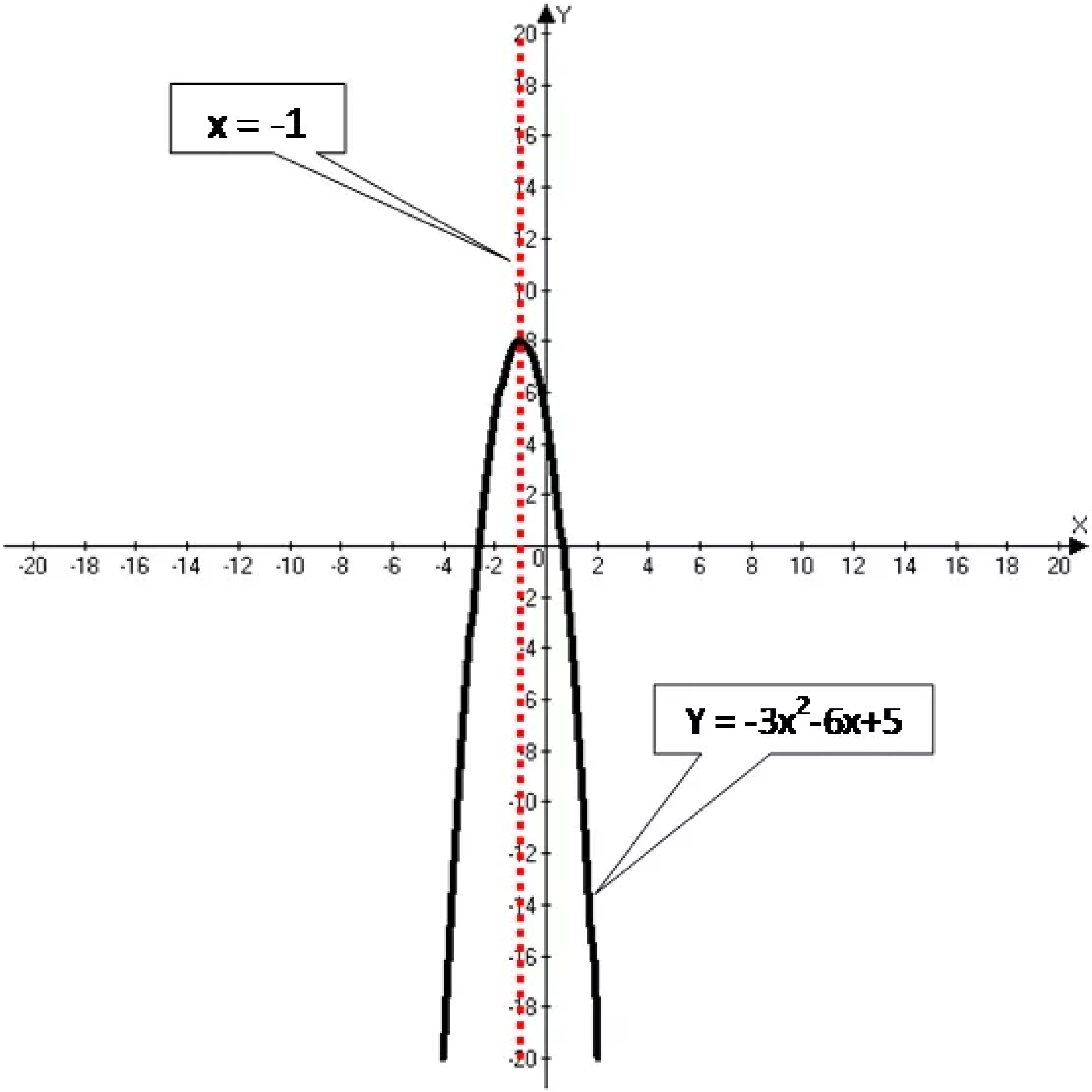
Hence, the maximum points of the parabola is $(-1, 8)$

Step4: Graph the function $y = -3x^2 - 6x + 5$

Now, we consider the table for $y = -3x^2 - 6x + 5$. We can substitute the different values of *x* is $y = -3x^2 - 6x + 5$, we get the *y* – values. Graph these ordered pairs and connect them, we get the smooth curve.

x	$-3x^2 - 6x + 5$	y	(x, y)
−2	$-3(-2)^2 - 6(-2) + 5 = 5$	5	$(-2, 5)$
−1	$-3(-1)^2 - 6(-1) + 5 = 8$	8	$(-1, 8)$
0	$-3(0)^2 - 6(0) + 5 = 5$	5	$(0, 5)$
1	$-3(1)^2 - 6(1) + 5 = -4$	−4	$(1, -4)$
2	$-3(2)^2 - 6(2) + 5 = -19$	−19	$(2, -19)$

Add these all ordered pairs; we get a parabola open downward.



Answer 4CU.

Consider the equation $b^2-6b+9=25$

Step-1:- first rewrite the equation $b^2-6b+9=25$ as a common square.

$$b^2-6b+9=25$$

$$b^2-2\cdot b+3+3^2=25 \qquad [\text{Write } 6b \text{ as } 2\cdot b\cdot 3]$$

$$(b-3)^2=25$$

Step-2:- Now solve the equation $(b-3)^2 = 25$ by taking the square root of each side. We obtain '*b*' values $(b-3)^2 = 25$

$\sqrt{(b-3)^2} = \sqrt{25}$ taking the square root of each side

$b-3 = \pm 5$ $\sqrt{25} = \pm 5$

$b-3+3 = 3 \pm 5$ Add 3 to each side

$b = 3 \pm 5$

$b = 3+5$ or $b = 3-5$

$b = 8$ or $b = -2$

There for, $b = 8$ or $b = -2$

Step-3:- Substitute each b value in the original equation $b^2 - 6b + 9 = 25$

$b^2 - 6b + 9 = 25$ [original equation]

$(8)^2 - 6(8) + 9 \stackrel{?}{=} 25$ [Replace b by 8]

$64 - 48 + 9 \stackrel{?}{=} 25$

$73 - 48 \stackrel{?}{=} 25$

$25 = 25$ True

$b^2 - 6b + 9 = 25$ [original equation]

$(-2)^2 - 6(-2) + 9 \stackrel{?}{=} 25$ [Replace b by 8]

$4 + 12 + 9 + 9 \stackrel{?}{=} 25$

$25 \stackrel{?}{=} 25$ True

$b = 8$ and $b = -2$ statics the original equation $b^2 - 6b + 9 = 25$

Hence, the solution set is $\boxed{\{-2, 8\}}$

Answer 4GCI.

Consider the equation x^2-4x+8

Claim: Write the equation x^2-4x+8 in vertex form

To rewrite the equation x^2-4x+8 in the standard form of parabola with vertex (h,k) is

$$y=a(x-h)^2+k$$

$$\begin{aligned} y &= x^2-4x+8 && \text{(original equation)} \\ &= x^2-2\cdot x\cdot 2+8 && (\text{Write } 4x \text{ as } 2\cdot x\cdot 2) \\ &= x^2-2\cdot x\cdot 2+2^2-2^2+8 \\ &= (x-2)^2+4 \end{aligned}$$

Therefore x^2-4x+8 vertex form is $\boxed{y=(x-2)^2+4}$

Answer 4PQ.

Consider the equation:

$$x^2+6x+10=0$$

The objective is to solve the equation $x^2+6x+10=0$ using graph.

To graph the function $f(x)=x^2+6x+10$,construct the table for $f(x)=x^2+6x+10$.

Now, substitute the different values of 'x' is the original function $f(x)=x^2+6x+10$.

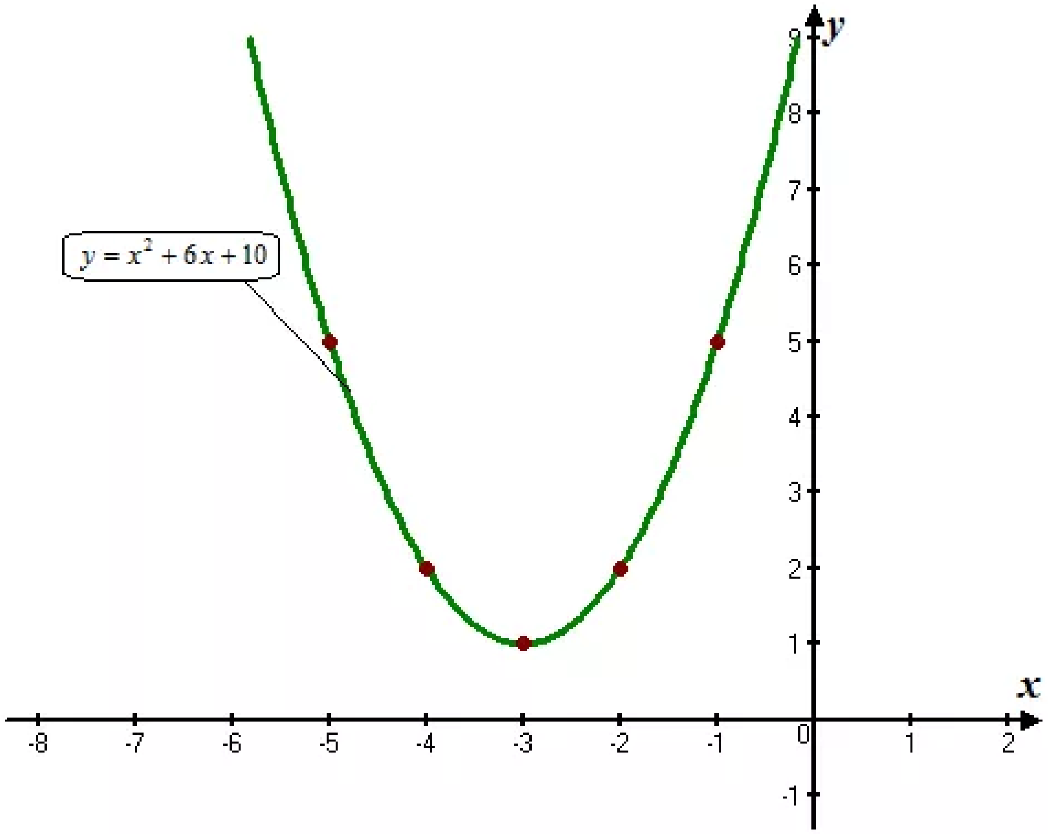
We obtain the $f(x)$ values as follows.

x	$x^2+6x+10$	$f(x)$	$(x,f(x))$
-5	$(-5)^2+6(-5)+10=5$	5	$(-5,5)$
-4	$(-4)^2+6(-4)+10=2$	2	$(-4,2)$
-3	$(-3)^2+6(-3)+10=1$	1	$(-3,1)$
-2	$(-2)^2+6(-2)+10=2$	2	$(-2,2)$
-1	$(-1)^2+6(-1)+10=5$	5	$(-1,5)$

Plot all the above ordered pairs and connected them.

We obtain the smooth curve.

The following is the figure:



We observe that the graph has no *x* – intercept. Thus, there are no real number solutions for this equation $x^2 + 6x + 10 = 0$

Therefore, the solution set of the equation $x^2 + 6x + 10 = 0$ is $\boxed{\{\phi\}}$.

Answer 5CU.

Consider the equation $m^2+14m+49=20$

Claim solve the equation $m^2+14m+49=20$

Step-1 :- Rewrite the equation $m^2+14m+49=20$ as a completing square form.

$$m^2+14m+49=20 \qquad [\text{original equation.}]$$
$$m^2+2\cdot m\cdot 7+49=20 \qquad [\text{write } 14 \text{ as } 2\cdot m\cdot 7]$$
$$m^2+2\cdot m\cdot 7+7^2=20$$
$$(m+7)^2=20 \qquad \left[\text{factor } m^2+2m+7^2 \text{ as } (m+7)^2\right]$$

Step-2 :- Now solve the equation $(m+7)^2=20$ by taking the square root of each side. We obtain '*m*' values

$$(m+7)^2=20$$
$$\sqrt{(m+7)^2}=\sqrt{20} \qquad [\text{taking the square root of each side}]$$
$$m+7=\pm\sqrt{20}$$
$$m+7-7=-7\sqrt{20} \qquad [\text{substract 7 to each side}]$$
$$m=-7\pm\sqrt{20}$$
$$m=-7+\sqrt{20} \text{ or } m=-7-\sqrt{20}$$
$$m=-7+4.5 \text{ or } m=-7-4.5 \qquad \left[\text{since} \sqrt{20}\approx 4.5\right]$$
$$m=-2.5 \text{ or } m=-11.5$$

Hence, the solution set is $\boxed{\{-2.5,-11.5\}}$

Answer 5GCI.

Consider the equation $y = x^2 + 6x - 1$

Claim: Write the equation $y = x^2 + 6x - 1$ in vertex form

To rewrite the equation $y = x^2 + 6x - 1$ in the standard form of parabola with vertex (h, k) is

$y = a(x-h)^2 + k$

$$\begin{aligned} y &= x^2 + 6x - 1 && \text{(original equation)} \\ &= x^2 + 2 \cdot x \cdot 3 - 1 && (\text{Write } 6x \text{ as } 2 \cdot x \cdot 3) \\ &= x^2 + 2 \cdot x \cdot 3 + 3^2 - 3^2 - 1 \\ &= x^2 + 2 \cdot x \cdot 3 + 3^2 - 9 - 1 \\ &= (x+3)^2 - 10 \end{aligned}$$

Therefore $y = x^2 + 6x - 1$ vertex form is $\boxed{y = (x+3)^2 - 10}$

Answer 5PQ.

Let us consider the equation $x^2 - 2x - 1 = 0$

Step1: Rewrite the consider related function $f(x) = x^2 - 2x - 1$

Now we construct the table for the function $f(x)$

x	$f(x) = x^2 - 2x - 1 =$
-4	$f(-4) = (-4)^2 - 2(-4) - 1 = 14$
-3	$f(-3) = (-3)^2 - 2(-3) - 1 = 7$
-2	$f(-2) = (-2)^2 - 2(-2) - 1 = 2$
-1	$f(-1) = (-1)^2 - 2(-1) - 1 = 2$
0	$f(0) = (0)^2 - 2(0) - 1 = -1$
1	$f(1) = (1)^2 - 2(1) - 1 = -2$
2	$f(2) = (2)^2 - 2(2) - 1 = -1$
3	$f(3) = (3)^2 - 2(3) - 1 = 2$
4	$f(4) = (4)^2 - 2(4) - 1 = 7$
5	$f(5) = (5)^2 - 2(5) - 1 = 14$

From the table, we can observe that $f(-1) = 2 > 0$ $f(0) = -1 < 0$ and -1, 0 are consecutive integers and also $f(2) = -1 < 0, f(3) = 2 > 0$ and 2, 3 are consecutive integers.

We have the rule "The roots of equation $f(x) = 0$ lies between any two consecutive integers 'b' and 'c'. If $f(v) < 0 \text{ and } f(c) > 0$ (or) $f(c) < 0 \text{ and } f(b) > 0$"

By using this rule the roots of the equation lies between -1, 0 and 2, 3

Let us take the number -0.5 and 2.5 which lies between -1, 0 and 2, 3 respectively.

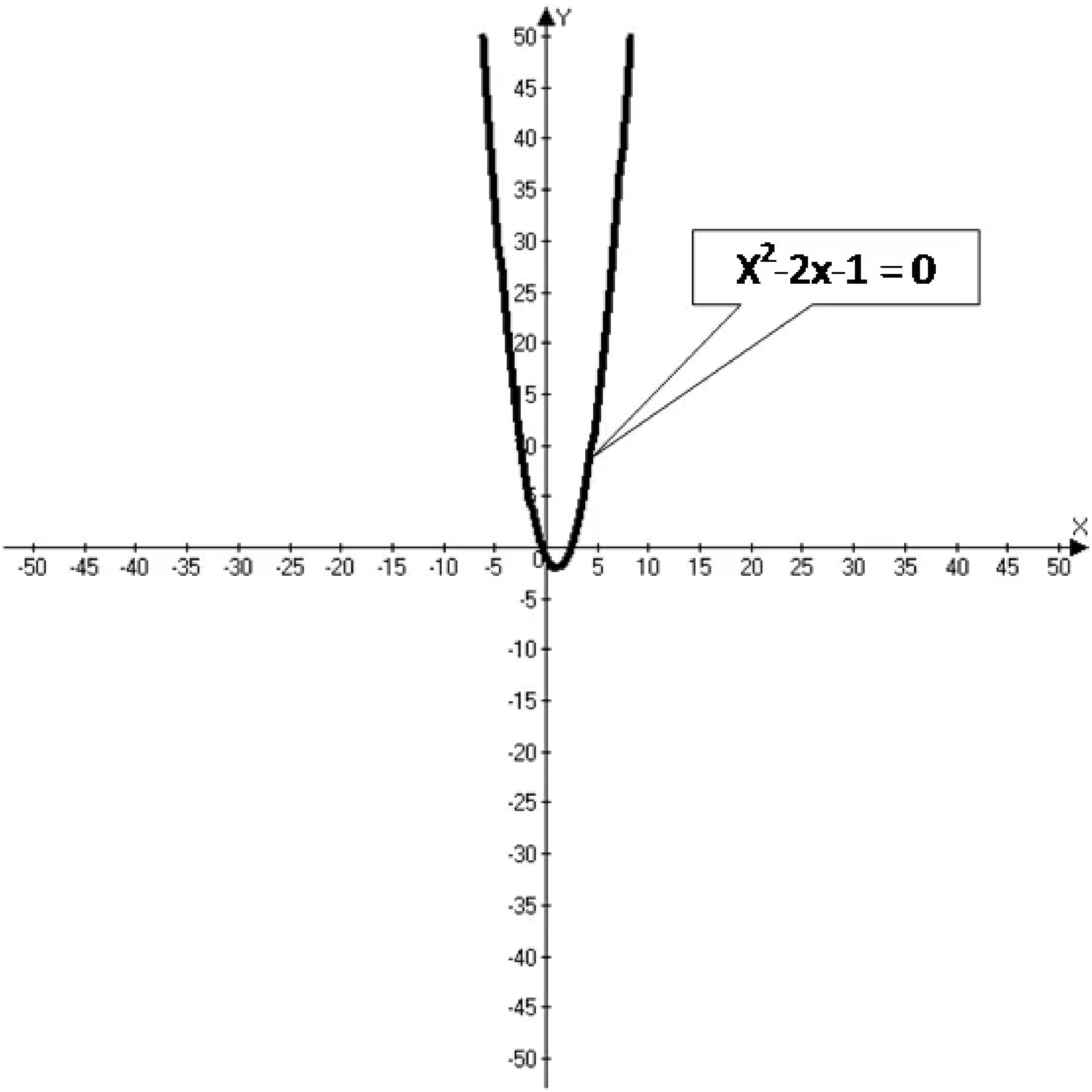
$$\begin{aligned} f(-0.5) &= (-0.5)^2 - 2\cdot(-0.5) - 1 \\ &= 0.25 + 1 - 1 \\ &= 0.25 \\ &\simeq 0 \end{aligned}$$

Similarly,

$$\begin{aligned} f(2.5) &= (2.5)^2 - 2\cdot(2.5) - 1 \\ &= 6.25 - 5 - 1 \\ &= 6.25 - 6 \\ &= 0.25 \end{aligned}$$

$$\simeq 0$$

By the above rule we can say that the roots of the equation $f(x) = x^2 - 2x - 1$ is -0.5, 2.5 approximately



Answer 6CU.

Consider the trinomial $a^2 - 12a + c$

Claim: To find the value of *c* that makes the trinomial $a^2 - 12a + c$ is a perfect sqaure

Step-1:- Find $\frac{1}{2}$ of -12 *i.e.* $\frac{-12}{2} = -6$

Step-2: Square the result of step 1 *i.e.* $(-b)^2 = 3b$

Step 3:- Add the result step 2 to $a^2 - 12a$ *i.e.* $a^2 - 12a + 3b$

Thus, $\boxed{c = 36}$ Notice that $a^2 - 12a + 36 = (a-6)^2$

Answer 6PQ.

Let us consider the equation $x^2 - 5x - 6 = 0$

Step1: Rewrite the consider related function $f(x) = x^2 - 5x - 6$

Now we construct the table for the function $f(x)$

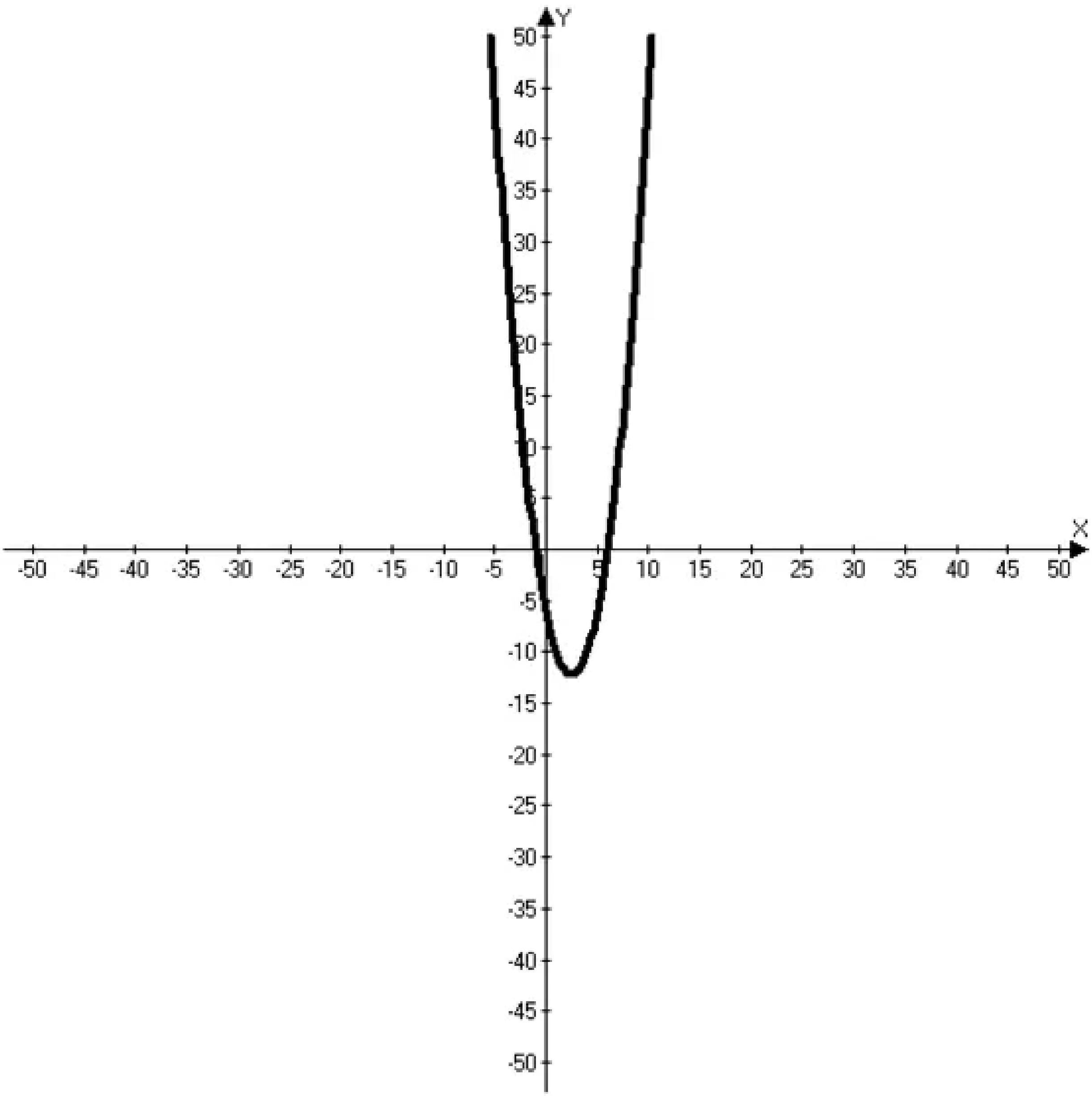
x	$f(x) = x^2 - 5x - 6$
-4	$f(-4) = (-4)^2 - 5(-4) - 6 = 30$
-3	$f(-3) = (-3)^2 - 5(-3) - 6 = 18$
-2	$f(-2) = (-2)^2 - 5(-2) - 6 = 8$
-1	$f(-1) = (-1)^2 - 5(-1) - 6 = 0$
0	$f(0) = (0)^2 - 5(0) - 6 = -6$
1	$f(1) = (1)^2 - 5(1) - 6 = -10$
2	$f(2) = (2)^2 - 5(2) - 6 = -12$
3	$f(3) = (3)^2 - 5(3) - 6 = -12$
4	$f(4) = (4)^2 - 5(4) - 6 = -10$
5	$f(5) = (5)^2 - 5(5) - 6 = -6$
6	$f(6) = (6)^2 - 5(6) - 6 = 0$

We use the rule that, '*a*' is a solution of the equation. If $f(a) = 0$. Graphically, $f(a)$ intersect *x* – axis at *x* = *a*, then '*a*' is solution of $f(a)$.

From the table, $f(-1) = 0, f(6) = 0$

-1 and 6 are the roots of $f(a) = 0$ in the graph $f(x)$ intersect the *x* – axis at $x = -1, x = 6$

The roots of the equation $\{-1, 6\}$



Answer 7CU.

Consider the trinomial t^2+5t+c

Claim: To find the value of c that makes the trinomial t^2+5t+c is a perfect square

Step-1:- Find $\frac{1}{2}$ of 5*i.e.* $\frac{5}{2}$

Step-2: Square the result of step 1 *i.e.* $\left(\frac{5}{2}\right)^2=\frac{25}{4}$

Step 3:- Add the result step 2 to t^2+5t *i.e.* $t^2+5t+\frac{25}{4}$

Thus, $\boxed{c=\frac{25}{4}}$ Notice that $t^2+5t+\frac{25}{4}=\left(t+\frac{5}{2}\right)^2$

Answer 7PQ.

Consider the equation $s^2+8s=-15$

Claim: To solve the equation $s^2+8s=-15$ by completing the square.

Step1: Write the equation $s^2+8s=-15$ as a perfect square form

$$s^2+8s=-15 \quad \text{(Original equation)}$$
$$s^2+2\cdot s\cdot 4=-15 \quad (\text{Factors:}8s=2\cdot s\cdot 4)$$
$$s^2+2\cdot s\cdot 4+4^2=-15+4^2 \quad (\text{Add } 4^2 \text{ on both sides})$$
$$s^2+2\cdot s\cdot 4+4^2=-15+16$$
$$s^2+2\cdot s\cdot 4+4^2=1$$
$$(s+4)^2=1 \quad \left(\text{Factors:}s^2+2\cdot s\cdot 4+4^2=(s+4)^2\right)$$

Step2: Now solve the equation $(s+4)^2=1$ by facing the square root of each side, we obtain the ‘*s* ’values

$$(s+4)^2=1 \quad \text{(Original equation)}$$
$$\sqrt{(s+4)^2}=\sqrt{1} \quad \text{(Taking the square root of each side)}$$
$$s+4=\pm 1 \quad \text{(Cancellation of the square root and square)}$$
$$s+4-4=\pm 1-4 \quad \text{(Substract 4 on both sides)}$$

$$s=\pm 1-4$$
$$s=1-4 \quad \text{or } s=-1-4$$
$$s=-3 \quad \text{or } s=-5$$

Step3: Substitute each value of 's' is the original equation $s^2+8s=-15$

$$s^2+8s=-15 \qquad \text{(Original equation)}$$

$$(-3)^2+8(-3)\overset{?}{=}-15 \qquad \text{(Replace } s \text{ by } -3\text{)}$$

$$9-24\overset{?}{=}-15$$

$$-15=-15 \quad \text{True}$$

$$s^2+8s=-15 \qquad \text{(Original equation)}$$

$$(-5)^2+8(-5)\overset{?}{=}-15 \qquad \text{(Replace } s \text{ by } -5\text{)}$$

$$25-40\overset{?}{=}-15$$

$$-15=-15 \quad \text{True}$$

Therefore $s = -3$ and $s = -5$ satisfies the equation $s^2+8s=-15$

Hence, the solution set is $\boxed{\{-3,-5\}}$

Answer 8CU.

Consider the trinomial $c^2-6c=7$

Claim Solve the equation $c^2-6c=7$ by completing the square.

Step 1: Rewrite the equation $c^2-6c=7$ as a completing the square

$$c^2-6c=7 \qquad \text{[original equation]}$$

$$c^2-2\cdot c\cdot 3=7 \qquad [\text{Write } -6c \text{ as } -2\cdot c\cdot 3]$$

$$c^2-2\cdot c\cdot 3+3^2=7+3^3 \qquad [\text{Add to each side by } 3^2]$$

$$c^2-2\cdot c\cdot 3+3^2=7+9$$

$$(x-3)^2=16 \qquad [\text{Factors } c^2-2\cdot c\cdot 3+3 \text{ as } (c-3)^2]$$

Step 2: Now, solve the equation $(x-3)^2 = 16$ by taking the square root of each side. We obtain 'c' values $(x-3)^2 = 16$

$$c-3=\sqrt{16} \quad \text{[Taking the square root of each side]}$$
$$c-3=\pm 4$$
$$c-3+3=3\pm 4 \quad \text{[Add 3 to each side]}$$
$$c=3\pm 4$$
$$c=3+4 \text{ or } c=3-4$$
$$c=7 \quad \text{or} c=-1$$

Step3: Substitute each value of c in the original equation $c^2-6c=7$

$$c^2-6c=7 \text{ [original equation]}$$
$$(7)^2-6(7)\overset{?}{=}7 \text{ [Replace } c \text{ by 7]}$$
$$49-42\overset{?}{=}7$$
$$7=7 \text{ True}$$
$$c^2-6c=7 \quad \text{[original equation]}$$
$$(-1)^2-6(-1)\overset{?}{=}7 \quad \text{[Replace } c \text{ by } -1\text{]}$$
$$1+6\overset{?}{=}7$$
$$7=7 \text{ True}$$

Therefore, $c=7$, and $c=-1$ satisfies the original equation $c^2-6c=7$

Hence, the solution set is $\boxed{\{-1,7\}}$

Answer 8PQ.

Consider the equation $a^2-10a=-24$

Claim: To solve the equation $a^2-10a=-24$ by completing the square.

Step1: Write the equation $a^2 - 10a = -24$ as a perfect square form

$$a^2 - 10a = -24 \quad \text{(Original equation)}$$

$$a^2 - 2 \cdot a \cdot 5 = -24 \quad (\text{Factors:} 10a = 2 \cdot a \cdot 5)$$

$$a^2 - 2 \cdot a \cdot 5 + 5^2 = -24 + 5^2 \quad (\text{Add } 5^2 \text{ on both sides})$$

$$a^2 - 2 \cdot a \cdot 5 + 5^2 = -24 + 25$$

$$(a-5)^2 = -24 + 25 \quad \left(\begin{array}{l}\text{Factors:} a^2 - 2 \cdot a \cdot 5 + 5^2 = (a-5)^2 \\ \text{and } 5^2 = 25\end{array}\right)$$

$$(a-5)^2 = 1 \quad \text{(Combine like terms)}$$

Step2: Now solve the equation $(a-5)^2 = 1$ by facing the square root of each side, we obtain the '*a* 'values

$$(a-5)^2 = 1 \quad \text{(Original equation)}$$

$$\sqrt{(a-5)^2} = \sqrt{1} \quad \text{(Taking the square root of each side)}$$

$$a - 5 = \pm 1 \quad \text{(Cancellation of the square root and square)}$$

$$a - 5 + 5 = \pm 1 + 5 \quad \text{(Add 5 on both sides)}$$

$$a = \pm 1 + 5$$

$$a = 1 + 5 \quad \text{or} \quad a = -1 + 5$$

$$a = 6 \quad \text{or} \quad a = 4$$

Step3: Substitute each value of '*a*' is the original equation $a^2 - 10a = -24$

$$a^2 - 10a = -24 \quad \text{(Original equation)}$$

$$(6)^2 - 10(6) \stackrel{?}{=} -24 \quad (\text{Replace } a \text{ by } 6)$$

$$36 - 60 \stackrel{?}{=} -24$$

$$-24 = -24 \quad \text{True}$$

$$a^2-10a=-24 \qquad \text{(Original equation)}$$

$$(4)^2-10(4)\stackrel{?}{=}-24 \qquad \text{(Replace } a \text{ by 4)}$$

$$16-40\stackrel{?}{=}-24$$

$$-24=-24 \quad \text{True}$$

Therefore a = 6 and a =4 satisfies the equation $a^2-10a=-24$

Hence, the solution set is $\boxed{\{6,4\}}$

Answer 9CU.

Consider the trinomial $x^2+7x=-12$

Claim Solve the equation $x^2+7x=-12$ by completing the square.

Step 1 : Rewrite the equation $x^2+7x=-12$ as a completing the square

$$x^2+7x=-12 \qquad [\text{original equation}]$$

$$x^2+1\cdot 7\cdot x=-12$$

$$x^2+\frac{2}{2}\cdot 7\cdot x=-12$$

$$x^2+2\cdot x\cdot\frac{7}{2}=-12$$

$$x^2+2\cdot x\cdot\frac{7}{2}+\left(\frac{7}{2}\right)^2=-12+\left(\frac{7}{2}\right)^2$$

$$\left(x+\frac{7}{2}\right)^2=-12+\frac{49}{4}$$

$$\left(x+\frac{7}{2}\right)^2=\frac{-48+48}{4}$$

$$\left(x+\frac{7}{2}\right)^2=\frac{1}{4}$$

$$\left(x+\frac{7}{2}\right)^2=0.25$$

Step 2: Now, solve the equation $\left(x+\frac{7}{2}\right)^2=0.25$ by taking the square root of each side. We obtain '*x*' values

$$\left(x+\frac{7}{2}\right)^2=0.25$$

$$\sqrt{\left(x+\frac{7}{2}\right)^2}=\sqrt{0.25} \qquad \left[\text{Taking the square root of each side}\right]$$

$$x+\frac{7}{2}=\pm 0.5 \qquad \left[\text{Since } \sqrt{0.25}=0.5\right]$$

$$x+3.5=\pm 0.5 \qquad \left[\text{Write } \frac{7}{2} \text{ as } 3.5\right]$$

$$x+3.5-3.5=-3.5\pm 0.5$$

$$x=-3.5\pm 0.5$$

$$x=-3.5+0.5 \text{ or } x=-3.5-0.5$$

$$x=-3 \qquad \text{or } x=-4$$

Therefore $x=-3$ or $x=-4$

Step3: Substitute each value of *x* in the original equation $x^2+7x=-12$

$$x^2+7x=-12 \quad \left[\text{original equation}\right]$$

$$(-3)^2+7(-3)\stackrel{?}{=}-12 \quad \left[\text{Replace } x \text{ by } -3\right]$$

$$9-21\stackrel{?}{=}-12$$

$$-12=-12 \text{ True}$$

$$x^2+7x=-12 \quad \left[\text{original equation}\right]$$

$$(-4)^2+7(-4)\stackrel{?}{=}-12 \quad \left[\text{Replace } x \text{ by } -4\right]$$

$$16-28\stackrel{?}{=}-12$$

$$-12=-12 \text{ True}$$

Therefore, $x=-3$, and $x=-4$ satisfies the original equation $x^2+7x=-12$

Hence, the solution set is $\boxed{\{-3,-4\}}$

Answer 9PQ.

Consider the equation $y^2 - 14y + 49 = 5$

Claim: To solve the equation $y^2 - 14y + 49 = 5$ by completing the square.

Step1: Write the equation $y^2 - 14y + 49 = 5$ as a perfect square form

$$y^2 - 14y + 49 = 5 \quad \text{(Original equation)}$$
$$y^2 - 14y + 49 - 49 = 5 - 49 \quad \text{(Sabstract 49 on both sides)}$$
$$y^2 - 14y = -44$$
$$y^2 - 2\cdot y\cdot 7 = -44 \quad (\text{Factors:} 14y = 2\cdot y\cdot 7)$$
$$y^2 - 2\cdot y\cdot 7 + 7^2 = -44 + 7^2 \quad (\text{Add } 7^2 \text{ on both sides})$$
$$y^2 - 2\cdot y\cdot 7 + 7^2 = -44 + 49$$
$$(y-7)^2 = 5 \quad \left(\text{Factorts:} y^2 - 2\cdot y\cdot 7 + 7^2 = (y-7)^2\right)$$

Step2: Now solve the equation $(y-7)^2 = 5$ by facing the square root of each side, we obtain the '*y*' values

$$(y-7)^2 = 5 \quad \text{(Original equation)}$$
$$\sqrt{(y-7)^2} = \sqrt{5} \quad \text{(Taking the square root of each side)}$$
$$y - 7 = \sqrt{(2.23)^2} \quad \left(\sqrt{5} \approx \sqrt{(2.23)^2}\right)$$
$$y - 7 = \pm 2.23$$
$$y - 7 + 7 = \pm 2.23 + 7 \quad \text{(Add 7 on both sides)}$$
$$y = \pm 2.23 + 7$$
$$y = 2.23 + 7 \quad \text{or} \quad y = -2.23 + 7$$
$$y = 9.23 \quad \text{or} \quad y = 4.77$$

Step3: Substitute each value of '*y*' is the original equation $y^2 - 14y + 49 = 5$

$$^2 - 14y + 49 = 5 \quad \text{(Original equation)}$$
$$(9.23)^2 - 14(9.23) + 49 \stackrel{?}{=} 5 \quad \text{(Replace } y \text{ by } 9.23)$$
$$85.19 - 129.22 + 49 \stackrel{?}{=} 5$$
$$4.97 \approx 5 \quad \text{True}$$

$$y^2-14y+49=5 \quad \text{(Original equation)}$$

$$(4.77)^2-14(4.77)+49 \stackrel{?}{=} 5 \quad \text{(Replace } y \text{ by } 4.77\text{)}$$

$$71.75-66.78+49 \stackrel{?}{=} 5$$

$$4.97 \approx 5 \quad \text{True}$$

Therefore *y* = 9.23 and *y* = 4.97 satisfies the equation $y^2-14y+49=5$

Hence, the solution set is $\boxed{\{9.23, 4.97\}}$

Answer 10CU.

Consider the trinomial $v^2+14v-9=6$

Claim Solve the equation $v^2+14v-9=6$ by completing the square.

Step 1 : Rewrite the equation $v^2+14v-9=6$ as a completing the square

$$v^2+14v-9=6 \quad [\text{original equation}]$$

$$v^2+14v-9+9=6+9 \quad [\text{Add to each side by } 6]$$

$$v^2+14v=15$$

$$v^2+2\cdot v\cdot 7=15 \quad [\text{Write } 14v \text{ as } 2\cdot v\cdot 7]$$

$$v^2+2\cdot v\cdot 7+7^2=15+7^2 \quad [\text{Add to each side by } 7^2]$$

$$(v+7)^2=15+49 \quad [\text{Factors } v^2+2\cdot v\cdot 7+49 \text{ as } (v+7)^2]$$

$$(v+7)^2=64$$

Step 2: Now, solve the equation $(v+7)^2=64$ by taking the square root of each side. We obtain '*v*' values

$$(v+7)^2=64$$

$$\sqrt{(v+7)^2}=\sqrt{64} \quad [\text{Taking the square root of each side}]$$

$$v+7=\pm 8 \quad [\text{Since } \sqrt{64}=\pm 8]$$

$$v+7=-7\pm 8 \quad [\text{Substract each side by } 7]$$

$$v=-7+8 \text{ or } v=-7-8$$

$$v=1 \quad \text{or } v=-15$$

<u>Step3:</u> Substitute each value of x in the original equation $x^2+7x=-12$

$$x^2+7x=-12 \quad [\text{original equation}]$$

$$(-3)^2+7(-3)\overset{?}{=}-12 \quad [\text{Replace } x \text{ by } -3]$$

$$9-21\overset{?}{=}-12$$

$$-12=-12 \text{ True}$$

$$x^2+7x=-12 \quad [\text{original equation}]$$

$$(-4)^2+7(-4)\overset{?}{=}-12 \quad [\text{Replace } x \text{ by } -4]$$

$$16-28\overset{?}{=}-12$$

$$-12=-12 \text{ True}$$

Therefore $v=1$ and $v=-15$. Satisfies the original equation $v^2+14v-9=6$

Hence, the solution set is $\boxed{\{-15,1\}}$

Answer 10PQ.

Consider the equation $2b^2 - b - 7 = 14$

Claim: To solve the equation $2b^2 - b - 7 = 14$ by completing the square.

Step1: Write the equation $2b^2 - b - 7 = 14$ as a perfect square form

$$2b^2 - b - 7 = 14 \qquad \text{(Original equation)}$$

$$2b^2 - b - 7 + 7 = 14 + 7 \qquad \text{(Add 7 on both sides)}$$

$$2b^2 - b = 21$$

$$\frac{1}{2}\left(2b^2 - b\right) = \frac{1}{2}\cdot 21 \qquad \text{(Divided by 2 on both sides)}$$

$$\frac{1}{2}\cdot 2\cdot b^2 - \frac{1}{2}\cdot b = \frac{1}{2}\cdot 21$$

$$b^2 - \frac{1}{2}\cdot b = \frac{21}{2}$$

$$b^2 - 1\cdot\frac{1}{2}\cdot b = \frac{21}{2}$$

$$b^2 - \frac{2}{2}\cdot\frac{1}{2}\cdot b = \frac{21}{2} \qquad \left(\text{Replace 1 by } \frac{2}{2}\right)$$

$$b^2 - 2\cdot b\cdot\frac{1}{4} = \frac{21}{2}$$

$$b^2 - 2\cdot b\cdot\frac{1}{4} + \left(\frac{1}{4}\right)^2 = \frac{21}{2} + \left(\frac{1}{4}\right)^2 \qquad \left(\text{Add } \left(\frac{1}{4}\right)^2 \text{ on both sides}\right)$$

$$\left(b - \frac{1}{4}\right)^2 = \frac{21}{2} + \frac{1}{16} \qquad \left(\text{Factors:} b^2 - 2\cdot b\cdot\frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(b - \frac{1}{4}\right)^2\right)$$

$$\left(b - \frac{1}{4}\right)^2 = \frac{21\cdot 8 + 1}{2} \qquad \left(\begin{array}{l}\text{keep as common denominator}\\ \text{and combine numerator}\end{array}\right)$$

$$\left(b - \frac{1}{4}\right)^2 = \frac{168 + 1}{2}$$

$$\left(b - \frac{1}{4}\right)^2 = \frac{169}{2}$$

Step2: Now solve the equation $\left(b-\frac{1}{4}\right)^2=\frac{169}{2}$ by facing the square root of each side, we obtain the 'b 'values

$$\left(b-\frac{1}{4}\right)^2=\frac{169}{16} \qquad \text{(Original equation)}$$

$$\sqrt{\left(b-\frac{1}{4}\right)^2}=\sqrt{\frac{169}{16}} \qquad \text{(Taking the square root of each side)}$$

$$b-\frac{1}{4}=\sqrt{\frac{13^2}{4^2}} \qquad \begin{pmatrix}169=13^2\\16=4^2\end{pmatrix}$$

$$b-\frac{1}{4}=\sqrt{\left(\frac{13}{4}\right)} \qquad \text{(Cancellation of the square root and square)}$$

$$b-\frac{1}{4}=\pm\frac{13}{4}$$

$$b-\frac{1}{4}+\frac{1}{4}=\pm\frac{13}{4}+\frac{1}{4} \qquad \left(\text{Add } \frac{1}{4} \text{ on both sides}\right)$$

$$b=\pm\frac{13}{4}+\frac{1}{4}$$

$$b=\frac{13}{4}+\frac{1}{4} \quad \text{or} \quad b=-\frac{13}{4}+\frac{1}{4}$$

$$b=\frac{13+1}{4} \quad \text{or} \quad b=\frac{-13+1}{4}$$

$$b=\frac{14}{4} \quad \text{or} \quad b=\frac{-12}{4}$$

$$b=\frac{7\cdot 2}{4} \quad \text{or} \quad b=\frac{-4\cdot 3}{4}$$

$$b=\frac{7}{2} \quad \text{or} \quad b=-3$$

Hence, the solution set is $\boxed{\left\{\frac{7}{2},-3\right\}}$

Answer 11CU.

Consider the trinomial $r^2 - 4r = 2$

Claim Solve the equation $r^2 - 4r = 2$ by completing the square.

Step 1 : Rewrite the equation $r^2 - 4r = 2$ as a completing the square

$$r^2 - 4r = 2 \quad [\text{original equation}]$$

$$r^2 - 2 \cdot r \cdot 2 = 2 \quad [\text{write } 4r \text{ as } 2 \cdot r \cdot 2]$$

$$r^2 - 2 \cdot r \cdot 2 + 2^2 = 2 + 2^2 \quad \left[\text{Add to each side by } 2^2\right]$$

$$(r-2)^2 = 2 + 4$$

$$(r-2)^2 = 6$$

Step 2: Now, solve the equation $(r-2)^2 = 6$ by taking the square root of each side. We obtain 'r' values

$$(r-2)^2 = 6$$

$$\sqrt{(r-2)^2} = \sqrt{6} \quad [\text{Taking the square root of each side}]$$

$$r - 2 \approx \pm 2.4 \quad \left[\text{Since } \sqrt{6} \approx 2.4\right]$$

$$r - 2 + 2 \approx 2 \pm 2.4 \quad [\text{Add each side by } 2]$$

$$r \approx 2 + 2.4 \text{ or } r \approx 2 - 2.4$$

$$r \approx 4.4 \quad \text{or} \quad r \approx -0.4$$

Therefore, $r \approx 4.4$ or $r \approx -0.4$

Hence, the solution set is $\boxed{\{-0.4, 4.4\}}$

Answer 12CU.

Consider the trinomial $a^2-24a+9=0$

Claim Solve the equation $a^2-24a+9=0$ by completing the square.

Step 1 : Rewrite the equation $a^2-24a+9=0$ as a completing the square

$$a^2-24a+9=0 \quad \text{[original equation]}$$

$$a^2-24a+9-9=0-9 \quad \text{[Substract to each side by 9]}$$

$$a^2-24a=-9$$

$$a^2-2\cdot a\cdot 12=-9 \quad [\text{Write } 24a \text{ as } 2\cdot a\cdot 12]$$

$$a^2-2\cdot a\cdot 12+(12)^2=-9+(12)^2 \quad [\text{Add to each side by } 12^2]$$

$$a^2-2\cdot a\cdot 12+(12)^2=-6+144$$

$$(a-12)^2=135 \quad \left[\text{Factors } a^2-2\cdot a\cdot 12+(12)^2 \text{ as } (a-12)^2\right]$$

Step 2: Now, solve the equation $(a-12)^2=135$ by taking the square root of each side. We obtain 'a' values

$$(a-12)^2=135$$

$$\sqrt{(a-12)^2}=\sqrt{135} \quad \text{[Taking the square root of each side]}$$

$$a-12\approx\pm 11.6$$

$$a-12\pm 12\approx 12\pm 11.6$$

$$a=12\pm 11.6$$

$$a=12+11.6 \text{ or } a=12-11.6$$

$$a=23.6 \quad \text{or } a=0.4$$

Step 3: Substitute each value of a in the original equation $a^2-24a+9=0$

$$a^2-24a+9=0$$

$$(23.6)^2-24(23.6)+9\overset{?}{=}0 \quad [\text{Replace } a \text{ by } 23.6]$$

$$55.7-566+9\overset{?}{=}0$$

$$566-566=0 \text{ True}$$

$$a^2-24a+9=0$$

$$(0.4)^2-24(0.4)+9\overset{?}{=}0 \qquad [\text{Replace } a \text{ by } 0.4]$$

$$0.16-9.6+9\overset{?}{=}0$$

$$9.16-9.6=0 \text{ True}$$

Therefore, $a\approx 23.6$ and $a=0.4$ satisfies the original equation $a^2-24a+9=0$

Hence, the solution set is $\boxed{\{0.4, 23.6\}}$

Answer 13CU.

Consider the trinomial $2p^2-5p+8=7$

Claim Solve the equation $2p^2-5p+8=7$ by completing the square.

Step 1 : Rewrite the equation $2p^2-5p+8=7$ as a completing the square

$$2p^2-5p+8=7$$

$$\frac{2p^2-5p+8=7}{2}=\frac{7}{2} \qquad [\text{divide each side by } 2]$$

$$p^2-\frac{5}{2}p+4=3.5 \qquad [\text{Simplification}]$$

$$p^2-\frac{5}{2}p+4-4=3.5-4 \qquad [\text{Substract each side by } 4]$$

$$p^2-\frac{5}{2}p=-0.5$$

$$p^2-1\cdot\frac{5}{2}p=-0.5$$

$$p^2-\frac{2}{2}\cdot\frac{5}{2}p=-0.5$$

$$p^2-2\cdot p\cdot\frac{5}{4}=-0.5$$

$$p^2-2\cdot p\frac{5}{4}+\left(\frac{5}{4}\right)^2=-0.5+\left(\frac{5}{4}\right)^2$$

$$\left(p-\frac{5}{4}\right)^2=-0.5+\frac{25}{16} \qquad \left[\text{Factors } p^2-2\cdot p\cdot\frac{5}{4}+\left(\frac{5}{4}\right)^2\right]$$

$$\left(p-\frac{5}{4}\right)^2=\frac{-8+25}{16} \qquad \left[\text{as } \left(p-\frac{5}{4}\right)^2\right]$$

$$\left(p-\frac{5}{4}\right)^2=\frac{17}{16}$$

Step 2: Now, solve the equation $\left(p-\frac{5}{4}\right)^2=\frac{17}{16}$ by taking the square root of each side. We obtain '*p*' values

$$\left(p-\frac{5}{4}\right)^2=\frac{17}{16}$$

$$\sqrt{\left(p-\frac{5}{4}\right)^2}=\sqrt{\frac{17}{16}} \qquad \left[\text{Taking the square root of each side}\right]$$

$$p-\frac{5}{4}=\pm 1 \qquad \left[\text{Since } \sqrt{\frac{17}{16}}\approx 1\right]$$

$$p-1.25=\pm 1$$

$$p-1.25+1.25=1.25\pm 1$$

$$p=1.25\pm 1$$

$$p=1.25+1 \text{ or } p=1.25-1$$

$$p=2.25 \quad \text{or } p=0.25$$

$$p\approx 2.2 \quad \text{or } p\approx 0.2$$

Step 3: Substitute each value of *p* in the original equation $2p^2-5p+8=7$

$$2p^2-5p+8=7$$

$$2(2.2)^2-5(2.2)+8\stackrel{?}{=}0 \qquad \left[\text{Replace } p \text{ by } (2.2)\right]$$

$$10-11+8\stackrel{?}{=}7$$

$$18-11=7$$

$$7=7 \text{ True}$$

$$2p^2-5p+8=7$$

$$(0.2)^2-24(0.2)+9\stackrel{?}{=}7 \qquad \left[\text{Replace } p \text{ by } 0.2\right]$$

$$0.01-1+8\stackrel{?}{=}7$$

$$-1+8=7$$

$$7=7 \text{ True}$$

Therefore, $p=2.2$ and $p=0.2$ satisfies the original equation $2p^2-5p+8=7$

Hence, the solution set is $\boxed{\{0.2, 2.2\}}$

Answer 14CU.

Let us consider a square having side the formula 'x'

Area of the square $= \text{side} \times \text{side} = x \cdot x = x^2$

According to the problem, the given length of the square increased by 6 inches and width by 4 inches.

And given the area of newly constructed outer rectangle is equal to the two time of the area of the given square.

__Claim:-__ To find the sides of the given square.

Length of the outer rectangle $= x+6$ inches (increased by 6 inches)

Width of the outer rectangle $= x+4$ inches (increased by 4 inches)

Area of the outer rectangle

$$= \text{length} \cdot \text{width}$$
$$= (x+6)\cdot(x+4)$$
$$= x\cdot(x+4)\cdot(x+4) \qquad [\text{Distributive law on addition}]$$
$$= x\cdot x + x\cdot 4 + 6\cdot x + 6\cdot 4 \qquad [\text{Distributive law on addition}]$$
$$= x^2 + 4x + 6x + 24 \qquad [\text{like terms additon}]$$

According to the problem area of the outer rectangle is equal to the area of the given square.

i.e. $x^2 + 10x + 24 = 2x^2$

$$-\cancel{x}\cdot\cancel{x} + 10x + 24 = 2x^2 \qquad [\text{Substract } x^2 \text{on both sides}]$$
$$10x + 24 = x^2 \qquad [\text{Addition} -10x - 24 \,\text{on both side}]$$
$$\cancel{-10x} - 24 \cancel{+10x} + 24 = x^2 - 10x - 24$$
$$0 = x^2 - 10x - 24 \qquad [\text{Substract like terms}]$$
$$x^2 - 10x - 24 = 0$$
$$x^2 - 12x + 2x - 24 = 0 \qquad [-10x \text{can be written as} = 12x + 2x]$$
$$x\cdot x - 12\cdot x + 2\cdot x - 2\cdot 12 = 0$$
$$x(x-12) + 2(x-12) = 0 \qquad [\text{Distributive law}]$$
$$(x-12)(x+2) = 0 \qquad [\text{Distributive}]$$
$$x - 12 = 0 \quad \& \quad x + 2 = 0$$
$$x = 12 \;\&\; x = -2$$

Length of the square will not be negative.

So we neglect $x = -2$

Let us take the side of square =12

The side of the given square =12 inches.

Verification: -

According to the problem given condition

Area of the outer rectangle $= 2x + \text{Area of the square}$

Length of the outer rectangle $= x + 6 = 12 + 6 = 18\,\text{inches}$

Width of the outer rectangle $= x + 4 = 12 + 4 = 16\,\text{inches}$

Area of the outer rectangle $= \text{length} \cdot \text{width}$

$$= 18 \cdot 16 = 288 \text{ inches}$$

$$\text{Area of the square} = x^2 = 12^2 144$$

$$2 \times \text{Area of the square} = 2 \times 144 = 288$$

$$\text{Area of theouter rectangle} = 288$$

$$= 2 \times \text{Area of the square}$$

Hence verified

Answer 15PA.

Consider the trinomial $b^2 - 4b + 4 = 16$

Claim Solve the equation $b^2 - 4b + 4 = 16$ by completing the square.

Step 1 : Rewrite the equation $b^2 - 4b + 4 = 16$ as a completing the square

$$b^2 - 4b + 4 = 16 \qquad [\text{original equation}]$$

$$b^2 - 2 \cdot b \cdot 2 + 4 = 16 \qquad [\text{Write } 4b \text{ as } 2 \cdot b \cdot 2]$$

$$(b-2)^2 = 16 \qquad \left[\text{factors } b^2 - 2 \cdot b \cdot 2 + 2^2 \text{ as } (b-2)^2\right]$$

Step 2: Now, solve the equation $(b-2)^2 = 16$ by taking the square root of each side. We obtain '*b*' values

$$(b-2)^2 = 16$$

$$\sqrt{(b-2)^2} = \sqrt{16} \qquad [\text{Taking the square root of each side}]$$

$$b - 2 = \pm 4 \qquad [\text{Add to each side by } 2]$$

$$b = 2 \pm 4$$

$$b = 2 + 4 \text{ or } b = 2 - 4$$

$$b = 6 \quad \text{or } b = -2$$

Therefore, $b = 6$ or $b = -2$

Step 3: Substitute each value of *b* in the original equation $b^2 - 4b + 4 = 16$

$$b^2 - 4b + 4 = 16$$

$$b^2 - 4b + 4 \overset{?}{=} 16 \qquad [\text{Replace } b \text{ by } 6]$$

$$36 - 24 + 4 \overset{?}{=} 16$$

$$40 - 24 = 16$$

$$16 \overset{?}{=} 16 \text{ True}$$

$$b^2-4b+4=16$$

$$(-2)^2-4(-2)+4\overset{?}{=}7 \quad [\text{Replace } b \text{ by } -2]$$

$$4+8+4\overset{?}{=}16$$

$$12+4\overset{?}{=}16$$

$$16=16 \text{ True}$$

Therefore, $b=6$ and $b=-2$ satisfies the original equation $b^2-4b+4=16$

Hence, the solution set is $\boxed{\{-2,6\}}$

Answer 16PA.

Consider the trinomial $t^2+2t+1=25$

Claim Solve the equation $t^2+2t+1=25$ by completing the square.

Step 1 : Rewrite the equation $t^2+2t+1=25$ as a completing the square

$$t^2+2t+1=25 \qquad [\text{original equation}]$$

$$t^2+2\cdot t\cdot 1+1=25 \qquad [\text{Write } 2t \text{ as } 2\cdot t\cdot 1]$$

$$(t+1)^2=25 \qquad \left[\text{factors } t^2+2\cdot t\cdot 1+1^2 \text{ as } (t+1)^2\right]$$

Step 2: Now, solve the equation $(t+1)^2=25$ by taking the square root of each side. We obtain 't' values

$$(t+1)^2=25$$

$$\sqrt{(t+1)^2}=\sqrt{25} \qquad [\text{Taking the square root of each side}]$$

$$t+1=\pm 5$$

$$t+1-1=-1\pm 5 \qquad [\text{Substract to each side by 1}]$$

$$t=-1+5 \text{ or } t=-1-5$$

$$t=4 \qquad \text{or } t=-6$$

Therefore, $t=4$ or $t=-6$

Step 3: Substitute each value of *b* in the original equation $t^2+2t+1=25$

$$t^2+2t+1=25$$
$$(4)^2+2(4)+1\overset{?}{=}25 \qquad [\text{Replace } t \text{ by } 4]$$
$$16+8+1\overset{?}{=}25$$
$$25=25 \text{ True}$$

$$t^2+2t+1=25$$
$$(-6)^2+2(-6)+1\overset{?}{=}25 \qquad [\text{Replace } t \text{ by } -6]$$
$$36-12+1\overset{?}{=}25$$
$$25=25 \text{ True}$$

Therefore, $t=-6$ and $t=4$ satisfies the original equation $t^2+2t+1=25$

Hence, the solution set is $\boxed{\{-6,4\}}$

Answer 17PA.

Consider the trinomial $g^2-8g+16=2$

Claim Solve the equation $g^2-8g+16=2$ by completing the square.

Step 1: Rewrite the equation $g^2-8g+16=2$ as a completing the square

$$g^2-8g+16=2 \qquad [\text{original equation}]$$
$$g^2-2\cdot g\cdot 4+4^2=2 \qquad [\text{Write } 8g \text{ as } 2\cdot g\cdot 4]$$
$$(g-4)^2=2 \qquad \left[\text{factors } g^2-2\cdot g\cdot 4+4^2 \text{as } (g-4)^2\right]$$

Step 2: Now, solve the equation $(g-4)^2=2$ by taking the square root of each side. We obtain '*g*' values

$$(g-4)^2=2$$
$$\sqrt{(g-4)^2}=\sqrt{2} \qquad [\text{Taking the square root of each side}]$$
$$g-4\approx\pm1.4 \qquad \left[\sqrt{2}\approx1.4\right]$$
$$g-4+4\approx4\pm1.4 \qquad [\text{Add to each side by } 4]$$
$$g=4+1.4 \text{ or } g=4-1.4$$
$$g=5.4 \quad \text{or} \quad g=2.6$$

Therefore, $g=5.4$ or $g=2.6$

Step 3: Substitute each value of g in the original equation $g^2 - 8g + 16 = 2$

$$g^2 - 8g + 16 = 2$$

$$(5.4)^2 - 8(5.4) + 16 \stackrel{?}{=} 2 \qquad [\text{Replace } g \text{ by } 5.4]$$

$$46 - 44 \stackrel{?}{=} 2$$

$$2 = 2 \text{ True}$$

$$g^2 - 8g + 16 = 2$$

$$(2.6)^2 - 8(2.6) + 16 \stackrel{?}{=} 2 \qquad [\text{Replace } g \text{ by } 2.6]$$

$$7 - 20 + 16 \stackrel{?}{=} 2$$

$$22 - 20 \stackrel{?}{=} 2$$

$$2 = 2 \text{ True}$$

Therefore, $g = 2.6$ and $g = 5.4$ satisfies the original equation $g^2 - 8g + 16 = 2$

Hence, the solution set is $\boxed{\{2.6, 5.4\}}$

Answer 18PA.

Consider the trinomial $y^2 - 12y + 36 = 5$

Claim Solve the equation $y^2 - 12y + 36 = 5$ by completing the square.

Step 1 : Rewrite the equation $y^2 - 12y + 36 = 5$ as a completing the square

$$y^2 - 12y + 36 = 5 \qquad [\text{original equation}]$$

$$y^2 - 2 \cdot y \cdot 6 + 6^2 = 5 \qquad [\text{Write } 12y \text{ as } 2 \cdot y \cdot 6]$$

$$(y-6)^2 = 5 \qquad \left[\text{factors } y^2 - 2 \cdot y \cdot 6 + 6^2 \text{ as } (y-6)^2\right]$$

Step 2: Now, solve the equation $(y-6)^2 = 5$ by taking the square root of each side. We obtain 'y' values

$$(y-6)^2 = 5$$

$$\sqrt{(y-6)^2} = \sqrt{5} \qquad [\text{Taking the square root of each side}]$$

$$y - 6 = \pm\sqrt{5}$$

$$y - 6 + 6 = 6 \pm \sqrt{5} \qquad [\text{Add to each side by } 6]$$

$$y = 6 + \sqrt{5} \text{ or } y = 6 - \sqrt{5}$$

$$y \approx 8.2 \quad \text{or} \quad y \approx 3.8$$

Therefore, $y = 8.2$ or $y = 3.8$

Step 3: Substitute each value of *y* in the original equation $y^2-12y+36=5$

$$y^2-12y+36=5$$

$$(8.2)^2-12(8.2)+36\overset{?}{=}5 \qquad [\text{Replace } y \text{ by } 8.2]$$

$$67-98+36\overset{?}{=}5$$

$$5=5 \text{ True}$$

$$y^2-12y+36=5$$

$$(3.8)^2-12(3.8)+36\overset{?}{=}5 \qquad [\text{Replace } y \text{ by } 3.8]$$

$$14-45+36\overset{?}{=}5$$

$$5=5 \text{ True}$$

Therefore, $y=3.8$ and $y=8.2$ satisfies the original equation $y^2-12y+36=5$

Hence, the solution set is $\boxed{\{3.8,8.2\}}$

Answer 19PA.

Consider the trinomial $w^2+16w+64=18$

Claim Solve the equation $w^2+16w+64=18$ by completing the square.

Step 1: Rewrite the equation $w^2+16w+64=18$ as a completing the square

$$w^2+16w+64=18 \qquad [\text{original equation}]$$

$$w^2+2\cdot w\cdot 8+8^2=18 \qquad [\text{Write } 16w \text{ as } 2\cdot w\cdot 8]$$

$$(w+8)^2=18 \qquad \left[\text{factors } w^2+2\cdot w\cdot 8+8^2 \text{as } (w+8)^2\right]$$

Step 2: Now, solve the equation $(w+8)^2=18$ by taking the square root of each side. We obtain '*w*' values

$$(w+8)^2=18$$

$$\sqrt{(w+8)^2}=\sqrt{18} \qquad [\text{Taking the square root of each side}]$$

$$w+8=\pm\sqrt{18}$$

$$w+8-8=-8\pm\sqrt{18} \qquad [\text{Substract to each side by } 8]$$

$$w=-8\pm\sqrt{18}$$

$$w=-8+\sqrt{18} \text{ or } w=-8-\sqrt{18}$$

$$w=-3.8 \quad \text{or} \quad w=-12.2$$

Therefore, $w=-3.8$ or $w=-12.2$

Step 3: Substitute each value of *w* in the original equation $w^2+16w+64=18$

$$w^2+16w+64=18$$

$$(-3.8)^2+16(-3.8)+64\overset{?}{=}18 \qquad [\text{Replace } w \text{ by } -3.8]$$

$$14-60+64\overset{?}{=}18$$

$$78-60\overset{?}{=}18$$

$18=18$ True

$$w^2+16w+64=18$$

$$(-12.2)^2+16(-12.2)+64\overset{?}{=}18 \qquad [\text{Replace } w \text{ by } -12.2]$$

$$149-195+64\overset{?}{=}18$$

$$213-195\overset{?}{=}18$$

$18=18$ True

Therefore, $w=-12.2$ and $w=-3.8$ satisfies the original equation $w^2+16w+64=18$

Hence, the solution set is $\boxed{\{-12.2,-3.8\}}$

Answer 20PA.

Consider the trinomial $a^2+18a+81=90$

Claim Solve the equation $a^2+18a+81=90$ by completing the square.

Step 1 : Rewrite the equation $a^2+18a+81=90$ as a completing the square

$$a^2+18a+81=90 \qquad [\text{original equation}]$$

$$a^2+2\cdot a\cdot 9+9^2=90 \qquad [\text{Write } 18a \text{ as } 2\cdot a\cdot 9]$$

$$(a+9)^2=90 \qquad \left[\text{factors } a^2+2\cdot a\cdot 9+9^2 \text{as } (a+9)^2\right]$$

Step 2: Now, solve the equation $(a+9)^2=90$ by taking the square root of each side. We obtain '*a*' values

$$(a+9)^2=90$$

$$\sqrt{(a+9)^2}=\sqrt{90} \qquad [\text{Taking the square root of each side}]$$

$$a+9=\pm\sqrt{90}$$

$$a+9-9=-9\pm\sqrt{90} \qquad [\text{Substract to each side by } 9]$$

$$a=-9\pm\sqrt{90}$$

$$a=-9+\sqrt{90} \text{ or } a=-9-\sqrt{90}$$

$$a=0.5 \quad \text{or} \quad a=-18.5$$

Therefore, $a=-18.5$ or $a=0.5$

Step 3: Substitute each value of *w* in the original equation $a^2+18a+81=90$

$$a^2+18a+81=90$$

$$(-18.5)^2+18(-18.5)+81\stackrel{?}{=}90 \qquad [\text{Replace } a \text{ by } (-18.5)]$$

$$342-333+81\stackrel{?}{=}90$$

$$90\stackrel{?}{=}90 \text{ True}$$

$$a^2+18a+81=90$$

$$(0.5)^2+18(0.5)+81\stackrel{?}{=}90 \qquad [\text{Replace } a \text{ by } 0.5]$$

$$0.25+9+81\stackrel{?}{=}90$$

$$90\approx 90 \text{ True}$$

Therefore, $a=-18.5$ and $a=0.5$ satisfies the original equation $a^2+18a+81=90$

Hence, the solution set is $\boxed{\{-18.5, 0.5\}}$

Answer 21PA.

Consider the trinomial $s^2-16s+c$

Claim To find the value of *c* that makes the trinomial $s^2-16s+c$ is a perfect square.

Step 1: Find $\frac{1}{2}$ of -16 *i.e.* $-\frac{16}{2}=-8$

Step 2: Square the result of step 1 *i.e.* $(-8)^2=64$

Step 3: Add the result step 2 to s^2-16s

i.e. $s^2-16s+64$

Thus $\boxed{c=64}$ notice that $s^2-16s+64=(s-8)^2$

Answer 22PA.

Consider the trinomial $y^2-10y+c$

Claim To find the value of *c* that makes the trinomial $y^2-10y+c$ is a perfect square.

Step 1: Find $\frac{1}{2}$ of -10 *i.e.* $-\frac{10}{2}=-5$

Step 2: Square the result of step 1 *i.e.* $(-5)^2=25$

Step 3: Add the result step 2 to $s^2 - 10s$

i.e. $s^2 - 10s + 25$

Thus $\boxed{c = 25}$ notice that $s^2 - 10s + 25 = (s-5)^2$

Answer 23PA.

Consider the trinomial $w^2 + 22w + c$

Claim To find the value of *c* that makes the trinomial $w^2 + 22w + c$ is a perfect square.

Step 1 : Find $\frac{1}{2}$ of 22 *i.e.* $\frac{22}{2} = 11$

Step 2: Square the result of step 1 *i.e.* $(1)^2 = 121$

Step 3: Add the result step 2 to $w^2 + 22w$

i.e. $w^2 + 22w + 121$

Thus $\boxed{c = 121}$ notice that $w^2 + 22w + 121 = (w+1)^2$

Answer 24PA.

Consider the trinomial $a^2 + 4a + c$

Claim To find the value of *c* that makes the trinomial $a^2 + 4a + c$ is a perfect square.

Step 1 : Find $\frac{1}{2}$ of 4 *i.e.* $\frac{4}{2} = 2$

Step 2: Square the result of step 1 *i.e.* $(2)^2 = 4$

Step 3: Add the result step 2 to $a^2 + 4a$

i.e. $a^2 + 4a + 4$

Thus $\boxed{c = 4}$ notice that $a^2 + 4a + 4 = (a+2)^2$

Answer 25PA.

Consider the trinomial $p^2 - 7p + c$

Claim To find the value of *c* that makes the trinomial $p^2 - 7p + c$ is a perfect square.

Step 1 : Find $\frac{1}{2}$ of -7 *i.e.* $-\frac{7}{2} = -3.5$

Step 2: Square the result of step 1 *i.e.* $(-3.5)^2 = 12.25$

Step 3: Add the result step 2 to $p^2 - 7p$

i.e. $p^2 - 7p + 12.25$

Thus $\boxed{c = 12.25}$ notice that $p^2 - 7p + 12.25 = (p - 3.5)^2$

Answer 26PA.

Consider the trinomial $k^2 + 11k + c$

Claim To find the value of *c* that makes the trinomial $k^2 + 11k + c$ is a perfect square.

Step 1: Find $\frac{1}{2}$ of 11 *i.e.* $\frac{11}{2} = 5.5$

Step 2: Square the result of step 1 *i.e.* $(5.5)^2 = 30.25$

Step 3: Add the result step 2 to $k^2 + 11k$

i.e. $k^2 + 11k + 30.25$

Thus $\boxed{c = 30.25}$ notice that $k^2 + 11k + 30.25 = (p + 5.5)^2$

Answer 27PA.

Consider the trinomial $x^2 + cx + 81$

Claim To find the value of *c* that makes the trinomial $x^2 + cx + 81$ is a perfect square.

Step 1: Find $\frac{1}{2}$ of 81. *i.e.* $\sqrt{81} = \pm 9$

Step 2: Multiply the result of step 1 by 2*x*. *i.e.* $2x \cdot (\pm 9) = \pm 18x$

Step 3: Add the result step 2 to $x^2 + 81$

i.e. $x^2 + 81 + 81$

Thus $\boxed{c = \pm 18}$ notice that $x^2 + 81 + 81 = (x + 9)^2$ or $(x - 9)^2$

Answer 28PA.

Consider the trinomial $x^2 + cx + 144$

Claim To find the value of *c* that makes the trinomial $x^2 + cx + 144$ is a perfect square.

Step 1: To Find square root of 144. *i.e.* $\sqrt{144} = \pm 12$

Step 2: Multiply the result of step 1 by 2*x*. *i.e.* $2x \cdot (\pm 12) = \pm 24x$

Step 3: Add the result step 2 to x^2+144

i.e. $x^2 \pm 24x+144$

Thus $\boxed{c=\pm 24}$ notice that $x^2 \pm 24x+144=(x\pm 12)^2$

Answer 29PA.

Consider the trinomial $s^2-4s-12=0$

Claim To solve the equation $s^2-4s-12=0$ by completing the square.

Step 1 : Write $s^2-4s-12=0$ as a perfect square form.

$$s^2-4s-12=0 \quad [\text{original equation}]$$
$$s^2-4s-12+12=12 \quad [\text{Add 12 on each side}]$$
$$s^2-4s=12$$
$$s^2-2\cdot s\cdot 2=12 \quad [\text{Write } 4s \text{ as } 2\cdot s\cdot 2]$$
$$s^2-2\cdot s\cdot 2+2^2=12+2^2 \quad [\text{Add } 2^2 \text{ on each side}]$$
$$(s-2)^2=16 \quad [\text{Factors } s^2-2\cdot s\cdot 2+2^2=(s-2)^2]$$

Step 2: Now solve the equation $(s-2)^2=16$ by satiating the square root of each side. We obtain the *s* values.

$$(s-2)^2=16$$
$$\sqrt{(s-2)^2}=\sqrt{16} \quad [\text{Taking the square root of each side}]$$
$$s-2=\pm 4$$
$$s-2+2=2\pm 4 \quad [\text{Add 2 to each side}]$$
$$s=2+4 \quad \text{or} \quad s=2-4$$
$$s=6 \quad \text{or} \quad s=-2$$

Step 3: Substitute each value of *s* in the original equation $s^2-4s-12=0$

$$s^2-4s-12=0 \quad [\text{original equation}]$$
$$(-2)^2-4(-2)-12\overset{?}{=}0$$
$$12-12\overset{?}{=}0$$
$$0=0 \text{ True}$$

$s^2 - 4s - 12 = 0$ [original equation]

$(6)^2 - 4(6) - 12 \stackrel{?}{=} 0$

$36 - 24 - 12 \stackrel{?}{=} 0$

$36 - 36 \stackrel{?}{=} 0$

$0 = 0$ True

Therefore, $s = -2$ and $s = 6$ satisfies the equation $s^2 - 4s - 12 = 0$

Hence, the solution set is $\boxed{\{-2, 6\}}$

Answer 30PA.

Consider the equation $d^2 + 3d - 10 = 0$

Claim: To solve the equation $d^2 + 3d - 10 = 0$ by completing the square.

Step1: Write $d^2 + 3d - 10 = 0$ as a perfect square form

$d^2 + 3d - 10 = 0$ (original equation)

$d^2 + 3d - 10 + 10 = 0 + 10$ (Add 10 on both sides)

$d^2 + 3d = 10$

$d^2 + 1 \cdot 3d = 10$

$d^2 + \frac{2}{2} \cdot 3d = 10$

$d^2 + 2 \cdot \left(\frac{3}{2}\right) d = 10$

$d^2 + 2 \cdot d \cdot \left(\frac{3}{2}\right) = 10$

$d^2 + 2 \cdot d \cdot \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 = 10 + \left(\frac{3}{2}\right)^2$ $\left(\text{Add } \left(\frac{3}{2}\right)^2 \text{ on both sides}\right)$

$\left(d + \frac{3}{2}\right)^2 = 10 + \frac{9}{4}$

$\left(d + \frac{3}{2}\right)^2 = \frac{49}{4}$ $\left(\text{Factors} \left(d + \frac{3}{2}\right)^2 = d^2 + 2 \cdot d \cdot \frac{3}{2} + \left(\frac{3}{2}\right)^2\right)$

Step2: Now solve the equation $\left(d+\frac{3}{2}\right)^2=\frac{49}{4}$ by facing the square root of each side, we obtain the 'd 'values

$$\left(d+\frac{3}{2}\right)^2=\frac{49}{4} \qquad \text{(Original equation)}$$

$$\sqrt{\left(d+\frac{3}{2}\right)^2}=\sqrt{\frac{49}{4}} \qquad \text{(Taking the square root of each side)}$$

$$d+\frac{3}{2}=\pm\frac{7}{2} \qquad \left(\text{Written as } \sqrt{49}=7 \text{ and } \sqrt{4}=2\right)$$

$$d=\pm\frac{7}{2}-\frac{3}{2}$$

$$d=\frac{7}{2}-\frac{3}{2} \quad \text{or} \quad d=-\frac{7}{2}-\frac{3}{2}$$

$$d=\frac{7-3}{2} \quad \text{or} \quad d=\frac{-7-3}{2}$$

$$d=\frac{4}{2} \quad \text{or} \quad d=\frac{-10}{2}$$

$$d=2 \quad \text{or} \quad d=-5$$

Step3: Substitute each value of d is the original equation $d^2+3d-10=0$

$$d^2+3d-10=0 \qquad \text{(Original equation)}$$

$$(2)^2+3(2)-10\overset{?}{=}0 \qquad (\text{Replace } d \text{ by } 2)$$

$$4+6-10\overset{?}{=}0$$

$$10-10\overset{?}{=}0$$

$$0=0 \qquad \text{True}$$

$$d^2+3d-10=0 \qquad \text{(Original equation)}$$

$$(-5)^2+3(-5)-10\overset{?}{=}0 \qquad (\text{Replace } d \text{ by } (-5))$$

$$25-15-10\overset{?}{=}0$$

$$25-25\overset{?}{=}0$$

$$0=0 \qquad \text{True}$$

Therefore d = 2 and d = −5 satisfies the equation $d^2+3d-10=0$

Hence, The solution set is $\boxed{\{2,-5\}}$

Answer 31PA.

Consider the equation $y^2 - 19y + 4 = 70$

Claim: To solve the equation $y^2 - 19y + 4 = 70$ by completing the square.

Step1: Write $y^2 - 19y + 4 = 70$ as a perfect square form

$$y^2 - 19y + 4 = 70 \qquad \text{(original equation)}$$
$$y^2 - 19y + 4 - 4 = 70 - 4 \qquad \text{(Subtract 4 on both sides)}$$
$$y^2 - 19y = 66$$
$$y^2 - 1 \cdot 19y = 66$$
$$y^2 - \frac{2}{2} \cdot 19y = 66$$
$$y^2 - 2 \cdot y\left(\frac{19}{2}\right) = 66$$
$$y^2 - 2 \cdot y\left(\frac{19}{2}\right) + \left(\frac{19}{2}\right)^2 = 66 + \left(\frac{19}{2}\right)^2 \qquad \left(\text{Add } \left(\frac{19}{2}\right)^2 \text{ on both sides}\right)$$
$$\left(y - \frac{19}{2}\right)^2 = 66 + \frac{361}{4} \qquad \left(\begin{array}{l}\text{Factors} \\ y^2 - 2 \cdot y\left(\frac{19}{2}\right) + \left(\frac{19}{2}\right)^2 = \left(y - \frac{19}{2}\right)^2\end{array}\right)$$
$$\left(y - \frac{19}{2}\right)^2 = \frac{66 \cdot 4 + 361}{4}$$
$$\left(y - \frac{19}{2}\right)^2 = \frac{264 + 361}{4}$$
$$\left(y - \frac{19}{2}\right)^2 = \frac{625}{4}$$

Step2: Now solve the equation $\left(y-\frac{19}{2}\right)^2=\frac{625}{4}$ by facing the square root of each side, we obtain the 'y 'values

$$\left(y-\frac{19}{2}\right)^2=\frac{625}{4} \qquad \text{(Original equation)}$$

$$\sqrt{\left(y-\frac{19}{2}\right)^2}=\sqrt{\frac{625}{4}} \qquad \text{(Taking the square root of each side)}$$

$$y-\frac{19}{2}=\frac{\sqrt{(25)^2}}{\sqrt{(2)^2}} \qquad \left(\text{Written as } \sqrt{625}=\sqrt{25^2} \text{ and } \sqrt{4}=\sqrt{(2)^2}\right)$$

$$y-\frac{19}{2}=\pm\frac{25}{2}$$

$$y-\frac{19}{2}+\frac{19}{2}=\pm\frac{25}{2}+\frac{19}{2} \qquad \left(\text{Add } \frac{19}{2} \text{ on both sides}\right)$$

$$y=\pm\frac{25}{2}+\frac{19}{2}$$

$$y=\frac{25}{2}+\frac{19}{2} \quad \text{or} \quad y=-\frac{25}{2}+\frac{19}{2}$$

$$y=\frac{25+19}{2} \quad \text{or} \quad y=\frac{-25+19}{2}$$

$$y=\frac{44}{2} \quad \text{or} \quad y=\frac{-6}{2}$$

$$y=\frac{2\cdot 22}{2\cdot 1} \quad \text{or} \quad y=\frac{-3\cdot 2}{2\cdot 1}$$

$$y=22 \quad \text{or} \quad y=-3$$

Step3: Substitute each value of y is the original equation $y^2-19y+4=70$

$$y^2-19y+4=70 \qquad \text{(Original equation)}$$

$$(22)^2-19(22)+4\stackrel{?}{=}70 \qquad \text{(Replace } y \text{ by 22)}$$

$$484-418+4\stackrel{?}{=}70$$

$$488-418\stackrel{?}{=}70$$

$$70=70 \qquad \text{True}$$

$$y^2-19y+4=70 \qquad \text{(Original equation)}$$

$$(-3)^2-19(-3)+4\stackrel{?}{=}70 \qquad \text{(Replace } y \text{ by } -3)$$

$$9+57+4\stackrel{?}{=}70$$

$$57+13\stackrel{?}{=}70$$

$$70=70 \qquad \text{True}$$

Therefore y =2 2 and y = -3 satisfies the equation $y^2-19y+4=70$

Hence, the solution set is $\boxed{\{22,-3\}}$

Answer 32PA.

Consider the equation $d^2+20d+11=200$

Claim: To solve the equation $d^2+20d+11=200$ by completing the square.

Step1: Write $d^2+20d+11=200$ as a perfect square form

$$d^2+20d+11=200 \qquad \text{(original equation)}$$

$$d^2+20d+11-11=200-11 \qquad \text{(Subtract 11 on both sides)}$$

$$d^2+20d=189$$

$$d^2+2\cdot d\cdot 10=189 \qquad (\text{Written } 20d=2\cdot d\cdot 10)$$

$$d^2+2\cdot d\cdot 10+(10)^2=189+(10)^2 \qquad \left(\text{Add } +(10)^2 \text{ on both sides}\right)$$

$$d^2+2\cdot d\cdot 10+(10)^2=189+100$$

$$(d+10)^2=289 \qquad \left(\text{Factor} d^2+2\cdot d\cdot 10+(10)^2=(d+10)^2\right)$$

Step2: Now solve the equation $(d+10)^2=289$ by facing the square root of each side, we obtain the 'd 'values

$$(d+10)^2=289 \qquad \text{(Original equation)}$$

$$\sqrt{(d+10)^2}=\sqrt{289} \qquad \text{(Taking the square root of each side)}$$

$$d+10=\sqrt{(17)^2}$$

$$d+10=\pm 17$$

$$d+10-10=\pm 17-10 \qquad \text{(Subtract 10 on both sides)}$$

$$d=\pm 17-10$$

$$d=17-10 \quad \text{or} \quad d=-17-10$$

$$d=7 \quad \text{or} \quad d=-27$$

Step3: Substitute each value of d is the original equation $d^2+20d+11=200$

$$d^2+20d+11=200 \qquad \text{(Original equation)}$$

$$(7)^2+20(7)+11\stackrel{?}{=}200 \qquad \text{(Replace } d \text{ by } 7)$$

$$49+140+11\stackrel{?}{=}200$$

$$60+140\stackrel{?}{=}200$$

$$200=200 \qquad \text{True}$$

$$d^2+20d+11=200 \qquad \text{(Original equation)}$$

$$(-27)^2+20(-27)+11\stackrel{?}{=}200 \qquad \text{(Replace } d \text{ by } -27)$$

$$729-540+11\stackrel{?}{=}200$$

$$740-540\stackrel{?}{=}200$$

$$200=200 \qquad \text{True}$$

Therefore d = 7 and d = -27 satisfies the equation $d^2+20d+11=200$

Hence, the solution set is $\boxed{\{7,-27\}}$

Answer 33PA.

Consider the equation $a^2-5a=-4$

Claim: To solve the equation $a^2-5a=-4$ by completing the square.

Step1: Write $a^2-5a=-4$ as a perfect square form

$$a^2-5a=-4$$

$$a^2-1\cdot 5a=-4$$

$$a^2-\frac{2}{2}\cdot 1\cdot 5a=-4$$

$$a^2-2\cdot a\cdot\frac{5}{2}=-4$$

$$a^2-2\cdot a\cdot\frac{5}{2}+\left(\frac{5}{2}\right)^2=-4+\left(\frac{5}{2}\right)^2 \qquad \left(\text{Add } \left(\frac{5}{2}\right)^2 \text{ on both sides}\right)$$

$$a^2-2\cdot a\cdot\frac{5}{2}+\left(\frac{5}{2}\right)^2=-4+\frac{25}{4}$$

$$\left(a-\frac{5}{2}\right)^2=\frac{-16+25}{4}$$

$$\left(a-\frac{5}{2}\right)^2=\frac{9}{4} \qquad \left(\text{Factors} a^2-2\cdot a\cdot\frac{5}{2}+\left(\frac{5}{2}\right)^2=\left(a-\frac{5}{2}\right)^2\right)$$

Step2: Now solve the equation $\left(a-\frac{5}{2}\right)^2=\frac{9}{4}$ by facing the square root of each side, we obtain the 'a 'values

$$\left(a-\frac{5}{2}\right)^2=\frac{9}{4} \qquad \text{(Original equation)}$$

$$\sqrt{\left(a-\frac{5}{2}\right)^2}=\sqrt{\frac{9}{4}} \qquad \text{(Taking the square root of each side)}$$

$$a-\frac{5}{2}=\frac{\sqrt{9}}{\sqrt{4}}$$

$$a-\frac{5}{2}=\pm\frac{3}{2}$$

$$a-\frac{5}{2}+\frac{5}{2}=\pm\frac{3}{2}+\frac{5}{2} \qquad \left(\text{Add } \frac{5}{2} \text{ on both sides}\right)$$

$$a=\pm\frac{3}{2}+\frac{5}{2}$$

$$a=\frac{3}{2}+\frac{5}{2} \quad \text{or} \quad a=-\frac{3}{2}+\frac{5}{2}$$

$$a=\frac{3+5}{2} \quad \text{or} \quad a=\frac{-3+5}{2}$$

$$a=\frac{8}{2} \quad \text{or} \quad a=\frac{2}{2}$$

$$a=\frac{2\cdot 4}{2} \quad \text{or} \quad a=1$$

$$a=4 \quad \text{or} \quad a=1$$

Step3: Substitute each value of a is the original equation $a^2-5a=-4$

$$a^2-5a=-4 \qquad \text{(Original equation)}$$

$$(4)^2-5(4)\stackrel{?}{=}-4 \qquad (\text{Replace } a \text{ by } -4)$$

$$16-20\stackrel{?}{=}-4$$

$$-4=-4 \qquad \text{True}$$

Answer 34PA.

Consider the equation $p^2-4p=21$

Claim:- Solve the equation $p^2-4p=21$ completing the square.

Step-1:- Write the equation $p^2 - 4p = 21$ as perfect square

$$p^2 - 4p = 21$$
$$p^2 - 2\cdot p\cdot 2 = 21$$
$$p^2 - 2\cdot p\cdot 2 + 2^2 = 21 + 2^2 \qquad [\text{Add } 2^2 \text{one ach side}]$$
$$(p-2)^2 = 25 \qquad [\text{Factors } p^2 - 2\cdot p\cdot 2 + 2^2 \text{as } (\text{p}-2)^2]$$

Step-2:- Now solve the equation $(p-2)^2 = 25$ by taking square root of each side we obtain root "*p*"

$$(p-2)^2 = 25 \qquad [\text{Original equation}]$$
$$\sqrt{(p-2)^2} = \sqrt{25} \qquad [\text{Taking the square root pf each side}]$$
$$p - 2 = \pm 5$$
$$p - 2 + 2 = 2 \pm 5$$
$$p = 2 + 5 \quad \text{or} \quad p = 2 - 5$$
$$p = 7 \quad \text{or} \quad p = -3$$

Step-3:- Substitute each value of p in the original equation $p^2 - 4p = 21$

$$p^2 - 4p = 21$$
$$(-3)^2 - 4(-3) \stackrel{?}{=} 21 \qquad [\text{Replace } p \text{ by} -3]$$
$$9 + 12 \stackrel{?}{=} 21$$
$$21 = 21 \qquad \text{True}$$

$$p^2 - 4p = 21$$
$$7^2 - 4(7) \stackrel{?}{=} 21 \qquad [\text{Replace } p \text{ by} 7]$$
$$49 - 28 \stackrel{?}{=} 21$$
$$21 = 21 \qquad \text{True}$$

$$p = -3 \text{ and } p = 7$$

Therefore *p* statistics the equation $p^2 - 4p = 21$

Hence, the solution set is $\boxed{\{-3, 7\}}$

Answer 35PA.

Consider the equation $x^2 + 4x + 3 = 0$

Claim: To solve the equation $x^2 + 4x + 3 = 0$ by completing the square.

Step1: Write $x^2+4x+3=0$ as a perfect square form

$$x^2+4x+3=0 \quad \text{(Original equation)}$$

$$x^2+4x+3-3=0-3 \quad \text{(Substract 3 on both sides)}$$

$$x^2+4x=-3$$

$$x^2+2\cdot 2\cdot x=-3$$

$$x^2+2\cdot x\cdot 2+(2)^2=-3+(2)^2 \quad \text{(Add } 2^2 \text{ on both sides)}$$

$$(x+2)^2=-3+4 \quad \left(\text{Factor} x^2+2\cdot x\cdot 2+(2)^2=(x+2)^2\right)$$

$$(x+2)^2=1$$

Step2: Now solve the equation $(x+2)^2=1$ by facing the square root of each side, we obtain the 'x 'values

$$(x+2)^2=1 \quad \text{(Original equation)}$$

$$\sqrt{(x+2)^2}=\sqrt{1} \quad \text{(Taking the square root of each side)}$$

$$x+2=\pm 1$$

$$x+2-2=\pm 1-2 \quad \text{(Substract 2 on both sides)}$$

$$x=\pm 1-2$$

$$x=1-2 \quad \text{or} \quad x=-1-2$$

$$x=-1 \quad \text{or} \quad x=-3$$

Step3: Substitute each value of x is the original equation $x^2+4x+3=0$

$$x^2+4x+3=0 \quad \text{(Original equation)}$$

$$(-1)^2+4(-1)+3\overset{?}{=}0 \quad \text{(Replace } x \text{ by } -1\text{)}$$

$$1-4+3\overset{?}{=}0$$

$$4-4\overset{?}{=}0$$

$$0=0 \quad \text{True}$$

$$x^2+4x+3=0 \qquad \text{(Original equation)}$$

$$(-3)^2+4(-3)+3\overset{?}{=}0 \qquad \text{(Replace } x \text{ by } -3\text{)}$$

$$9-12+3\overset{?}{=}0$$

$$12-12\overset{?}{=}0$$

$$0=0 \qquad \text{True}$$

Therefore *x* = -1 and *x* = -3 satisfies the equation $x^2+4x+3=0$

Hence, the solution set is $\boxed{\{-1,-3\}}$

Answer 36PA.

Consider the equation $d^2-8d+7=0$

Claim: To solve the equation $d^2-8d+7=0$ by completing the square.

Step1: Write $d^2-8d+7=0$ as a perfect square form

$$d^2-8d+7=0 \qquad \text{(Original equation)}$$

$$d^2-8d+7-7=0-7 \qquad \text{(Substract 7 on both sides)}$$

$$d^2-8d=-7$$

$$d^2-2\cdot d\cdot 4=-7$$

$$d^2-2\cdot d\cdot 4+(4)^2=-7+(4)^2 \qquad \left(\text{Add } (4)^2 \text{ on both sides}\right)$$

$$(d-4)^2=-7+16 \qquad \left(\text{Factor} d^2-2\cdot d\cdot 4+(4)^2=(d-4)^2\right)$$

$$(d-4)^2=9$$

Step2: Now solve the equation $(d-4)^2=9$ by facing the square root of each side, we obtain the '*d* 'values

$$(d-4)^2=9 \qquad \text{(Original equation)}$$

$$\sqrt{(d-4)^2}=\sqrt{9} \qquad \text{(Taking the square root of each side)}$$

$$d-4=\pm 3$$

$$d-4+4=\pm 3+4 \qquad \text{(Add 4 on both sides)}$$

$$d=\pm 3+4$$

$$d=3+4 \qquad \text{or} \qquad d=-3+4$$

$$d=7 \qquad \text{or} \qquad d=1$$

Step3: Substitute each value of *d* is the original equation $d^2-8d+7=0$

$$d^2-8d+7=0 \quad \text{(Original equation)}$$

$$(7)^2-8(7)+7\overset{?}{=}0 \quad \text{(Replace } d \text{ by 7)}$$

$$49-56+7\overset{?}{=}0$$

$$56-56\overset{?}{=}0$$

$$0=0 \quad \text{True}$$

$$d^2-8d+7=0 \quad \text{(Original equation)}$$

$$(1)^2-8(1)+7\overset{?}{=}0 \quad \text{(Replace } d \text{ by 1)}$$

$$1-8+7\overset{?}{=}0$$

$$8-8\overset{?}{=}0$$

$$0=0 \quad \text{True}$$

Therefore *d* = 7 and *d* = 1 satisfies the equation $d^2-8d+7=0$

Hence, the solution set is $\boxed{\{7,1\}}$

Answer 37PA.

Consider the equation $s^2-10s=23$

Claim: To solve the equation $s^2-10s=23$ by completing the square.

Step1: Write $s^2-10s=23$ as a perfect square form

$$s^2-10s=23 \quad \text{(Original equation)}$$

$$s^2-2\cdot s\cdot 5=23$$

$$s^2-2\cdot s\cdot 5+5^2=23+5^2 \quad \text{(Add } 5^2 \text{ on both sides)}$$

$$(s-5)^2=23+25 \quad \left(\text{factors} s^2-2\cdot s\cdot 5+5^2=(s-5)^2\right)$$

$$(s-5)^2=48$$

Step2: Now solve the equation $(s-5)^2=48$ by facing the square root of each side, we obtain the '*s* 'values

$$(s-5)^2=48 \qquad \text{(Original equation)}$$
$$\sqrt{(s-5)^2}=\sqrt{48} \qquad \text{(Taking the square root of each side)}$$
$$s-5=\sqrt{48}$$
$$s-5=\pm 6.92 \qquad \left(\sqrt{48}\approx 6.92\right)$$
$$s-5+5=\pm 6.92+5 \qquad \text{(Add 5 on both sides)}$$
$$s=\pm 6.92+5$$
$$s=6.92+5 \quad \text{or} \quad s=-6.92+5$$
$$s=11.92 \quad \text{or} \quad s=-1.92$$

Hence, the solution set is $\boxed{\{11.92,-1.92\}}$

Answer 38PA.

Consider the equation $m^2-8m=4$

Claim: To solve the equation $m^2-8m=4$ by completing the square.

Step1: Write $m^2-8m=4$ as a perfect square form

$$m^2-8m=4 \qquad \text{(Original equation)}$$
$$m^2-2\cdot m\cdot 4=4$$
$$m^2-2\cdot m\cdot 4+(4)^2=4+(4)^2 \qquad \left(\text{Add } 4^2 \text{ on both sides}\right)$$
$$(m-4)^2=4+16 \qquad \left(\text{factors: } m^2-2\cdot m\cdot 4+(4)^2=(m-4)^2\right)$$
$$(m-4)^2=20$$

Step2: Now solve the equation $(m-4)^2=20$ by facing the square root of each side, we obtain the '*m* 'values

$$(m-4)^2=20 \qquad \text{(Original equation)}$$
$$\sqrt{(m-4)^2}=\sqrt{20} \qquad \text{(Taking the square root of each side)}$$
$$m-4=\pm 4.5$$
$$m-4+4=\pm 4.5+4 \qquad \text{(Add 4 on both sides)}$$
$$m=\pm 4.5+4$$
$$m=4.5+4 \quad \text{or} \quad m=-4.5+4$$
$$m=8.5 \quad \text{or} \quad m=-0.5$$

Hence, the solution set is $\boxed{\{8.5,-0.5\}}$

Answer 39PA.

Consider the equation $9r^2+49=42r$

Claim: To solve the equation $9r^2+49=42r$ by completing the square.

Step1: Write the equation $9r^2+49=42r$ as a perfect square form

$$9r^2+49=42r \quad \text{(Original equation)}$$
$$9r^2+49-42r=42r-42r \quad \text{(Substract } -42r \text{ on both sides)}$$
$$9r^2-42r+49=0$$
$$9r^2-42r+49-49=0-49 \quad \text{(Substract } -49 \text{ on both sides)}$$
$$9r^2-42r=-49$$
$$9r^2-2\cdot r\cdot 21=-49 \quad \text{(Factor } 42=2\cdot 21\text{)}$$
$$(3r)^2-2\cdot 3r\cdot 7=-49 \quad \text{(Writen as } 21=3\cdot 7\text{)}$$
$$(3r)^2-2\cdot 3r\cdot 7+7^2=-49+7^2 \quad \text{(Add } 7^2 \text{ on both sides)}$$
$$(3r-7)^2=-49+49 \quad \left(\text{factors: } (3r)^2-2\cdot 3r\cdot 7+7^2=(3r-7)^2\right)$$
$$(3r-7)^2=0$$

Step2: Now solve the equation $(3r-7)^2=0$ by facing the square root of each side, we obtain the 'r 'values

$$(3r-7)^2=0 \quad \text{(Original equation)}$$
$$\sqrt{(3r-7)^2}=\sqrt{0} \quad \text{(Taking the square root of each side)}$$
$$3r-7=\pm 0$$
$$3r-7=0 \quad \text{or} \quad 3r-7=-0$$
$$3r-7+7=0+7 \quad \text{or} \quad 3r-7+7=-0+7$$
$$\text{(Add 7 on both sides)}$$
$$3r=7 \quad \text{or} \quad 3r=7$$
$$\frac{3r}{3}=\frac{7}{3} \quad \text{or} \quad \frac{3r}{3}=\frac{7}{3}$$
$$\text{(Dividing 3 on both sides)}$$
$$r=\frac{7}{3} \quad \text{or} \quad r=\frac{7}{3}$$

Step3: Substitute each value of r is the original equation

$$9r^2+49=42r \quad \text{(Original equation)}$$

$$9\left(\frac{7}{3}\right)^2+49\stackrel{?}{=}42\left(\frac{7}{3}\right) \quad \left(\text{Replace } r \text{ by } \frac{7}{3}\right)$$

$$9\left(\frac{49}{9}\right)+49\stackrel{?}{=}\frac{42.7}{3}$$

$$49+49\stackrel{?}{=}14.7$$

$$98=98 \quad \text{True}$$

$$9r^2+49=42r \quad \text{(Original equation)}$$

$$9\left(\frac{7}{3}\right)^2+49\stackrel{?}{=}42\left(\frac{7}{3}\right) \quad \left(\text{Replace } r \text{ by } \frac{7}{3}\right)$$

$$9\left(\frac{49}{9}\right)+49\stackrel{?}{=}\frac{42.7}{3}$$

$$49+49\stackrel{?}{=}14.7$$

$$98=98 \quad \text{True}$$

Therefore $r=\frac{7}{3}$ and $r=\frac{7}{3}$ satisfies the equation $9r^2+49=42r$

Hence, the solution set is $\boxed{\left\{\frac{7}{3},\frac{7}{3}\right\}}$

Answer 40PA.

Consider the equation $4h^2+25=20h$

Claim: To solve the equation $4h^2+25=20h$ by completing the square.

Step1: Write the equation $4h^2+25=20h$ as a perfect square form

$$4h^2+25=20h \quad \text{(Original equation)}$$

$$4h^2+25-25=20h-25 \quad \text{(Substract 25 on both sides)}$$

$$4h^2=20h-25$$

$$4h^2-20h=20h-25-20h \quad \text{(Substract } -20h \text{ on both sides)}$$

$$4h^2-20h=-25$$

$$4h^2-2\cdot(2h)\cdot 5=-25 \quad \left(\text{Factors: } 20=2\cdot 2\cdot 5 \text{ and } 4=2^2\right)$$

$$(2h)^2-2\cdot(2h)\cdot 5+5^2=-25+5^2 \quad \left(\text{Add } 5^2 \text{ on both sides}\right)$$

$$(2h)^2-2\cdot(2h)\cdot 5+5^2=-25+25$$

$$(2h-5)^2=0 \quad \left(\text{Factots:} (2h)^2-2\cdot(2h)\cdot 5+5^2 =(2h-5)^2\right)$$

Step2: Now solve the equation $(2h-5)^2=0$ by facing the square root of each side, we obtain the 'h 'values

$$(2h-5)^2=0 \qquad \text{(Original equation)}$$

$$\sqrt{(2h-5)^2}=\sqrt{0} \qquad \text{(Taking the square root of each side)}$$

$$2h-5=\pm 0$$

$$2h-5=0 \quad \text{or} \quad 2h-5=-0$$

$$2h-5+5=0+5 \quad \text{or} \quad 2h-5+5=-0+5$$

$$\text{(Add 5 on both sides)}$$

$$2h=5 \quad \text{or} \quad 2h=5$$

$$\frac{2h}{2}=\frac{5}{2} \quad \text{or} \quad \frac{2h}{2}=\frac{5}{2}$$

$$\text{(Divide 2 on both sides)}$$

$$h=\frac{5}{2} \quad \text{or} \quad h=\frac{5}{2}$$

Step3: Substitute each value of h is the original equation $(2h-5)^2=0$

$$4h^2+25=20h \qquad \text{(Original equation)}$$

$$4\left(\frac{5}{2}\right)^2+25\overset{?}{=}20\left(\frac{5}{2}\right) \qquad \left(\text{Replace } h \text{ by } \frac{5}{2}\right)$$

$$4\cdot\frac{25}{4}+25\overset{?}{=}\frac{20\cdot 5}{2}$$

$$25+25\overset{?}{=}\frac{2\cdot 5\cdot 10}{2}$$

$$50=50 \qquad \text{True}$$

$$4h^2+25=20h \qquad \text{(Original equation)}$$

$$4\left(\frac{5}{2}\right)^2+25\overset{?}{=}20\left(\frac{5}{2}\right) \qquad \left(\text{Replace } h \text{ by } \frac{5}{2}\right)$$

$$4\cdot\frac{25}{4}+25\overset{?}{=}\frac{20\cdot 5}{2}$$

$$25+25\overset{?}{=}\frac{2\cdot 5\cdot 10}{2}$$

$$50=50 \qquad \text{True}$$

Therefore $h=\frac{5}{2}$ and $h=\frac{5}{2}$ satisfies the equation $4h^2+25=20h$

Hence, the solution set is $\boxed{\left\{\frac{5}{2},\frac{5}{2}\right\}}$

Answer 41PA.

Consider the equation $0.3t^2 + 0.1t = 0.2$

Claim: To solve the equation $0.3t^2 + 0.1t = 0.2$ by completing the square.

Step1: Write the equation $0.3t^2 + 0.1t = 0.2$ as a perfect square form

$$0.3t^2 + 0.1t = 0.2 \qquad \text{(Original equation)}$$

$$0.3t^2 + 1\cdot(0.1)t = 0.2$$

$$\frac{0.3t^2}{0.3} + 1\cdot\frac{(0.1)}{0.3}t = \frac{0.2}{0.3} \qquad \text{(Divide 0.3 on both sides)}$$

$$t^2 + \frac{2}{2}\left(\frac{0.1}{0.3}\cdot t\right) = \frac{0.2}{0.3}$$

$$t^2 + 2\cdot t\cdot\frac{0.1}{0.6} = \frac{0.2}{0.3}$$

$$t^2 + 2\cdot t\cdot\frac{1}{6} = \frac{2}{3}$$

$$t^2 + 2\cdot t\cdot\frac{1}{6} + \left(\frac{1}{6}\right)^2 = \frac{2}{3} + \left(\frac{1}{6}\right)^2 \qquad \left(\text{Add } \left(\frac{1}{6}\right)^2 \text{ on both sides}\right)$$

$$\left(t + \frac{1}{6}\right)^2 = \frac{2}{3} + \frac{1}{36} \qquad \left(\text{Factors:} t^2 + 2\cdot t\cdot\frac{1}{6} + \left(\frac{1}{6}\right)^2 = \left(t + \frac{1}{6}\right)^2\right)$$

$$\left(t + \frac{1}{6}\right)^2 = \frac{25}{36}$$

Step2: Now solve the equation $\left(t+\frac{1}{6}\right)^2=\frac{25}{36}$ by facing the square root of each side, we obtain the 't 'values

$$\left(t+\frac{1}{6}\right)^2=\frac{25}{36} \qquad \text{(Original equation)}$$

$$\sqrt{\left(t+\frac{1}{6}\right)^2}=\sqrt{\frac{25}{36}} \qquad \text{(Taking the square root of each side)}$$

$$t+\frac{1}{6}=\pm\frac{5}{6}$$

$$t+\frac{1}{6}-\frac{1}{6}=\pm\frac{5}{6}-\frac{1}{6} \qquad \left(\text{Substract } \frac{1}{6} \text{ on both sides}\right)$$

$$t=\pm\frac{5}{6}-\frac{1}{6}$$

$$t=\frac{5}{6}-\frac{1}{6} \quad \text{or} \quad t=-\frac{5}{6}-\frac{1}{6}$$

$$t=\frac{5-1}{6} \quad \text{or} \quad t=\frac{-5-1}{6}$$

$$t=\frac{4}{6} \quad \text{or} \quad t=\frac{-6}{6}$$

$$t=\frac{2\cdot 2}{2\cdot 3} \text{or} \quad t=-1$$

$$t=\frac{2}{3} \quad \text{or} \quad t=-1$$

Step3: Substitute each value of t is the original equation $0.3t^2+0.1t=0.2$

$$0.3t^2+0.1t=0.2 \qquad \text{(Original equation)}$$

$$0.3\left(\frac{2}{3}\right)^2+0.1\left(\frac{2}{3}\right)\stackrel{?}{=}0.2 \qquad \left(\text{Replace } t \text{ by} \frac{2}{3}\right)$$

$$0.13+0.15\stackrel{?}{=}0.2$$

$$0.2=0.2 \qquad \text{True}$$

$$0.3t^2 + 0.1t = 0.2 \qquad \text{(Original equation)}$$

$$0.3(-1)^2 + 0.1(-1) \stackrel{?}{=} 0.2 \qquad (\text{Replace } t \text{ by } -1)$$

$$0.3 - 0.1 \stackrel{?}{=} 0.2$$

$$0.2 = 0.2 \qquad \text{True}$$

Therefore $t = \frac{2}{3}$ and $t = -1$ satisfies the equation $0.3t^2 + 0.1t = 0.2$

Hence, the solution set is $\boxed{\left\{\frac{2}{3}, -1\right\}}$

Answer 42PA.

Consider the equation $0.4v^2 + 2.5 = 2v$

Claim: To solve the equation $0.4v^2 + 2.5 = 2v$ by completing the square.

Step1: Write the equation $0.4v^2 + 2.5 = 2v$ as a perfect square form

$$0.4v^2 + 2.5 = 2v \qquad \text{(Original equation)}$$

$$0.4v^2 + 2.5 - 2v = 2v - 2v \qquad (\text{Substract } 2v \text{ on both sides})$$

$$0.4v^2 - 2v + 2.5 = 0$$

$$0.4v^2 - 2v + 2.5 - 2.5 = 0 - 2.5 \qquad (\text{Substract } 2.5 \text{ on both sides})$$

$$0.4v^2 - 2v = -2.5$$

$$\frac{0.4v^2}{0.4} - \frac{2v}{0.4} = \frac{-2.5}{0.4} \qquad (\text{Divide } 0.4 \text{ on both sides})$$

$$v^2 - 5v = \frac{-25}{4} \qquad \text{(Simplification)}$$

$$v^2 - 1 \cdot 5v = \frac{-25}{4}$$

$$v^2 - \frac{2}{2} \cdot 5v = \frac{-25}{4}$$

$$v^2 - 2 \cdot v \cdot \frac{5}{2} = \frac{-25}{4}$$

$$v^2 - 2 \cdot v \cdot \frac{5}{2} + \left(\frac{5}{2}\right)^2 = \frac{-25}{4} + \left(\frac{5}{2}\right)^2$$

$$\left(v - \frac{5}{2}\right)^2 = \frac{-25}{4} + \frac{25}{4} \qquad \left(\text{Written as} \left(\frac{5}{2}\right)^2 = \frac{25}{4}\right)$$

$$\left(v-\frac{5}{2}\right)^2=0$$

$$\left(v-\frac{5}{2}\right)\left(v-\frac{5}{2}\right)=0 \qquad \left(\text{Written as}\left(v-\frac{5}{2}\right)^2=\left(v-\frac{5}{2}\right)\left(v-\frac{5}{2}\right)\right)$$

$$v-\frac{5}{2}=0 \qquad \text{or} \qquad v-\frac{5}{2}=0$$

$$v-\frac{5}{2}+\frac{5}{2}=0+\frac{5}{2} \qquad \text{or} \quad v-\frac{5}{2}+\frac{5}{2}=0+\frac{5}{2}$$

$$\left(\text{Add }\frac{5}{2}\text{ on both sides}\right) \quad \left(\text{Add }\frac{5}{2}\text{ on both sides}\right)$$

$$v=\frac{5}{2} \qquad \text{or} \qquad v=\frac{5}{2}$$

Hence, the solution set is $\boxed{\left\{\frac{5}{2},\frac{5}{2}\right\}}$

Answer 43PA.

Consider the equation $5x^2+10x-7=0$

Claim: To solve the equation $5x^2+10x-7=0$ by completing the square.

Step1: Write the equation $5x^2+10x-7=0$ as a perfect square form

$$5x^2+10x-7=0 \qquad (\text{Original equation})$$

$$5x^2+10x-7+7=0+7 \qquad (\text{Add 7 on both sides})$$

$$5x^2+10x=7$$

$$\frac{5x^2}{5}+\frac{10x}{5}=\frac{7}{5} \qquad (\text{Divide 5 on both sides})$$

$$x^2+2x=\frac{7}{5}$$

$$(1\cdot x)^2+2\cdot x\cdot 1+1^2=\frac{7}{5}+1^2 \qquad (\text{Add 1 on both sides})$$

$$(x+1)^2=\frac{7}{5}+1$$

$$(x+1)^2=\frac{7+5}{5}$$

$$(x+1)^2=\frac{12}{5}$$

Step2: Now solve the equation $(x+1)^2 = \frac{12}{5}$ by facing the square root of each side, we obtain the 'x 'values

$$(x+1)^2 = \frac{12}{5} \qquad \text{(Original equation)}$$

$$\sqrt{(x+1)^2} = \sqrt{\frac{12}{5}} \qquad \text{(Taking the square root of each side)}$$

$$x+1 = \frac{\sqrt{12}}{\sqrt{5}}$$

$$x+1 = \pm\frac{3.46}{2.23}$$

$$x+1 = \pm 1.55$$

$$x+1-1 = \pm 1.55 - 1 \qquad \text{(Substract 1 on both sides)}$$

$$x = \pm 1.55 - 1$$

$$x = 1.55 - 1 \quad \text{or} \quad x = -1.55 - 1$$

$$x = 0.55 \quad \text{or} \quad x = 0.55$$

Hence, the solution set is $\boxed{\{0.55, -2.55\}}$

Answer 44PA.

Consider the equation $9w^2 - 12w - 1 = 0$

Claim: To solve the equation $9w^2 - 12w - 1 = 0$ by completing the square.

Step1: Write the equation $9w^2 - 12w - 1 = 0$ as a perfect square form

$$9w^2 - 12w - 1 = 0 \qquad \text{(Original equation)}$$

$$3^2 w^2 - 12w - 1 + 1 = 0 + 1 \qquad \text{(Add 1 on both sides)}$$

$$(3w)^2 - 2\cdot(3w)\cdot 2 = 1 \qquad (\text{factors } 12 = 2\cdot 3\cdot 2)$$

$$(3w)^2 - 2\cdot(3w)\cdot 2 + 2^2 = 1 + 2^2 \qquad (\text{Add } 2^2 \text{ on both sides})$$

$$(3w)^2 - 2\cdot(3w)\cdot 2 + 2^2 = 1 + 4$$

$$(3w)^2 - 2\cdot(3w)\cdot 2 + 2^2 = 5$$

$$(3w-2)^2 = 5 \qquad \left(\text{factors:}(3w)^2 - 2\cdot(3w)\cdot 2 + 2^2 = (3w-2)^2\right)$$

Step2: Now solve the equation $(3w-2)^2 = 5$ by facing the square root of each side, we obtain the '*w*'values

$$(3w-2)^2 = 5 \qquad \text{(Original equation)}$$

$$\sqrt{(3w-2)^2} = \sqrt{5} \qquad \text{(Taking the square root of each side)}$$

$$3w-2 = \pm 2.23$$

$$3w-2+2 = \pm 2.23+2 \qquad \text{(Add 2 on both sides)}$$

$$3w = \pm 2.23+2$$

$$\frac{3w}{3} = \pm\frac{2.23}{3}+\frac{2}{3} \qquad \text{(Divide 3 on both sides)}$$

$$w = \pm\frac{2.23}{3}+\frac{2}{3}$$

$$w = \frac{2.23}{3}+\frac{2}{3} \quad \text{or} \quad w = -\frac{2.23}{3}+\frac{2}{3}$$

$$w = \frac{4.23}{3} \quad \text{or} \quad w = \frac{-0.23}{3}$$

$$w = 1.41 \quad \text{or} \quad w = -0.08$$

Hence, the solution set is $\boxed{\{1.41, -0.08\}}$

Answer 45PA.

Consider the equation $\frac{1}{2}d^2-\frac{5}{4}d-3=0$

Claim: To solve the equation $\frac{1}{2}d^2-\frac{5}{4}d-3=0$ by completing the square.

Step1: Write the equation $\frac{1}{2}d^2-\frac{5}{4}d-3=0$ as a perfect square form

$$\frac{1}{2}d^2-\frac{5}{4}d-3=0 \qquad \text{(Original equation)}$$

$$2\cdot\frac{1}{2}\cdot d^2-\frac{5}{4}\cdot 2\cdot d-2\cdot 3=0 \qquad \text{(Multiply 2 on both sides)}$$

$$d^2-\frac{5}{2}d-6=0$$

$$d^2-\frac{5}{2}d-6+6=0+6 \qquad \text{(Add 6 on both sides)}$$

$$d^2-\frac{5}{2}d=6$$

$$d^2-1\cdot\frac{5}{2}\cdot d=6$$

$$d^2-\frac{2}{2}\cdot\frac{5}{2}\cdot d=6$$

$$d^2-2\cdot d\cdot\frac{5}{4}=6$$

$$d^2-2\cdot d\cdot\frac{5}{4}+\left(\frac{5}{4}\right)^2=6+\left(\frac{5}{4}\right)^2 \qquad \left(\text{Add } \left(\frac{5}{4}\right)^2 \text{ on both sides}\right)$$

$$d^2-2\cdot d\cdot\frac{5}{4}+\left(\frac{5}{4}\right)^2=6+\frac{25}{16}$$

$$\left(d-\frac{5}{4}\right)^2=\frac{16\cdot 6+25}{16} \qquad \left(\text{factors:} d^2-2\cdot d\cdot\frac{5}{4}+\left(\frac{5}{4}\right)^2=\left(d-\frac{5}{4}\right)^2\right)$$

$$\left(d-\frac{5}{4}\right)^2=\frac{96+25}{16}$$

$$\left(d-\frac{5}{4}\right)^2=\frac{121}{16}$$

Step2: Now solve the equation $\left(d-\frac{5}{4}\right)^2=\frac{121}{16}$ by facing the square root of each side, we obtain the 'd 'values

$$\left(d-\frac{5}{4}\right)^2=\frac{121}{16} \qquad \text{(Original equation)}$$

$$\sqrt{\left(d-\frac{5}{4}\right)^2}=\sqrt{\frac{121}{16}} \qquad \text{(Taking the square root of each side)}$$

$$d-\frac{5}{4}=\sqrt{\frac{11^2}{4^2}}$$

$$d-\frac{5}{4}=\sqrt{\left(\frac{11}{4}\right)^2}$$

$$d-\frac{5}{4}=\pm\frac{11}{4} \qquad \text{(Cancellation of the square root and square)}$$

$$d-\frac{5}{4}+\frac{5}{4}=\pm\frac{11}{4}+\frac{5}{4} \qquad \left(\text{Add } \frac{5}{4} \text{ on both sides}\right)$$

$$d=\pm\frac{11}{4}+\frac{5}{4}$$

$$d=\frac{11}{4}+\frac{5}{4} \quad \text{or} \quad d=-\frac{11}{4}+\frac{5}{4}$$

$$d=\frac{11+5}{4} \quad \text{or} \quad d=\frac{-11+5}{4}$$

$$d=\frac{16}{4} \quad \text{or} \quad d=\frac{-6}{4}$$

$$d=\frac{4\cdot 4}{4} \quad \text{or} \quad d=\frac{-3\cdot 2}{2\cdot 2}$$

$$d=4 \quad \text{or} \quad d=\frac{-3}{2}$$

Hence, the solution set is $\boxed{\left\{4,-\frac{3}{2}\right\}}$

Answer 46PA.

Consider the equation $\frac{1}{3}f^2-\frac{7}{6}f+\frac{1}{2}=0$

Claim: To solve the equation $\frac{1}{3}f^2-\frac{7}{6}f+\frac{1}{2}=0$ by completing the square.

Step1: Write the equation $\frac{1}{3}f^2-\frac{7}{6}f+\frac{1}{2}=0$ as a perfect square form

$$\frac{1}{3}f^2-\frac{7}{6}f+\frac{1}{2}=0 \qquad (\text{Original equation})$$

$$3\cdot\left(\frac{1}{3}f^2-\frac{7}{6}f+\frac{1}{2}\right)=3\cdot(0) \qquad (\text{Multiply 3 on both sides})$$

$$3\cdot\frac{1}{3}\cdot f^2-3\cdot\frac{7}{6}\cdot f+3\cdot\frac{1}{2}=0$$

$$f^2-\frac{7}{2}\cdot f+\frac{3}{2}=0$$

$$f^2-\frac{7}{2}\cdot f+\frac{3}{2}-\frac{3}{2}=0-\frac{3}{2} \qquad \left(\text{Substract } \frac{3}{2} \text{ on both sides}\right)$$

$$f^2-\frac{7}{2}\cdot f=-\frac{3}{2}$$

$$f^2-1\cdot\frac{7}{2}\cdot f=-\frac{3}{2}$$

$$f^2-\frac{2}{2}\cdot\frac{7}{2}\cdot f=-\frac{3}{2}$$

$$f^2-2\cdot\frac{7}{4}\cdot f=-\frac{3}{2}$$

$$f^2-2\cdot\frac{7}{4}\cdot f+\left(\frac{7}{4}\right)^2=-\frac{3}{2}+\left(\frac{7}{4}\right)^2 \qquad \left(\text{Add } \left(\frac{7}{4}\right)^2 \text{ on both sides}\right)$$

$$\left(f-\frac{7}{4}\right)^2=-\frac{3}{2}+\frac{49}{16} \qquad \left(\text{Factors:} f^2-2\cdot\frac{7}{4}\cdot f+\left(\frac{7}{4}\right)^2 = \left(f-\frac{7}{4}\right)^2\right)$$

$$\left(f-\frac{7}{4}\right)^2=\frac{-8\cdot 3+49}{16} \qquad \left(\begin{array}{l}\text{Take a common denominator}\\ \text{and combine the numerator}\end{array}\right)$$

$$\left(f-\frac{7}{4}\right)^2=\frac{-24+49}{16}$$

$$\left(f-\frac{7}{4}\right)^2=\frac{25}{16}$$

Step2: Now solve the equation $\left(f-\frac{7}{4}\right)^2=\frac{25}{16}$ by facing the square root of each side, we obtain the 'f 'values

$$\left(f-\frac{7}{4}\right)^2=\frac{25}{16} \qquad \text{(Original equation)}$$

$$\sqrt{\left(f-\frac{7}{4}\right)^2}=\sqrt{\frac{25}{16}} \qquad \text{(Taking the square root of each side)}$$

$$f-\frac{7}{4}=\sqrt{\frac{5^2}{4^2}}$$

$$f-\frac{7}{4}=\sqrt{\left(\frac{5}{4}\right)^2}$$

$$f-\frac{7}{4}=\pm\frac{5}{4} \qquad \left(\begin{array}{l}\text{Cancellation of the square root}\\ \text{and square on both sides}\end{array}\right)$$

$$f-\frac{7}{4}+\frac{7}{4}=\pm\frac{5}{4}+\frac{7}{4} \qquad \left(\text{Add } \frac{7}{4} \text{ on both sides}\right)$$

$$f=\pm\frac{5}{4}+\frac{7}{4}$$

$$f=\frac{5}{4}+\frac{7}{4} \quad \text{or} \quad f=-\frac{5}{4}+\frac{7}{4}$$

$$f=\frac{5+7}{4} \quad \text{or} \quad f=\frac{-5+7}{4}$$

$$f=\frac{12}{4} \quad \text{or} \quad f=\frac{2}{4}$$

$$f=\frac{4\cdot 3}{4\cdot 1} \quad \text{or} \quad f=\frac{2\cdot 1}{2\cdot 2}$$

$$f=3 \quad \text{or} \quad f=\frac{1}{2}$$

Hence, the solution set is $\boxed{\left\{3,\frac{1}{2}\right\}}$

Answer 47PA.

Consider the trinomial $x^2+4x+c=0$

__Claim__ To solve the equation $x^2+4x+c=0$ by completing the square.

__Step 1__: Rewrite the equation $x^2+4x+c=0$ as perfect square.

$$x^2+4x+c=0 \quad \text{[original equation]}$$
$$x^2+4x+c-c=0-c \quad \text{[Substract to each side by } c\text{]}$$
$$x^2+4x=-c$$
$$x^2+2\cdot x\cdot 2=-c \quad \text{[Write } 4x \text{ as } 2\cdot x\cdot 2\text{]}$$
$$x^2+2\cdot x\cdot 2+2^2=-c+2^2 \quad \left[\text{Add to each side by } 2^2\right]$$
$$(x+2)^2=4-c \quad \left[\text{Factors } x^2+2\cdot x\cdot 2+2^2 \text{ as } (x+2)^2\right]$$

__Step 2:__ Now solve the equation $(x+2)^2=4-c$ by satiating the square root of each side. We obtain the *s* values.

$$(x+2)^2=4-c$$
$$\sqrt{(x+2)^2}=\pm\sqrt{4-c} \quad \text{[Taking the square root of each side]}$$
$$x+2=\pm\sqrt{4-c}$$
$$x+2-2=-2\pm\sqrt{4-c} \quad \text{[Substract to each side by 2]}$$
$$x=-2\pm\sqrt{4-c}$$
$$x=-2+\sqrt{4-c} \text{ or } x=-2+\sqrt{4-c}$$

Hence, the solution set is $\boxed{\left\{-2-\sqrt{4-c},-2+\sqrt{4-c}\right\}}$

Answer 48PA.

Consider the trinomial $x^2 - 6x + c = 0$

Claim To solve the equation $x^2 - 6x + c = 0$ by completing the square.

Step 1 : Rewrite the equation $x^2 - 6x + c = 0$ as perfect square.

$$x^2 - 6x + c = 0 \quad \text{[original equation]}$$

$$x^2 - 6x + c - c = 0 - c \quad [\text{Substract to each side by } c]$$

$$x^2 - 6x = -c$$

$$x^2 - 2 \cdot x \cdot 3 = -c \quad [\text{Write } 6x \text{ as } 2 \cdot x \cdot 3]$$

$$x^2 - 2 \cdot x \cdot 3 + 3^2 = 3^2 - c \quad \left[\text{Add to each side by } 3^2\right]$$

$$(x-3)^2 = 9 - c \quad \left[\text{Factors } x^2 - 2 \cdot x + 3^2 \text{ as } (x-3)^2\right]$$

Step 2: Now solve the equation $(x-3)^2 = 9 - c$ by satiating the square root of each side. We obtain the *s* values.

$$(x-3)^2 = 9 - c$$

$$\sqrt{(x-3)^2} = \pm\sqrt{9-c} \quad [\text{Taking the square root of each side}]$$

$$x - 3 = \pm\sqrt{9-c}$$

$$x - 3 + 3 = 3 \pm \sqrt{9-c} \quad [\text{Add to each side by } 3]$$

$$x = -2 \pm \sqrt{4-c}$$

$$x = 3 + \sqrt{9-c} \text{ or } x = 3 - \sqrt{9-c}$$

Hence, the solution set is $\boxed{\left\{3+\sqrt{9-c}, 3-\sqrt{9-c}\right\}}$

Answer 49PA.

Consider a rectangular park which has plot of wild flowers

Length of the park = 9 meters

Width of the park = 6 meters

According to the problem then is a path around the park with constant width

And given area of the park = Area of the path

Claim To find the width of the path around the park.

Step 1 : We find the area of the park.

By use the formula area of the rectangular.

$$= \boxed{(\text{Length} \times \text{Width})}$$

Area of the park $= 9 \times 6 = \underline{54}$ square meters.

Step 2 Let us consider the width of the path $= 'x'$ meters

Then length of the outer rectangular $=$ length of the park $+2x$

$$= \underline{9 + 2x}$$

Width of the outer rectangle $=$ width of the park $+2x$

$$= \underline{6 + 2x}$$

The area of outer rectangle $= (9+2x)(6+2x)$

$$= 9(6+2x) + 2x(6+2x)$$
$$= 9 \cdot 6 + 6 \cdot 2x + 2x \cdot 6 + 2x \cdot 2x \quad [\text{use distributive property}]$$
$$= 54 + 18x + 12x + 4x^2$$
$$= 54 + 30x + 4x^2 \qquad [\text{Adding like}]$$
$$= 4x^2 + 30x + 54$$

Step 3:

Area of the path $=$ Area of the outer rectangle $-$ Area of the park

$$= 4x^2 + 3x + 54 - 54$$
$$= 4x^2 + 30x$$

Step 4: According to the problem area of the path = Area of park.

$$4x^2+30x=54$$

$$x^2+\frac{30}{4}x=\frac{54}{4} \qquad [\text{Divide by 4 on both sides}]$$

$$x^2+2\cdot\frac{15}{4}\cdot x=\frac{27}{2}$$

$$x^2+2\cdot x\cdot\frac{15}{4}+\left(\frac{15}{4}\right)^2=\frac{27}{2}+\left(\frac{15}{4}\right)^2 \qquad \left[\text{Adding } \left(\frac{15}{4}\right)^2 \text{ on both sides}\right]$$

$$\left(x+\frac{15}{4}\right)^2=\frac{27}{2}+\frac{225}{16} \qquad \left[\begin{array}{l}\text{Factors the } x^2+2\cdot x\cdot\frac{15}{4}+\left(\frac{15}{4}\right)^2 \\ \text{as } \left(x+\frac{15}{4}\right)^2\end{array}\right]$$

$$\left(x+\frac{15}{4}\right)^2=\frac{216+225}{16}$$

$$\left(x+\frac{15}{4}\right)^2=\frac{441}{16}$$

Now solve the equation $\left(x+\frac{15}{4}\right)^2=\frac{441}{16}$ by taking the square root on both sides to obtain the '*x*' value

$$\left(x+\frac{15}{4}\right)^2=\frac{441}{16}$$

$$\sqrt{\left(x+\frac{15}{4}\right)^2}=\sqrt{\frac{441}{16}}$$

$$x+\frac{15}{4}=\pm\frac{21}{4}$$

$$x+\frac{15}{4}-\frac{15}{4}=-\frac{15}{4}\pm\frac{21}{4} \qquad \left[\text{Subtract } \frac{15}{4} \text{ on both sides}\right]$$

$$x=\frac{-15}{4}\pm\frac{21}{4}$$

$$x=\frac{-15}{4}+\frac{21}{4} \quad \text{or} \quad x=\frac{-15}{4}-\frac{21}{4}$$

$$x=\frac{6}{4} \quad \text{or} \quad x=\frac{-36}{4}$$

$$x=\frac{3}{2} \quad \text{or} \quad x=-9$$

$$x=1.5$$

$$x=-9 \text{ or } x=1.5$$

Since the width is always the neglect negative value.

The width of the path is $\boxed{1.5\text{ meters}}$

Verifications:- According to the problem

Area of path = Area of park

Now area of the path $= 4x^2 + 30x$

Substitute $x = (1.5)m$

Area of the path $= 4(1.5)^5 + 30(1.5)$

$= 9 + 45$

$= 54$

According the problem area of the park $= 9 \times 6$

$= 54$

Area of the path = Area of the park

Hence, verified.

Answer 50PA.

Consider, In early 1900s, the average American ate 300 pounds of bread and cereal every year.

By the 1960s, Americans were eating half that amount. However, eating cereal and bread is on the rise again. The consumption of these types of foods can be moduld by the function.

$y = 0.059x^2 - 7.423x + 362.1$ where *y* represents the bread and cereal consumption in pounds and *x* represents the number of year since 1900.

Claim To find the, In what future year will the average American consume 300 pounds of bread and cereal

i.e. Substitute *y* by 300 in the original equation

$$y = 0.059x^2 - 7.423x + 362.1 \qquad [\text{original equation}]$$
$$300 = 0.059x^2 - 7.423x + 362.1 \qquad [\text{Replace } y \text{ by } 300]$$

Step 1: Re write the equation $0.059x^2 - 7.423x + 362.1 = 300$ as perfect square form.

$$0.059x^2 - 7.423x + 362.1 = 300 \qquad [\text{original equation}]$$
$$\frac{0.059x^2 - 7.423x + 362.1}{0.059} = \frac{300}{0.059} \qquad [\text{Divide to each side by } 0.059]$$
$$\frac{0.059}{0.059}x^2 - \frac{7.423}{0.059}x + \frac{362.1}{0.059} = \frac{300}{0.059}$$
$$x^2 - 126x + 6137 = 5085$$
$$x^2 - 126x + 6137 - 6137 = 5085 - 6137 \qquad [\text{Substract to each side by } 6137]$$
$$x^2 - 126x = -1052$$
$$x^2 - 2\cdot x\cdot 63 = -1052 \qquad [\text{Write } -126x \text{ as } -2\cdot x\cdot 63]$$
$$x^2 - 2\cdot x\cdot 63 + (63)^2 = -1052 + (63)^2$$
$$(x-63)^2 = -1052 + 3969 \qquad \left[\begin{array}{l}\text{Factor} x^2 - 2\cdot x\cdot 63 + (63)^2 \\ = (x-63)^2\end{array}\right]$$
$$(x-63)^2 = 2917$$

Step 2: Now, solve the equation $(x-63)^2 = 2917$ by taking the square root of each side. We obtain the 'x'. values.

$$(x-63)^2 = 2917$$

$$\sqrt{(x-63)^2} = \pm\sqrt{2917} \quad \text{[Taking the square roots of each side]}$$

$$x-63 = \pm\sqrt{2917}$$

$$x-63+63 = 63\pm\sqrt{2917} \quad \text{[Add to each side by 63]}$$

$$x = 63\pm\sqrt{2917}$$

$$x = 63+\sqrt{2917} \text{ or } x = 63-\sqrt{2917}$$

$$x = 117 \qquad \text{or } x = 9$$

Therefore, $x = 9$ or $x = 117$

Suppose $x = 9$

The average year of the American consumer 300 pounds of bread and cereal is $(1900+9)$ years =1909

Hence, 1909 years, the average American consume 300 pounds of bread and cereal

Suppose $x = 117$

The average year of the American consume 300 pounds of bread and cereal is $(1900+117) = 2017$

Hence, 2017 year, the average American consume 300 pounds of bread and cereal.

Answer 51PA.

Consider the equation $x^2+4x+12=0$

Claim Solve the equation $x^2+4x+12=0$

Step 1: Rewrite the equation $x^2+4x+12=0$ as a perfect square

$$x^2+4x+12=0 \quad \text{[original equation]}$$

$$x^2+4x+12-12=0-12 \quad \text{[Substract to each side by 12]}$$

$$x^2+4x=-12$$

$$x^2+2\cdot x\cdot 2=-12 \quad \text{[Write } 4x \text{ as } 2\cdot x\cdot 2\text{]}$$

$$x^2+2x\cdot 2+2^2=-12+2^2 \quad \text{[Add to each side by } 2^2\text{]}$$

$$(x+2)^2=-12+4$$

$$(x+2)^2=-8$$

Step 2: Now, solve the equation $(x+2)^2=-8$ by taking the square root of each side. We obtain 'x' values.

$$(x+2)^2=-8$$
$$\sqrt{(x+2)^2}=\pm\sqrt{-8}$$
$$x+2=\pm\sqrt{-1}\cdot\sqrt{8}$$
$$x+2=-2\pm i\sqrt{8} \qquad [\text{Substract to each side by } 2]$$
$$x=-2\pm i\sqrt{8}$$

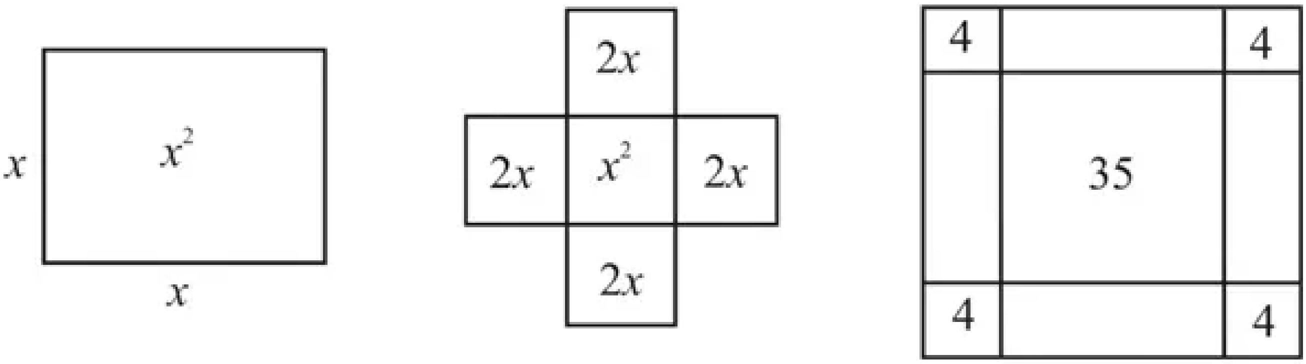
Therefore, x is not real solution

Hence, there are no real solution, since completing the square $(x+2)^2=-8$ and the square of a number cannot be negative.

Answer 53PA.

Al – Khwarizmi used squares to geometrically represents quadratic equation. Answers should include the following steps.

Step 1: Al – Khwarizmi represented x^2 by a square whose sides were each x units long. To this square, he added. He rectangles with length. X units long and width $\frac{8}{4}$ or 2 units long. This area represents 35. To make this a square, four 4×4 squares must by added.



Step 2 : To solve $x^2+8x=35$ by completing the square. First we have to write the equation, $x^2+8x=35$ as a perfect square

$$x^2+8x=35 \qquad [\text{original equation}]$$
$$x^2+2\cdot x\cdot 4=35 \qquad [\text{Write } 8x \text{ as } 2\cdot x\cdot 4]$$
$$x^2+2\cdot x\cdot 4+4^2=35+4^2 \qquad [\text{Add } 4^2 \text{ to each side}]$$
$$(x+4)^2=35+16$$
$$(x+4)^2=51$$

Step 3: Now, solve the equation $(x+4)^2 = 51$ by taking the square root of each side. We get the *x* values.

$$(x+4)^2 = 51$$
$$\sqrt{(x+4)^2} = \pm\sqrt{51} \qquad [\text{Taking the square root of each side}]$$
$$x+4 = \pm\sqrt{51}$$
$$x+4-4 = -4\pm\sqrt{51}$$
$$x = -4\pm\sqrt{51}$$
$$x = -4+\sqrt{51} \text{ or } x = -4-\sqrt{51}$$
$$x \approx 3.14 \qquad \text{or } x \approx -11.4$$

The solution set is $\boxed{\{-11.14, 3.14\}}$

Answer 54PA.

Consider the terminal $a^2 - 26a + 169$

Step 1:

$$a^2 - 26a + 169 = a^2 - 2\cdot a\cdot 13 + 13^3$$
$$= (a-13)^2 \qquad \left[a^2 - 2\cdot a\cdot 13 + 13^2 = (a-13)^2\right]$$

Hence, $a^2 - 26a + 169$ is a perfect square.

Now, we can eliminate the choice *A*.

Step 2: Consider terminal $a^2 + 32a + 256$

$$a^2 + 32a + 256 = a^2 + 2\cdot a\cdot 16 + 16^2$$
$$= (a+16)^2 \qquad [\text{Factor}]$$
$$= a^2 + 2\cdot a\cdot 16 + 16^2$$

Hence, the terminal $a^2 + 32a + 256$ is a perfect squares.

Now, we can eliminate the choice *B*

Step 3: Consider $a^2 - 44a + 484$

$$a^2 - 44a + 484 = a^2 - 2 \cdot a \cdot 22 + (22)^2$$
$$= (a-22)^2 \quad \left[\text{FActors } a^2 - 2 \cdot a \cdot 22 + (22)^2\right]$$

Hence, $a^2 - 44a + 484$ is a perfect square

Now, we can eliminate choice *D*.

Hence, from step 1, step 2 and step 3, we have

$a^2 - 26a + 169, a^2 + 32a + 256$ and $a^2 - 44a + 484$ are perfect square.

Therefore, $a^2 + 30a - 225$ is not a perfect square.

Answer $\boxed{C}$

Answer 55PA.

Consider the equation $x^2 + 5x = 14$

Claim: To find the equivalent equation $x^2 + 5x = 14$

Step 1:

Rewrite the equation $x^2 + 5x = 14$ as a perfect square form

$$x^2 + 5x = 14 \quad [\text{Original equation}]$$
$$x^2 + 1 \cdot 5x = 14$$
$$x^2 + \frac{2}{2} \cdot 5x = 14$$
$$x^2 + 2 \cdot x \cdot \frac{5}{2} = 14$$
$$x^2 + 2 \cdot x \cdot \frac{5}{2} + \left(\frac{5}{2}\right)^2 = 14 + \left(\frac{5}{2}\right)^2$$
$$\left(x + \frac{5}{2}\right)^2 = 14 + \frac{25}{4} \quad \left[\text{Factor } x^2 + 2 \cdot x \cdot \frac{5}{2} + \left(\frac{5}{2}\right)^2\right]$$
$$\left(x + \frac{5}{2}\right)^2 = \frac{56+25}{4}$$
$$\left(x + \frac{5}{2}\right)^2 = \frac{81}{4}$$

Hence, the equal equivalent equation $x^2 + 5x = 4$ is $\left(x + \frac{5}{2}\right)^2 = \frac{81}{4}$

Answer $\boxed{A}$

Answer 56MYS.

Consider the equation $x^2+7x+12=0$.

Claim: Solve the equation $x^2+7x+12=0$ graphing

Let graph the related function $f(x)=x^2+7x+12$

Step 1: Write the equation of the axis of symmetry.

Now, compare the function $y=f(x)=x^2+7x+12$ with the standard quadratic function.

$f(x)=ax^2+bx+c$, where $a\neq 0$

$a=1, b=7, c=12$

Use the rule "The equation of the axis of symmetry for graph of $f(x)=ax^2+bx+c$

Where $a\neq 0$ is $x=\frac{-b}{2a}$"

$$
\begin{aligned}
x &= \frac{-b}{2a} && \text{[Equation for the aixs of symmetry of a parabola]}\\
&= -\frac{7}{2\cdot 1} && \text{[Replace } a \text{ by 1 and } b \text{ by 7]}\\
x &= \frac{-7}{2}\\
x &= -3\cdot 5
\end{aligned}
$$

The equation of the axis of symmetry is $\boxed{x=-3.5}$.

Step 2: Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x=-1$ and vertex lies on the axis, the x – coordinate for the vertex is -3.5

$$
\begin{aligned}
f(x) &= x^2+7x+12 && \text{[original equation]}\\
f(x) &= (-3.5)^2+7(3.5)+12 && \text{[Replace } x \text{ by } -3.5\text{]}\\
&= 12.25-24.5+12\\
f(x) &= -0.25
\end{aligned}
$$

The vertex is $(-3.5,-0.25)$

Step 3: Identity the vertex as a maximum or minimum

Use the rule "The quadric function $f(x) = ax^2 + bx + c$ where $a \neq 0$.

Suppose the coefficient of x^2 term is positive, the parabola is open upward and vertex is the minimum point.

Suppose the coefficient of x^2 term is negative, the parabola is open downward.

And vertex is the maximum point".

Since the coefficient of x^2 term is positive, the parabola opens upward and vertex $(-3.5, -0.25)$.

Step 4: Graph the function

Now, construct the table for $f(x) = x^2 + 7x + 12$ (green curve).

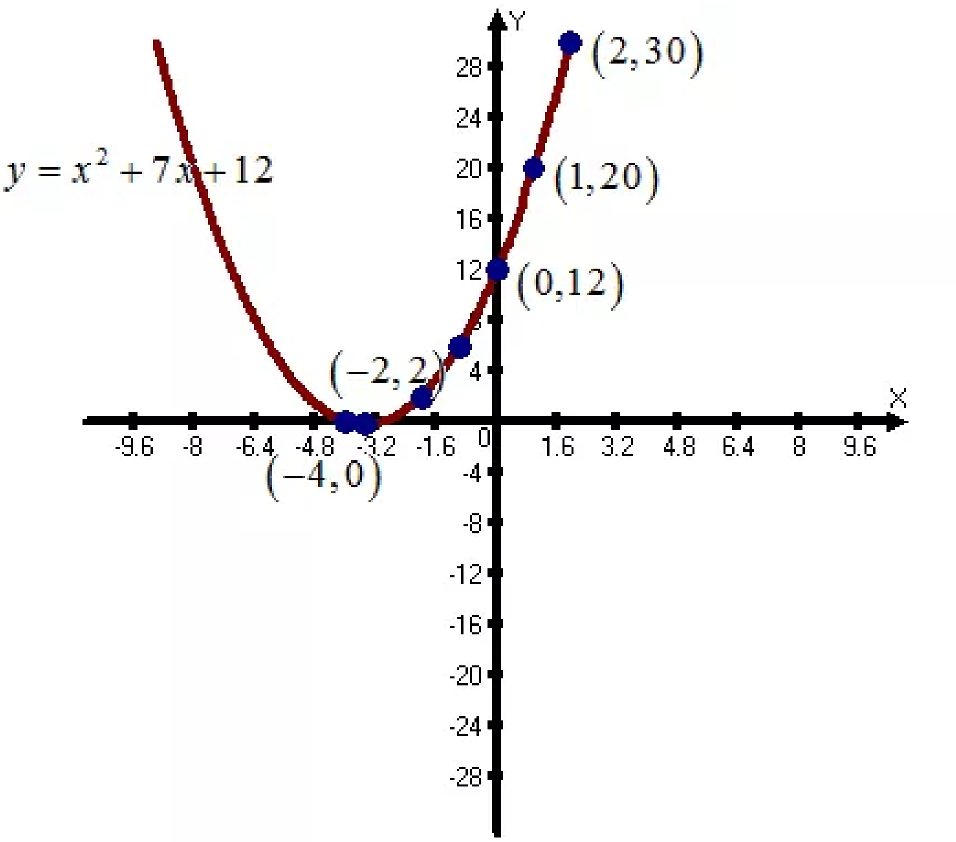
Substitute the different values of *x* in the original function $y = x^2 + 7x + 12$.

We obtain the different value of *y*. plot these all ordered pairs and connected them, get a smooth curve.

Table for $y = x^2 + 7x + 18$

x	$x^2 + 7x + 12$	y	(x, y)
-4	$(-4)^2 + 7(-4) + 12 = 0$	0	$(-4, 0)$
-3.5	$(3.5)^2 + 7(3.5) + 12 = -0.25$	-0.25	$(-3.5, -0.25)$
-2	$(-2)^2 + 7(-2) + 12 = 2$	2	$(-2, 2)$
-1	$(-1)^2 + 7(-1) + 12 = 6$	6	$(-1, 6)$
0	$(0)^2 + 7(0) + 12 = 12$	12	$(0, 12)$
1	$(1)^2 + 7(1) + 12 = 20$	20	$(1, 20)$
2	$(2)^2 + 7(2) + 12 = 30$	30	$(2, 30)$

Now, add these all ordered pairs (brown dots), get the parabola.



Answer 57MYS.

Let us consider the equation $x^2 - 16 = 0$

Claim: Solve the equation $x^2 - 16 = 0$ by graphing I

Let graph the related function $f(x) = x^2 - 16$

Step 1: Write the equation of the axis of symmetry.

Now comparing he gaaph $y = x^2 + 0x - 16$ with the standard quadratic function.

$f(x) = ax^2 + bx + c$, where $a \neq 0$. We obtain

$a = 1, b = 0, c = -16$

Use the rule "The equation of the axis of symmetry for graph of $f(x) = ax^2 + bx + c$

Where $a \neq 6$ is $x = \frac{-b}{2a}$"

$$x = \frac{-b}{2a} \quad \text{[Equation for the aixs of symmetry of a parabola]}$$

$$= -\frac{0}{2 \cdot 1} \quad \text{[Replace } a \text{ by 1 and } b \text{ by 0]}$$

$$x = \frac{0}{2 \cdot 1}$$

$$x = 0$$

The equation of the axis of symmetry is $\boxed{x = 0}$

Step 2: Find the coordinates of the vertex. Since the equation of the axis of symmetry is $x = 0$ and vertex lies on the axis, the *x* – coordinate for the vertex is 0

$$f(x) = x^2 - 16 \quad \text{[original equation]}$$
$$y = 0 - 16 \quad [\text{Replace } x \text{ by } 0]$$
$$y = -16$$

The vertex is $(0, -16)$

Step 3: Identity the vertex as a maximum or minimum

Since the coefficient of the x^2 term is positive the parabola open upward and the vertex is $(0, -16)$ maximum point.

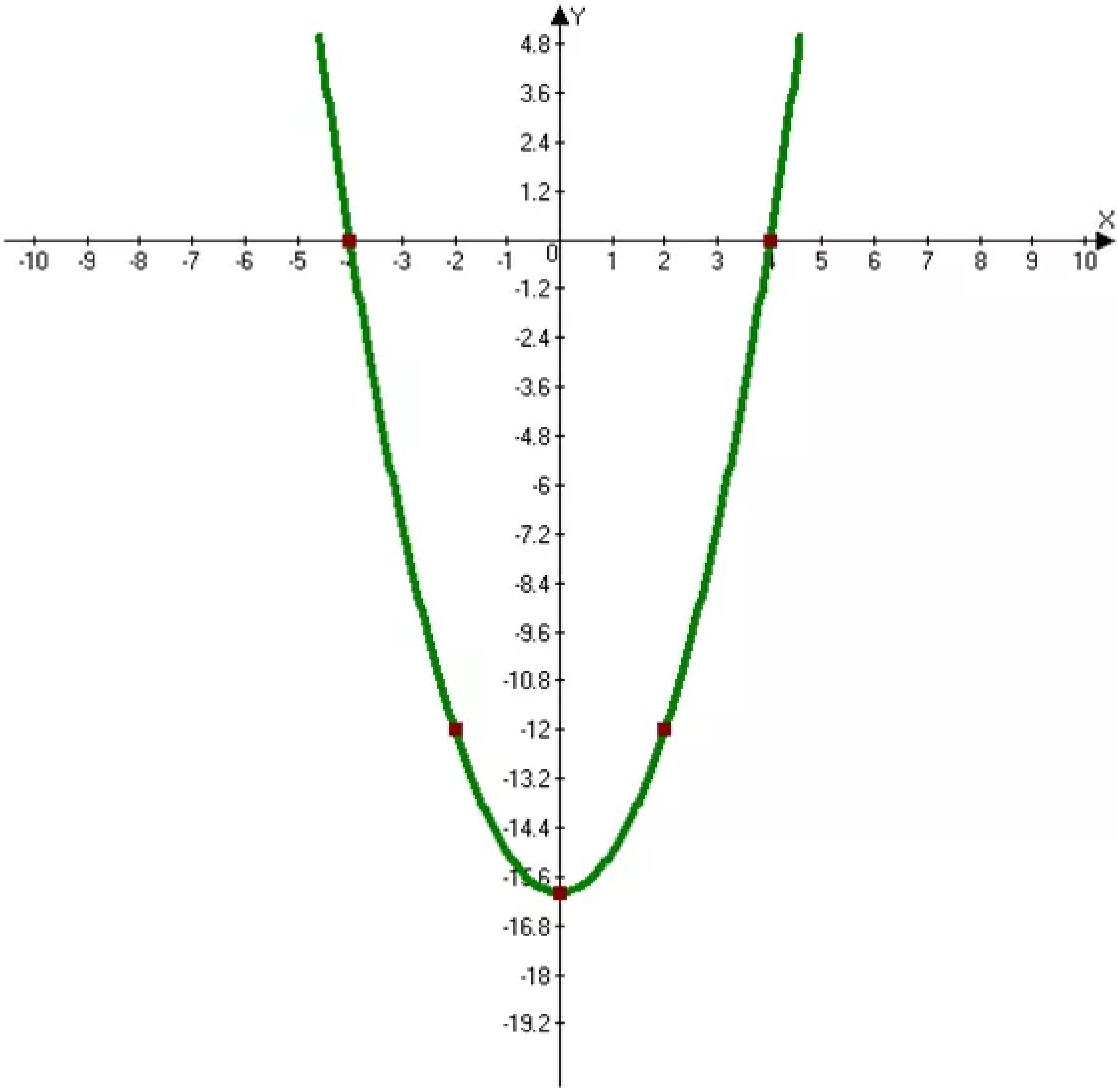
Step 4: Graph the function $y = x^2 - 16$ (green curve)

Now we constant the table for $y = x^2 - 16$. Now, we can substitute different values of *x* in the original function $y = x^2 - 16$ we obtain different *y* values. Plotting these all ordered pairs and connect them we get a smooth curve.

Table for $y = x^2 - 16$

x	$x^2 - 16$	y	(x, y)
-4	$(-4)^2 - 16 = 0$	0	$(-4, 0)$
-2	$(-2)^2 - 16 = -12$	-12	$(-2, -12)$
0	$(0)^2 - 16 = -16$	-16	$(0, -16)$
2	$(2)^2 - 16 = -12$	-12	$(2, -12)$
4	$(4)^2 - 16 = 0$	0	$(4, 0)$

Now add these all ordered pairs (brown dots), we get the parabola.



Answer 58MYS.

Consider the equation $x^2 - 2x + 6 = 0$

Claim: Solve the equation $x^2 - 2x + 6 = 0$ by graphing, the graph related function

$y = x^2 - 2x + 6$

Step 1: Write the equation of the axis of symmetry.

Now compare the function $y = x^2 - 2x + 6$ with the standard quadratic function.

$f(x) = ax^2 + bx + c$, where $a \neq 0$. We obtain

$a = 1, b = -2, c = 6$

Use the rule "The equation of the axis of symmetry for graph of $f(x) = ax^2 + bx + c$

Where $a \neq 0$ is $x = \frac{-b}{2a}$"

$$x = \frac{-b}{2a} \qquad [\text{Equation for the aixs of symmetry of a parabola}]$$

$$= -\frac{-2}{2 \cdot 1} \qquad [\text{Replace } a \text{ by } 1 \text{ and } b \text{ by } -2]$$

$$x = \frac{2}{2}$$

$$x = 1$$

The equation of the axis of symmetry is $\boxed{x = 1}$

Step 2: Find the coordinates of the vertex. Since the equation of the axis of symmetry is $x = 1$ and vertex lies on the axis, the *x* – coordinate for the vertex is 1

$$y = x^2 - 2x + 6 \qquad [\text{original equation}]$$

$$y = (1)^2 - 2(1) + 6 \qquad [\text{Replace } x \text{ by } 1]$$

$$y = 1 - 2 + 6s$$

$$y = 7 - 2$$

$$y = 5$$

The vertex is $(1, 5)$

Step 3: Identity the vertex as a maximum or minimum

Use the rule "The standard form of the quadratic function $f(x)=ax^2+bx+c$ where $a \neq 0$. Suppose the coefficient of x^2 term is positive, the parabola is open upward and vertex is the minimum point. Suppose the coefficient of x^2 term is negative, the parabola is open downward. And vertex is the maximum point". Since the coefficient of x^2 term is positive , the parabola open upward and vertex $(1,5)$ is the minimum point.

Hence the minimum point of the parabola $y=x^2-2x+6$ is $\boxed{(1,5)}$

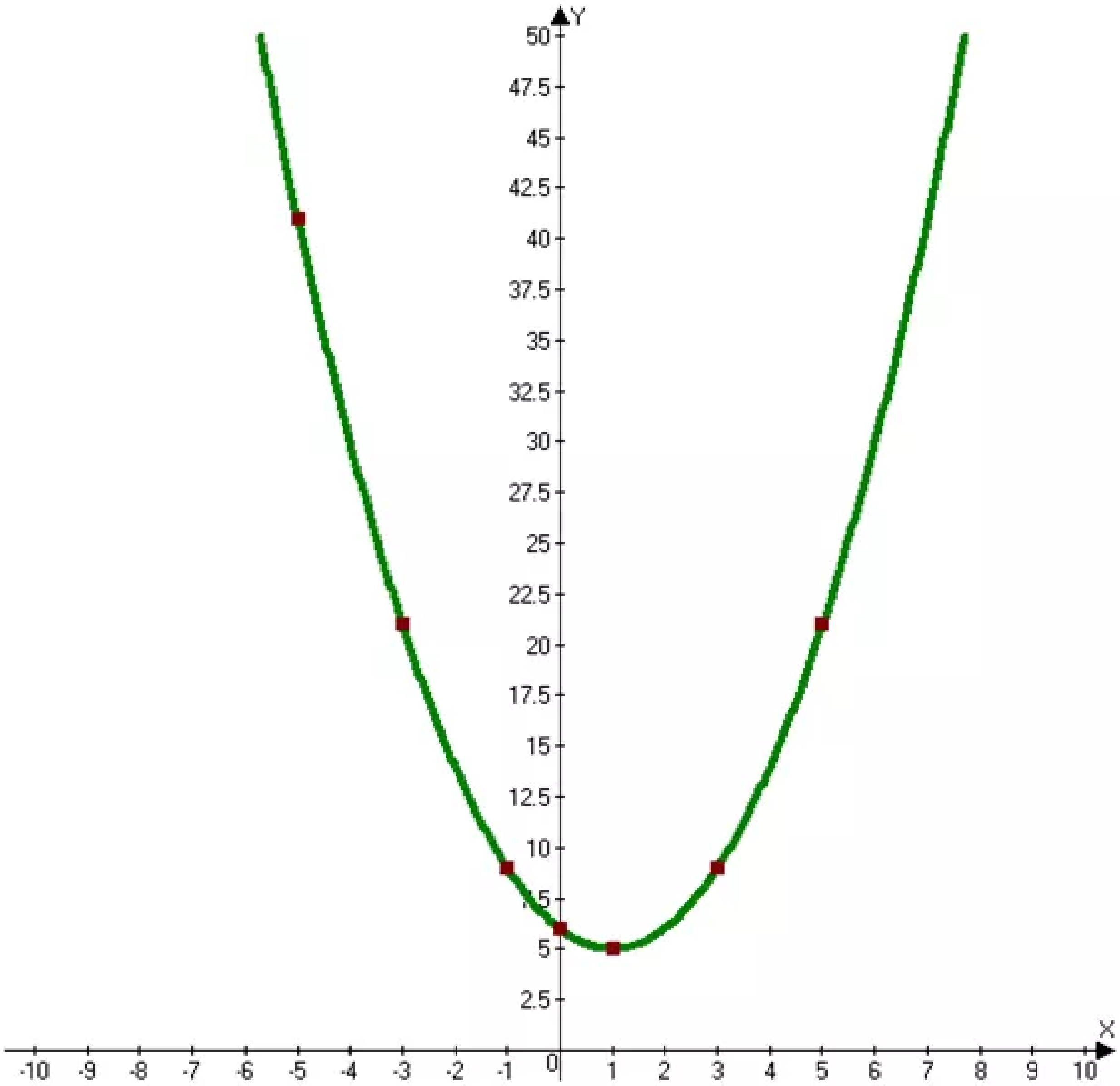
Step 4: Graph the function $y=x^2-2x+6$

Now we construct the table for $y=x^2-2x+6$ (green curve). We can substitute the different values of *x* in the original function $y=x^2+7x+12$. We obtain the different value of *y*. plotting these all ordered pairs and connected them, we get a smooth curve.

Table for $y=x^2-2x+6$

x	x^2-2x+6	y	(x,y)
-5	$(-5)^2-2(-5)+6=41$	41	$(-5,41)$
-3	$(-3)^2-2(-3)+6=21$	21	$(-3,21)$
-1	$(-1)^2-2(-1)+6=9$	9	$(-1,9)$
0	$(0)^2-2(0)+6=6$	6	$(0,6)$
1	$(1)^2-2(1)+6=5$	5	$(1,5)$
3	$(3)^2-2(3)+6=9$	9	$(3,9)$
5	$(5)^2-2(5)+6=21$	21	$(5,21)$

Now add these all ordered pairs (brown dots), we get the parabola.



Answer 59MYS.

Consider the equation $y = 4x^2 + 16$

Claim: Solve the equation $y = 4x^2 + 16$ by table values to graph

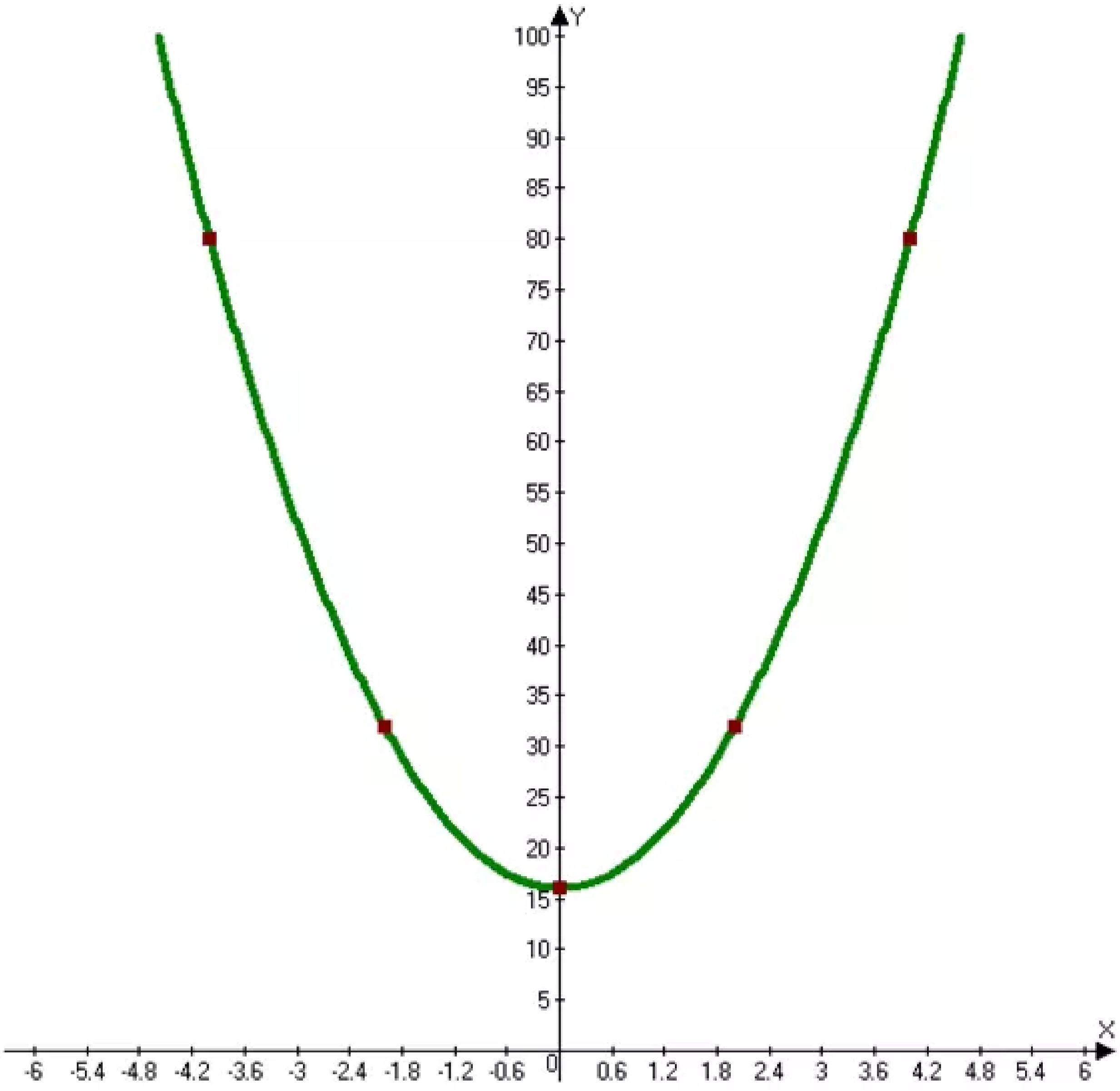
Now, we to construct table for $y = 4x^2 + 16$ (green curve)

Now, we substitute different value of x in the original equation $y = 4x^2 + 16$, we get the different" y "value plotting these all ordered pairs and connect them we get a smooth curve.

Table for $y = 4x^2 + 16$

x	$4x^2 + 16$	y	(x, y)
-4	$4(-4)^2 + 16 = 80$	80	$(-4, 80)$
-2	$4(-2)^2 + 16 = 80$	32	$(-2, 32)$
0	$4(0)^2 + 16 = 80$	16	$(0, 16)$
2	$4(2)^2 + 16 = 80$	32	$(2, 32)$
4	$4(4)^2 + 16 = 80$	80	$(4, 80)$

Now, add these all ordered pairs (brown dots), we obtain the parabola. These parabola open up ward and minimum point is $(0,16)$.



Answer 60MYS.

Consider the equation $y = x^2 - 3x - 10$

Claim: Solve the equation $y = x^2 - 3x - 10$ by table to graph

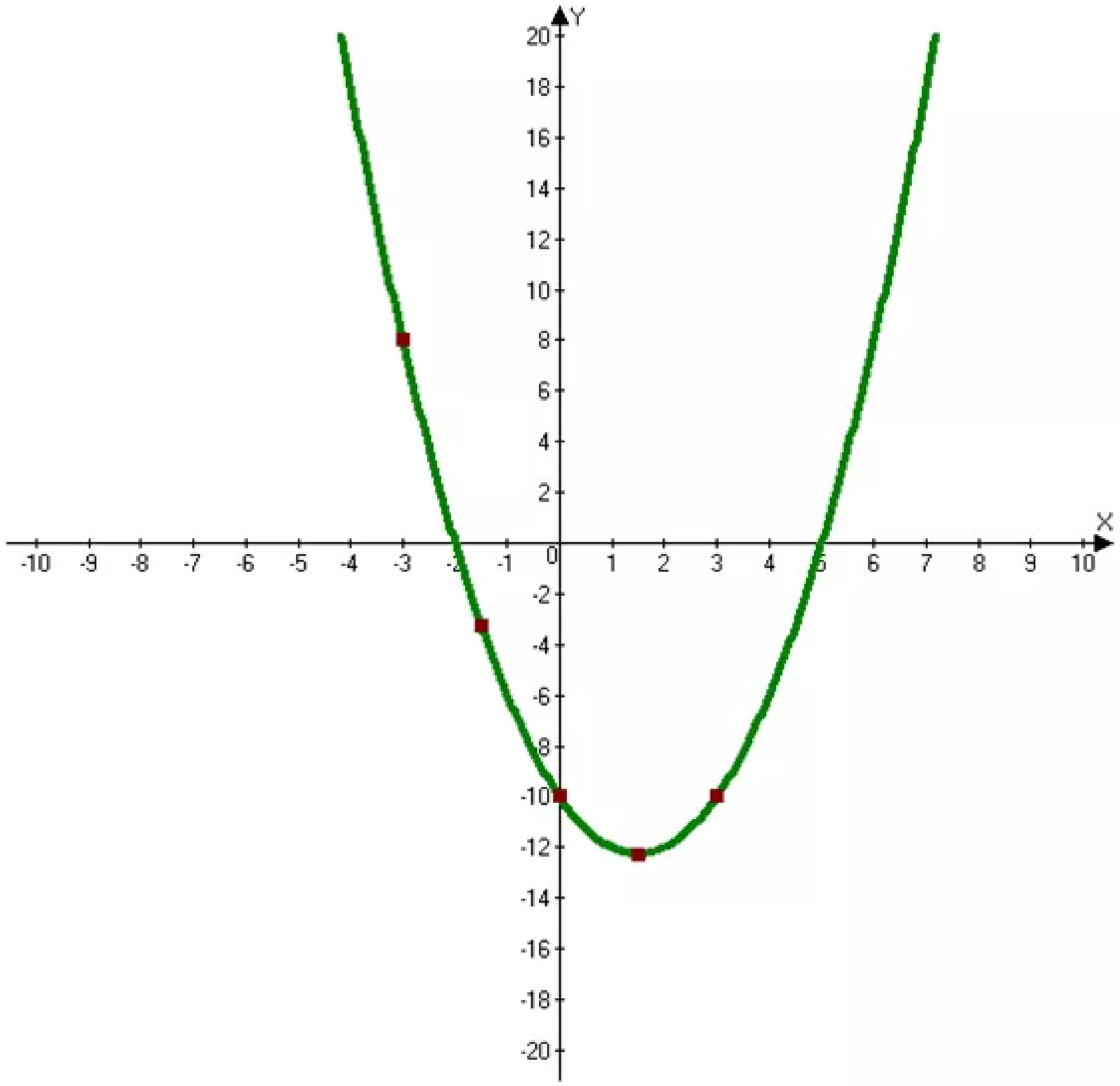
Now, we to construct table for $y = x^2 - 3x - 10$ (green curve)

We can substitute different value of x in the original equation $y = x^2 - 3x - 10$ .we obtained the different" y "value plotting these ordered pairs and connect them we get a smooth curve.

Table for $y = x^2 - 3x - 10$

x	$x^2 - 3x - 10$	y	(x, y)
-3	$(-3)^2 - 3\cdot(3) - 10 = 8$	8	$(-3, 8)$
$-1\cdot 5$	$(-1\cdot 5)^2 - 3\cdot(-1\cdot 5) - 10 = -3\cdot 25$	-3.25	$(-1.5, -3.25)$
0	$(0)^2 - 3\cdot(0)^2 - 10 = -10$	-10	$(0, -10)$
$1\cdot 5$	$(1\cdot 5)^2 - 3(1\cdot 5) - 10 = -12\cdot 25$	-12.25	$(1.5, -12.25)$
3	$(3)^2 - 3\cdot(3) - 10 = -10$	-10	$(3, -10)$

Now, add these all ordered pairs (brown dots), we get the parabola open up ward and minimum point is $(-1.5,-3.25)$.



Answer 61MYS.

Consider the equation $y=-x^2+3x-4$

Claim: Solve the equation $y=-x^2+3x-4$ by table of given value of graph

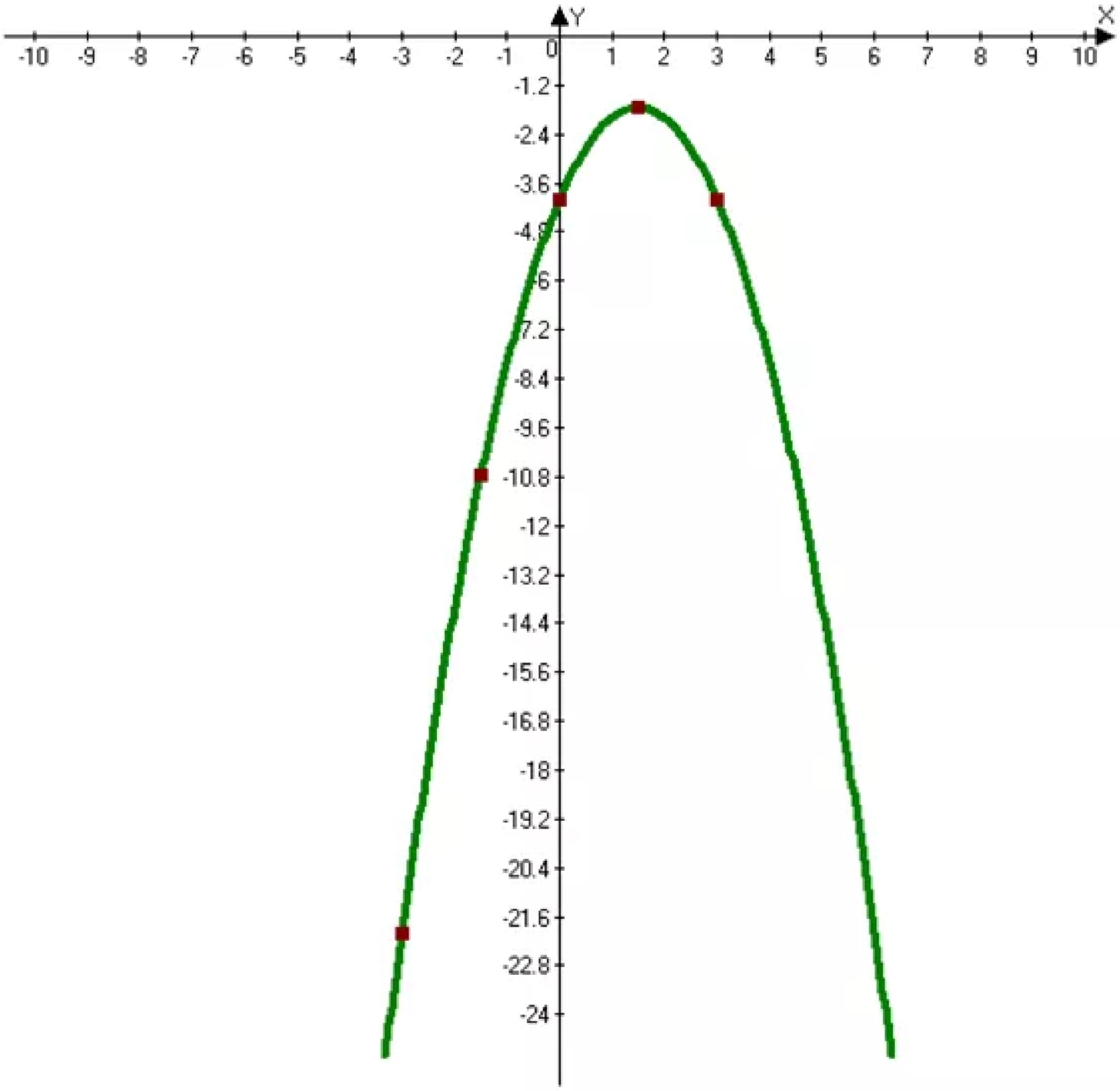
Now, we can construct the table for $y=-x^2+3x-4$ (green curve)

We can substitute the different value of x in the original function $y=-x^2+3x-4$ .we obtained the different function y-axis or y-value plotting these ordered pairs and connect them we get a smooth curve.

Table for $y=-x^2+3x-4$

x	$-x^2+3x-4$	y	(x,y)
-3	$-(-3)^2+3\cdot(-3)-4=-22$	-22	$(-3,22)$
-1.5	$-(-1\cdot5)^2+3\cdot(-1\cdot5)-4=-10.75$	-10.75	$(-1.5,-10.75)$
0	$-(0)^2+3\cdot(0)-4=-4$	-4	$(0,-4)$
$1\cdot5$	$-(1\cdot5)^2+3(1\cdot5)-4=-1.75$	-1.75	$(1.5,-1.75)$
3	$-(3)^2+3\cdot(3)-4=-4$	-4	$(3,-4)$

Add these all ordered pairs (brown dots), we get the parabola these parabolas open down ward and maximum point is $(1.5, -1.75)$.



Answer 62MYS.

Consider the monomials $14a^2b^3, 20a^3b^2c, 35ab^3c,$

Claim: To find GCF of $14a^2b^3, 20a^3b^2c, 35ab^3c,$

Step:- factor each monomials completely

$$14a^2b^3 = 7\cdot 2\cdot a\cdot a\cdot b\cdot b\cdot b$$
$$20a^3b^2c = 5\cdot 2\cdot 2\cdot a\cdot a\cdot a\cdot b\cdot b\cdot c$$
$$35ab^3c = 7\cdot 5\cdot a\cdot b\cdot b\cdot c$$

Now we have out the common factors form each monomial

i.e.

from $14a^2b^3 = 7\cdot 2\cdot \quad b \quad b \quad b \quad a \quad a -$

from $20a^3b^2c = 5\cdot 2\cdot 2 \quad b \quad b \quad c \quad a \quad a \quad a$

from $35ab^2c = 5\cdot 7\cdot a \quad b \quad b \quad c \quad a$

$b \quad b \quad a$

Common factor $= b\cdot b\cdot a$

$= b^2\cdot a$

Greatest common factor of $14a^2b^3, 20a^3b^2c, 35ab^3c$ is $\boxed{b^2a}$

Answer 63MYS.

Consider the monomials $32m^2n^3$

$8m^2n$

$56m^3n^2$

Factorization of each monomial complete

$$\begin{aligned}32m^2n^3 &= 32\cdot m\cdot n\\ &= 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot m\cdot m\cdot n\cdot n\cdot n\end{aligned}$$

$$\boxed{32m^2n^3 = 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot m\cdot m\cdot n\cdot n\cdot n}$$

$$\begin{aligned}8m^2n &= 8\cdot m^2\cdot n\\ &= 2\cdot 2\cdot 2\cdot m\cdot m\cdot n\end{aligned}$$

$$\boxed{8m^2n = 2\cdot 2\cdot 2\cdot m\cdot m\cdot n}$$

$$\begin{aligned}56m^3n^2 &= 56\cdot m^3\cdot n^2\\ &= 2\cdot 2\cdot 2\cdot 7\cdot m\cdot m\cdot m\cdot n\cdot n\end{aligned}$$

$$\boxed{56m^3n^2 = 2\cdot 2\cdot 2\cdot 7\cdot m\cdot m\cdot m\cdot n\cdot n}$$

Take out the common factors from each monomial

$$
\begin{array}{rccccccccccc}
32m^2n^3 = & 2 & 2 & 2 & 2 & 2 & \cdot & m & m & n & n & n \\
8m^2n = & 2 & 2 & 2 & & & & m & m & n & & \\
56m^3n^2 = & 2 & 2 & 2 & 7 & m & & m & m & n & n & \\
 & \downarrow & \downarrow & \downarrow & & & & \downarrow & \downarrow & \downarrow & & \\
 & 2 & 2 & 2 & & & & m & m & n & &
\end{array}
$$

Multiply together the common factor.

Common factor $2 \cdot 2 \cdot 2 \cdot m \cdot m \cdot n = \underline{8m^2n}$

$$\boxed{G.C.F = 8m^2n}$$

Answer 64MYS.

Consider the system of equation is $y = 2x$ and $x + y = 9$

Claim: - solve the equation by usist substitution

Step-1 :- To find x value

i.e. substitute $y = 2x$ in $x + y = 9$

$x + y = 9$ [Original equation]

$x + 2x = 9$ [Replace y by $2x$]

$3x = 9$ [Combine like term]

$\frac{3x}{3} = \frac{9}{3}$ [Divide esch side by 3]

$x = 3$

$\boxed{x = 3}$

Step-2:- Now to find y value substitute

$x = 3x$ in $y = 2x$

$y = 2x$ [Original equation]

$y = 3 \cdot 2$ [Repalce x by 3]

$y = 6$

$y = \boxed{6}$

Answer 65MYS.

Consider the system of equation is $x = y + 3$ and $2x - 3y = 5$

Claim: - Solve the equation by using substitution

Step-1:- To find x values

Substitute $x = y + 3$ is $2x - 3y = 5$

$$2x - 3y = 5 \quad [\text{Original equation}]$$

$$2(y+3) - 3y = 5 \quad [\text{Replace } x \text{ by } 9 + 3x]$$

$$2y + 2 \cdot 3 - 3y = 5 \quad [\text{Use disributive property}]$$

$$2y + 6 - 3y = 5$$

$$2y - 3y + 6 - 6 = 5 - 6 \quad [\text{Substract 6 to each side}]$$

$$(2-3)y = 5 - 6$$

$$-1y = -1$$

$$y = 1 \quad [\text{Multiply} -1 \text{ to each side}]$$

$$\boxed{y = 1}$$

Step-2:- To find x value substitute

$y = 1$ In $x = y + 3$

$$x = 9 + 3 \quad [\text{Original equation}]$$

$$x = 1 + 3 \quad [\text{Repalce } x \text{ by } 3]$$

$$x = 4$$

$$x = \boxed{4}$$

Step-3:- substitute $x=4$ and $y=1$ in the original equation $x=9+3$ and $2x-3y=5$

$$x=y+3 \quad [\text{Orihginal equation }]$$

$$4\overset{?}{=}1+3 \quad [\text{Replace } x \text{ by } 4 \text{ and } y \text{ by } 1]$$

$$4=4 \text{ True}$$

$$2x-3y=5 \quad [\text{Orihginal equation }]$$

$$2(4)-3(1)\overset{?}{=}5 \quad [\text{Replace } x \text{ by } 4 \text{ and } y \text{ by } 1]$$

$$8-3\overset{?}{=}5$$

$$5=5 \text{ True}$$

There $x=4$ and $y=1$ statistics the original equation $x=y+3$ and $2x-3y=5$

Hence, the solution set is $\boxed{(4,1)}$

Answer 66MYS.

Consider the system of equation is $x-2y=3$ and $3x+y=23$

Claim: - Solve the equation by using substitution

Step-1:- To find *x* is $x-2y=3$

$$x-2y=2 \quad [\text{ Original equation }]$$

$$x-2y+2y=3+2y \quad [\text{Add } 2y \text{ to each side }]$$

$$x=3+2y$$

Step-2:- To find *y* value

Now substitute $x=3+2y$ and $3x+y=23$

$$3x+y=23 \quad [\text{Original equation }]$$

$$3(3+2y)+y=23 \quad [\text{Repalce } x \text{ by } 3+2y]$$

$$3\cdot3+3\cdot2y+y=23 \quad [\text{Use distributive property}]$$

$$9+6y+9=23$$

$$6y+y+9-9=23-9 \quad [\text{Substreact } 9 \text{ to ecah side}]$$

$$7y=14$$

$$\frac{7y}{7}=\frac{14}{7} \quad [\text{Divide to each side by } 7]$$

$$y=2$$

$$\boxed{y=2}$$

Step-3:- To find x value

Substitute $y=2$ in $x=3+2y$

$x=3+2y$ in $x=3+2y$

$x=3+2y$ [Original equation]

$x=3+2(2)$ [Replace y by 2]

$x=3+4$

$x=7$

$\boxed{x=7}$

Step-4:- Substitute $x=7$ and $y=2$ in the original equation $x-2y=3$ and $3x+y=23$

$x-2y=3$ [Orihginal equation]

$7-2(2)\overset{?}{=}3$ [Replace x by 2 and y by 7]

$7-4\overset{?}{=}3$

$3=3$ True

$3x+y=23$ [Orihginal equation]

$3(7)+2\overset{?}{=}23$ [Replace x by 7 and y by 2]

$21+2\overset{?}{=}23$

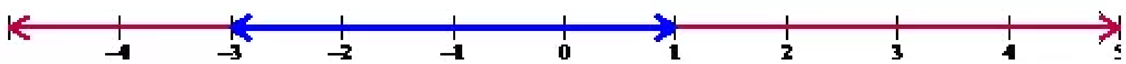
$23=23$ True

There $x=7$ and $y=3$ statistics the original equation $x-2y=3$ and $3x+y=23$

Hence, the solution set is $\boxed{(7,3)}$

Answer 67MYS.

Consider the graph

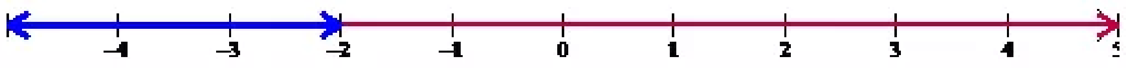


In the graph, the shaded area lies between -3 to 1

i.e. $\boxed{-3<x<1}$

Answer 68MYS.

Consider the graph



In the graph, the shaded area lies between $-\infty$ to -2

i.e. $\boxed{-\infty < x < -2}$

Answer 69MYS.

Consider the function $5x-3y=7$

Claim:. To find the perpendicular to the graph $5x-3y=7$ and passes thought the point $(8,-2)$ rewrite the given function $5x-3y=7$

$5x-3y-7=7-7=0$

Step-1:- To find the perpendicular to the graph $5x-3y=7$ and passes thought the point $(8,-2)$ use the rule u .The perpendicular to the graph $ax+by+c=0$ is $-bx+uy+k=0$

The perpendicular to the graph $5x-3y-7=0$ is $-(-3)x+5y+k=0$

Step-2:- To find the k value in $3x+5y+k=0$.

The perpendicular graph passes the point $(8,-2)$ *i.e.* the point $(8,-2)$ statistics the equation

$$3x+5y+k=0$$

$$3x+5y+k=0 \qquad [\text{Original equaton}]$$

$$3(8)+5(-2)+k=0 \qquad [\text{Repleace } x \text{ by and } y \text{ by} -2\ ]$$

$$24-10+k=0$$

$$14+k=0$$

$$k=-14$$

Now substitute $k=14$ is $3x+5y+k=0$.

We obtain $3x+5y+14=0$

Step-3:- Now write slop intersect form the equation $3x+5y-14=0$.

$$3x+5y-14=0 \qquad [\text{Original equaton}]$$

$$3x+5y-14+14=0+14 \qquad [\text{Add14to each side}]$$

$$3x+5y=14$$

$$-3x+3x+5y=-3x+14 \qquad [\text{Substract 14 to each side}]$$

$$5y=-3x14$$

$$\frac{5y}{5}=\frac{-3x+14}{5} \qquad [\text{Divide to each side by 5}]$$

$$y=\frac{-3}{5}x+\frac{14}{5}$$

The slope intersects form the equation $3x+5y-14=0$ is $\boxed{y=\frac{-3}{5}x+\frac{14}{5}}$

Step-3:- Now write slop intersect form the equation $3x+5y-14=0$.

$$3x+5y-14=0 \qquad [\text{Original equaton}]$$

$$3x+5y-14+14=0+14 \qquad [\text{Add14to each side}]$$

$$3x+5y=14$$

$$-3x+3x+5y=-3x+14 \qquad [\text{Substract 14 to each side}]$$

$$5y=-3x14$$

$$\frac{5y}{5}=\frac{-3x+14}{5} \qquad [\text{Divide to each side by 5}]$$

$$y=\frac{-3}{5}x+\frac{14}{5}$$

The slope intersects form the equation $3x+5y-14=0$ is $\boxed{y=\frac{-3}{5}x+\frac{14}{5}}$

Answer 70MYS.

In the graph, the straight line passes though the point $(1,1)$ and $(-1,-3)$

Claim:- To find the line equation.

Use the formula "The equation of the line and passes thought the (x_1, y_1) and (x_2, y_2) is

$$\frac{(y-y_1)}{y_2-y_1}=\frac{(x-x_1)}{x_2-x_1}$$

$$\frac{(y-y_1)}{y_2-y_1}=\frac{(x-x_1)}{x_2-x_1} \quad \begin{bmatrix}\text{Equation for line and passes} \\ \text{through the tow point} \\ (x_1,y_1)\ (x_2,y_2)\end{bmatrix}$$

$$\frac{y-1}{-3-1}=\frac{x-1}{-1-1} \quad \begin{bmatrix}\text{Replace } y_1 \text{ by } 1,\ y_2 \text{by} -3, \\ x_1 \text{by} 1 \text{ and } x_2 \text{by} 1\end{bmatrix}$$

$$\frac{(y-1)}{-4}=\frac{(x-1)}{-2}$$

$$\frac{-4(y-1)}{-4}=\frac{-4(x-1)}{-2} \quad [\text{Multiply} -4 \text{ on each side}]$$

$$y-1=2(x-1)$$

$$y-1=2x-2 \quad [\text{Use distributive peoperty }]$$

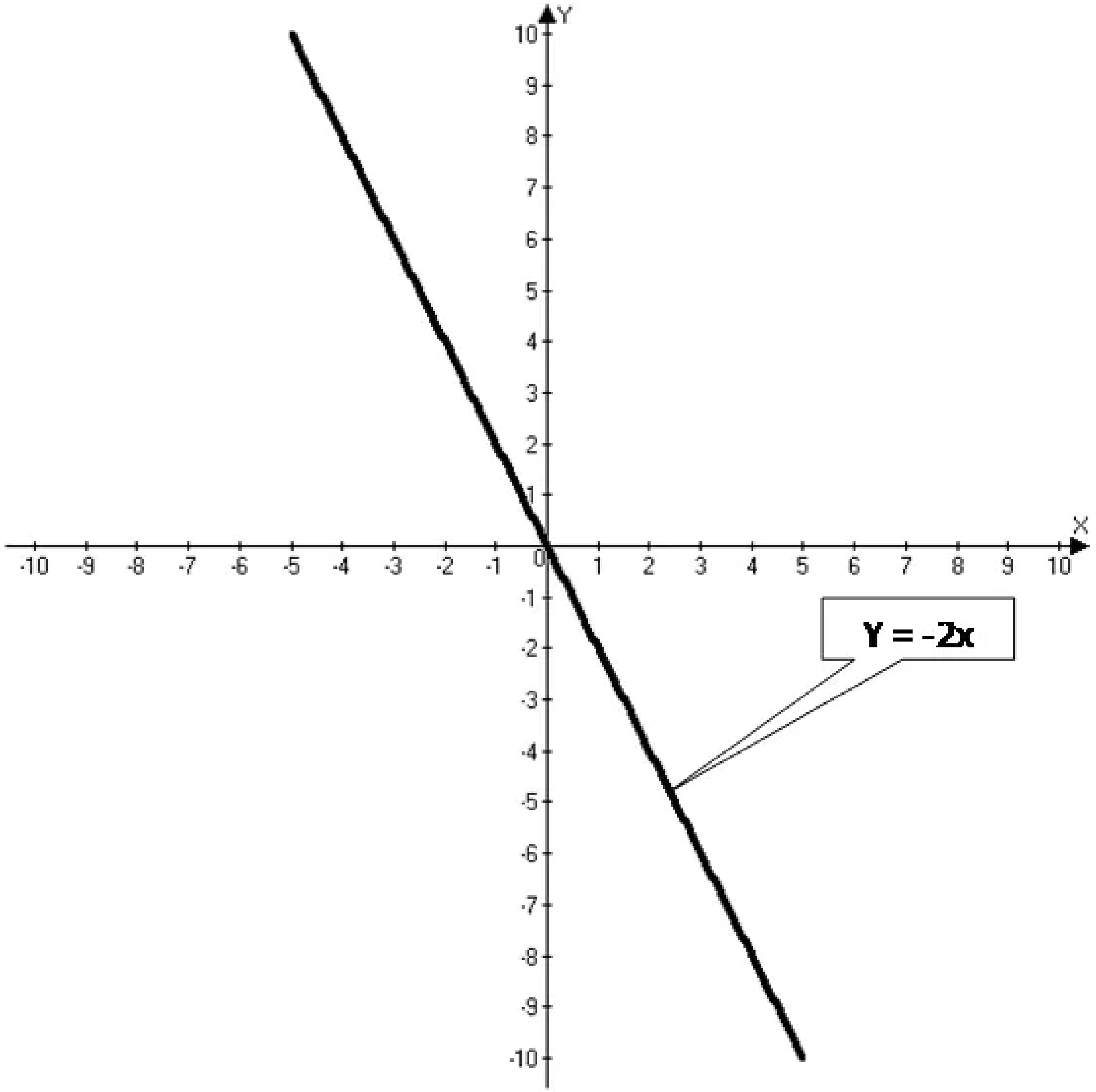
$$y-1+1=2x-2+1 \quad [\text{Add 1to each side}]$$

$$y=2x-1$$

Hence the line equation is $y=2x-1$

Answer 71MYS.

Consider the following graph



In the graph, the straight line passes though the point $(-1,2)$ and $(1,-2)$

Claim:- To find the line equation.

Use the formula "The equation of the line and passes thought the (x_1,y_1) and (x_2,y_2) is

$$\frac{(y-y_1)}{y_2-y_1}=\frac{(x-x_1)}{x_2-x_1}$$

$$\frac{(y-y_1)}{y_2-y_1}=\frac{(x-x_1)}{x_2-x_1} \qquad \left[\begin{array}{l}\text{Equation for line and passes}\\ \text{through the tow point}\\ (x_1,y_1)\ \&(x_2,y_2)\end{array}\right]$$

$$\frac{y-2}{-2-2}=\frac{x-(-1)}{1-(-1)} \qquad \left[\begin{array}{l}\text{Replace } y_1 \text{by} 2,\ y_2 \text{by} -2,\\ x_1 \text{by} -1 \text{ and } x_2 \text{by } 1\end{array}\right]$$

$$\frac{y-2}{-4}=\frac{x+1}{1+1}$$

$$\frac{y-2}{-4}=\frac{x+1}{2}$$

$$\frac{-4(y-2)}{-4}=\frac{-4(x+1)}{2} \qquad [\text{Multiply} -4 \text{ on each side}]$$

$$y-2=-2(x+1)$$

$$y-2=2x-2 \qquad [\text{Use distributive peoperty}]$$

$$y-2+2=2x-2+2 \qquad [\text{Add 2to each side}]$$

$$y=-2x$$

Hence the line equation is $\boxed{y=-2x}$

Answer 72MYS.

Consider the formula $\sqrt{b^2-4ac}$

Claim:- To find value of $\sqrt{b^2-4ac}$,When

$a=1, b=-2, \text{and } c=-15$

$$\sqrt{b^2-4ac}=\sqrt{(-2)^2-4\cdot1\cdot(-15)} \qquad \left[\begin{array}{l}\text{Replace } a \text{ by } 1,\\ b \text{ by} -2, c \text{ by} -15\end{array}\right]$$

$$=\sqrt{4+60}$$

$$=\sqrt{64}$$

$$=8$$

Therefore $\sqrt{b^2-4ac}$ is $\boxed{8}$ when $a = 1, b=-2, c=-15$

Answer 73MYS.

Consider the formula $\sqrt{b^2 - 4ac}$

Claim:- To find value of $\sqrt{b^2 - 4ac}$,When

$a = 2, b = 7, \text{and } c=3$

$$\sqrt{b^2 - 4ac} = \sqrt{(7)^2 - 4 \cdot 2 \cdot (3)} \qquad \begin{bmatrix} \text{Replace } a \text{ by } 2 \\ b \text{ by} 7, c \text{ by} 3 \end{bmatrix}$$

$$= \sqrt{49 - 24}$$

$$= \sqrt{25}$$

$$= 5$$

There for $\sqrt{b^2 - 4ac}$ is $\boxed{5}$ When $a = 2, b = 7, c = 3$

Consider the formula $\sqrt{b^2 - 4ac}$

Claim:- To find value of $\sqrt{b^2 - 4ac}$,When

$a = 1, b = 5, \text{and } c= -2$

$$\sqrt{b^2 - 4ac} = \sqrt{(5)^2 - 4 \cdot 1 \cdot (-2)} \qquad \begin{bmatrix} \text{Replace } a \text{ by } 1 \\ b \text{ by } 5, c \text{ by} -2 \end{bmatrix}$$

$$= \sqrt{25 + 8}$$

$$= \sqrt{33}$$

$$= 5.8$$

There for $\sqrt{b^2 - 4ac}$ is $\boxed{5.8}$ When $a = 1, b = 5, c = -2$

Answer 75MYS.

Consider the formula $\sqrt{b^2 - 4ac}$

Claim:- To find value of $\sqrt{b^2 - 4ac}$,When

$a = -2, b = 7, \text{and } c=5$

$$\sqrt{b^2 - 4ac} = \sqrt{(7)^2 - 4 \cdot (-2) \cdot 5} \qquad \begin{bmatrix} \text{Replace } a \text{ by } -2 \\ b \text{ by} 7, c \text{ by} 5 \end{bmatrix}$$

$$= \sqrt{49 + 40}$$

$$= \sqrt{89}$$

$$= 9.4$$

There for $\sqrt{b^2 - 4ac}$ is $\boxed{9.4}$ When $a = -2, b = 7, c = 5$