

21. Surface Area, Volume and Capacity (Cuboid, Cube and Cylinder)

EXERCISE 21 (A)

Question 1.

Find the volume and the total surface area of a cuboid, whose :

(i) length = 15 cm, breadth = 10 cm and height = 8 cm.

(ii) $l = 3.5$ m, $b = 2.6$ m and $h = 90$ cm,

Solution:

(i) Length = 15 cm, Breadth = 10 cm, Height = 8 cm.

Volume of a cuboid = Length x Breadth x Height = $15 \times 10 \times 8 = 1200$ cm³.

Total surface area of a cuboid $2(l \times b + b \times h + h \times l) = 2(15 \times 10 + 10 \times 8 + 8 \times 15) = 2(150 + 80 + 120) = 2 \times 350 = 700$ cm²

(ii) Length = 3.5 m Breadth = 2.6 m, Height = 90 cm = $\frac{90}{100}$ m = 0.9 m.

Volume of a cuboid = $l \times b \times h = 3.5 \times 2.6 \times 0.9 = 8.19$ m³

Total surface area of a cuboid = $2(l \times b + b \times h + h \times l) = 2(3.5 \times 2.6 + 2.6 \times 0.9 + 0.9 \times 3.5) = 2(9.10 + 2.34 + 3.15) = 2(14.59) = 29.18$ m²

Question 2.

(i) The volume of a cuboid is 3456 cm³. If its length = 24 cm and breadth = 18 cm ; find its height.

(ii) The volume of a cuboid is 7.68 m³. If its length = 3.2 m and height = 1.0 m; find its breadth.

(iii) The breadth and height of a rectangular solid are 1.20 m and 80 cm respectively. If the volume of the cuboid is 1.92 m³; find its length.

Solution:

(i) Volume of the given cuboid = 3456 cm³.

Length of the given cuboid = 24 cm.

Breadth of the given cuboid = 18 cm

We know,

Length x Breadth x Height = Volume of a cuboid

$\Rightarrow 24 \times 18 \times \text{Height} = 3456$

$\Rightarrow \text{Height} = \frac{3456}{24 \times 18}$

$\Rightarrow \text{Height} = \frac{3456}{432}$

$\Rightarrow \text{Height} = 8$ cm

(ii) Volume of a cuboid = 7.68 m³

Length of a cuboid = 3.2 m

Height of a cuboid = 1.0 m

We know

Length x Breadth x Height = Volume of a cuboid

$3.2 \times \text{Breadth} \times 1.0 = 7.68$

$\Rightarrow \text{Breadth} = \frac{7.68}{3.2 \times 1.0}$

$$\Rightarrow \text{Breadth} = \frac{7.68}{3.2}$$

$$\Rightarrow \text{Breadth} = 2.4 \text{ m}$$

(iii) Volume of a rectangular solid = 1.92 m^3

Breadth of a rectangular solid = 1.20 m

Height of a rectangular solid = $80 \text{ cm} = 0.8 \text{ m}$

We know

Length x Breadth x Height = Volume of a rectangular solid (cubical)

$$\text{Length} \times 1.20 \times 0.8 = 1.92$$

$$\text{Length} \times 0.96 = 1.92$$

$$\Rightarrow \text{Length} = \frac{1.92}{0.96}$$

$$\Rightarrow \text{Length} = \frac{192}{96}$$

$$\Rightarrow \text{Length} = 2 \text{ m}$$

Question 3.

The length, breadth and height of a cuboid are in the ratio $5 : 3 : 2$. If its volume is 240 cm^3 ; find its dimensions. (Dimensions means : its length, breadth and height). Also find the total surface area of the cuboid.

Solution:

Let length of the given cuboid = $5x$

Breadth of the given cuboid = $3x$

Height of the given cuboid = $2x$

Volume of the given cuboid = Length x Breadth x Height

$$= 5x \times 3x \times 2x = 30x^3$$

But we are given volume = 240 cm^3

$$30x^3 = 240 \text{ cm}^3$$

$$\Rightarrow x^3 = \frac{240}{30}$$

$$\Rightarrow x^3 = 8$$

$$\Rightarrow x = 8^{\frac{1}{3}}$$

$$\Rightarrow x = (2 \times 2 \times 2)^{\frac{1}{3}}$$

$$\Rightarrow x = 2 \text{ cm}$$

Length of the given cube = $5x = 5 \times 2 = 10 \text{ cm}$

Breadth of the given cube = $3x = 3 \times 2 = 6 \text{ cm}$

Height of the given cube = $2x = 2 \times 2 = 4 \text{ cm}$

Total surface area of the given cuboid = $2(l \times b + b \times h + h \times l)$

$$= 2(10 \times 6 + 6 \times 4 + 4 \times 10) = 2(60 + 24 + 40) = 2 \times 124 = 248 \text{ cm}^2$$

Question 4.

The length, breadth and height of a cuboid are in the ratio $6 : 5 : 3$. If its total surface area is 504 cm^2 ; find its dimensions. Also, find the volume of the cuboid.

Solution:

Let length of the cuboid = $6x$

Breadth of the cuboid = $5x$

Height of the cuboid = $3x$

Total surface area of the given cuboid = $2(l \times b + b \times h + h \times l)$

$$= 2(6x \times 5x + 5x \times 3x + 3x \times 6x) = 2(30x^2 + 15x^2 + 18x^2)$$
$$= 2 \times 63x^2 = 126x^2$$

But we are given total surface area of the given cuboid = 504 cm^2

$$126x^2 = 504 \text{ cm}^2$$

$$\Rightarrow x^2 = \frac{504}{126}$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \sqrt{4}$$

$$\Rightarrow x = 2 \text{ cm.}$$

Length of the cuboid = $6x = 6 \times 2 = 12 \text{ cm}$

Breadth of the cuboid = $5x = 5 \times 2 = 10 \text{ cm}$

Height of the cuboid = $3x = 3 \times 2 = 6 \text{ cm}$

Volume of the cuboid = $l \times b \times h = 12 \times 10 \times 6 = 720 \text{ cm}^3$

Question 5.

Find the volume and total surface area of a cube whose each edge is :

(i) 8 cm

(ii) 2 m 40 cm.

Solution:

(i) Edge of the given cube = 8 cm

Volume of the given cube = $(\text{Edge})^3 = (8)^3 = 8 \times 8 \times 8 = 512 \text{ cm}^3$

Total surface area of a cube = $6(\text{Edge})^2 = 6 \times (8)^2 = 384 \text{ cm}^2$

(ii) Edge of the given cube = 2 m 40 cm = 2.40 m

Volume of a cube = $(\text{Edge})^3$

Volume of the given cube = $(2.40)^3 = 2.40 \times 2.40 \times 2.40 = 13.824 \text{ m}^3$

Total surface area of the given cube = $6 \times 2.4 \times 2.4 = 34.56 \text{ m}^2$

Question 6.

Find the length of each edge of a cube, if its volume is :

(i) 216 cm^3

(ii) 1.728 m^3

Solution:

(i) $(\text{Edge})^3 = \text{Volume of a cube}$

$(\text{Edge})^3 = 216 \text{ cm}^3$

$\Rightarrow \text{Edge} = (216)^{1/3}$

$\Rightarrow \text{Edge} = (3 \times 3 \times 3 \times 2 \times 2 \times 2)^{1/3}$

$\Rightarrow \text{Edge} = 3 \times 2$

$\Rightarrow \text{Edge} = 6 \text{ cm. Ans.}$

(ii) $(\text{Edge})^3 = \text{Volume of a cube}$

$\therefore (\text{Edge})^3 = 1.728 \text{ m}^3$

$\Rightarrow (\text{Edge})^3 = \frac{1.728}{1.000} = \frac{1728}{1000}$

$\Rightarrow \text{Edge} = \left(\frac{1728}{1000} \right)^{1/3}$

$\Rightarrow \text{Edge} = \left(\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}{10 \times 10 \times 10} \right)^{1/3}$

$\Rightarrow \text{Edge} = \frac{2 \times 2 \times 3}{10}$

$\Rightarrow \text{Edge} = \frac{12}{10} \text{ m}$

$\Rightarrow \text{Edge} = 1.2 \text{ m Ans.}$

Question 7.

The total surface area of a cube is 216 cm². Find its volume.

Solution:

$6(\text{Edge})^2 = \text{Total surface area of a cube}$

$6(\text{Edge})^2 = 216 \text{ cm}^2$

$\Rightarrow (\text{Edge})^2 = \frac{216}{6}$

$\Rightarrow (\text{Edge})^2 = 36$

$\Rightarrow \text{Edge} = \sqrt{36}$

$\Rightarrow \text{Edge} = 6 \text{ cm}$

Volume of the given cube = $(\text{Edge})^3 = (6)^3 = 6 \times 6 \times 6 = 216 \text{ cm}^3$

Question 8.

A solid cuboid of metal has dimensions 24 cm, 18 cm and 4 cm. Find its volume.

Solution:

Length of the cuboid = 24 cm

Breadth of the cuboid = 18 cm

Height of the cuboid = 4 cm

Volume of the cuboid = $l \times b \times h = 24 \times 18 \times 4 = 1728 \text{ cm}^3$

Question 9.

A wall 9 m long, 6 m high and 20 cm thick, is to be constructed using bricks of dimensions 30 cm, 15 cm and 10 cm. How many bricks will be required.

Solution:

Length of the wall = 9 m = $9 \times 100 \text{ cm} = 900 \text{ cm}$

Height of the wall = 6 m = $6 \times 100 \text{ cm} = 600 \text{ cm}$

Breadth of the wall = 20 cm

Volume of the wall = $900 \times 600 \times 20 \text{ cm}^3 = 10800000 \text{ cm}^3$

Volume of one Brick = $30 \times 15 \times 10 \text{ cm}^3 = 4500 \text{ cm}^3$

Number of bricks required to construct the wall = $\frac{\text{Volume of wall}}{\text{Volume of one brick}}$
 $= \frac{10800000}{4500}$
 $= 2400$

Question 10.

A solid cube of edge 14 cm is melted down and recasted into smaller and equal cubes each of edge 2 cm; find the number of smaller cubes obtained.

Solution:

Edge of the big solid cube = 14 cm

Volume of the big solid cube = $14 \times 14 \times 14 \text{ cm}^3 = 2744 \text{ cm}^3$

Edge of the small cube = 2 cm

Volume of one small cube = $2 \times 2 \times 2 \text{ cm}^3 = 8 \text{ cm}^3$

Number of smaller cubes obtained = $\frac{\text{Volume of big cube}}{\text{Volume of one small cube}}$
 $= \frac{2744}{8} = 343$

Question 11.

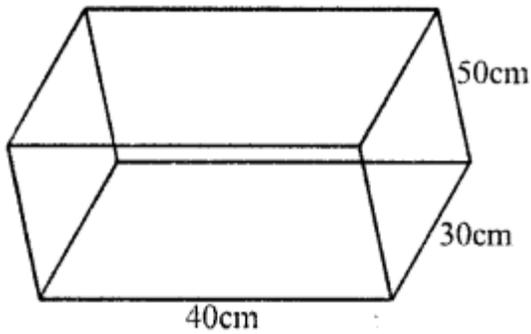
A closed box is cuboid in shape with length = 40 cm, breadth = 30 cm and height = 50 cm. It is made of thin metal sheet. Find the cost of metal sheet required to make 20 such boxes, if 1 m² of metal sheet costs Rs. 45.

Solution:

Length of closed box (l) = 40 cm

Breadth (b) = 30 cm

and height (h) = 50 cm



$$\begin{aligned} \text{Total surface area} &= 2(lb + bh + hl) \\ &= 2(40 \times 30 + 30 \times 50 + 50 \times 40) \text{ cm}^2 \\ &= 2(1200 + 1500 + 2000) \text{ cm}^2 \\ &= 2 \times 4700 = 9400 \text{ cm}^2 \end{aligned}$$

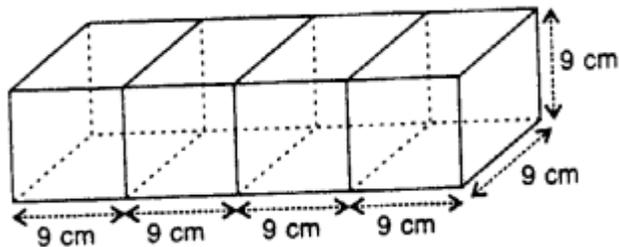
Surface area of sheet used for 20 such boxes = $9400 \times 20 = 188000 \text{ cm}^2$

Cost of 1 m^2 sheet = Rs. 45

$$\begin{aligned} \text{Total cost} &= \frac{18000 \times 45}{100 \times 100 \times 100} \\ &= \text{Rs. } 846 \end{aligned}$$

Question 12.

Four cubes, each of edge 9 cm, are joined as shown below :



Write the dimensions of the resulting cuboid obtained. Also, find the total surface area and the volume of the resulting cuboid.

Solution:

Edge of each cube = 9 cm

(i) Length of the cuboid formed by 4 cubes (l) = $9 \times 4 = 36 \text{ cm}$

Breadth (b) = 9 cm and height (h) = 9 cm

(ii) Total surface area of the cuboid = $2(lb + bh + hl)$

$$= 2(36 \times 9 + 9 \times 9 + 9 \times 36) \text{ cm}^2$$

$$= 2(324 + 81 + 324) \text{ cm}^2$$

$$= 2 \times 729 \text{ cm}^2$$

$$= 1458 \text{ cm}^2$$

(iii) Volume = $l \times b \times h = 36 \times 9 \times 9 \text{ cm}^3 = 2916 \text{ cm}^3$

EXERCISE 21 (B)

Question 1.

How many persons can be accommodated in a big-hall of dimensions 40 m, 25 m and 15 m ; assuming that each person requires 5 m^3 of air?

Solution:

[No. of persons

$$= \frac{\text{Vol. of the hall}}{\text{Vol. of air required for each person}}]$$

Sol. Length of the hall = 40 m

Breadth ,, ,, ,, = 25 m

Height ,, ,, ,, = 15 m

$$\begin{aligned}\text{Volume of the hall} &= L \times B \times H \\ &= 40 \times 25 \times 15 \\ &= 15000 \text{ m}^3\end{aligned}$$

Volume of the air required for each person
= 5 m^3

No. of persons who can be accommodated

$$\begin{aligned}&= \frac{\text{Volume of the hall}}{\text{Volume of air required for each person}} \\ &= \frac{15000 \text{ m}^3}{5 \text{ m}^3} \\ &= 3000 \text{ Ans.}\end{aligned}$$

Question 2.

The dimension of a class-room are; length = 15 m, breadth = 12 m and height = 7.5 m. Find, how many children can be accommodated in this class-room ; assuming 3.6 m^3 of air is needed for each child.

Solution:

Length of the room = 15 m

Breadth of the room = 12 m

Height of the room = 7.5 m

Volume of the room = $L \times B \times H = 15 \times 12 \times 7.5 \text{ m}^3 = 1350 \text{ m}^3$

Volume of air required for each child = 3.6 m^3

No. of children who can be accommodated in the class room.

$$\begin{aligned} &= \frac{\text{Volume of class room}}{\text{Volume of air needed for each child}} \\ &= \frac{1350\text{m}^3}{3.6\text{m}^3} \\ &= 375 \text{ Ans.} \end{aligned}$$

Question 3.

The length, breadth and height of a room are 6 m, 5.4 m and 4 m respectively. Find the area of :

- (i) its four-walls
- (ii) its roof.

Solution:

Length of the room = 6 m

Breadth of the room = 5.4 m

Height of the room = 4 m

(i) Area of four walls = $2(L+B) \times H$
 $= 2(6 + 5.4) \times 4 = 2 \times 11.4 \times 4 = 91.2 \text{ m}^2$

(ii) Area of the roof = $L \times B = 6 \times 5.4 = 32.4 \text{ m}^2$

Question 4.

A room 5 m long, 4.5 m wide and 3.6 m high has one door 1.5 m by 2.4 m and two windows, each 1 m by 0.75 m. Find :

- (i) the area of its walls, excluding door and windows ;
- (ii) the cost of distempering its walls at the rate of Rs.4.50 per m^2 .
- (iii) the cost of painting its roof at the rate of Rs.9 per m^2 .

Solution:

Length of the room = 5 m

Breadth of the room = 4.5 m

Height of the room = 3.6 m

$$\begin{aligned}\text{Area of the roof} &= L \times B \\ &= 5 \times 4.5 \text{ m}^2 \\ &= 22.5 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of four walls} &= 2[L+B] \times H \\ &= 2[5+4.5] \times 3.6 \\ &= 2(9.5) \times 3.6 \\ &= 19 \times 3.6 \\ &= 68.4 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of one door} &= 1.5 \times 2.4 \text{ m}^2 \\ &= 3.60 \text{ m}^2 \\ &= 3.6 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of one window} &= 1 \times 0.75 \text{ m}^2 \\ &= 0.75 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of 2 windows} &= 0.75 \times 2 \text{ m}^2 \\ &= 1.5 \text{ m}^2\end{aligned}$$

(i) Area of four walls excluding door and windows

$$\begin{aligned}&= 68.4 - (3.6 + 1.5) \\ &= 68.4 - 5.1 \\ &= 63.3 \text{ m}^2 \text{ Ans.}\end{aligned}$$

(ii) Cost of distempering four walls @ Rs.4.50 per m²

$$\begin{aligned}&= 63.3 \times 4.50 \\ &= \text{Rs.}284.85 \text{ Ans.}\end{aligned}$$

(iii) Cost of painting the roof @ Rs.9 per m²

$$\begin{aligned}&= 22.5 \times 9 \\ &= \text{Rs.}202.50 \text{ Ans.}\end{aligned}$$

Question 5.

The dining-hall of a hotel is 75 m long ; 60 m broad and 16 m high. It has five – doors 4 m by 3 m each and four windows 3 m by 1.6 m each. Find the cost of :

(i) papering its walls at the rate of Rs.12 per m²;

(ii) carpetting its floor at the rate of Rs.25 per m².

Solution:

Length of the dining hall of a hotel = 75 m

Breadth of the dining hall of a hotel = 60 m

Height of the dining hall of a hotel = 16 m

(i) Area of four walls of the dining hall = $2[L+B] \times H = 2(75 + 60) \times 16$

$$= 2(135) \times 16$$

$$= 270 \times 16$$

$$= 4320 \text{ m}^2$$

$$\text{Area of one door} = 4 \times 3 \text{ m}^2$$

$$= 12 \text{ m}^2$$

$$\text{Area of 5 doors} = 12 \times 5 = 60 \text{ m}^2$$

$$\text{Area of one window} = 3 \times 1.6 = 4.8 \text{ m}^2$$

$$\text{Area of 4 windows} = 4.8 \times 4 = 19.2 \text{ m}^2$$

Area of the walls to be papered

$$= 4320 - (60 + 19.2)$$

$$= 4320 - 79.2$$

$$= 4240.8 \text{ m}^2$$

Cost of papering the walls @ Rs.12 per m^2

$$= 4240.8 \times 12$$

$$= \text{Rs.}50889.60 \text{ Ans.}$$

(ii) Area of floor = $L \times B$

$$= 75 \times 60$$

$$= 4500 \text{ m}^2$$

Cost of carpetting the floor @ Rs.25 per m^2

$$= 4500 \times 25$$

$$= \text{Rs.}112500 \text{ Ans.}$$

Question 6.

Find the volume of wood required to make a closed box of external dimensions 80 cm, 75 cm and 60 cm, the thickness of walls of the box being 2 cm throughout.

Solution:

External length of the closed box = 80 cm

External Breadth of the closed box = 75 cm

External Height of the closed box = 60 cm

External volume of the closed box = $80 \times 75 \times 60 = 360000 \text{ cm}^3$

Internal length of the closed box = $80 - 4 = 76 \text{ cm}$

Internal Breadth of the closed box = $75 - 4 = 71 \text{ cm}$

Internal Height of the closed box = $60 - 4 = 56 \text{ cm}$

Internal volume of the closed box = $76 \times 71 \times 56 \text{ cm} = 302176 \text{ cm}^3$

Volume of wood required to make the closed box = $360000 - 302176 = 57824 \text{ cm}^3$

Question 7.

A closed box measures 66 cm, 36 cm and 21 cm from outside. If its walls are made of metal-sheet, 0.5 cm thick ; find :

- (i) the capacity of the box ;
- (ii) volume of metal-sheet and
- (iii) weight of the box, if 1 cm³ of metal weighs 3.6 gm.

Solution:

External length of the closed box = 66cm.

External breadth of the closed box = 36 cm

External height of the closed box = 21 cm

External volume of the closed box = $66 \times 36 \times 21 = 49896 \text{ cm}^3$

Internal length of the box = $(66 - 2 \times 0.5) = 66 - 1 = 65 \text{ cm}$

Internal breadth of the box = $(36 - 2 \times 0.5) = 36 - 1 = 35 \text{ cm}$

Internal height of the box = $(21 - 2 \times 0.5) = 21 - 1 = 20 \text{ cm}$

Internal Volume of the box = $65 \times 35 \times 20 = 45500 \text{ cm}^3$

(i) Capacity of the box = 45500 cm^3

(ii) Volume of metal sheet of the box = External volume – Internal volume
 $= 49896 - 45500 = 4396 \text{ cm}^3$

(iii) 1 cm³ of metal weigh 3.6 grams.

Weight of the box = $4396 \times 3.6 \text{ gm} = 15825.6 \text{ gm}$

Question 8.

The internal length, breadth and height of a closed box are 1 m, 80 cm and 25 cm. respectively. If its sides are made of 2.5 cm thick wood ; find :

(i) the capacity of the box

(ii) the volume of wood used to make the box.

Solution:

Internal length of the closed box = 1m = 100 cm

.. breadth = 80 cm

.. height = 25 cm

.. volume = $100 \times 80 \times 25$
 $= 200000 \text{ cm}^3$

External length of the box = $(100 + 2 \times 2.5)$
 $= 100 + 5 = 105 \text{ cm}$

External breadth = $(80 + 2 \times 2.5)$
 $= 80 + 5 = 85 \text{ cm}$

External height " " " = $(25 + 2 \times 2.5)$
 $= 25 + 5 = 30 \text{ cm}$

External volume = $105 \times 85 \times 30 \text{ cm}^3$
 $= 267750 \text{ cm}^3$

(i) The capacity of the box

$$\begin{aligned}
 &= 100 \times 80 \times 25 \text{ cm}^3 \\
 &= 200000 \text{ cm}^3 \\
 &= \frac{200000}{100 \times 100 \times 100} \text{ m}^3 \\
 &= 0.2 \text{ m}^3 \text{ Ans.}
 \end{aligned}$$

(ii) The volume of wood used to make the box

$$\begin{aligned}
 &= \text{External volume} - \text{Internal volume} \\
 &= 267750 - 200000 \\
 &= 67750 \text{ cm}^3 \\
 &= \frac{67750}{100 \times 100 \times 100} \text{ m}^3 \\
 &= 0.06775 \text{ m}^3 \text{ Ans.}
 \end{aligned}$$

Question 9.

Find the area of metal-sheet required to make an open tank of length = 10 m, breadth = 7.5 m and depth = 3.8 m.

Solution:

Length of the tank = 10 m

Breadth of the tank = 7.5 m

Depth of the tank = 3.8 m

Area of four walls = $2[L+B] \times H = 2(10 + 7.5) \times 3.8$
 $= 2 \times 17.5 \times 3.8 = 35 \times 3.8 = 133 \text{ m}^2$

Area of the floor = $L \times B = 10 \times 7.5 = 75 \text{ m}^2$

Area of metal sheet required to make the tank = Area of four walls + Area of floor = $133 \text{ m}^2 + 75 \text{ m}^2 = 208 \text{ m}^2$

Question 10.

A tank 30 m long, 24 m wide and 4.5 m deep is to be made. It is open from the top. Find the cost of iron-sheet required, at the rate of ₹ 65 per m^2 , to make the tank.

Solution:

Length of the tank = 30 m

Width of the tank = 24 m

Depth of the tank = 4.5 m

Area of four walls of the tank = $2[L+B] \times H = 2(30 + 24) \times 4.5 = 2 \times 54 \times 4.5 \text{ m}^2 = 486 \text{ m}^2$

Area of the floor of the tank = $L \times B = 30 \times 24 = 720 \text{ m}^2$

Area of Iron sheet required to make the tank = Area of four walls + Area of floor = $486 + 720 = 1206 \text{ m}^2$

Cost of iron sheet required @ Rs. 65 per $\text{m}^2 = 1206 \times 65 = \text{Rs. } 78390$

EXERCISE 21(C)

Question 1.

The edges of three solid cubes are 6 cm, 8 cm and 10 cm. These cubes are melted and recast into a single cube. Find the edge of the resulting cube.

Solution:

Edge of first solid cube = 6 cm

$$\text{Volume} = (6)^3 = 216 \text{ cm}^3$$

Edge of second cube = 8 cm

$$\text{Volume} = (8)^3 = 512 \text{ cm}^3$$

Edge of third cube = 10 cm

$$\text{Volume} = (10)^3 = 1000 \text{ cm}^3$$

$$\text{Sum of volumes of three cubes} = 216 + 512 + 1000 = 1728 \text{ cm}^3$$

Let a be the edge of so formed cube volume = a^3

$$a^3 = 1728 = (12)^3$$

$$a = 12 \text{ cm}$$

Question 2.

Three solid cubes of edges 6 cm, 10 cm and x cm are melted to form a single cube of edge 12 cm, find the value of x .

Solution:

Edge of first cube = 6 cm

$$\text{Volume} = (6)^3 = 216 \text{ cm}^3$$

Edge of second cube = 10 cm

$$\text{Volume} = (10)^3 = 1000 \text{ cm}^3$$

Edge of third cube = x

$$\text{Volume} = x^3$$

Edge of resulting cube = 12 cm

$$\text{Volume} = (12)^3 = 1728 \text{ cm}^3$$

$$216 + 1000 + x^3 = 1728$$

$$x^3 = 1728 - 216 - 1000 = 512 = (8)^3$$

$$x = 8$$

Edge of third cube = 8 cm

Question 3.

The length of the diagonals of a cube is $8\sqrt{3}$ cm.

Find its:

- (i) edge
- (ii) total surface area
- (iii) Volume

Solution:

(i) Length of diagonal of a cube = $8\sqrt{3}$ cm

Length of edge = $\frac{8\sqrt{3}}{\sqrt{3}} = 8$ cm

(ii) Total surface area = $6a^2 = 6 \times 8^2 = 6 \times 64 \text{ cm}^2 = 384 \text{ cm}^2$

(iii) Volume = $a^3 = (8)^3 = 512 \text{ cm}^3$

Question 4.

A cube of edge 6 cm and a cuboid with dimensions 4 cm x x cm x 15 cm are equal in volume. Find:

- (i) the value of x.
- (ii) total surface area of the cuboid.
- (iii) total surface area of the cube.
- (iv) which of these two has greater surface and by how much?

Solution:

Edge of a cube = 6 cm

Volume = $a^3 = (6)^3 = 216 \text{ cm}^3$

Dimensions of a cuboid = 4 cm x x cm x 15 cm

Volume = $60x \text{ cm}^3$

Volume of both is equal

$$(i) \therefore 60x = 216 \Rightarrow x = \frac{216}{60} = \frac{36}{10}$$

$$\therefore x = 3.6 \text{ cm}$$

(ii) Total surface area of cuboid

$$\begin{aligned} &= 2[lb + bh + hl] \\ &= 2[4 \times 3.6 + 3.6 \times 15 + 15 \times 4] \text{ cm}^2 \\ &= 2[14.4 + 54.0 + 60] \text{ cm}^2 \\ &= 128.4 \times 2 = 256.8 \text{ cm}^2 \end{aligned}$$

(iii) Total surface area of cube

$$= 6a^2 = 6(6)^2 = 6 \times 36 = 216 \text{ cm}^2$$

(iv) Difference of surface areas = $256.8 - 216$

$$= 40.8 \text{ cm}^2$$

\therefore Surface area of cuboid is greater

Question 5.

The capacity of a rectangular tank is 5.2 m^3 and the area of its base is $2.6 \times 10^4 \text{ cm}^2$; find its height (depth).

Solution:

Capacity of a tank = 5.2 m^3

and area of its base = $2.6 \times 10^4 \text{ cm}^2$

$$= \frac{2.6 \times 10000}{100 \times 100} = 2.6 \text{ m}^2$$

$$\Rightarrow lb = 2.6 \text{ m}^2$$

$$\text{and } lbh = 5.2 \text{ m}^3$$

$$\therefore \text{Height } (h) = \frac{5.2}{2.6} = 2 \text{ m}$$

Question 6.

The height of a rectangular solid is 5 times its width and its length is 8 times its height. If the volume of the wall is 102.4 cm^3 , find its length.

Solution:

Height of rectangular solid = $5 \times \text{width}$

and length = $8 \times \text{height} = 8 \times 5 \times \text{width} = 40 \times \text{width}$

Volume = 102.4 cm^3

Let width = w

Then height = $40w$

and height = $5w$

$$\therefore w \times 40w \times 5w = 102.4$$

$$w^3 = \frac{102.4}{40 \times 5} = 0.512$$

$$= (0.8)^3$$

$$\therefore w = 0.8$$

$$\therefore \text{Length} = 40w = 40 \times 0.8 = 32 \text{ cm}$$

Question 7.

The ratio between the lengths of the edges of two cubes are in the ratio $3 : 2$. Find the ratio between their:

(i) total surface area

(ii) volume.

Solution:

Ratio in edges of two cubes = $3:2$

Let edge of first cube = $3x$

Then edge of second cube = $2x$

(i) Now total surface area of first cube = $6 \times (3x)^2 = 6 \times 9x^2 = 54x^2$

and of surface area of second cube = $6 \times (2x)^2 = 6 \times 4x^2 = 24x^2$

$$\text{Ratio} = 54x^2 : 24x^2 = 9:4$$

$$\text{(ii) Volume of first cube} = (3x)^3 = 27x^3$$

$$\text{and second cube} = (2x)^3 = 8x^3$$

$$\text{Ratio} = 27x^3 : 8x^3 = 27 : 8$$

Question 8.

The length, breadth and height of a cuboid (rectangular solid) are 4 : 3 : 2.

(i) If its surface area is 2548 cm², find its volume.

(ii) If its volume is 3000 m³, find its surface area.

Solution:

$$\text{Surface area of cuboid} = 2548 \text{ cm}^2$$

Ratio in length, breadth and height of a cuboid = 4 : 3 : 2

Let length = 4x, Breadth = 3x and height = 2x

$$\therefore \text{Surface area} = 2(4x \times 3x + 3x \times 2x + 2x \times 4x)$$

$$= 2(12x^2 + 6x^2 + 8x^2)$$

$$= 2 \times 26x^2 = 52x^2$$

$$\therefore 52x^2 = 2548$$

$$x^2 = \frac{2548}{52} = 49 = (7)^2$$

$$\therefore x = 7$$

$$\therefore \text{Length} = 4x = 4 \times 7 = 28 \text{ cm}$$

$$\text{Breadth} = 3x = 3 \times 7 = 21 \text{ cm}$$

$$\text{and height} = 2x = 2 \times 7 = 14 \text{ cm}$$

$$\therefore \text{Volume} = lbh$$

$$= 28 \times 21 \times 14 \text{ cm}^3 = 8232 \text{ cm}^3$$

(ii) If volume = 3000 m³

$$\Rightarrow 4x \times 3x \times 2x = 3000$$

$$\Rightarrow 24x^3 = 3000$$

$$\Rightarrow x^3 = \frac{3000}{24} = 125 = (5)^3$$

$$\therefore x = 5 \text{ m}$$

$$\text{Length} = 5 \times 4 = 20, \text{ breadth} = 5 \times 3 = 15 \text{ m}$$

$$\text{and height} = 5 \times 2 = 10 \text{ m}$$

$$\therefore \text{Surface area} = 2[lb + bh + hl]$$

$$= 2[20 \times 15 + 15 \times 10 + 10 \times 20] \text{ m}^2$$

$$= 2 [300 + 150 + 200] \text{ m}^2$$

$$= 2 \times 650 = 1300 \text{ m}^2$$

EXERCISE 21(D)

Question 1.

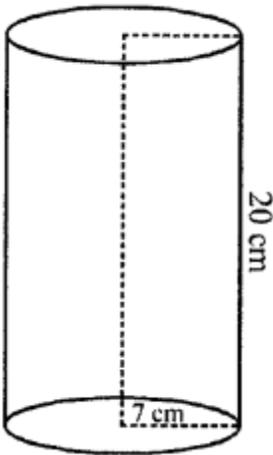
The height of a circular cylinder is 20 cm and the diameter of its base is 14 cm. Find:

- (i) the volume
- (ii) the total surface area.

Solution:

Height of cylinder (h) = 20 cm
and diameter of its base (d) = 14 cm
and radius of its base (r) = $14/2 = 7$ cm

(i) Volume = $\pi r^2 h$
 $= 22/7 \times 7 \times 7 \times 20 \text{ cm}^3 = 3080 \text{ cm}^3$



(ii) Total surface area = $2\pi r(h + r)$
 $= 2 \times 22/7 \times 7 (20 + 7) \text{ cm}^2 = 44 \times 27 = 1188 \text{ cm}^2$

Question 2.

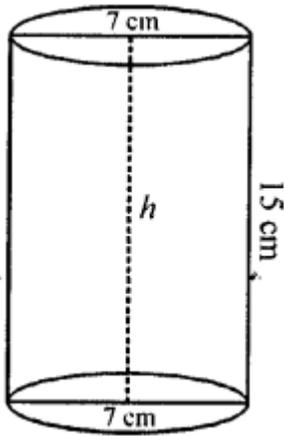
Find the curved surface area and the total surface area of a right circular cylinder whose height is 15 cm and the diameter of the cross-section is 14 cm.

Solution:

Diameter of the base of cylinder = 14 cm

Radius (r) = $14/2$ cm = 7 cm

Height (h) = 15 cm



$$\begin{aligned} \text{Curved surface area} &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 7 \times 15 = 660 \text{ cm}^2 \\ \text{Total surface area} &= 2\pi r (h + r) \\ &= 2 \times \frac{22}{7} \times 7(15 + 7) \\ &= 2 \times \frac{22}{7} \times 7 \times 22 = 968 \text{ cm}^2 \end{aligned}$$

Question 3.

Find the height of the cylinder whose radius is 7 cm and the total surface area is 1100 cm².

Solution:

Total surface area = 1100 cm²
 Radius = 7 cm
 Let height of the cylinder = h
 Then, total surface area = $2\pi r(h + r)$

$$\Rightarrow 2 \times \frac{22}{7} \times 7(h + 7) = 1100$$

$$\Rightarrow 44(h + 7) = 1100$$

$$\Rightarrow 44h + 308 = 1100$$

$$\Rightarrow 44h = 1100 - 308$$

$$h = \frac{792}{44} = 18 \text{ cm}$$

Question 4.

The curved surface area of a cylinder of height 14 cm is 88 cm². Find the diameter of the base of the cylinder.

Solution:

Height (h) = 14 cm
 Curved surface area ($2\pi rh$) = 88 cm²

Then, $2\pi rh = 88 \text{ cm}^2$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 14 = 88 \text{ cm}^2$$

$$\Rightarrow 88r = 88$$

$$\Rightarrow r = \frac{88}{88} = 1 \text{ cm}$$

Then diameter = $1 \times 2 = 2 \text{ cm}$

Question 5.

The ratio between the curved surface area and the total surface area of a cylinder is 1 : 2. Find the ratio between the height and the radius of the cylinder.

Solution:

Let r be the radius and h be the height of a right circular cylinder, then Curved surface area = $2\pi rh$

and total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

But their ratio is 1 : 2

$$\therefore \frac{2\pi r}{2\pi r(h+r)} = \frac{1}{2} \Rightarrow \frac{h}{h+r} = \frac{1}{2}$$

$$\Rightarrow 2h = h + r \Rightarrow 2h - h = r$$

$$\Rightarrow h = r = 1 : 1$$

Hence their radius and height are equal.

Question 6.

Find the capacity of a cylindrical container with internal diameter 28 cm and height 20 cm.

Solution:

Diameter = 28 cm

Radius = $28/2 \text{ cm} = 14 \text{ cm}$

Height = 20 cm

Volume = $\pi r^2 h = 22/7 \times 14 \times 14 \times 20$

Volume = 12320 cm^3

Question 7.

The total surface area of a cylinder is 6512 cm^2 and the circumference of its bases is 88 cm. Find:

(i) its radius

(ii) its volume

Solution:

Let r be the radius and h be the height of the given cylinder.

Circumference = $2\pi r = 88 \text{ cm}$ (Given)

$$\Rightarrow 2 \times \frac{22}{7} \times r = 88 \text{ cm}$$

$$\Rightarrow r = 88 \times \frac{7}{22} \times \frac{1}{2}$$

$$\Rightarrow r = 14 \text{ cm}$$

$$\text{Total surface area} = 2\pi r(h + r) = 6512 \text{ cm}^2$$

(Given)

$$\Rightarrow 88(h + 14) = 6512$$

$$(\because 2\pi r = 88 \text{ cm and } r = 14 \text{ cm})$$

$$\Rightarrow h + 14 = \frac{6512}{88}$$

$$\Rightarrow h + 14 = 74 \Rightarrow h = 60 \text{ cm}$$

$$\therefore \text{Volume of the cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times (14)^2 \times 60 \text{ cm}^3 = 36960 \text{ cm}^3$$

Question 8.

The sum of the radius and the height of a cylinder is 37 cm and the total surface area of the cylinder is 1628 cm². Find the height and the volume of the cylinder.

Solution:

Let r and h be the radius and height of the solid cylinder respectively.

Given, $r + h = 37 \text{ cm}$

Total surface area of the cylinder = 1628 cm² (Given)

$$\therefore 2\pi r(r + h) = 1628 \text{ cm}^2$$

$$\Rightarrow 2\pi r \times 37 = 1628 \text{ cm}^2$$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 37 = 1628 \text{ cm}^2$$

$$\Rightarrow r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7 \text{ cm}$$

$$r + h = 37 \text{ cm} \Rightarrow 7 + h = 37 \text{ cm} \Rightarrow h = 30 \text{ cm}$$

$$\text{Volume of the cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

Question 9.

A cylindrical pillar has radius 21 cm and height 4 m. Find :

(i) the curved surface area of the pillar

(ii) cost of polishing 36 such cylindrical pillars at the rate of Rs. 12 per m².

Solution:

Radius of the cylinder = 21 cm

$$= \frac{21}{100} = 2.1 \text{ m}$$

Height of the cylinder = 4 m

(i) Curved surface area of the cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1 \times 4 = 52.8 \text{ m}^2$$

(ii) Cost of polishing 1 m² = ₹12

Cost of polishing (36 × 52.8 m²)

$$= ₹12 \times 1900.80 = ₹22,809.60$$

Question 10.

If radii of two cylinders are in the ratio 4 : 3 and their heights are in the ratio 5 : 6, find the ratio of their curved surfaces.

Solution:

Ratio in radii of two cylinders = 4 : 3

and ratio in their heights = 5 : 6

Let r_1 and r_2 be the radii and h_1, h_2 be their heights respectively.

$$\therefore r_1 : r_2 = 4 : 3 \text{ and } h_1 : h_2 = 5 : 6$$

$$\therefore r_1 = \frac{4}{3} \text{ and } \frac{h_1}{h_2} = \frac{5}{6}$$

\therefore Surface area of the first cylinder = $2\pi r_1 h_1$

and area of second cylinder = $2\pi r_2 h_2$

$$\frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{r_1}{r_2} \times \frac{h_1}{h_2} = \frac{4}{3} \times \frac{5}{6} = \frac{20}{18}$$

$$= \frac{10}{9} = 10 : 9$$

\therefore Ratio in their surface areas = 10 : 9

EXERCISE 21(E)

Question 1.

A cuboid is 8 m long, 12 m broad and 3.5 high, Find its

- (i) total surface area
- (ii) lateral surface area

Solution:

Length of a cuboid = 8 m

Breadth of a cuboid = 12 m

Height of a cuboid = 3.5 m

(i) Total surface area = $2(lb + bh + hl)$

$$= 2(8 \times 12 + 12 \times 3.5 + 3.5 \times 8)$$

$$= 2(96 + 42 + 28)$$

$$= 2 \times 166 = 332 \text{ m}^2$$

(ii) Lateral surface area = $2h(l + b)$

$$= 2 \times 3.5(8 + 12) = 7 \times 20 = 140 \text{ m}^2$$

Question 2.

How many bricks will be required for constructing a wall which is 16 m long, 3 m high and 22.5 cm thick, if each brick measures 25 cm x 11.25 cm x 6 cm ?

Solution:

Length of the wall = 16 m = $16 \times 100 \text{ cm} = 1600 \text{ cm}$

Height of the wall = 3 m = $3 \times 100 \text{ cm} = 300 \text{ cm}$

Breadth of the wall = 22.5 cm

Volume of the wall = $1600 \times 300 \times 22.5 \text{ cm}^3 = 1,08,00,000 \text{ cm}^3$

Volume of one brick = $25 \times 11.25 \times 6 \text{ cm}^3 = 1687.5 \text{ cm}^3$

Number of bricks required to construct the

$$\text{wall} = \frac{\text{Volume of wall}}{\text{Volume of one brick}}$$

$$= \frac{1,08,00,000}{1687.5} = 6400$$

Question 3.

The length, breadth and height of cuboid are in the ratio 6 : 5 : 3. If its total surface area is 504 cm^2 , find its volume.

Solution:

Let length of the given cuboid = $6x$

Breadth of the given cuboid = $5x$

Height of the given cuboid = $3x$

Total surface area of the given cuboid

$$\begin{aligned}
 &= 2(lb + bh + hl) \\
 &= 2(6x \times 5x + 5x \times 3x + 3x \times 6x) \\
 &= (30x^2 + 15x^2 + 18x^2) \\
 &= 2 \times 63x^2 = 126x^2
 \end{aligned}$$

But, we are given total surface area

$$= 504 \text{ cm}^2$$

$$\therefore 126x^2 = 504$$

$$\Rightarrow x^2 = \frac{504}{126}$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x^2 = (2)^2$$

$$\Rightarrow x = 2 \text{ cm}$$

\therefore Length of the given cuboid = $6x$

$$= 6 \times 2 \text{ cm} = 12 \text{ cm}$$

Breadth of the given cuboid = $5x$

$$= 5 \times 2 \text{ cm} = 10 \text{ cm}$$

Height of the given cuboid = $3x$

$$= 3 \times 2 \text{ cm} = 6 \text{ cm}$$

Now, volume of the cuboid = $l \times b \times h$

$$= 12 \times 10 \times 6 = 720 \text{ cm}^3$$

Question 4.

The external dimensions of an open wooden box are 65 cm, 34 cm and 25 cm. If the box is made up of wood 2 cm thick, find the capacity of the box and the volume of wood used to make it.

Solution:

External length of the open box = 65 cm

External breadth of the open box = 34 cm

External height of the open box = 25 cm

External volume of the open box = $65 \times 34 \times 25 \text{ cm}^3 = 55250 \text{ cm}^3$

Internal length of open box = $65 - (2 \times 2) \text{ cm} = 61 \text{ cm}$

Internal breadth of an open box = $34 - (2 \times 2) \text{ cm} = 30 \text{ cm}$

Internal height of open box = $25 - 2 \text{ cm} = 23 \text{ cm}$

Internal volume of open box or capacity of the box = $61 \times 30 \times 23 \text{ cm}^3 = 42090 \text{ cm}^3$

Volume of wood required to make the closed box = $55250 - 42090 \text{ cm}^3 = 13160 \text{ cm}^3$

Question 5.

The curved surface area and the volume of a toy, cylindrical in shape, are 132 cm^2 and 462 cm^3 respectively. Find, its diameter and its length.

Solution:

Let the radius of a toy = r and

height of the toy = h

Curved surface area of a toy = 132 cm^2

$$\Rightarrow 2\pi rh = 132 \text{ cm}^2$$

$$\Rightarrow 2\pi rh = 132 \text{ cm}^2$$

$$\Rightarrow r = \frac{132}{2\pi \times h} \text{ cm}^2 \quad \dots(i)$$

Also, volume of a toy = 462 cm^3

$$\Rightarrow \pi r^2 h = 462 \text{ cm}^3$$

$$\Rightarrow r^2 = \frac{462}{\pi \times h} \quad \dots(ii)$$

Now, substitute the value of r , we get

$$\frac{(132)^2}{(2)^2 \times (\pi)^2 \times h^2} = \frac{462}{\pi \times h}$$

$$\Rightarrow \frac{132^2}{4 \times \pi \times h} = 462$$

$$\Rightarrow 4 \times \pi \times h = \frac{132 \times 132}{462}$$

$$\Rightarrow h = \frac{132 \times 132}{462 \times \pi \times 4}$$

$$\Rightarrow h = \frac{132 \times 132 \times 7}{462 \times 22 \times 4} = 3 \text{ cm}$$

Now, put the value of h in eq. (i), we get

$$r = \frac{132 \times 7}{2 \times 22 \times 3} = 7 \text{ cm}$$

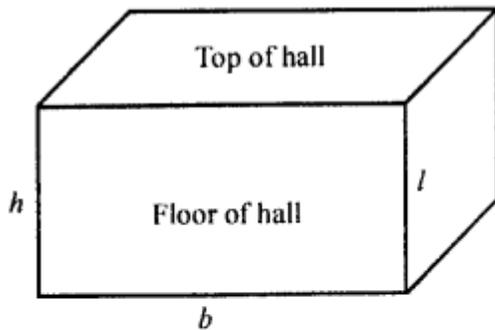
$$\begin{aligned} \therefore \text{Diameter of the toy} &= 2 \times r \\ &= 2 \times 7 \text{ cm} = 14 \text{ cm} \end{aligned}$$

Question 6.

The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs. 10 per m^2 is Rs. 15,000, find the height of the hall.

Solution:

Let length, breadth and height of the rectangular hall be l m, b m and h m respectively.



$$\text{Perimeter of the floor of hall} = 2(l + b)$$

$$250 \text{ m} = 2(l + b)$$

$$(l + b) = \frac{250}{2} = 125 \text{ cm} \quad \dots(i)$$

$$\begin{aligned} \text{Area of four walls} &= \text{Area of cuboid} - \text{Area of floor} - \text{Area of top} \\ &= 2(lb + bh + hl) - (l \times b) - (l \times b) \\ &= 2(lb) + 2(bh) + 2(hl) - 2lb = 2lh + 2bh \\ &= 2h(l + b) \\ &= 2h \times 125 \text{ [From (i)]} \\ &= 250h \text{ m}^2 \end{aligned}$$

$$\text{Area of four walls} = 250h \text{ m}^2$$

$$\text{Cost of painting } 1 \text{ m}^2 \text{ area} = \text{Rs. } 10$$

$$\text{Cost of painting } 250h \text{ m}^2 \text{ area} = \text{Rs. } 10 \times 250h = 2500h$$

$$15000 = 2500h$$

$$h = 15000/2500$$

The height of the hall is 6 m.

Question 7.

The length of a hall is double its breadth. Its height is 3 m. The area of its four walls (including doors and windows) is 108 m^2 , find its volume.

Solution:

Let the breadth be x

and the length be $2x$

Height = 3 m

Area of four walls = 108 m^2

$$\Rightarrow 2(l + b)h = 108$$

$$\Rightarrow 2(2x + x)3 = 108$$

$$\Rightarrow 6 \times 3x = 108$$

$$\Rightarrow 3x = \frac{108}{6}$$

$$\Rightarrow x = \frac{18}{3} = 6 \text{ m}$$

\therefore Breadth = $x = 6 \text{ m}$

and length = $2x = 12 \text{ m}$

Hence, Volume = $l \times b \times h$

$$= 12 \times 6 \times 3$$

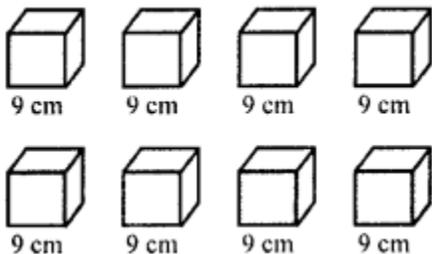
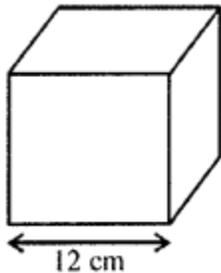
$$= 216 \text{ m}^3$$

Question 8.

A solid cube of side 12 cm is cut into 8 identical cubes. What will be the side of the new cube? Also, find the ratio between the surface area of the original cube and the total surface area of all the small cubes formed.

Solution:

Here, cube of side 12 cm is divided into 8 cubes of side 9 cm.



Given that,

Their volumes are equal.

Volume of big cube of 12 cm = Volume of 8 cubes of side a cm

$$(\text{Side of big cube})^3 = 8 \times (\text{Side of small cube})^3$$

$$(12)^3 = 8 \times a^3$$

$$\Rightarrow a^3 = \frac{12 \times 12 \times 12}{8}$$

$$\Rightarrow a^3 = 6^3 \text{ cm}^3$$

$$\Rightarrow a = 6 \text{ cm}$$

\therefore Side of small cube = 6 cm

Ratio of their surface

$$= \frac{\text{Surface area of the original cube}}{\text{Total surface area of the small cube}}$$

$$= \frac{6(\text{side of big cube})^2}{8 \times 6(\text{side of small cube})^2}$$

$$= \frac{6 \times 12 \times 12}{8 \times 6 \times 6 \times 6} = \frac{4}{8} = 1 : 2$$

So, the ratio is 1 : 2

Question 9.

The diameter of a garden roller is 1.4 m and it 2 m long. Find the maximum area covered by it 50 revolutions?

Solution:

Diameter of the roller = 1.4 m

Radius (r) = $1.4/2 = 0.7$ m

and length (h) = 2m

Curved surface area = $2\pi rh = 2 \times \frac{22}{7} \times 0.7 \times 2 \text{ m}^2 = 8.8 \text{ m}^2$

Area covered in 50 complete revolutions = $8.8 \times 50 \text{ m}^2 = 440 \text{ m}^2$

Area of the playground = 440 m^2

Question 10.

In a building, there are 24 cylindrical pillars. For each pillar, radius is 28 m and height is 4 m. Find the total cost of painting the curved surface area of the pillars at the rate of ₹ 8 per m^2 .

Solution:

Radius (r) of each pillar = 28 m

Height (h) = 4 m

Curved surface area of each pillar = $2\pi rh$

= $2 \times \frac{22}{7} \times 28 \times 4 \text{ m}^2 = 704 \text{ m}^2$

Surface area of 24 pillars = $704 \times 24 \text{ m}^2 = 16,896 \text{ m}^2$

Cost of cleaning = Rs. 8 per m^2

Total cost = Rs. $16,896 \times 8 = \text{Rs. } 1,35,168$