Surface Areas and Volumes

MATHEMATICAL REASONING

1. If the height of a cylinder is doubled, by what number must the radius of the base be multiplied so that the resulting cylinder has the same volume as the original cylinder?

(a) 4 (b) $\frac{1}{\sqrt{2}}$

(c) 2 (d) $\frac{1}{2}$

2. A metal sheet 27 cm long, 8 cm broad and 1 cm thick is melted into a cube. Find the difference between surface areas of two solids.

(a) 280 cm^2 (b) 284 cm^2

(c) 296 cm^2 (d) 286 cm^2

3. The height of a cone is equal to its base diameter. Then slant height of the cone is

(a) $\sqrt{r^2+h^2}$ (b) $r\sqrt{5}$

(c) $h\sqrt{5}$ (d) $rh\sqrt{5}$

4. The length of the longest rod that can be kept in a cuboidal room of dimensions $10m\times10m\times5m$ is ___.

(a) 16 m (b) 10 m

(c) 15 m (d) 12 m

5. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, then the amount of the beverage that can be poured from the bowl into the cylindrical vessel is ____.

(a) $66\frac{2}{3}\%$ (b) $78\frac{1}{2}\%$

(c) 100% (d) None of these

6. If the length of diagonal of a cube is $\sqrt{12}\,cm$, then the volume of the cube is

(a) $8\sqrt{12}\,cm^3$ (b) $8\,cm^3$

(c) $16\sqrt{2}\,cm^3$ (d) $16\,cm^3$

7. The volume of a cylinder of radius r is 1/4 of the volume of a rectangular box with a square base of side length x. If the cylinder and the box have equal heights, what is the value of r in terms of x ?

(a) $\frac{x^2}{2\pi}$ (b) $\frac{x}{2\sqrt{\pi}}$

(c) $\frac{\sqrt{2x}}{\pi}$ (d) $\frac{x}{2\sqrt{\pi}}$

8. The edge of a cube is 20 cm. How many small cubes of edge 5 cm can be formed from this cube?

(a) 4 (b) 32

(c) 64 (d) 100

9. The volume of two spheres are in the ratio $64:27$. The difference of their surface areas, if the sum of their radii is 7 units, is ___.

(a) $28\pi\,sq.$ units (b) 88 sq. units

(c) $88\pi\,sq.$ units (d) $4\pi\,sq.$ units

10. The radii of two cylinders are in the ratio $2:3$ and their heights are in the ratio of $5:3$. The ratio of their volumes is ___.

(a) $10:17$ (b) $20:27$

(c) $17:27$ (d) $20:37$

EVERYDAY MATHEMATICS

11. A covered wooden box has the inner measures as 115 cm, 75 cm, 35 cm and the thickness of wood is 2.5 cm. Then the volume of the wood is ____

(a) 80000 cu. cm (b) 82125 cu. cm

(c) 84000 cu. cm (d) 85000 cu. cm

12. A spherical ball of lead, 3 cm in diameter is melted and recast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm respectively. The diameter of the third ball is ____.

(a) 2.66 cm (b) 2.5 cm

(c) 3 cm (d) 3.5 cm

13. How many metres of cloth, 5 m wide, will be required to make a conical tent, the radius of whose base is 7 m and height is 24 m?

(a) 550 m (b) 168 m

(c) 110 m (d) 33.6 m

14. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.

(a) 266.11 cm^3 (b) 301.12 cm^3

(c) 242.36 cm^3 (d) 278.34 cm^3

15. Water flows in a tank $150m \times 100m$ mat the base, through a pipe whose cross-section is 2 dm by 1.5 dm at the speed of 15 km per hour. In what time, will the water be 3 metres deep?

(a) 50 hours (b) 150 hours
(c) 100 hours (d) 200 hours

16. A teak wood log is first cut in the form of a cuboid of length 2.3 m, width 0.75 m and of a certain thickness. Its volume is 1.104 m^3. How many rectangular planks of size $2.3\ m \times 0.75\ m \times 0.04\ m$ can be cut from the cuboid?

(a) 16 (b) 64
(c) 68 (d) 4

17. How many bricks, each measuring $25cm \times 11.25\ cm \times 6\ cm$, will be needed to build a wall $8m \times 6m \times 22.5cm$?

(a) 5600 (b) 6000
(c) 6400 (d) 7200

18. A circus tent is cylindrical to a height of 3 metres and conical above it. if its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.

(a) 1996 m (b) 2096 m
(c) 1947 m (d) 1800 m

19. A school provides milk to the students daily in a cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto an height of 12 cm, then how many litres of milk is needed to serve 1600 students?

(a) 739.2 litres (b) 538 litres
(c) 740 litres (d) 400 litres

20. A small village, having a population of 5000, requires 75 litres of water per head per day. The village has got an overhead tank of measurement $40m \times 25m \times 15\ m.$ For how many days will the water of this tank last?

(a) 30 days (b) 32 days
(c) 40 days (d) 45 days

ACHIEVERS SECTION (HOTS)

21. Match the following.

	Column-I		Column-II
(p)	A cylinder of radius 3 cm is inscribed in a sphere of radius 5 cm, then volume of cylinder is ___.	(1)	$38.5cm^3$
(q)	A conical pit of top diameter 3.5 cm is 12 cm deep, the capacity of pit is ___.	(2)	$512\ cm^3$
(r)	The length of a diagonal of a cube is $8\sqrt{3}\,cm.$ then volume of cube is ___.	(3)	$72\pi\ cm^3$
(s)	The capacity of a conical vessel with height 12 cm and slant height 13 cm is___.	(4)	$100\pi\,cm^3$

(a) $(p) \to (2); (q) \to (3); (r) \to (4); (s) \to (1)$
(b) $(p) \to (1); (q) \to (3); (r) \to (2); (s) \to (4)$
(c) $(p) \to (3); (q) \to (1); (r) \to (2); (s) \to (4)$
(d) $(p) \to (4); (q) \to (1); (r) \to (3); (s) \to (2)$

22. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of the water is raised by 6.75 cm. What is the radius of the sphere?

(a) 9 cm (b) 13 cm
(c) 11 cm (d) 15 cm

23. Read the statement carefully and write T for true and 'F' for false.

(i) Volume of a cylinder is three times the volume of a cone on the same base and of same height.
(ii) Volume of biggest sphere in cube of edge 6 cm is $36\pi\ cm^3$.
(iii) Cuboids and cubes are special forms of right prisms.

	(i)	(ii)	(iii)
(a)	T	F	T
(b)	T	T	T
(c)	F	T	F
(d)	F	T	T

24. The internal and external radii of a hollow hemispherical bowl are 15 cm and 16 cm respectively, find the cost of painting the owl at the rate of 35 paise per cm^2, if

(i) the area of the edge of the bowl is ignored.

(ii) the area of the edge of the bowl is taken into account.

	(i)	(ii)
(a)	₹ 1058.20	₹ 1092.30
(b)	₹ 1020.50	₹ 1045
(c)	₹ 1092.50	₹ 1058.20
(d)	₹ 1086.20	₹ 1095.2

25. Study the statements carefully.

Statement-I: If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

Statement-II: Curved surface area is increased when diameter decreases.

Which of the following options hold?

(a) Both Statement-I and Statement-II are true.

(b) Statement-I is true but Statement-II is false.

(c) Statement-I is false but Statement-II is true.

(d) Both Statement-I and Statement-II are false.

Hints & Explanations

1. (b): Let radius and height of original cylinder be r_1 and h_1 respectively

$\therefore$ Volume of original cylinder $= \pi r_1^2 h_1$

Also, let radius of new cylinder be r_2

and height of new cylinder

$= 2\times$(height of original cylinder)

$= 2\times h_1 = 2h_1$

$\therefore$ Volume of new cylinder $= \pi r_2^2.2h_1$

According to question,

Volume of original cylinder

= Volume of new cylinder

$\Rightarrow \pi r_1^2 h_1 = \pi r_2^2.2h_1 \Rightarrow r_1^2 = 2r_2^2 \Rightarrow r_2 = \frac{1}{\sqrt{2}}r_1$

Hence, radius of base of new cylinder must be multiplied by $\frac{1}{\sqrt{2}}$ so that the new cylinder has same volume as original.

2. (d): Volume of the metal sheet

$(27\times 8\times 1)cm^3 = 216 cm^3$

Surface area of the metal sheet

$= (27\times 8 + 8\times 1 + 1\times 27)$

$= 2(216 + 8 + 27) = 502 cm^2$

Volume of cube = Volume of metal sheet

$(\text{side})^2 = 216 cm^3 \Rightarrow side = 6 cm$

Now, surface area of cube $= 6(6)^2 = 216 cm^2$

$\therefore$ Required difference $= (520 - 216)cm^2$

$= 286 cm^2$

3. (b): Let radius of the cone = r

$\therefore$ Height of the cone (h) = diameter = 2r

$\therefore$ Slant height of the cone $(l) = \sqrt{h^2 + r^2}$

$= \sqrt{(2r)^2 + r^2} = \sqrt{5r^2} = \sqrt{5}r$

4. (c) : Given, length of cuboid $(l) = 10m$

Breath of cuboid (b) = 10 m

Height of cuboid (h) = 5 m

As, length of longest rod in cuboid

= Diagonal of cuboid

$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(10)^2 + (10)^2 + (5)^2}$

$= \sqrt{100 + 100 + 25} = \sqrt{225} = 15$

5. (c) : Let radius of hemispherical bowl and cylindrical vessel be r.

Also, r = 50% more than h

$\Rightarrow$ r = 50% of h + h

$\Rightarrow r = \frac{3h}{2}$ ...(i)

Now, volume of bowl $(V_1) = \frac{2}{3}\pi r^3$ ...(ii)

and volume of vessel $(V_2) = \pi r^2 h$ ...(iii)

Dividing eqn. (iii) by (ii), we get

$$\frac{V_1}{V_2} = \frac{\frac{2}{3}\pi r^3}{\pi r^2 h} = \frac{2r}{3h} = \frac{2}{3h}\left(\frac{3h}{2}\right) \quad [\text{ by (i)}]$$

$= 1$

$\Rightarrow \quad V_1 = V_2$

$\therefore$ Volume of bowl = Volume of vessel

The amount of beverage that can be poured into vessel is 100%

6. (b): Let 'a' be the side of cube.

Since length of diagonal $\sqrt{12} cm$

$\therefore \sqrt{a^2 + a^2 + a^2} = \sqrt{12}$

Squaring both sides, we get

$3a^2 = 12 \Rightarrow a^2 = 4 \Rightarrow a = 2$

$\therefore$ Volume of the cube $= 2\times 2\times 2 = 8 cm^3$.

7. (b): Let the height of cylinder and rectangular box be h.

Volume of cylinder $= \pi r^2 h$

$\therefore$ Volume of rectangular box $= x\times x\times h = x^2 h$

According to question,

Volume of cylinder $= \frac{1}{4} \times$ Volume of rectangular box

$\Rightarrow \pi r^2 h = \frac{1}{4} \times x^2 h \Rightarrow r^2 = \frac{x^2}{4\pi}$ or $r = \frac{x}{2\sqrt{\pi}}$

8. (c) : Let 'n' be the number of cubes which can be formed from the given cube.

Volume of big cube = Volume of n smaller abes

$\Rightarrow 20 \times 20 \times 20 = n \times 5 \times 5 \times 5$

$\Rightarrow n = \frac{20 \times 20 \times 20}{5 \times 5 \times 5} = 4 \times 4 \times 4 = 64$

9. (a) : Let r_1 and r_2 be radii of two spheres.

According to question,

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \qquad \ldots(i)$$

Given, $r_1 + r_2 = 7$

From (i) and (ii), we get $r_1 = 4$ units, $r_2 = 3$ units

$\therefore$ Required difference $= 4\pi r_1^2 - 4\pi r_2^2$

$= 4\pi(4^2 - 3^2) = 4\pi \times 7 = 28\pi\, sq.$ units.

10. (b) : let r_1, r_2 be the radius of the radius of two cylinders

$\therefore \frac{r_1}{r_2} = \frac{2}{3}$

Let h_1 and h_2 be height of two cylinders

$\therefore \frac{h_1}{h_2} = \frac{5}{3}$

Now, $\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \cdot \frac{h_1}{h_2}$

$= \left(\frac{2}{3}\right)^2 \cdot \frac{5}{3} = \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$

$\therefore$ Required ration $= 20 : 27$

11. (b) : The inner dimensions of box are 115 cm, 75 cm and 35 cm

$\therefore$ Volume of inner box $= 115 \times 75 \times 35$

$= 301875\, cm^3$

Also, the outer dimensions of box are $(115 + 2 \times 2.5)$cm, $(75 + 2 \times 2.5)$cm and $(35 + 2 \times 2.5)$cm

i.e., 120 cm, 80 and 40 cm

$\therefore$ Volume of outer box $= 120 \times 80 \times 40$

$= 384000\, cm^3$

Now, Volume of wood

= Volume of outer box – Volume of inner box

$= 384000 = 301857 = 82125\, cm^3$.

12. (b) :

13. (c) : Radius of cone (r) = 7 m

Height of cone (h) = 24 m

$\therefore$ Now slant height of the cone, $l = \sqrt{h^2 + r^2}$

$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625}$

$\therefore l = 25m$

Curved surface area of conical tent $= \pi rl$

$= \frac{22}{7} \times 7 \times 25 = 550\, m^2$

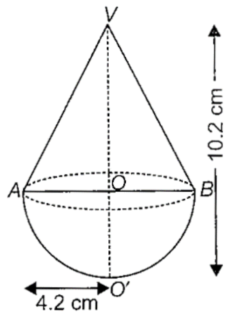
$\therefore$ Required length of cloth

$= \frac{\text{Curved surfce area of conical tent}}{\text{Width of cloth}}$

$= \frac{550}{5} = 110\, m$

14. (a) : Let the radius of the hemisphere and base of cone be rand the height of the conical part of the toy be h. Then, r = OA = 4.2 cm, h = VO = VO' – OO' = (10.2 – 4.2)cm

$\Rightarrow$ h = 6 cm



$\therefore$ Volume of the wooden toy = Volume of the conical part + Volume of the hemi – spherical part

$\left(\frac{1}{3}\pi r^2 h + \frac{2\pi}{3} r^3\right) cm^3$

$= \left[\frac{\pi r^2}{3}(h + 2r)\right] cm^3$

$= \left[\frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times (6 + 2 \times 4.2)\right] cm^2$

$= \left[\frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 14.4\right] cm^3$

$= 266.11\, cm^3$

15. (c) : Suppose in x hours water will be 3 metres deep in the tank.

Volume of water in the tank in x hours

$= (150 \times 100 \times 3)\text{m}^3 = 45000\, m^3$

Area of the cross-section of the pipe

$= \left(\frac{2}{10} \times \frac{1.5}{10}\right) m^2 = \frac{3}{100} m^2$

Volume of water that flows in the tank is x hours = (Area of cross-section of the pipe) × (Speed of water) × (Time)

$= \left(\frac{3}{100} \times 15000 \times x\right) m^3$

[∵ Speed = 15 km/hr = 15000 m/hr]

$= (450x) m^3$

Since, the volume of water in the tank is equal to the volume of water that flows in the tank in x hours.

∴ $450x = 45000 \Rightarrow x = 100$ hours.

∴

16. (a) : Let the thickness of the log be x metres.

Since, volume $= 1.104 m^3$

$\Rightarrow 2.3 \times 0.75 \times x = 1.104$

$\Rightarrow x = \frac{1.104}{2.3 \times 0.75} = 0.64 m$

Since the length and breadth of each rectangular plank is the same as that of the cuboid.

∴ No. of rectangular planks

$= \frac{\text{Thickness of cuboid}}{\text{Thickness of each plank}} = \frac{0.64}{0.04} = \frac{64}{4} = 16$

17. (c) : Length of wall = 8m = 800 cm

Breadth of wall = 6m = 600 cm

Height of wall = 22.5 cm

∴ Volume of wall $= (800 \times 600 \times 22.5) \text{cm}^3$

Also, volume of each brick $= (25 \times 11.25 \times 6) cm^3$

Let 'n' be number of bricks required to build the wall.

∴ Volume of wall $= n \times$ volume of each brick

$\Rightarrow n = \frac{800 \times 600 \times 22.5}{25 \times 11.25 \times 6} = 6400$

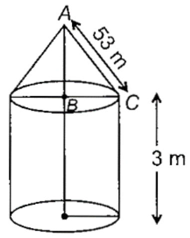
18. (c) : For cylindrical part,

Radius (r) $= \frac{105}{2} m$

Height (h) = 3 m

For conical part. Slant height $(l) = 53 m$

Radius (r) $= \frac{105}{2} m$



∴ Total curved surface area of tent $= 2\pi rh + \pi rl$

$= \pi r(2h + l) = \frac{22}{7} \times \frac{105}{2} \times (6 + 53)$

$= (11 \times 15 \times 59) m^2$

Hence, length of canvas

$= \frac{\text{Total curved surface area of tent}}{\text{Widt of cloth}}$

$= \frac{11 \times 5 \times 59}{5} = 1947 m$

19. (a) : Demeter of a glass = 7 cm

$\Rightarrow$ Radius of the glass (r) $= \frac{7}{2} cm$

Height of a glass filled with milk (h) = 12 cm

∴ Milk contained in the cylindrical glass

$= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 cm^3 = 462 cm^3$

Now, quantity of milk required for 1600 students

$= 462 \times 1600 cm^3 = 739200 cm^3$

$= \frac{739200}{1000}$ litres $\left[\because 1 cm^3 = \frac{1}{1000} \text{litre}\right]$

= 739.2 lires

20. (c) : Total population of village = 5000

Water required per head per day = 75 litres

∴ Volume of water required for a small village

Per day $= 5000 \times 75$ lires

$= 375000 \text{ litres} = \frac{375000}{1000} m^3 = 375 m^3$

$[\because 1 \text{m}^3 = 1000 \text{ litres}]$

Volume of an overhead tank

$= (40 \times 25 \times 15) m^3 = 15000 m^3$

∴ Number of days

$= \frac{\text{Volume of overhead tank}}{\text{Volume of water required for a small village per day}}$

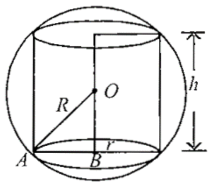
$= \frac{15000}{375} = 40$

Hence, water of the tank will last for 40 days.

21. (c) : (P) Let height and radius of the cylinder be h an r respectively.

Let R be the radius of sphere.

In ΔOAB



$(OA)^2 = (OB)^2 + (AB)^2$

$\Rightarrow R^2 = \left(\frac{h}{2}\right)^2 + r^2$

$\Rightarrow R^2 - r^2 = \frac{h^2}{4}$

$\Rightarrow h = 2\sqrt{R^2 - r^2}$

$= 2\sqrt{(5)^2 - (3)^2} = 2 \times 4 = 8\,cm$

$\therefore$ Volume of cylinder $= \pi r^2 h$

$= \pi \times 3 \times 3 \times 8 = 72\pi\,cm^3$

(Q) Radius of conical pit (r) $= \frac{3.5}{2}\,cm$

Depth of conical pit (h) = 12 cm

Volume of conical pit $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 12 = 38.5\,cm^3$

(R) Length of diagonal of cube of side 'a' $= \sqrt{3a}$

$\therefore\ \sqrt{3a} = 8\sqrt{3} \Rightarrow a = 8\,cm$

Volume of cube $= (\text{side})^3 = 512\,cm^3$

(S) Volume of conical vessel

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (l^2 - h^2) \times 12$

$= \frac{1}{3} \times \pi \times ((13)^2 - (12)^2) \times 12$

$= \frac{1}{3} \times \pi \times 25 \times 12 = 100\pi\,cm^3$

22. (a) : Radius of the cylindrical tub (r) = 12 cm.
Height of water level in the tub (h) = 20 cm.
When the Spherion iron ball is dropped into the tub, the level of water rises by 6.75 cm.

$\therefore$ Volume increased in the cylindrical part

$= \pi r^2 h = \pi \times 12 \times 12 \times 6.75\,cm^3$

Thus, the volume of the spherical ball

$= \pi \times 12 \times 12 \times 6.75\,cm^3$

Let R be the radius of the spherical ball.

$\therefore$ Volume of spherical ball $= \frac{4}{3}\pi R^3$

Now, $\frac{4}{3}\pi R^3 = \pi \times 12 \times 12 \times 6.75$

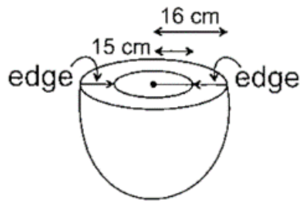
$\Rightarrow R^3 = \frac{3 \times 12 \times 12 \times 6.75}{4} = R^3 = 729$

$\Rightarrow R = 9$

Hence, the radius of the sphere = 9 cm.

23. (b) :

24. (a) : Internal radius of the bowl (r) = 15 cm
External radius of the bowl (R) = 16 cm
(i) If area of edge is ignored, then Surface area of bowl



$= 2\pi R^2 + 2\pi r^2$

$= 2\pi(R^2 + r^2)$

$= 2\pi(16^2 + 15^2)$

$= 22 \times \frac{22}{7} \times 481$

$= \frac{21164}{7}\,cm^2$

Cost of painting $1\,cm^2$ area of the bowl

= ₹ 0.35

Cost of painting $\frac{21164}{7}\,cm^2$ area of the bowl

$= (0.35 \times \frac{21164}{7}\,\text{cm}^2) =$ ₹ 1058.20

(ii) If area of edge is counted, then Surface area of bowl

$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= 2\pi(R^2 + r^2) + \pi(R^2 - r^2)$

$= \pi[(2(16^2 + 15^2) + (16^2 - 15^2)]$

$= \pi(2 \times 481 + 31) = \frac{22}{7} \times 993$

$= \frac{21846}{7}\,cm^2$

Cost of painting area $\frac{21846}{7}\,cm^2$

$= \frac{21846}{7} \times 0.35 =$ ₹ 1092.3

25. (b) : Statement – I Let the diameter of sphere be 2r.

Decreased diameter $= \left(2r - \frac{25}{100} \times 2r\right)$

$= 2r - \frac{r}{2} = \frac{3r}{2}$

New diameter $= \frac{3r}{2}$.

So, radius $(r) = \frac{3r}{4}$

Curved surface area of sphere $= 4\pi r^2$

New curved surface area $= 4\pi\left(\frac{3r}{4}\right)^2$

Now, decreased in surface area

$$= \frac{4\pi r^2 - 4\pi\left(\frac{3r}{4}\right)^2}{4\pi r^2} \times 100\%$$

$$= \frac{4\pi r^2\left(1 - \frac{9}{16}\right)}{4\pi r^2} \times 100\%$$

$$= \frac{7}{16} \times 100\% = 43.75\%$$

Statement II: If diameter is decreased, then curved surface area is also decreased.