23. Probability

EXERCISE 23

Question 1.
A die is thrown, find the probability of getting:
(i) a prime number
(ii) a number greater than 4
(iii) a number not greater than 4.

Solution:
A die has six numbers : 1, 2, 3, 4, 5, 6
∴ Number of possible outcomes = 6

(*i*) a prime number

Number of favourable outcomes = a prime number = 1, 3, 5 which are 3 in numbers

$$P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

(*ii*) Number of favourable outcome = greater than four i.e. two number 5 and 6

$$\therefore\ P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

(*iii*) Number of favourable outcome = not greater than 4 or numbers will be 1, 2, 3, 4 which are 4 in numbers

$$\therefore\ P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{4}{6} = \frac{2}{3}$$

Question 2.
A coin is tossed. What is the probability of getting:
(i) a tail? (ii) ahead?
Solution:
On tossing a coin once,
Number of possible outcome = 2

(i) Favourable outome getting a tail = 1

$\Rightarrow$ number of favourable outcome = 2

$$\therefore \text{P(E)} = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{1}{2}$$

(ii) a head

Similarly, favourable outcome getting a head = 1

But number of possible outcome = 2

$$\therefore \text{P(E)} = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{1}{2}$$

Question 3.
A coin is tossed twice. Find the probability of getting:
(i) exactly one head (ii) exactly one tail
(iii) two tails (iv) two heads
Solution:

(i) Exactly one head

Possible number of favourable outcomes = 2

(*i.e.* TH and HT)

Total number of possible outcomes = 4

$$\therefore \text{P(E)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{2}{4} = \frac{1}{2}$$

(ii) Exactly one tail

Possible number of favourable outcomes = 2

(*i.e.* TH and HT)

Total number of possible outcomes = 4

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{2}{4} = \frac{1}{2}$$

(*iii*) Two tails

Possible number of favourable outcomes = 1

(*i.e.* TT)

Total number of possible outcomes = 4

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{1}{4}$$

(*iv*) Two heads

Possible number of favourable outcomes = 1

(*i.e.* HH)

Total number of possible outcomes = 4

$$\therefore P(E) = \frac{1}{4}$$

Question 4.

A letter is chosen from the word 'PENCIL' what is the probability that the letter chosen is a consonant?

Solution:

Total no. of letters in the word 'PENCIL = 6

Total Number of Consonant = 'PNCL' i.e. 4

$$P(E) = \frac{\text{Total No. of consonants}}{\text{Total No. of letters in the word PENCIL}}$$

$$= \frac{4}{6} = \frac{2}{3}$$

Question 5.
A bag contains a black ball, a red ball and a green ball, all the balls are identical in shape and size. A ball is drawn from the bag without looking into it. What is the probability that the ball drawn is:
(i) a red ball
(ii) not a red ball
(iii) a white ball.
Solution:
Total number of possible outcomes = 3

$$\therefore \ P(E) = \frac{1}{3}$$

(*ii*) Not a red ball

Number of favourable outcomes

= Green ball + Black ball

= 1 + 1 + 2

$$\therefore \ P(E) = \frac{2}{3}$$

(*iii*) A white ball

Number of favourable outcomes = 0

$$\therefore \ P(E) = \frac{0}{3} = 0$$

Question 6.
6. In a single throw of a die, find the probability of getting a number
(i) greater than 2
(ii) less than or equal to 2
(iii) not greater than 2.
Solution:
A die has six numbers = 1, 2, 3, 4, 5, 6

(*i*) Exactly one head

Possible number of favourable outcomes = 2

(*i.e.* TH and HT)

Total number of possible outcomes = 4

$$\therefore \ P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{2}{4} = \frac{1}{2}$$

(*ii*) Exactly one tail

Possible number of favourable outcomes = 2

(*i.e.* TH and HT)

∴ Number of possible outcomes = 6

$$\therefore \text{P(E)} = \frac{4}{6} = \frac{2}{3}$$

(*ii*) Less than or equal to 2

Number of favourable outcomes = 1, 2

$$\therefore \text{P(E)} = \frac{2}{6} = \frac{1}{3}.$$

(*iii*) Not greater than 2

Number of favourable outcomes = 1, 2

$$\therefore \text{P(E)} = \frac{2}{6} = \frac{1}{3}$$

Question 7.

A bag contains 3 white, 5 black and 2 red balls, all of the same shape and size. A ball is drawn from the bag without looking into it, find the probability that the ball drawn is:

(i) a black ball.

(ii) a red ball.

(iii) a white ball.

(iv) not a red ball.

(v) not a black ball.

Solution:

In a bag, 3 balls are white

2 balls are red

5 balls are black

Total number of balls = 3 + 2 + 5 = 10

(i) Number of possible outcome of one black ball = 10

and number of favouable outcome of one black ball = 5

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{5}{10} = \frac{1}{2}$$

(*ii*) Number of possible outcome of one red ball = 10

and number of favourable outcome = 2

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{2}{10} = \frac{1}{5}$$

(*iii*) Number of possible outcome of white ball = 10

and number of favourable outcome = 3

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{3}{10}$$

(*iv*) Number of possible outcome = 10

Number of favourable outcome

= 3 + 5 = 8

not a red ball

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{8}{10} = \frac{4}{5}$$

(v) Number of possible outcomes = 10
Number of favourable outcome
not a black ball = 3 + 2 = 5

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{5}{10} = \frac{1}{2}$$

Question 8.
In a single throw of a die, find the probability that the number:
(i) will be an even number.
(ii) will be an odd number.
(iii) will not be an even number.
Solution:
A die has six numbers : 1, 2, 3, 4, 5, 6
∴ Number of possible outcome = 6
(i) Number of favourable outcome = an even number i.e. 2, 4, 6 which are 3 in numbers

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

(*ii*) & (*iii*) Number of favourable outcome
= not an even number i.e. odd numbers
: 1, 3, 5 which are 3 in numbers

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

Question 9.
In a single throw of a die, find the probability of getting :
(i) 8
(ii) a number greater than 8
(iii) a number less than 8
Solution:
On a die the numbers are 1, 2, 3, 4, 5, 6 i.e. six.
∴ Number of possible outcome = 6

(*i*) Number of favourable outcome = 0
(∵ 8 is not possible)

$$\therefore \text{P(E)} = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{0}{6} = 0$$

(*ii*) Number greater than 8 will be 0

$$\therefore \text{P(E)} = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{0}{6} = 0$$

(*iii*) Number less than 8 will be 1, 2, 3, 4, 5, 6

$$\therefore \text{P(E)} = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{6}{6} = 1$$

Question 10.
Which of the following can not be the probability of an event?

(*i*) $\frac{2}{7}$ (*ii*) 3.8

(*iii*) 37% (*iv*) –0.8

(*v*) 0.8 (*vi*) $\frac{-2}{5}$

(*vii*) $\frac{7}{8}$

Solution:
The probability of an event cannot be
(ii) 3.8 i.e. the probability of an even cannot exceed 1.
(iv) i.e. -0.8 and
(vi) -2/5, This is because probability of an even can never be less than 1.

Question 11.
A bag contains six identical black balls. A child withdraws one ball from the bag without looking into it. What is the probability that he takes out:
(i) a white ball,
(ii) a black ball
Solution:
∵ There are 6 black balls in a bag
∴ number of possible outcome = 6
(i) A white ball
As there is no white ball in the bag
∴ Its probability is zero (0) = or P(E) = 0
(ii) a black ball
∴ Number of favourable outcome = 1

$$\therefore \ P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{1}{6}$$

Question 12.
Three identical coins are tossed together. What is the probability of obtaining:
all heads?
exactly two heads?
exactly one head?
no head?

Solution:

Total outcomes = 8

i.e. (H, H, H), (H, H, T), (H, T, H), (T, T, T), (T, H, H), (T, T, H), (H, T, T), (T, H, T)

(*i*) Favourable outcome = *i.e.* (H, H, H)

$\therefore$ P (of getting all heads) = $\frac{1}{8}$

(*ii*) Favourable outcomes = 3 (H, H, T), (H, T, H), (T, H, H)

$\therefore$ P(E) = $\frac{3}{8}$

(*iii*) Favourable outcomes = 3 (H, T, H), (T, T, H), (H, T, T)

$\therefore$ P(E) = $\frac{3}{8}$

(*iv*) Favourable outcomes = 1 *i.e.* (T, T, T)

$\therefore$ P(E) = $\frac{1}{8}$

Question 13.

A book contains 92 pages. A page is chosen at random. What is the probability that the sum of the digits in the page number is 9?

Solution:

Number of pages of the book = 92

Which are from 1 to 92

Number of possible outcomes = 92

$\therefore$ Number of pages whose sum of its page is 9 = 10

i.e. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90

$\therefore$ P(E) = $\frac{10}{92} = \frac{5}{46}$

Question 14.

Two coins are tossed together. What is the probability of getting:
(i) at least one head
(ii) both heads or both tails.

Solution:

∵ A coins has two faces Head and Tail or H, T
∴ Two coins are tossed
∴ Number of coins = 2 x 2 = 4
which are HH, HT, TH, TT

(*i*) At least one head, then

Number of outcomes = 3

$$\therefore \text{P(E)} = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{3}{4}$$

(*ii*) When both head or both tails, then

Number of outcomes = 2

$$\therefore \text{P(E)} = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{2}{4} = \frac{1}{2}$$

Question 15.
From 10 identical cards, numbered 1, 2, 3, …… , 10, one card is drawn at random. Find the probability that the number on the card drawn is a multiple of:
(i) 2 (ii) 3
(iii) 2 and 3 (iv) 2 or 3
Solution:
Total outcomes = 10

i.e. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

(*i*) Favourable outcomes = 5 *i.e.* 2, 4, 6, 8, 10

$$P(E) = \frac{5}{10} = \frac{1}{2}$$

(*ii*) Favourable outcomes = 3 *i.e.* 3, 6, 9

$$P(E) = \frac{3}{10}$$

(*iii*) Favourable outcomes = 1 *i.e.* 6

$$P(E) = \frac{1}{10}$$

(*iv*) Favourable outcomes = 7
i.e. 2, 3, 4, 6, 8, 9, 10

$$P(E) = \frac{7}{10}$$

Question 16.
Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is:
(i) 0
(ii) 12
(iii) less than 12
(iv) less than or equal to 12
Solution:
Total outcomes = 36 i.e.

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

(*i*) Favourable outcomes = 0

$$P(E) = \frac{0}{36} = 0$$

(*ii*) Favourable outcomes = 1 *i.e.* (6, 6)

$$P(E) = \frac{1}{36}$$

(*iii*) Favourable outcomes = 35 [Except (6, 6)]

$$P(E) = \frac{35}{36}$$

(*iv*) Favourable outcomes = 36

$$P(E) = \frac{36}{36} = 1$$

Question 17.

A die is thrown once. Find the probability of getting:
(i) a prime number
(ii) a number greater than 3
(iii) a number other than 3 and 5
(iv) a number less than 6
(v) a number greater than 6.

Solution:

Total outcomes = 6
i.e., 1, 2, 3, 4, 5 and 6

(*i*) Favourable outcomes = 3 *i.e.*, 2, 3, 5

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

(*ii*) Favourable outcomes = 3 *i.e.*, 4, 5, 6

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

(*iii*) Favourable outcomes = 4 *i.e.*, 1, 2, 4, 6

$$P(E) = \frac{4}{6} = \frac{2}{3}$$

(*iv*) Favourable outcomes = 5
i.e., 1, 2, 3, 4, and 5

$$P(E) = \frac{5}{6}$$

(*v*) Favourable outcomes = 0

$$P(E) = \frac{0}{6} = 0$$

Question 18.

Two coins are tossed together. Find the probability of getting:
(i) exactly one tail
(ii) at least one head
(iii) no head
(iv) at most one head

Solution:

Total outcomes = 4
i.e., HH, HT, TT, TH

(*i*) Favourable outcomes = 2 *i.e.*, HT and TH

$$P(E) = \frac{2}{4} = \frac{1}{2}$$

(*ii*) Favourable outcomes = 3
i.e., HH, HT and TH

$$P(E) = \frac{3}{4}$$

(*iii*) Favourable outcomes = 1 *i.e.*, TT

$$P(E) = \frac{1}{4}$$

(*iv*) Favourable outcomes = 3
i.e., HH, HT and TH

$$P(E) = \frac{3}{4}$$