CHAPTER 06

Principle of Effective Stress, Capillarity and Permeability

6.1 Introduction

Terzaghi was the first to enunciate the effective stress principle. It can be said that in doing that, he opened the flood gates to the discipline of soil mechanics. Some of the basic developments on shear strength, compressibility and lateral earth pressures are a direct offshoot of the effective stress concept. The effective stress principle consists of two parts:

(i) Definition of the effective stress

(ii) Importance of effective stress in engineering behavior of soil.

6.2 Total Stress, Pore Pressure and Effective Stress

6.2.1 Total Stress

Normal stress is defined as the sum of the normal component of the forces (ΣN) over a plane ($X - X$) divided by the area of plane (A). Generally, it is denoted by σ.

$$\therefore \quad \sigma = \frac{\Sigma N}{A}$$

Let us consider a stratified soil sample of unit width and unit length for the plane over which normal stresses to be computed.

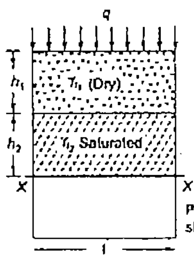


Fig. 6.1

$$\therefore \quad \sigma = \frac{q \cdot A + \gamma_{t_1} \times h_1 \times A + \gamma_{t_2} \times h_2 \times A}{A}$$

$$= q + \gamma_d h_1 + \gamma_{sat} h_2$$

If external pressure q is zero, then total stress caused due to the over burden alone are known as 'Geostatic stress'.

then, $\quad \sigma = \gamma_d h_1 + \gamma_{sat} h_2 \quad$ ...(i)

Remember: Total stress is a parameter which can be measured with suitable instrument, such as a pressure cell.

6.2.2 Pore Pressure

It is the pressure exerted by the pore water filled in the void of the soil skeleton. It is denoted by u and equal to the depth (h_2) below the ground water table upto the point (A) where it is measured and multiplied by the unit weight of water γ_w.

$$\therefore \quad u = h_2 \gamma_w \quad \text{...(ii)}$$

Pore pressure is also called the neutral stress because it acts on all sides of the particle, hence does not cause the soil particles to press against adjacent particles.

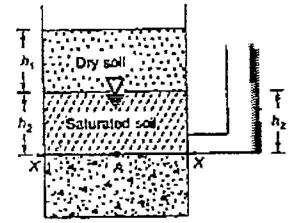


Fig. 6.2

Remember: Pore water pressure, like the total stress is also a measurable parameter. A standpipe, piezometer or pore water pressure transducer may be used to measure the pore water pressure.

6.2.3 Principle of Effective Stress

Total stress (σ) is made up of two parts

(i) one part is due to pore water pressure (u)

(ii) the other part is due to pressure exerted by the soil skeleton which is called effective stress ($\bar{\sigma}$).

i.e. $\quad \sigma = u + \bar{\sigma}$

$\therefore \quad \bar{\sigma} = \sigma - u$

Substituting value of σ and u from equation (i) and (ii), we have

$$\bar{\sigma} = (\gamma_d h_1 + \gamma_{sat} h_2) - \gamma_w h_2$$

$$= \gamma_d h_1 + (\gamma_{sat} - \gamma_w) h_2 = \gamma_d h_1 + \gamma' h_2$$

where γ' is submerged unit weight.

Remember

- Effective stress in soils is a grain to grain contact pressure which a soil particle can exhibit.
- The effective stress is not a physical parameter and cannot be measured.
- Increase in effective stress causes the particles to pack more closely, decreases the void ratio, leads to a decrease in compressibility and increases in the shearing resistance of the soil.
- When there is an equal increase in the total stress and the pore pressure, then effective stress remain unchanged.
- Principle of effective stress applies only to normal stresses not to the shear stress.

6.3 Effective Stress in Partially Saturated Soils

Effective stress, $\bar{\sigma} = \sigma - u_a + \chi(u_a - u_w)$

where, σ = Total stress

u_a = Pore air pressure

u_w = Pore water pressure

χ = The parameter to be determined experimentally

= 0 (for dry soil)

= 1 (for saturated soil)

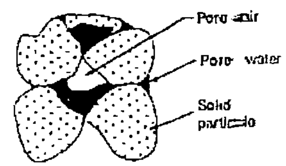


Fig. 6.3

It means that, if the soil is partially saturated, pore air pressure is considered along with pore water pressure to analyze the stresses in the soil mass.

6.4 Capillarity in Soils

As we know, the rain water percolates into the ground under the influence of gravity and gets stored in the soil pores, over an impervious stratum, in the form of ground water reservoir.

The upper surface of the zone of full saturation of the soil is called the water table or phreatic surface. At the water table, the ground water is subjected to atmospheric pressure. In other words, the pore pressure is zero.

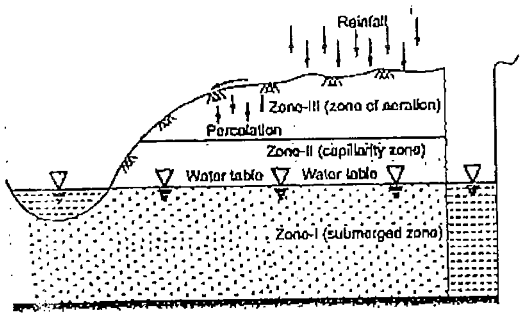


Fig. 6.4 Zones of soil water below ground surface

Soil water can exist in three zones as shown below:

- **Zone-I (Submerged Zone):** This zone exist below the ground water table. The soil in this zone is in submerged condition and pore pressure is hydrostatic i.e. positive.
- **Zone-II (Capillarity Zone):** If gravity was the only force, acting on the percolating water and taking it downward, then the soil above the water table would be completely dry. But it is not so in actual practice. Soil in this zone is completely saturated upto some height above the water table. This phenomenon of rising water in soil is known as capillarity in soils.
- **Zone-III (Zone of Aeration):** Above the capillarity zone and upto a certain height, there exist a zone called the zone of aeration, in which the soil is able to retain small water droplets, surrounded on all sides by air. This water is known as contact moisture or hygroscopic water. The contact moisture is retained by the soil against the gravity drainage by the action of capillary.

6.4.1 Capillary Rise in Soils

We can get an understanding of capillarity in soil by idealizing the continuous void spaces as capillary tubes. Consider a single idealized tube as shown in figure below.

The surface tension (force) pulls the water upward till the height h_c, at which the weight of water in the column is in equilibrium with the magnitude of the surface tension force.

For equilibrium

$$h_c \cdot \frac{\pi d^2}{4} \gamma_w = (\pi \cdot d) T \cos\alpha$$

$$h_c = \frac{4T\cos\alpha}{\gamma_w \cdot d}$$

where, α = contact angle

T = surface tension

d = diameter of capillary tube

γ_w = unit weight of water

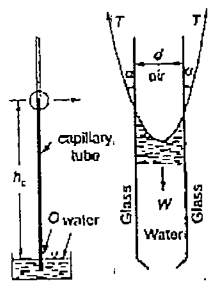


Fig. 6.5 Capillary Rise in Soils

Remember

- $h_c(\text{cm}) = \frac{0.3084}{d(\text{cm})}$ ....at 4°C
- $h_c(\text{cm}) = \frac{0.2975}{d(\text{cm})}$ ....at 20°C

Because of the complex nature of the soil, a theoretical prediction of capillary rise in soil is not possible. Terzaghi and Peck suggested an approximate relationship to find capillary rise in-situ.

$$h_c(\text{cm}) = \frac{C}{eD_{10}}$$

where, C = Empirical constant; the value of which depends on the shape and surface impurities of the grains and lies between 0.1 and 0.5 cm^2.

e = void ratio

D_{10} = Effective grain size

Do you know? Capillarity involves both adhesive and cohesive forces.

6.4.2 Capillary Pressure

Capillary water rises against gravity and is held by the surface tension. Therefore the capillary water exerts a tensile force on soil and resulting negative pressure of water (Capillary pressure) creates attraction between the particles.

From the effective stress equation,

$$\bar{\sigma} = \sigma - u$$

When the pore water is in compression, as in case of hydrostatic pressure, u is taken positive whereas when it is in tension, u is taken negative.

Therefore effective stress in capillary zone

$$\bar{\sigma} = \sigma - (-u_c) = \sigma + u_c$$

It means that due to capillarity in soils, effective stress increases.

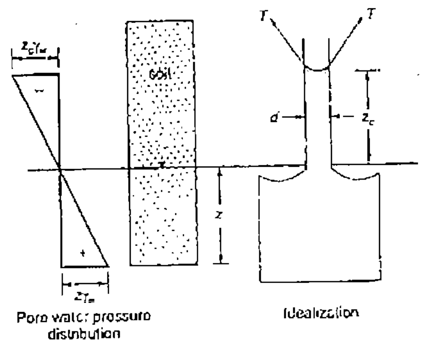


Fig. 6.6

Do You Know

- Negative pressure of water (capillary pressure) held above water table results in attractive forces between particles. It is called as soil suction.
- Bulking of sand also occurs because of capillarity. Capillarity produces apparent cohesion which holds the particles in cluster, enclosing honey combs.
- Water may flow over the crest of an impermeable core in a dam even though, the free surface may be lower than the crest of core. This effect of capillarity is known as capillary siphoning.

Example 6.1 Compute the maximum capillarity tension for a tube of 0.12 mm in diameter at 4°C.

Solution:

Using $h_c = \dfrac{0.3084}{d(\text{cm})} = \dfrac{0.3084}{0.012}$

$= 25.7$ cm or 0.257 m

$\therefore$ Capillary Tension $= h_c \gamma_w$

$= 0.257 \times 9.81$

$= 2.521 \text{ kN/m}^2$

Example 6.2 The capillary rise in a soil having $D_{10} = 0.07$ mm is 49 cm. Estimate the capillary rise in another soil having its $D_{10} = 0.12$ mm. Assume the same void ratios in both the soils.

Solution:

Soil I	Soil II
$D_{10} = 0.07$ mm	$D'_{10} = 0.12$ mm
$= 0.007$ cm	$= 0.012$ cm
$h_c = 49$ cm	$h'_c = ?$

Using $h_c \cdot D_{10} = \text{Constant}$

$\therefore$ $h_c \cdot D_{10} = h'_c \cdot D'_{10}$

$$h'_c = \left(\frac{D_{10}}{D'_{10}}\right) h_c$$

$$= \left(\frac{0.017}{0.012}\right) \times 49 = 28.58 \text{ cm}$$

Hence, the capillary rise in the second soil would be 28.6 cm.

6.5 Geostatic Stresses in Soils

Stresses within the soil mass may be caused by the self weight of the soil and also by the external loads which may be applied to the soil. The pattern of stresses caused by the external loads is usually complicated one. However, there is one common situation in which the self-weight of soil gives rise to a very simple pattern of stresses, that is when the ground surface is horizontal and the nature of soil does not very significantly in the horizontal direction. Such a situation frequently occurs in the case of sedimentary deposits. The total stress caused in such a situation at a point in a soil mass is called the geostatic stress.

Do You Know

- In the geostatic situation, there are no shear stresses upon the horizontal or vertical planes within the soil.
- If external loading contributes to this stress, the stress will simply be called as the total stress.

The vertical geostatic stress at a point below the surface is equal to the weight of the soil lying directly above the point.

If the unit weight of the soil (γ) is constant with depth, then

Total stress $= \sigma$

$= \gamma \times (z \times 1 \times 1)$

$= \gamma \cdot z$

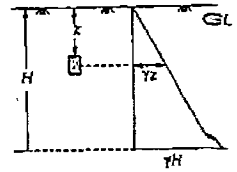


Fig. 6.7 Uniform Soil

If the soil is stratified with different unit weights for each stratum as shown in figure below:

Then σ can be calculated as

$$\sigma = \gamma_1 z_1 + \gamma_2 z_2 + \gamma_3 z_3 + \ldots\ldots$$

$$= \Sigma \gamma z$$

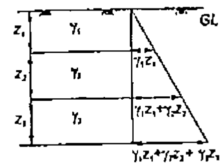


Fig. 6.8 Stratified soil

Case 1: When Soil Is Dry

Consider a situation when the water table is at large distance from the ground surface. Then, the stresses at a depth z is given as

Total stress, $\sigma = \gamma_d z$

Pore water pressure, $u = 0$

and effective stress, $\bar{\sigma} = \sigma - u = \gamma_d z - 0 = \gamma_d z$

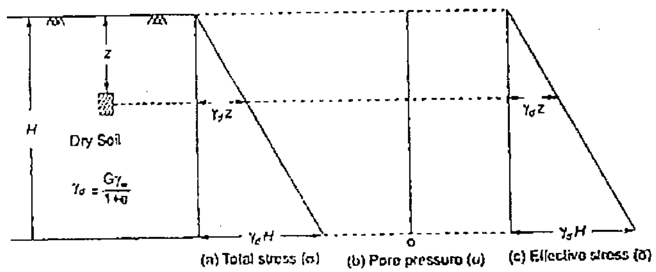


Fig. 6.9 Vertical stress distribution in dry soil

Case 2: When Soil Is Moist

There is a situation of partially saturated soil wherein it is difficult to predict the pore water pressure distribution. In this case pore pressure is neglected. Bulk unit weight of soil is calculated by

$$\gamma_t = \frac{(G + Se)}{1 + e} \gamma_w$$

Thus,

Total stress, $\sigma = \gamma_t \cdot z$

Pore water pressure, $u = 0$

$\bar{\sigma} = \sigma - u = \gamma_t z - 0 = \gamma_t \cdot z$

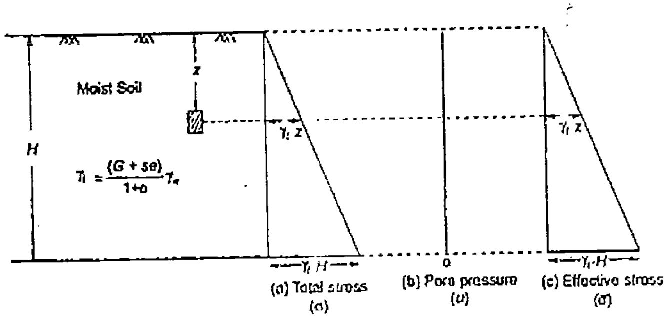


Fig. 6.10 Vertical stress distribution in moist soil

Case 3: When Soil Is Saturated

This is a situation of saturated soil above water table, which is due to some reasons like rain, watering or irrigation (except capillarity of soil). Hence pore water pressure is zero. Therefore, the stresses are governed by the saturated weight of the soil.

Thus,

Total stress, $\sigma = \gamma_{sat} \cdot z$

Pore water pressure, $u = 0$

Effective stress, $\bar{\sigma} = \sigma - u = \gamma_{sat} \cdot z - 0 = \gamma_{sat} \cdot z$

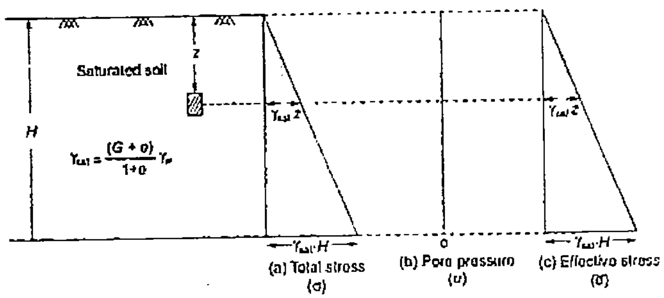


Fig. 6.11 Vertical stress distribution in saturated soil

Case 4: When Soil Is Completely Submerged

In this situation, soil is under the water table in submerged condition with water table at ground level. Therefore pore water pressure is hydrostatic. In this case, total stress is governed by saturated unit weight of soil and effective stress is governed by submerged unit weight.

Thus,

Total stress, $\sigma = \gamma_{sat} \cdot z$

Pore water pressure, $u = \gamma_w \cdot z$

Effective stress, $\bar{\sigma} = \sigma - u = \gamma_{sat} \cdot z - \gamma_w \cdot z = \gamma' z$

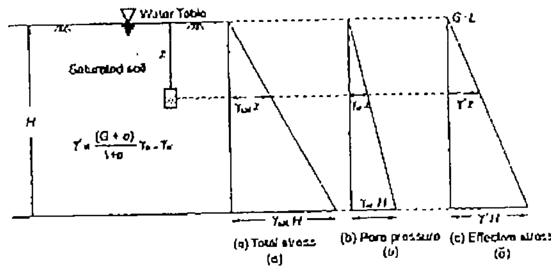


Fig. 6.12 Vertical stress distribution in submerged soil

Case 5: When Soil Is Completely Saturated by Capillarity

In this situation, soil is completely saturated but pore water pressure is negative and vary linearly from zero at water table to $\gamma_w(H-z)$ at any depth below ground level.

Total stress, $\sigma = \gamma_{sat} \cdot z$

Pore pressure, $u = -\gamma_w(H-z)$

Effective stress, $\bar{\sigma} = \sigma - u = \gamma_{sat} \cdot z + \gamma_w(H-z) = (\gamma_{sat} - \gamma_w)z + \gamma_w H$

$= \gamma' z + \gamma_w H$

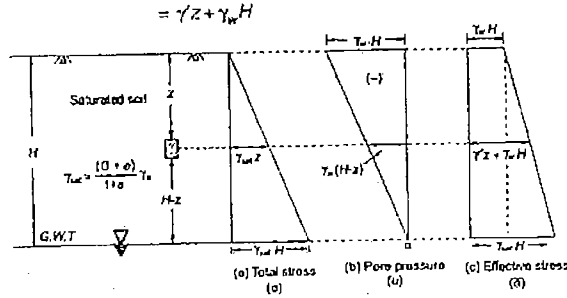


Fig. 6.13 Vertical stress distribution in saturated soil by capillarity

Special Case

When height of capillary rise is less than H (i.e. $0 < h_c < H$)

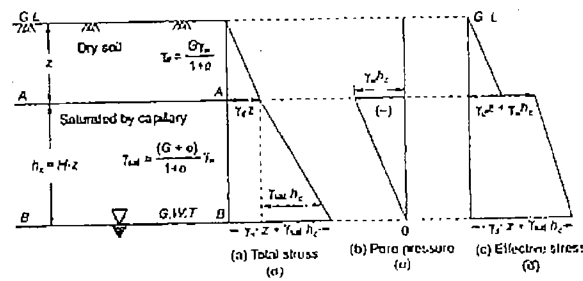


Fig. 6.14 Vertical stress distribution in partially saturated soil

At Ground Level

Total stress, $\sigma = 0$
Pore water pressure, $u = 0$
Effective stress, $\bar{\sigma} = 0$

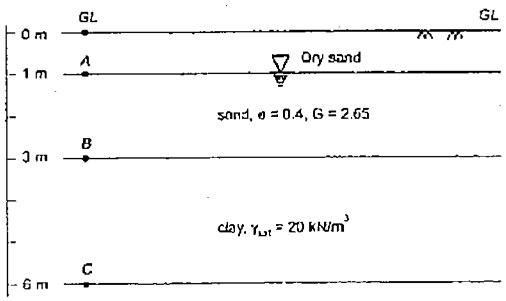
At the Level of *A-A*

Total stress, $\sigma_A = \gamma_d \cdot z$
Pore water pressure, $u_A = -\gamma_w h_c$
Effective stress, $\bar{\sigma} = \sigma - u = \gamma_d \cdot z - (-\gamma_w h_c) = \gamma_d \cdot z + \gamma_w h_c$

At the Level of *B-B*

Total stress, $\sigma_B = \gamma_d \cdot z + \gamma_{sat} h_c$
Pore water pressure, $u_B = 0$
Effective stress, $\bar{\sigma} = \sigma - u = \gamma_d \cdot z + \gamma_{sat} h_c$

Example 6.3 For the subsoil condition shown in figure below, calculate the total, neutral and effective stress at 1 m, 3 m and 6 m below the ground level. Assume $\gamma_w = 10$ kN/m^3.



Solution:

Note that sand above ground water table is dry and completely saturated below the water table. Therefore, we have to first determine γ_d and γ_{sat} for sand, using $e = 0.4$ and $G = 2.65$.

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 10}{1+0.4} = 18.93 \text{ kN/m}^3$$

Also,
$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.4)\times 10}{1+0.4} = 21.79 \text{ kN/m}^3$$

(i) Total, neutral and effective stress at 1 m depth

(a) Total stress, $\sigma_A = \gamma_d \cdot z = 18.93 \times 1 = 18.93$ kN/m^3

(b) Neutral stress, $u_A = 0$

(c) Effective stress, $\bar{\sigma}_A = \sigma_A - u_A = 18.93 - 0 = 18.93$ kN/m^2

(ii) Total, neutral and effective stress at 3 m depth

(a) Total stress, $\sigma_B = \gamma_d \times 1 + \gamma_{sat,\,sand} \times 2 = 18.93 \times 1 + 21.79 \times 2 = 62.51$ kN/m^2

(b) Neutral stress, $u_B = \gamma_w \times 2 = 10 \times 2 = 20$ kN/m^2

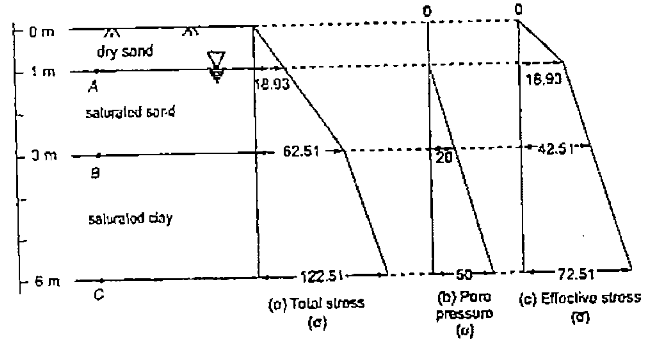
(c) Effective stress, $\bar{\sigma}_B = \sigma_B - u_B = 62.51 - 20 = 42.51$ kN/m^2

(iii) Total, neutral and effective stress at 6 m depth

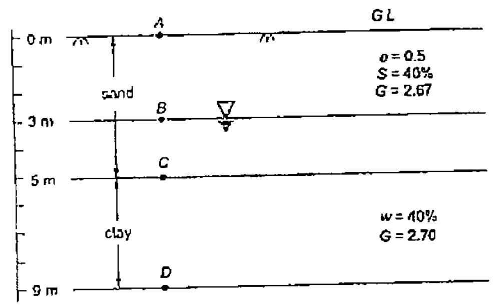
(a) Total stress, $\sigma_C = \gamma_d \times 1 + \gamma_{sat,\,sand} \times 2 + \gamma_{sat,\,clay} \times 3 = 18.93 \times 1 + 21.79 \times 2 + 20 \times 3 = 122.51$ kN/m^2

(b) Neutral stress, $u_C = \gamma_w \times 5 = 10 \times 5 = 50$ kN/m^2

(c) Effective stress, $\bar{\sigma}_C = \sigma_C - u_C = 122.51 - 50 = 72.51$ kN/m^2



Example 6.4 For the subsoil condition shown in figure. Draw the total, neutral and effective stress diagram upto the depth of 9 m, neglecting the capillary rise.



Solution:

For Sand

Given: $e = 0.5$, $S = 0.40$, $G = 2.67$

Using
$$\gamma_t = \frac{(G+Se)\gamma_w}{1+e} = \frac{(2.67+0.4\times 0.5)\times 9.81}{1+0.5} = 18.77 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.67+0.5)\times 9.81}{1+0.5} = 20.73 \text{ kN/m}^3$$

For Clay

Given: $w = 0.40$, $G = 2.70$

Clay is located well below the water table. Therefore, $S = 100\%$

Using $$e = \frac{w \cdot G}{S} = \frac{0.40 \times 2.70}{1} = 1.08$$

$$\therefore \quad \gamma_{sat,\,clay} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.7+1.08)}{1+1.08} \times 9.81 = 17.83 \text{ kN/m}^3$$

(I) Total, neutral and effective stress at *GL*

Total stress, $\sigma_A = 0$

Neutral stress, $u_A = 0$

Effective stress, $\bar{\sigma}_A = \sigma_A - u_A = 0$

(II) Total, neutral and effective stress at 3 m

Total stress, $\sigma_B = \gamma_t \times 3 = 18.77 \times 3 = 56.31 \text{ kN/m}^2$

Neutral stress, $u_B = 0$

Effective stress, $\bar{\sigma}_B = \sigma_B - u_B = 56.31 - 0 = 56.31 \text{ kN/m}^2$

(III) Total, neutral and effective stress at 5 m

Total stress, $\sigma_C = \gamma_t \times 3 + \gamma_{sat,\,sand} \times 2 = 18.77 \times 3 + 20.73 \times 2 = 97.77 \text{ kN/m}^2$

Neutral stress, $u_C = \gamma_w \times 2 = 9.81 \times 2 = 19.62 \text{ kN/m}^2$

Effective stress, $\bar{\sigma}_C = \sigma_C - u_C = 97.77 - 19.62 = 78.15 \text{ kN/m}^2$

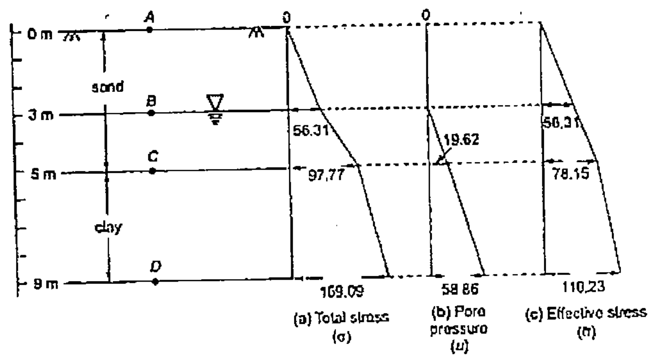
(IV) Total, neutral and effective stress at 9 m

Total stress, $\sigma_D = \gamma_t \times 3 + \gamma_{sat,\,sand} \times 2 + \gamma_{sat,\,clay} \times 4$

$= 18.77 \times 3 + 20.73 \times 2 + 17.83 \times 4 = 169.09 \text{ kN/m}^2$

Neutral stress, $u_D = \gamma_w \times 6 = 9.81 \times 6 = 58.86 \text{ kN/m}^2$

Effective stress, $\bar{\sigma}_D = \sigma_D - u_D = 169.09 - 58.86 = 110.23 \text{ kN/m}^2$

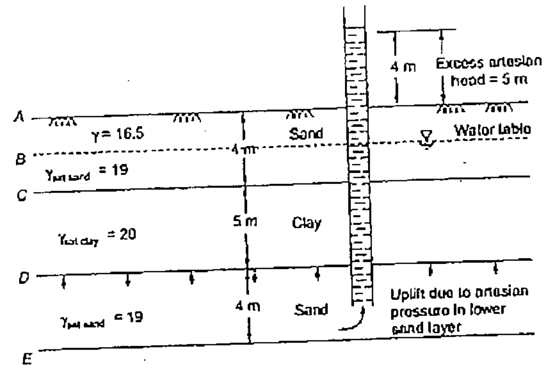


Variation of total, neutral and effective stress

Example 6.5 A 5 m thick clay layer lies between two layers of sand each 4 m thick, the top of the upper layer of sand being at ground level. The water table is 2 m below the ground level but the lower layer of sand is under artesian pressure, the piezometric surface being 4 m above ground level. The saturated unit weight of clay is 20 kN/m³ and that of sand is 19 kN/m³ above the water table unit weight of sand is 16.5 kN/m³. Calculate (a) the effective vertical stress at the top and bottom of the clay layer (b) Also draw the total vertical stress diagram in the given soil layers.

Solution:

Given conditions are shown in figure



(a) Effective stress at top clay layer

Total stress at *C*, $\sigma_C = \gamma \times z + \gamma_{sat,\,sand} \times 2$

$= 16.5 \times 2 + 19 \times 2 = 71 \text{ kN/m}^2$

Neutral stress at *C*, $u_C = \gamma_w \times 2 = 9.81 \times 2 = 19.62 \text{ kN/m}^2$

$\therefore$ Effective stress at *C*, $\bar{\sigma}_C = \sigma_C - u_C = 71 - 19.62 = 51.38 \text{ kN/m}^2$

Alternatively

Effective stress at *C*, $\bar{\sigma}_C = \gamma \times 2 + \gamma' \times 2$

$= 16.5 \times 2 + (19 - 9.81) \times 2 = 51.38 \text{ kN/m}^2$

Effective stress at bottom of clay layer

Total stress at *D*, $\sigma_D = \gamma \times z + \gamma_{sat,\,sand} \times 2 + \gamma_{sat,\,clay} \times 5$

$= 16.5 \times 2 + 19 \times 2 + 20 \times 5 = 171 \text{ kN/m}^2$

Neutral stress at *D*, $u_D = \gamma_w \times$ Total water head $= 9.81 \times 13 = 127.53 \text{ kN/m}^2$

$\therefore$ Effective stress at *D*, $\bar{\sigma}_D = \sigma_D - u_D = 171 - 127.53 = 43.47 \text{ kN/m}^2$

Alternatively

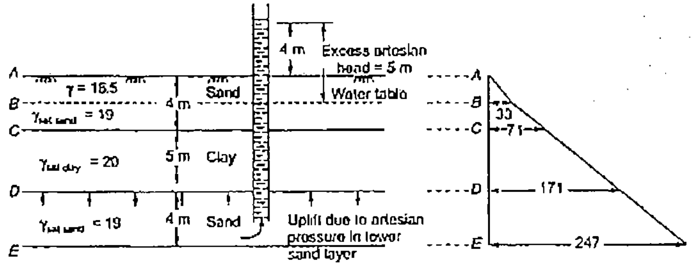
Effective stress at *D*, $\bar{\sigma}_D = \gamma \times 2 + \gamma_{sub,\,sand} \times 2 + \gamma_{sub,\,clay} \times 5 -$ uplift caused by excess artesian head

$= 16.5 \times 2 + (19 - 9.81) \times 2 + (120 - 9.81) \times 5 - 9.8 \times 6$

$= 33 + 18.38 + 50.95 - 58.86$

$= 43.47 \text{ kN/m}^2$

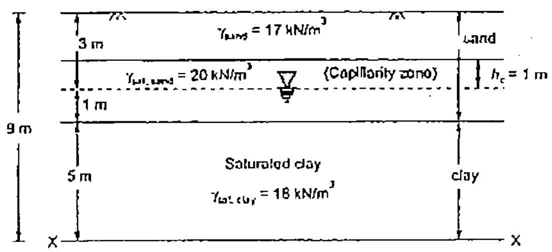
(b) Total stress diagram



Example 6.6 A layer of saturated clay 5 m thick is overlain by sand 4.0 m deep. The water table is 3 m below the top surface. The saturated unit weights of clay and sand are 18 kN/m³ and 20 kN/m³ respectively. Above the water table, the unit weight of sand is 17 kN/m³. Calculate the effective pressures on a horizontal plane at a depth of 9 m below the ground surface. What will be the increase in the effective pressure at 9 m, if the soil gets saturated by capillary, upto height of 1 m above the water table?

Solution:

The given condition is shown in figure



(a) Effective stress at X - X

Total stress at X - X, $\sigma_X = \gamma_{sand} \times 3 + \gamma_{sat,\ sand} \times 1 + \gamma_{sat,\ clay} \times 5$

$= 17 \times 3 + 20 \times 1 + 18 \times 5 = 161$ kN/m²

Neutral stress at X - X, $u_X = \gamma_w \times 6 = 9.81 \times 6 = 58.86$ kN/m²

∴ Effective stress at X - X,

$\bar{\sigma}_X = \sigma - u = 161 - 58.86 = 102.14$ kN/m²

Alternatively

$\bar{\sigma}_X = \gamma_{sand} \times 3 + \gamma_{sub,\ sand} \times 1 + \gamma_{sat,\ clay} \times 5$

$= 17 \times 3 + (20 - 9.81) \times 1 + (18 - 9.81) \times 5$

$= 51 + 10.19 + 8.19 \times 5$

$= 102.14$ kN/m²

(b) When soil gets saturated upto height of 1 m above the water table by capillarity, then

Total stress at X - X, $\sigma = \gamma_{sand} \times 2 + \gamma_{sat,\ sand} \times 2 + \gamma_{sat,\ clay} \times 5$

$= 17 \times 2 + 20 \times 2 + 18 \times 5 = 164$ kN/m²

Pore pressure a X - X, $u = \gamma_w \times 6 = 9.81 \times 6 = 58.86$ kN

∴ Effective stress at X - X,

$\bar{\sigma} = \sigma - u = 164 - 58.86 = 105.14$ kN/m²

Alternatively

$\bar{\sigma} = \gamma_{sand} \times 2 + \gamma_{sat,\ sand} \times 1 + \gamma_{sub,\ clay} \times 1 + \gamma_{sub,\ clay} \times 5$

$= 17 \times 2 + 20 \times 1 + (20 - 9.81) \times 1 + (18 - 9.81) \times 5$

$= 34 + 20 + 10.19 + 8.19 \times 5$

$= 105.14$ kN/m²

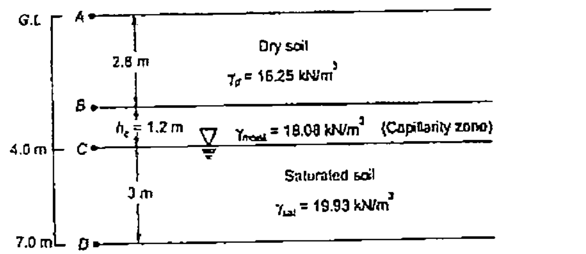
∴ Increase in pressure $= 105.14 - 102.14 = 3$ kN/m²

Example 6.7 Granular soil deposit is 7 m deep over an impermeable layer. The ground water table is 4 m below the ground surface. The deposit has a zone of capillary rise of 1.2 m with a saturation of 50%, plot the variation of total stress, pore water pressure and effective stress with the depth of deposit, $e = 0.6$ and $G = 2.65$.

Solution:

The given situation is shown in figure.

Assume that soil above capillary zone is completely dry.



Unit weight of dry soil above water table,

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.6} = 16.25 \text{ kN/m}^3$$

Unit weight of moist soil in capillary zone,

$$\gamma_{moist} = \frac{(G+se)}{1+e}\gamma_w = \frac{(2.65 + 0.50 \times 0.6) \times 9.81}{1+0.6} = 18.08 \text{ kN/m}^3$$

Unit weight of completely saturated soil,

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.6) \times 9.81}{1+0.6} = 19.93 \text{ kN/m}^3$$

(a) Total stresses

(i) Total stress at A, $\sigma_A = 0$

(ii) Total stress at B, $\sigma_B = \gamma_d \times 2.8 = 16.25 \times 2.8 = 45.5$ kN/m²

(iii) Total stress at C, $\sigma_C = \gamma_d \times 2.8 + \gamma_{moist, cap} \times 1.2$
$= 16.25 \times 2.8 + 18.08 \times 1.2 = 67.196$ kN/m²

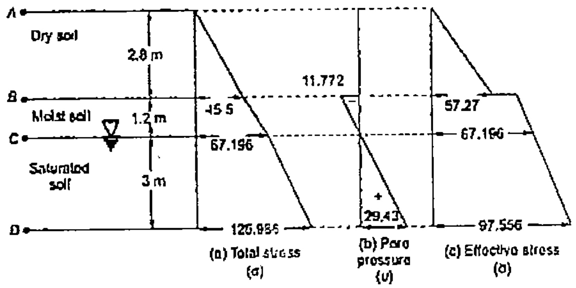
(iv) Total stress at D, $\sigma_D = \gamma_d \times 2.8 + \gamma_{moist, cap} \times 1.2 + \gamma_{sat} \times 3$
$= 16.25 \times 2.8 + 18.08 \times 1.2 + 19.93 \times 3 = 126.986$ kN/m²

(b) Pore pressure

(i) Pore pressure at A, $u_A = 0$
(ii) Pore pressure at B, $u_B = -h_c \times \gamma_w = -1.2 \times 9.81 = -11.72$ kN/m²
(iii) Pore pressure at C, $u_C = 0$
(iv) Pore pressure at D, $u_D = +\gamma_w \cdot z = 9.81 \times 3 = 29.430$ kN/m²

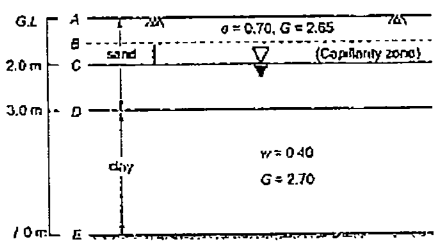
(c) Effective stress

(i) Effective stress at A, $\bar{\sigma}_A = \sigma_A - u_A = 0$
(ii) Effective stress at B, $\bar{\sigma}_B = \sigma_B - u_B = 45.5 - (11.772) = 57.272$ kN/m²
(ii) Effective stress at B, $\bar{\sigma}_B = \sigma_B - u_B = 45.5 - (11.772) = 57.272$ kN/m²
(iii) Effective stress at C, $\bar{\sigma}_C = \sigma_C - u_C = 67.196 - 0 = 67.196$ kN/m²
(iv) Effective stress at D, $\bar{\sigma}_D = \sigma_D - u_D = 126.986 - 29430 = 97.556$ kN/m²



Example 6.8 A soil profile consists of 3 m thick layer of dry sand ($G = 2.65$, $e = 0.7$ and effective size = 0.11 mm). A 4 m thick clay layer is present beneath sand ($w = 40\%$, $G = 2.70$). The ground water table is located at a depth of 2 m from the ground level. Plot variation of total, neutral and effective stress. Assume $C = 0.5$ cm².

Solution:
The given conditions are shown in figure



Height of capillary rise above water table,

$$h_c = \frac{C}{eD_{10}} = \frac{0.5}{0.70 \times (0.011)} = 65 \text{ cm} = 0.65 \text{ m}$$

Hence sand will be saturated upto 0.65 m above the water table and rest of the sand will completely dry.

$$\gamma_{d, sand} = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.70} = 15.30 \text{ kN/m}^3$$

$$\gamma_{sat, sand} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.70) \times 9.81}{1+0.70} = 19.33 \text{ kN/m}^3$$

As the clay layer is submerged under water table, clay will be completely saturated.
Using. $S \cdot e = w \cdot G$

$$e = \frac{w \cdot G}{S} = \frac{0.40 \times 2.70}{1} = 1.08$$

$$\therefore \quad \gamma_{sat, clay} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.70+1.08) \times 9.81}{1+1.08} = 17.83 \text{ kN/m}^3$$

(a) Total stress (σ)

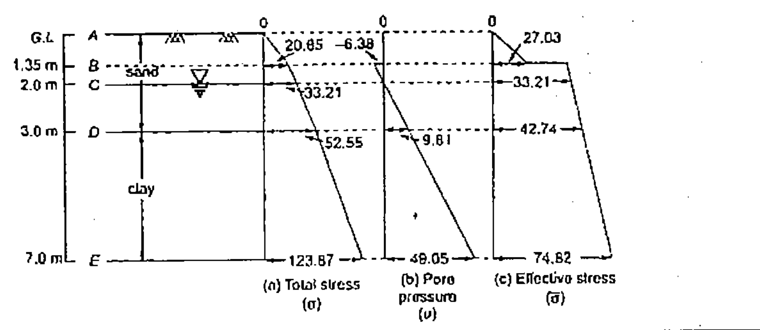
(i) Total stress at A, $\sigma_A = 0$
(ii) Total stress at B, $\sigma_B = \gamma_{d, sand}(2 - h_c) = 15.30 \times (2 - 0.65)$
$= 20.65$ kN/m²
(iii) Total stress at C, $\sigma_C = \gamma_{d, sand}(2 - h_c) + \gamma_{sat, sand} \times h_c$
$= 15.30 \times (2 - 0.65) + 19.33 \times 0.65$
$= 33.21$ kN/m²
(iv) Total stress at D, $\sigma_D = \gamma_{d, sand}(2 - h_c) + \gamma_{sat, sand}(h_c + 1)$
$= 15.30 (2 - 0.65) + 19.33 (0.65 + 1)$
$= 52.55$ kN/m²
(v) Total stress at E, $\sigma_E = \gamma_{d, sand}(2 - h_c) + \gamma_{sat, sand}(h_c + 1) + \gamma_{sat, clay} \times 4$
$= 15.30 (2 - 0.65) + 19.33 (0.65 + 1) + 17.83 \times 4$
$= 123.87$ kN/m²

(b) Pore pressure (u)

(i) Pore pressure at A, $u_A = 0$
(ii) Pore pressure at B, $u_B = -h_c\gamma_w = -0.65 \times 9.81 = -6.38$ kN/m²
(iii) Pore pressure at C, $u_C = 0$
(iv) Pore pressure at D, $u_D = \gamma_w \cdot z = 9.81 \times 1 = 9.81$ kN/m²
(v) Pore pressure at E, $u_E = \gamma_w \times 5 = 9.81 \times 5 = 49.05$ kN/m²

(c) Effective stress ($\bar{\sigma} = \sigma - u$)

(i) Effective stress at A, $\bar{\sigma}_A = 0$
(ii) Effective stress at B, $\bar{\sigma}_B = 27.03$ kN/m²
(iii) Effective stress at C, $\bar{\sigma}_C = 33.21$ kN/m²
(iv) Effective stress at D, $\bar{\sigma}_D = 42.74$ kN/m²
(v) Effective stress at E, $\bar{\sigma}_E = 74.82$ kN/m²



6.6 Effect of Water Table Fluctuations on Effective Stress

Figure shows a layered soil system consisting of soil-1 and soil-2.

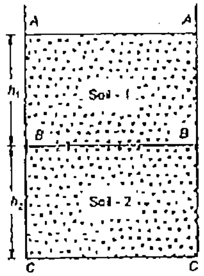


Fig. 6.15

Case 1: When water table is below C-C:

(a) Total, Neutral and Effective Stress at *A-A*:

$$\sigma_A = 0$$
$$u_A = 0$$
$$\bar{\sigma} = \sigma_A - u_A = 0$$

(b) Total, Neutral and Effective Stress at *B-B*:

$$\sigma_B = \gamma_{d_1} h_1$$
$$u_B = 0$$
$$\bar{\sigma} = \sigma_B - u_B = \gamma_{d_1} h_1 - 0 = \gamma_{d_1} h_1$$

(c) Total, Neutral and Effective Stress at *C-C*:

$$\sigma_C = \gamma_{d_1} h_1 + \gamma_{d_2} h_2$$
$$u_C = 0$$
$$\bar{\sigma} = \sigma_C - u_C = (\gamma_{d_1} h_1 + \gamma_{d_2} h_2) - 0 = \gamma_{d_1} h_1 + \gamma_{d_2} h_2$$

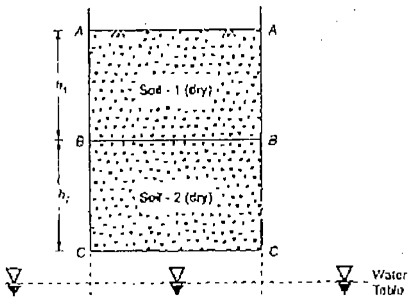


Fig. 6.16

Case 2: When water table is at *B-B*:

(a) Total, Neutral and Effective Stress at *A-A*:

$$\sigma_A = 0$$
$$u_A = 0$$
$$\bar{\sigma}_A = \sigma_A - u_A = 0$$

(b) Total, Neutral and Effective Stress at *B-B*:

$$\sigma_B = \gamma_{d_1} h_1$$
$$u_B = 0$$
$$\bar{\sigma}_B = \sigma_B - u_B = \gamma_{d1} \cdot h_1 - 0 = \gamma_{d1} \cdot h_1$$

(c) Total, Neutral and Effective Stress at *C-C*:

$$\sigma_C = \gamma_{d1} \cdot h_1 + \gamma_{(sat)2} \cdot h_2$$
$$u_C = \gamma_w \times h_2$$
$$\bar{\sigma}_C = \sigma_C - u_C = (\gamma_{d1} \times h_1 + \gamma_{(sat)2} \times h_2 - \gamma_w \times h_2)$$
$$= \gamma_{d1} h_1 + \gamma_{sub} h_2$$

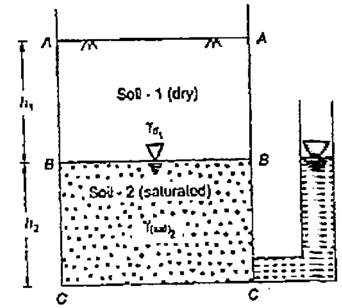


Fig. 6.17

Case 3: When Water Table is at Ground Level i.e. *A-A*:

(a) Total, Neutral and Effective Stress at *A - A*:

$$\sigma_A = 0$$
$$u_A = 0$$
$$\bar{\sigma}_A = \sigma_A - u_A = 0$$

(b) Total, Neutral and Effective Stress at *B-B*:

$$\sigma_B = \gamma_{(sat)_1} \cdot h_1$$
$$u_B = \gamma_w \cdot h_1$$
$$\bar{\sigma}_B = \sigma_B - u_B = \gamma_{(sat)_1} \cdot h_1 - \gamma_w \cdot h_1$$
$$= \left[\gamma_{(sat)_1} - \gamma_w\right] h_1$$
$$= \gamma_1' \cdot h_1$$

where, γ_1' = submerged unit weight for soil - 1

(c) Total, Neutral and Effective Stress at *C-C*:

$$\sigma_C = \gamma_{(sat)_1} \cdot h_1 + \gamma_{(sat)_2} \cdot h_2$$
$$u_C = \gamma_w \cdot (h_1 + h_2)$$
$$\bar{\sigma}_C = \sigma_C - u_C = \gamma_{(sat)_1} \cdot h_1 + \gamma_{(sat)_2} \cdot h_2 - \gamma_w (h_1 + h_2)$$
$$= \left[\gamma_{(sat)_1} - \gamma_w\right] \cdot h_1 + \left[\gamma_{(sat)_2} - \gamma_w\right] \cdot h_2$$
$$= \gamma_1' \cdot h_1 + \gamma_2' \cdot h_2$$

where, γ_1' = submerged unit weight for soil - 1

γ_2' = submerged unit weight for soil - 2

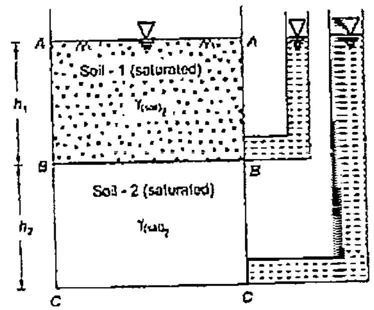


Fig. 6.18

Case 4: When Water Table Is Above Ground Level

(a) Total, Neutral and Effective Stress at A-A:

σ_A = Overburden pressure due to water column of h_w height

$= \gamma_w \times h_w$

$u_A = \gamma_w \cdot h_w$

$\bar{\sigma}_A = \sigma_A - u_A = \gamma_w h_w - \gamma_w h_w = 0$

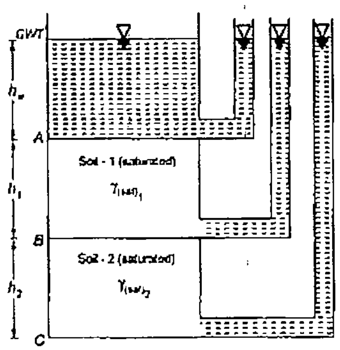


Fig. 6.19

(b) Total, Neutral and Effective stress at B-B:

$\sigma_B = \gamma_w \cdot h_w + \gamma_{(sat)1} \cdot h_1 + \gamma_{(sat)2} \cdot h_2$

$u_B = \gamma_w (h_w + h_1)$

$\bar{\sigma}_B = \sigma_B - u_B$

$= \gamma_w h_w + \gamma_{(sat)1} \cdot h_1 - \gamma_w (h_w + h_1)$

$= (\gamma_{(sat)1} - \gamma_w) \cdot h_1$

$= \gamma'_1 \cdot h_1$

(c) Total, Neutral and Effective stress at C-C:

$\sigma_C = \gamma_w \cdot h_w + \gamma_{(sat)1} \cdot h_1 + \gamma_{(sat)2} \cdot h_2$

$u_c = \gamma_w (h_w + h_1 + h_2)$

$\bar{\sigma}_c = \sigma_c - u_c$

$= \gamma_w \cdot h_w + \gamma_{(sat)1} \cdot h_1 + g_{(sat)2} \cdot h_2 - \gamma_w (h_w + h_1 + h_2)$

$= [\gamma_{(sat)1} - \gamma_w] \cdot h_1 + [\gamma_{(sat)2} - \gamma_w] \cdot h_2$

$= \gamma'_1 h_1 + \gamma'_2 h_2$

Where, γ'_1 = submerged unit weight for soil-1

γ'_2 = submerged unit weight for soil-2

6.7 Permeability of Soils

Soil is a particulate material and has pores that provides a passage for water. Such passages vary in size and are interconnected. The permeability of a soil is a property which describes quantitatively, the ease with which water flows through that soil.

Remember

- The same soil may exhibit different degrees of permeability, depending upon its structure. A loose sand is much more permeable than when it is dense.
- A clay soil with a flocculated structure is more permeable than the same soil with a dispersed structure.

6.7.1 Darcy's Law

The flow of free water through soil is governed by Darcy's Law. Darcy established that the flow occurring per unit time is directly proportional to the head causing flow and the area of cross-section of the soil sample but is inversely proportional to the length of the sample, i.e.

$$q \propto \frac{\Delta h}{L} . A$$

$$q = k . \frac{\Delta h}{L} . A = k . i . A$$

Δh = head causing flow

Where, $i = \frac{\Delta h}{L}$, hydraulic gradient

k = Coefficient of permeability of soil

A = Area of cross-section

Note: The Coefficient of permeability of soil (k) is defined as the average velocity of flow which will occur under unit hydraulic gradient. It has the unit same as velocity i.e. cm/sec, or m/day etc.

6.7.2 Discharge Velocity

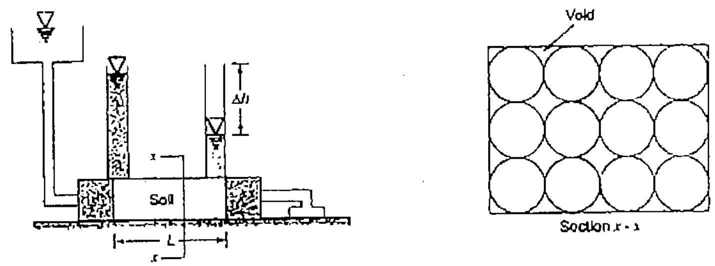


Fig. 6.20

Area of soil specimen at section $X - X = A$

From Darcy's Law

$$q = K . i . A$$

$$\frac{q}{A} = K . i = v$$

Here, V is average velocity based on gross area of cross section. The average velocity is also referred as superficial velocity. It is superficial because the actual flow is through pores in the cross section and not through entire cross-sectional area.

6.7.3 Seepage Velocity

The flow through soils, however occurs only through the interconnected pores. The velocity through the pore is called seepage velocity (v_s).

As the flow is continuous, discharge q must be same throughout the system. Thus

$$q = A . v = A_v . c_s$$

$$v = \frac{A_v}{A} . v_s$$

where, A_v = area of voids in total cross-sectional area A

Since, $\frac{A_v}{A} \simeq \frac{V_v \text{(Volume of void)}}{V \text{(Total volume)}} = n$ (n = porosity)

$\therefore \quad v = n \cdot v_s$

i.e., Seepage velocity $\quad v_s = \dfrac{v}{n}$

Remember: Since $n < 1$, the seepage velocity is always greater than the discharge or superficial velocity.

6.7.4 Limitations of Darcy's Law

- Flow should fulfill continuity conditions.
- The flow should be steady state laminar and one dimensional.
- Soil should be completely saturated.
- No volume changes should occur during or as a result of flow.

6.7.5 Coefficient of Permeability

The coefficient of permeability can be defined using Darcy's equation,

$$q = k \cdot i$$

Hence, the coefficient of permeability may be defined as the velocity of flow which would occur under unit hydraulic gradient. It is also called coefficient of hydraulic conductivity. Coefficient of permeability has dimension of velocity.

Table: Typical value of the coefficient of permeability

S.No.	Soil Type	Coefficient of Permeability (cm/sec)	Drainage Properties
1.	Clean gravel	>1	Very pervious
2.	Sand	1×10^{-3} cm/s to 1 cm/s	Pervious
3.	Silt	1×10^{-7} cm/s to 1×10^{-3} cm/s	Poorly pervious
4.	Clay	$< 10^{-7}$ cm/s	Impervious

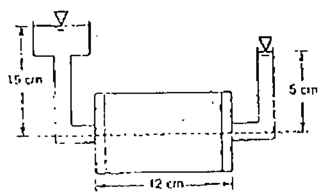
Remember

The coefficient of permeability divided by porosity of soil is known as coefficient of percolation (k_p).

$$k_p = \frac{k}{n}$$

where k_p = Coefficient of percolation
k = Coefficient of permeability
n = Porosity

Example 6.9 For the soil specimen shown in figure, calculate the soil permeability if rate of flow is 2 cc/min. Take area cross-section of soil specimen as 8 cm².



Solution:

Given,
L = 12 cm,
A = 8 cm²

Q = 2 cc per minute = $\dfrac{2}{60}$ cc/sec

$H_L = H_1 - H_2$ = 15 − 5 = 10 cm

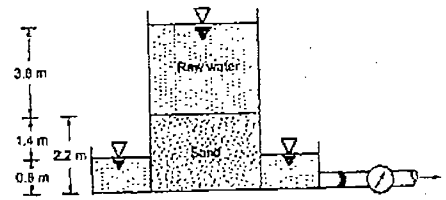
Hydraulic gradient, $\quad i = \dfrac{H_L}{L} = \dfrac{10}{12}$

Using Darcy's equation, $\quad Q = k \cdot i \cdot A$

$$\frac{2}{60} = k \cdot \frac{10}{12} \cdot 8$$

$$k = 5 \times 10^{-3} \text{ cm/sec}$$

Example 6.10 A slow sand filter of area 2.85 m² consists 2.2 m sandy layer as shown in figure. Calculate the amount of water supplied per day if permeability of sand is 4.8×10^{-5} m/s.



Solution:

Given,
A = 2.85 m²
$k = 4.8 \times 10^{-5}$ m/s

Let us assume that datum is at bottom of filter.

$\therefore$ Head loss,

ΔH = Total available at top of sand layer − total head after filtration through sand layer
= (2.2 m + 3.8 m) − (0 + 0.8 m)
= 5.2 m

$\therefore$ Hydraulic gradient, $\quad i = \dfrac{\Delta H}{\text{thickness of sand layer}} = \dfrac{5.2m}{2.2m} = 2.364$

From Darcy's law, $\quad Q = k \cdot i \cdot A$

$= 4.8 \times 10^{-5} \times 2.364 \times 285$

$= 3.234 \times 10^{-4}$ m³/s

$\therefore$ Discharge in a day $= Q \times 24 \times 60 \times 60$ m³/d

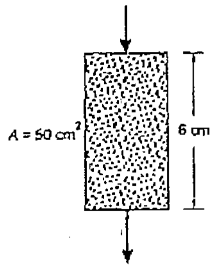
$= 3.234 \times 10^{-4} \times 86400$ m³/d

$= 24.94$ m³/d

Example 6.11 **Calculate the coefficient of permeability of a soil sample 6 cm in height and 50 cm² in cross sectional area, if a quantity of water equal 450 ml passed down in 10 minutes under an effective constant head of 40 cm. On oven drying, the test specimen weight 495 g. Taking the specific gravity of soil solids as 2.65, calculate the seepage velocity of water during the test.**

Solution:

Given,
$$A = 50\ cm^2$$
$$Q = 450\ ml/10\ min = 0.45\ l/600\ sec$$
$$= \frac{0.45 \times 1000 cm^3}{600 sec} = 0.75\ cm^3/sec$$
$$i = \frac{\Delta H}{L} = \frac{40cm}{6cm} = 6.67$$



From Darcy's Law,
$$Q = k.\ i.\ A$$
$$0.75\ cm^3/s = k \times 6.67 \times 50\ cm^2$$
$$k = \frac{0.75}{6.67 \times 50} = 2.25 \times 10^{-3}\ cm/sec$$

Note that on oven drying, the volume of sample remain the same.

$$\therefore \quad \gamma_d = \frac{W_d}{V} = \frac{495gm}{(50 \times 6)cm^3} = 1.65\ gm/cm^3$$

Using,
$$\gamma_d = \frac{G.\gamma_w}{1+e}.$$
$$\Rightarrow \quad 1.65 = \frac{1 \times 2.65}{1+e}$$
$$\therefore \quad e = 0.606$$

Using,
$$n = \frac{e}{1+e}.$$
$$\Rightarrow \quad n = \frac{0.606}{1+0.606} = 0.377$$

Now, seepage velocity $(v_s) = \frac{\text{Discharge velocity}}{\text{Porosity (n)}}$

where, Discharge velocity $(v) = \frac{\text{Discharge}(Q)}{\text{Area of specimen}(A)}$

$$= \frac{0.75 cm^3/s}{50 cm^2} = 0.015\ cm/s$$

$$\therefore \quad \text{Seepage velocity } (v_s) = \frac{0.015}{0.377}\ cm/s$$
$$= 0.0398\ cm/s \text{ or } 0.398\ mm/s$$

6.8 Permeability of Stratified Soils

In nature, soils are usually deposited in successive layers. Even if the different layers of the deposit are homogeneous within themselves, this may lead to a considerable disparity in the average permeability parallel to the bedding plant and that normal to the bedding planes.

Broadly stratification can be considered as horizontal and continuous, average coefficient for flow in horizontal and vertical directions can be estimated.

6.8.1 Horizontal Flow

Consider the solid profile, shown in figure below consisting of n number of layers with $H_1, H_2, H_3 \ldots H_n$ in thickness with their permeability $k_1, k_2, k_3 \ldots\ldots k_n$ respectively.

Let H = Total thickness of deposit

$= H_1 + H_2 + H_3 + H_4 \ldots H_n$

k_H = Average permeability in the horizontal direction.

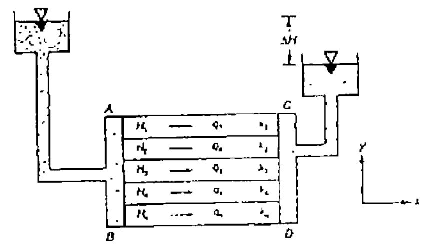


Fig. 6.21

Assume the total head along the line AB to be constant. Similarly, the total head along CD may also be taken as constant but value will be less than that along AB.

$$\therefore \quad i = i_1 = i_2 = i_3 = i_4 \ldots i_n$$

Total discharge through the soil deposit = sum of discharge through the individual layers.

$$\therefore \quad q = q_1 + q_2 + q_3 + q_4 + \ldots q_n$$
$$k_H . i . A = k_1 i A_1 + k_2 i A_2 + k_3 i A_3 + k_4 i A_4 + \ldots\ldots k_n i A_n$$
$$\Rightarrow \quad k_H . H = k_1 . H_1 + k_2 . H_2 + k_3 . H_3 + k_4 . H_4 + \ldots k_n . H_n$$
$$k_H = \frac{k_1 H_1 + k_2 H_2 + k_3 H_3 + k_4 H_4 + \ldots k_n H_n}{H}$$
$$k_H = \frac{k_1 H_1 + k_2 H_2 + k_3 H_3 + k_4 H_4 + \ldots k_n H_n}{H_1 + H_2 + H_3 + H_4 + \ldots\ldots H_n} = \frac{\Sigma kH}{\Sigma H}$$

6.8.2 Vertical Flow

In this case, flow takes place in the direction perpendicular to the stratification.

In this case, to satisfy the continuity condition.

$$q = q_1 = q_2 = q_3 = q_4 = \ldots q_n$$

Let head loss through different layers be $h_1, h_2, h_3, h_4 \ldots\ldots h_n$

∴ Total head, loss,

$$\Delta H = h_1 + h_2 + h_3 + h_4 + \ldots\ldots h_n \qquad \ldots(i)$$

Also let K_v be the average permeability in vertical direction

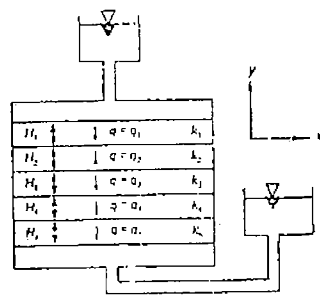


Fig. 6.22

Now,
$$q = k_v . i . A = k_V . \frac{\Delta H}{H} . A$$

or
$$\Delta H = \frac{q.H}{k_v . A}$$

Similarly,
$$h_1 = \frac{q.H_1}{k_1 . A}, \quad h_2 = \frac{q.H_2}{k_2 . A},$$

$$h_3 = \frac{qH_3}{k_3 \cdot A}, \; h_u = \frac{qH_v}{k_u \cdot A}$$

From equation (i), we get

$$\frac{qH}{k_v A} = \frac{qH_1}{k_1 A} + \frac{qH_2}{k_2 A} + \frac{qH_3}{k_3 A} + \frac{qH_4}{k_4 A} + \ldots \frac{qH_n}{k_n}$$

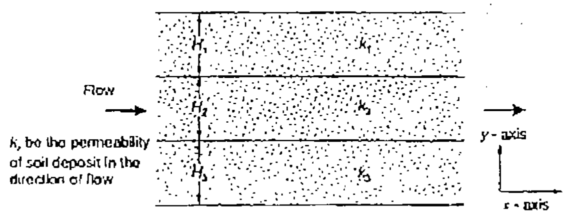
or

$$k_v = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3} + \frac{H_4}{k_4} + \ldots \frac{H_n}{k_n}}$$

Example 6.12 : A horizontal stratified soil deposit consists of three layers, each uniform in itself. The permeability of the layer are 8×10^{-4}, 50×10^{-4} and 15×10^{-4} cm/sec; and their thicknesses are 6 m, 3 m and 12 m respectively. Find the effective average permeability of the deposit in horizontal and vertical directions.

Solution:

Since it is a horizontal deposit, the k_x i.e the permeability parallel to bedding planes, will be the horizontal permeability



$$k_x = \frac{k_1H_1 + k_2H_2 + k_3H_3 + \ldots}{H_1 + H_2 + H_3}$$

Using the above values, we have

$$k_x = k_h = \frac{8\times10^{-4}\times6 + 50\times10^{-4}\times3 + 15\times10^{-4}\times12}{6+3+12}$$

$$= \frac{378}{21}\times10^{-4} = 18\times10^{-4} \text{ cm/sec}$$

Permeability in vertical direction, k_y, being the permeability of soil deposit in the direction of flow.

$$k_y = \frac{H_1 + H_2 + H_3}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}}$$

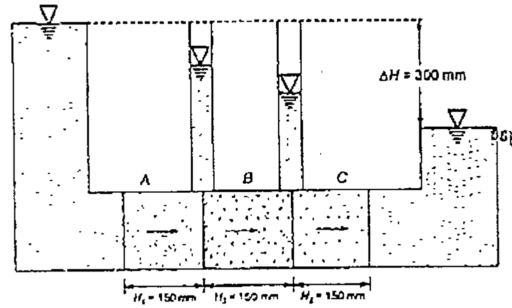
Using the above values, we get

$$k_v = k_y = \frac{6+3+12}{\frac{6}{8\times10^{-4}} + \frac{3}{50\times10^{-4}} + \frac{12}{15\times10^{-4}}}$$

$$= \frac{21\times10^{-4}}{0.75\times0.06+0.80} = \frac{21}{1.61}\times10^{-4}$$

$$= 13.04\times10^{-4} \text{ cm/sec}$$

Example 6.13 The soil layers below have a cross-sectional area of 100 mm x 100 mm each. The permeability of each soil is: $k_A = 10^{-2}$ cm/sec; $k_B = 3\times10^{-3}$ cm/sec; $k_C = 4.9\times10^{-4}$ cm/s. Find the rate of water supply in cm³/hr.



Solution:

Drawing it looks like a horizontal flow, but in reality it is a vertical flow because flow has to cross through every layer. Hence every layer has the same velocity.

$$v = v_1 = v_2 = v_3$$

$$\therefore \quad k_{eq} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}} = \frac{450}{\frac{150}{1\times10^{-2}} + \frac{150}{3\times10^{-3}} + \frac{150}{4.9\times10^{-4}}}$$

$$= 1.2\times10^{-3} \text{ cm/sec}$$

Head loss during flow. $H_L = 300$ mm

$\therefore$ Hydraulic gradient, $i = \frac{H_L}{H} = \frac{300}{450} = \frac{2}{3}$

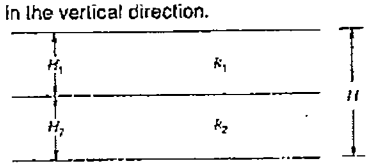
By Darcy's equation, $Q = k_{eq}\, i\, A = 1.2\times10^{-3}\times\frac{2}{3}\times(10\text{cm}\times10\text{cm})\times\frac{3600}{1}$ cm³/hr

$= 291$ cm³/hr

Example 6.14 Prove that for stratified deposits of soil, the average permeability in the horizontal direction is greater than the average permeability in the vertical direction.

Solution:

Consider stratified deposits consisting two layers of thicknesses H_1 and H_2 with coefficient of permeability k_1 and k_2 as shown in figure.



We know average permeability in Horizontal and Vertical directions are given by

$$k_H = \frac{k_1H_1 + k_2H_2}{H_1 + H_2} = \frac{k_1H_1 + k_2H_2}{H}$$

$$k_V = \frac{H_1 + H_2}{\frac{H_1}{k_1} + \frac{H_2}{k_2}} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2}}$$

Therefore, $$\frac{k_H}{k_V} = \frac{(k_1H_1 + k_2H_2)/H}{H/[(H_1/k_1) + (H_2/k_2)]}$$

$$= \frac{H_1^2 + H_2^2\left[(k_1^2 + k_2^2)/k_1k_2\right]H_1H_2}{H^2}$$

$$= \frac{H_1^2 + H_2^2 + \left[(k_1^2 + k_2^2)/k_1k_2\right]H_1H_2}{H_1^2 + H_1^2 + 2H_1H_2} \qquad [\because H = H_1 + H_2]$$

Since k_1 and k_2 are always positive,

$(k_1 - k_2)^2 \geq 0$

or $k_1^2 + k_2^2 \geq 2\,k_1k_2$

$$\frac{k_1^2 + k_2^2}{k_1k_2} \geq 2$$

The above ratio is equal to 2, only when $k_1 = k_2$ otherwise it is greater than 2. Here if we take different values of k_1 and k_2 then the above ratio will always be greater than 2.

Therefore, $\frac{k_H}{k_V}$ in relation, the numerator is greater than the denominator.

Hence, $k_H > k_V$

6.9 Determination of Coefficient of Permeability

The coefficient of permeability of a soil can be determined using following methods.

(1) Laboratory methods

(2) Field methods

(3) Indirect methods

6.9.1 Laboratory Methods

Laboratory tests can be performed on both undisturbed and remoulded samples. The coefficient of permeability can be determined in laboratory by the following methods.

(a) Constant head permeability test

(b) Variable head permeability test

(c) Capillary permeability test

(a) Constant head permeability test: This test is preferred for coarse grained soils, such as gravels and sands. The complete set up for constant head permeameter is shown in figure below.

An observation is taken by collecting a quantity of water in a graduated jar for a known time.

Let volume of water collected in time 't' is 'V'.

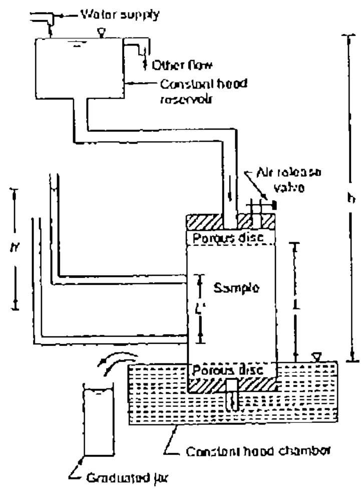


Fig. 6.23 Constant head permeameter

$\therefore$ Discharge, $Q = \frac{V}{t}$

$\Rightarrow \quad k.i.A = \frac{V}{t} \qquad$ [By Darcy's Law]

or $\quad k.\frac{h}{L}.A = \frac{Q}{t} \qquad \left[\because i = \frac{H_L}{L} = \frac{h}{L}\right]$

or $\quad k = \frac{QL}{Ah}$

where, L = distance between manometer taping point

A = Cross sectional area of sample

h = difference in manometric levels

Example 6.15 The following data were recorded in a constant head permeability test:

Internal diameter of permeameter = 7.5 cm

Head lost over a sample length of 18 cm = 24.7 cm

Quantity of water collected in 60 seconds = 626 ml.

Porosity of the soil sample = 44%

Calculate the coefficient of permeability of the soil. Also, determine the discharge velocity and the seepage velocity during the test.

Solution:

Area of soil sample, $A = \frac{\pi}{4}.d^2 = \frac{\pi}{4} \times (7.5)^2 = 44.18\ \text{cm}^2$

$H = 24.7$ cm, $L = 18$ cm, $t = 60$ sec

$\therefore$ Hydraulic gradient, $i = \frac{H_L}{L} = \frac{24.7}{18} = 1.372$

Discharge, $Q = \frac{V}{t} = \frac{626}{60} = 10.433\ \text{cm}^3/\text{s}$

From Darcy's equation, $Q = k.i.A$

$10.433 = k \times 1.372 \times 44.18$

$k = 0.172$ cm/s

Discharging velocity, $v = \frac{Q}{A} = \frac{10.433}{44.18} = 0.236$ cm/s

Given, porosity of soil sample, $n = 0.44$

∴ Seepage velocity, $v_s = \frac{v}{n} = \frac{0.236}{0.44} = 0.537$ cm/s

(b) Variable head permeability test: Variable head permeability test is used for fine sands and silts which have relatively low permeability. The complete setup for variable head permeability test is shown in figure.

After saturation, the stand pipe of area cross-section 'a' is filled with water and time cross pending to h_1 is noted down. Now water is allowed to fall to h_2, and time t_2 is noted.

Let at any intermediate stage water level be 'h' which falls by 'dh' in time 'dt'. So the head difference is h.

At that stage, let the discharge is 'q'.

$$\therefore \quad q = k.i.A = k.\frac{h}{L}.A$$

Volume of water collected in 'dt',

$$q.dt = a(-dh)$$

$$k.\frac{h}{L}A\,dt = -a\,dh$$

Integrating, $$k.\int_{t_1}^{t_2} dt = \frac{-aL}{A}\int_{h_1}^{h_2}\frac{dh}{h} = \frac{aL}{A}\int_{h_2}^{h_1}\frac{dh}{h}$$

$$k(t_2 - t_1) = \frac{aL}{A}(\log_e h_1 - \log_e h_2)$$

$$= \frac{aL}{A}\log_e\left(\frac{h_1}{h_2}\right)$$

$$k = \frac{aL}{At}\log_e\left(\frac{h_1}{h_2}\right)$$

Where $t = (t_2 - t_1)$

$$k = \frac{2.303aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right)$$

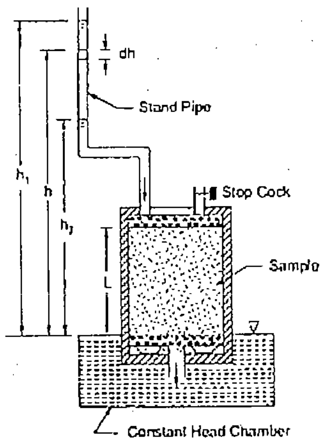


Fig.6.24 Falling head permeameter

NOTE

- Let variable head test is performed in two stages on same soil if, in first stage water level falls from h_1 to h_2 in time interval 't' and during second stage, water level falls from h_2 to h_3 in same time interval 't', then h_1, h_2 and h_3 shall be related as

$$h_2 = \sqrt{h_1 h_3}$$

- The above results can be used to check the consistency of test in a soil. If variable head test is performed in two stages at equal time intervals, then above relation should hold true in each case

Example 6.16 In a falling head permeability test, the head causing fall was initially 90 cm, and it drops 6 cm in 15 minutes. How much time is required for the head to fall to 45 cm.

Solution:

We know, the coefficient of permeability under falling head permeability test is given by

$$k = 2.303\frac{aL}{At}\log_{10}\frac{h_1}{h_2}$$

Here, $h_1 = 90$ cm,

$h_2 = 90 - 6 = 84$ cm

and $t_1 = 15$ minutes

Therefore $$t = 2.303\frac{aL}{kA}\log_{10}\frac{h_1}{h_2}$$

or $$\frac{2.303aL}{kA} = \frac{t}{\log_{10}\left(\frac{h_1}{h_2}\right)}$$

$$\Rightarrow \quad \frac{2.303aL}{kA} = \frac{15}{\log_{10}\left(\frac{90}{84}\right)} = 500.61 \text{ minutes} \quad ...(i)$$

Let 'T' be the time interval in which head falls from 90 cm to 45 cm.

Therefore, $$T = \frac{2.303aL}{kA}\log_{10}\frac{h_1}{h_3} = 500.61 \times \log_{10}\frac{90}{45}$$

$$= 150.70 \text{ minutes}$$

Example 6.17 A permeameter of 100 mm diameter with a sample length of 30 cm was used for constant head and falling head test. While conducting a constant head test, the loss of head was 120 cm for the soil sample and rate of flow was 3.2 cm³/s. Find the coefficient of permeability. If a falling head test was performed on the same sample at the same void ratio, find the time taken for the head to fall from 98 cm to 50 cm. The diameter of stand pipe in the falling head test was 25 mm.

Solution:

Constant head test:

Area of sample, $A = \pi\left(\frac{100^2}{4}\right) = 7854$ mm²

Hydraulic gradient, $i = \frac{h}{L} = \frac{120}{30} = 4$

Using, $q = k.\,i.\,A$

Therefore, $$k = \frac{q}{i.A} = \frac{3.2 \times 10^3}{4 \times 7854} = 0.102 \text{ mm/s}$$

Falling head test

Area of stand pipe, $a = \pi\left(\frac{25^2}{4}\right) = 490.9\ \text{mm}^2$

Using, $k = 2.303\frac{aL}{At}\log_{10}\frac{h_1}{h_2}$

or $t = \frac{2.303\,aL}{kA}\log_{10}\frac{h_1}{h_2}$

$$t = \frac{2.303\times 490.9\times 300}{0.102\times 7854}\log_{10}\left(\frac{98}{50}\right)$$

(c) Capillary permeability test: It is also known as horizontal capillary test. This test helps us to evaluate the coefficient of permeability (k) as well as value of capillary head (h_c) of the soil sample. The above two test are suitable for saturated soil conditions where as this test can be used for partially saturated soil also.

Procedure:

- In this test, dry or partially saturated soil is placed in a cylinder glass tube of 4 cm dia and 35 cm long. The tube is graduated. Water is allowed to flow from one and other end is open to atmosphere to permit the expulsion of air.

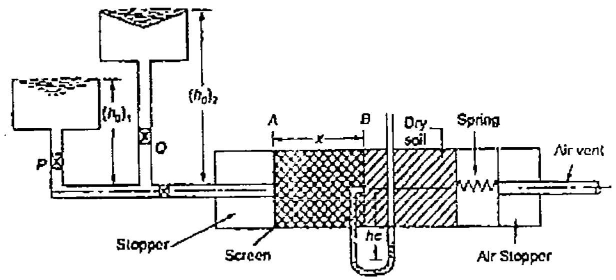


Fig. 6.25

- Let, any time interval 't' after the start of the test, the capillary water travels through a distance x from point A to point B. At point A, there is a pressure head h_0, while at point B, there is a negative pressure h_c.

  ∴ Head lost in flow from A to B,

$$H_L = h_0 - (-h_c) = h_0 + h_c$$

  ∴ Hydraulic gradient, $i = \frac{H_L}{L} = \frac{h_0 + h_c}{x}$

- The seepage velocity through the soil mass (v_s) is given by

$$v_s = \frac{k\,i}{n} \quad \text{... For saturated soil}$$

$$= \frac{k\,i}{Sn} \quad \text{... For partially saturated soil}$$

- Let under head h_0 (constant), water moves distance 'dx' in time 'dt' when it is at a distance x after time 't'.

∴ $v_s = \frac{dx}{dt} = \frac{ki}{Sn}$

or $\frac{dx}{dt} = \frac{k\left(\frac{h_0 + h_c}{x}\right)}{Sn}$

or $x\,dx = \frac{k}{Sn}(h_0 + h_c)dt$

Integrating between the limit x_1 and x_2 for x, and corresponding values of time t_1 and t_2, we get

$$\int_{x_1}^{x_2} x\,dx = \left(\frac{h_0 + h_c}{Sn}\right)k\int_{t_1}^{t_2} dt$$

$$\left[\frac{x^2}{2}\right]_{x_1}^{x_2} = \left(\frac{h_0 + h_c}{Sn}\right)k.[t]_{t_1}^{t_2}$$

$$\frac{x_2^2 - x_1^2}{2} = \frac{k(h_0 + h_c)}{Sn}(t_2 - t_1)$$

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}(h_0 + h_c) \quad \text{...(i)}$$

$S = 100\%$, the above expression becomes

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{n}(h_0 + h_c) \quad \text{...(ii)}$$

where, S = Degree of saturation
h_0 = Head at inlet
h_c = capillary head of soil
n = porosity

- Since k and h_c are the two unknown in above equation. Thus in order to overcome this difficulty, the test is performed in two stages.

Stage-I:

In first stage, soil sample is connected with head h_{0_1}, while value of h_{0_2} is kept closed.

Let under head h_{0_1}, wetted surface of soil moves from x_2 in time t_1 to t_2.

$$\therefore \quad \frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}(h_{0_1} + h_c) \quad \text{...(i)}$$

Stage-II:

In second stage, valve of h_{0_1} is closed and soil connected to constant head $h_{0_2}\,(h_{0_2} > h_{0_1})$ and Let under head h_{0_2}, wetted surface of soil moved from x_1' to x_2' in time t_1' to t_2'.

$$\frac{x_2'^2 - x_1'^2}{t_2' - t_1'} = \frac{2k}{Sn}(h_{o_2} + h_c) \qquad ...(ii)$$

On solving equation (i) and (ii), values of K and h_c computed.

Example 6.18 A capillary permeability test was conducted in two stages under a head of 60 cm and 180 cm. In the first stage, the wetted surface moved 1.5 cm to 7 cm in 7 minutes. In the second stage, it advanced from 7 cm to 18.5 cm in 24 minutes. The degree of saturation at the end of the test was 85% and the porosity was 35%. Determine the capillary head and coefficient of permeability.

Solution:

Stage-I	Stage-II
$h_{o_1} = 60$ cm	$x_2' = 18.5$ cm
$h_{o_2} = 180$ cm	$x_1' = 7$ cm
$x_2 = 7$ cm	$t'_2 - t'_1 = 24$ min
$x_1 = 1.5$ cm	$s = 85\%$
$t_2 - t_1 = 7$ min	$n = 35\%$

For stage I,

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}(h_{o_1} + h_c)$$

$$\frac{7^2 - 1.5^2}{7} = \frac{2k}{0.85 \times 0.35}(60 + h_c)$$

$$k(60 + h_c) = 0.9936 \qquad ...(i)$$

Similarly, for stage II,

$$\frac{x_2'^2 - x_1'^2}{t_2' - t_1'} = \frac{2k}{Sn}(h_{o_2} + h_c)$$

$$\frac{18.5^2 - 7^2}{24} = \frac{2k}{0.85 \times 0.35}(180 + h_c)$$

$$\frac{7^2 - 1.5^2}{7} = \frac{2k}{0.85 \times 0.35}(180 + h_c)$$

$$k(180 + h_c) = 1.819 \qquad ...(ii)$$

From equation (i) and (ii), we get

$$\frac{60 + h_c}{180 + h_c} = \frac{0.9936}{1.819}$$

$$1.819(60 + h_c) = 0.9936(180 + h_c)$$

$$109.14 + 1.819\, h_c = 178.848 + 0.9936\, h_c$$

$$0.8254\, h_c = 69.708$$

$$h_c = 84.45 \text{ cm}$$

Substituting value of h_c in equation (i), we get

$$k(60 + 84.45) = 0.9936$$

$$144.45\, k = 0.9936$$

$$k = 6.87 \times 10^{-3} \text{ cm/min}$$

6.9.2 Field Methods

Laboratory test must be performed, as far as possible, on undisturbed soil samples but obtaining undisturbed sampling is very difficult. The permeability values obtained from the laboratory test may often be quite different from the true value. So for more accurate, representative values, the field tests are conducted.

There are several ways of measuring permeability, in which, the pumping out and pumping in test are quite common. These tests are carried out in boreholes where subsurface explorations are out. These test can be done effectively upto a depth of 30 m and gives the most reliable results.

(a) **Pumping out test:** The pumping out test is more general and accurate method for permeability determination below the water table.

This test is carried out for two basic flow condition: unconfined flow (gravity flow) and confined flow (artesian flow). For confined as well as unconfined flow case, separate formulas have been derived for determining the yield of a well, in terms of drawdown in observation wells, radius of influence, depth of stratum and permeability (k) of soil stratum.

Remember

Aquifer: An aquifer is defined as a formation of a permeable material which have water holding and water seepage capacity. Examples-sand and gravels

Types of Aquifers:

(i) **Unconfined aquifers:** These are also called non-artesian aquifers. These are the top most water bearing strata having no impermeable overburden lying over them.

(ii) **Confined aquifers:** These are also called artesian aquifer. These are sandwiched between two impermeable strata, one at top and other at bottom. In these aquifers, water may be under artesian pressure.

Aquiclude: These are geological formation or stratum of impervious material which do not admit flow of ground water. Example - Clay

Aquitard: These are porous but low permeable. Their seepage capacity is low and discharge under gravity is very low. Example - Silt

Aquifuge: These are a soil mass of rock or an impervious formation which are neither porous nor permeable

Now known the yield and other factors, the permeability k can be easily determined in the field by performing the tests.

(i) Pumping Out Test in Unconfined Aquifer

Specific yield: Specific yield of an unconfined aquifer is the ratio of that volume of water which can flow under gravity to the total volume of soil mass when soil is saturated.

Let V = Total volume of aquifer

V_{wy} = Volume of water that can yielded under gravity

$$\therefore \text{ Specific yield, } \quad S_y = \frac{V_{wy}}{V} \times 100 \text{ (in \%)}$$

NOTE: Coarse grain soils have more specific yield.

Specific Retention: It is the ratio of that volume of water which can be retained in the voids against the gravity to the total volume aquifer.

$\therefore$ Specific retention, $S_R = \frac{V_{wR}}{V} \times 100$

Note that, $V_{wy} + V_{vR} = V_v$ [Under saturation condition]

or $\frac{V_{wR}}{V} + \frac{V_{vn}}{V} = \frac{V_v}{V}$

or $S_y + S_R = n$ [where n = porosity]

NOTE: Specific retention reduces with increase in particle size.

Analysis of Unconfined Aquifer:

Assumptions:

(i) Darcy's Law is valid and flow is laminar.

(ii) Flow is steady and draw down is stable.

(iii) The test well penetrate through the entire thickness of the aquifer. It means that flow is radial, not spherical i.e. no flow occurs from the base of well.

Two observation wells are made to steady position of water table which must be located within the radius of influence.

Consider an elementary cylinder of soil having radius r, thickness dr and height h. Let the water level falls in the observation well at the rate of dh. Hydraulic gradient,

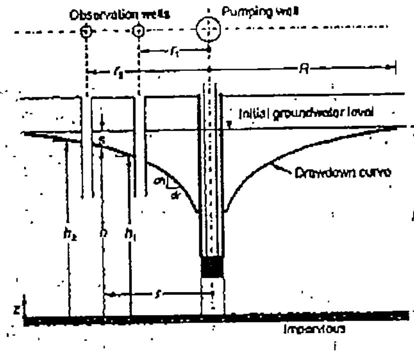


Fig. 6.26 Unconfined Aquifer

$$i = \frac{dh}{dr} \quad \text{and} \quad A = 2\pi rh$$

Discharge into the well = Discharge out of the well i.e pumping rate 'q'

$$\therefore \quad q = k.\,i.\,A = k.\left(\frac{dh}{dr}\right)2\pi rh$$

$$\Rightarrow \quad 2\pi k.\,h.\,dh = q.\frac{dr}{r}$$

Integrating both sides, $2\pi k\int_{h_1}^{h_2} h\,dh = q.\int_{r_1}^{r_2}\frac{dr}{r}$

$$2\pi k\frac{\left[h_2^2 - h_1^2\right]}{2} = q\log_e\left(\frac{r_2}{r_1}\right)$$

$$\therefore \quad k = \frac{2.303\,q\log_{10}\left(\frac{r_2}{r_1}\right)}{\pi(h_2^2 - h_1^2)} \quad \text{[Thiem's Equation]}$$

NOTE In Dupit's theory, observation wells are not taken. He took observation at main test well itself. then $h_1 = h_0$, $h_2 = H$, $r_1 = r_0$ (radius of main well), $r_2 = R$ (Radius of Influence) and H = depth of original ground water table from impervious stratum.

Thus, $k = \frac{2.303\,q\log_{10}\left(\frac{R}{r_0}\right)}{\pi(H^2 - H_0^2)}$ [Dupit's Equation]

Do You Know? Coefficient of transmissibility (T): It is defined as the rate of flow water through a vertical strip of aquifer having unit width extending to full saturation depth under unit hydraulic gradient.

$$\therefore \quad q = k\,.\,i\,.\,A$$

$$T = q = k \times 1 \times (1 \times H) = k.H$$

where, H is thickness of aquifer and k is coefficient of permeablity.

Example 6.19 • **A well penetrates into an unconfined aquifer having a saturated depth of 100 m. The discharge is 250 lit/min at 12 m drawdown. Assuming equilibrium flow conditions and homogenous aquifer, estimate the discharge at 18 m drawdown.**

Solution:

Given, H = 100 m, S_1 = 12 m

Drawdown, $S_1 = H - h_0$

$\therefore$ $h_0 = 100 - 12 = 88$ m

For first case,

Using, $k = \frac{2.303\,q\log_{10}\left(\frac{R}{r_0}\right)}{\pi(H^2 - H_0^2)}$

On rearranging,

$$\frac{\pi k}{2.303\log_{10}\left(\frac{R}{r_0}\right)} = \frac{q}{(H^2 - H_0^2)}$$

$$= \frac{250}{100^2 - 88^2} = \frac{250}{2256} \quad \text{...(i)}$$

For second case,

Drawdown, $S_2 = 18$ m

$\therefore$ $H_0 = H - S_2 = 100 - 18 = 82$ m

Again $q = \frac{\pi k(H^2 - H_0^2)}{2.303\log_{10}\left(\frac{R}{r_0}\right)} = \left[\frac{\pi k}{2.303\log_{10}\left(\frac{R}{r_0}\right)}\right] \times (100^2 - 82^2)$

$$= \frac{250}{2256} \times 182 \times 18$$

$$= 363 \text{ litres/minutes}$$

Example 6.20 A 30 cm diameter well penetrates to a depth of 25 m below the static water table. After 24 hours of pumping @ 5400 litres/minutes, the water level in a test well at 90 m is lowered by 0.53 m, and in a well 30 m away, the drawdown is 1.11 m.

(a) What is the transmissibility of the aquifer?

(b) Also determine the draw down in the main well.

Solution:
Since the well penetrates 25 m below the static water table, it evidently is the case of unconfined aquifer.

Given, $H = 25$ m

Drawdown, $S_2 = 0.53$ m

$r_0 = 30$ m,

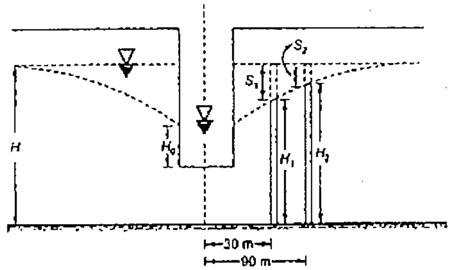
$r_2 = 90$ m

$S_1 = 1.11$ m

$h_2 = H - S_2$

$= 25 - 0.53$

$= 24.47$ m



$h_1 = H - S_1 = 25.11 = 23.89$ m

$q = 5400$ l/min $= 5.4$ m³/min $= 0.09$ m³/s

Using Thiem's equation, $$k = \frac{2.303 q \log_{10}\left(\frac{r_2}{r_1}\right)}{\pi(h_2^2 - h_1^2)}$$

Substituting values, we get, $$k = \frac{2.303 \times 0.09 \times \log\left(\frac{90}{30}\right)}{\pi(24.47^2 - 23.89^2)} = 1.121 \times 10^{-3} \text{ m/s}$$

(a) We know,
Transmissibility, $T = k.H = 1.121 \times 10^{-3} \times 25 = 0.028$ m²/s

(b) To determine the drawdown in the main well, use dupit's equal

$$k = \frac{2.303 \log_{10}\left(\frac{r_1}{r_0}\right)}{\pi(h_1^2 - H_0^2)}$$

where, r_0 = radius of main well = 25 cm

H_0 = Steady water level in main well

$$\therefore \quad 1.121 \times 10^{-3} = \frac{2.303 \log_{10}\left(\frac{30}{0.15}\right)}{\pi(23.89^2 - H_0^2)}$$

$H_0 = 12.08$ m

∴ Drawdown in main well,

$S_0 = H - H_0 = 25 - 12.08 = 12.92$ m

(ii) Pumping Out Test in Confined aquifer: An artesian well (confined) penetrating the full depth of the aquifer is shown in figure below.

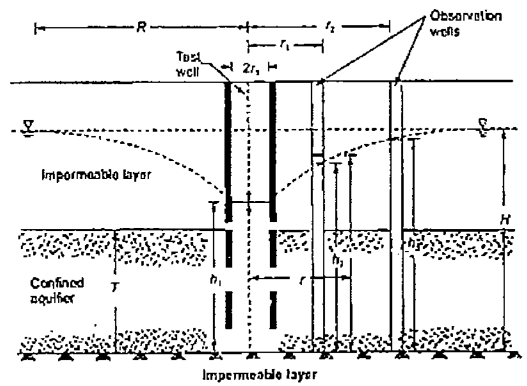


Fig.6.27 Confined Aquifer

In the steady state condition, consider an elementary cylinder of radius r, thickness 'dr' and height h.

The rate of flow is given by,

$$q = kiA = k.\frac{dh}{dr}.(2\pi rB)$$

on rearranging and integrating,

$$q.\int_{r_1}^{r_2}\frac{dr}{r} = 2\pi k.B\int_{h_1}^{h_2}.dh$$

$$k = \frac{2.303 q \log_{10}\left(\frac{r_2}{r_1}\right)}{2\pi B(h_2 - h_1)} \quad \text{[Thiem's equation]}$$

For Dupit' equation,

$r_1 = r_0$

$h_2 = H$

$H_1 = H_0$

$$\therefore \quad k = \frac{2.303 q \log_{10}\left(\frac{R}{r_0}\right)}{2\pi B(H - B)} \quad \text{[Dupit's equation]}$$

Special Case

Spherical Flow In Well: If test well just penetrating over impervious confined stratum. In this case, flow from the base is spherical but very low

The discharge q_s from such a well can, however, be calculated from the equation,

$$Q_s = 2\pi k . r_0 (H - H_0)$$
$$= 2\pi K . r_0 . S$$

where,

H_0 = steady water level in well

S = drawdown in well $= H - H_0$

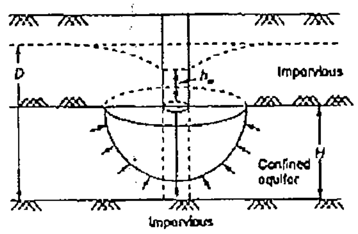
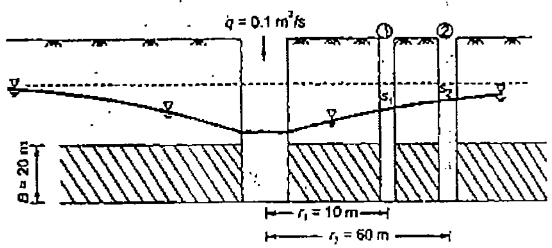


Fig. 28

Example 6.21 An aquifer of 20 m average thickness is overlain by impermeable layer of 30 m thickness. A test well of 0.5 m diameter and two observation wells at a distance of 10 m and 60 m the test well are drilled through the aquifer. After pumping at a rate of 0.1 m^3/sec for a long time, the following drawdowns were stabilized in these wells; first observation well, 4 m; second observation well, 3 m. Show the arrangement in a diagram. Determine the coefficient of permeability and the drawdown in the test well.

Solution:



Given that $s_1 = 4$ m, $s_2 = 3$ m

We know that
$$q = \frac{2\pi kB(s_1 - s_2)}{2.303\log_{10}\left(\frac{r_2}{r_1}\right)} \qquad (\because kB = T)$$

$$\Rightarrow \quad q = \frac{2\pi T(s_1 - s_2)}{2.303\log_{10}\left(\frac{r_2}{r_1}\right)}$$

$$\Rightarrow \quad 0.1 = \frac{2\pi T(4-3)}{2.303\log_{10}\left(\frac{60}{10}\right)}$$

$$\Rightarrow \quad T = \frac{2.303 \times 0.1 \times \log_{10} 6}{2\pi}$$

$$\Rightarrow \quad T = 0.0285 \text{ m}^2\text{/sec}$$

$$\therefore \quad k = \frac{T}{B} = \frac{0.0285}{20} = 1.425 \times 10^{-3} \text{ m/sec}$$

Again,
$$q = \frac{2\pi T(s_w - s_1)}{2.303\log_{10}\left(\frac{r_1}{r_w}\right)}$$

where s_w is drawdown in well

r_w is radius of well $= \frac{0.5}{2} = 0.25$m

$$\therefore \quad 0.1 = \frac{2\pi \times 0.0285 \times (s_w - 4)}{2.303\log_{10}\left(\frac{10}{0.25}\right)}$$

$$\Rightarrow \quad s_w = \frac{0.1 \times 2.303 \times \log_{10} 40}{2\pi \times 0.0285} + 4 = 6.06 \text{ m}$$

Thus coefficient of permeability of test well, $k = 1.425 \times 10^{-3}$ m/s and drawdown in the test well $s_w = 6.06$ m

(b) Pumping in Test: The pumping in test is suitable for low permeability and thin strata where adequate yield may not be available for pumping out test.

By these test, permeability of soil at the bottom of the bore hole is obtained. Hence these are best suited for permeability determination of stratified deposits.

There are basically two type of pumping-in test:

(i) Open end test
(ii) Packer's Test

(i) Open End Test

- In open-end test, the water flows out of the test hole through its bottom hole.
- An open end pipe is sink into the ground and then soil in the pipe is removed.
- The clean water having temperature greater than underground is added and discharge is measured.
- During test, head is kept constant.
- The coefficient of permeability is determined by the following equation.

$$k = \frac{q}{5.5rH}$$

where, H = difference of levels between the inlet of the causing and the water table.

r = inner radius of casing

q = the constant rate of flow (discharge)

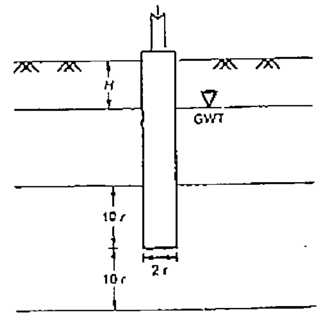


Fig. 6.29

NOTE: For accurate results, the lower end of the pipe should be at a distance of not less than 10r from the top as well as from the bottom of the stratum.

(ii) Packer's Test

- In Packer's test, the water flows out through the sides of the section of a hole enclosed between packers.

- If L length of pipe is perforated and lower end of the casing is plugged, then, the value of the coefficient of permeability is found by the following equations.

$$k = \frac{q}{2\pi LH}\log_e\left(\frac{L}{r}\right) \quad \text{... if } L \geq 10\,r$$

or $$k = \frac{q}{2\pi LH}\sinh^{-1}\left(\frac{L}{2r}\right) \quad \text{... if } \leq L \leq 10\,r$$

where, r = inner radius of hole
H = difference of water levels at the entry and the ground water table for the hole tested below the water table.

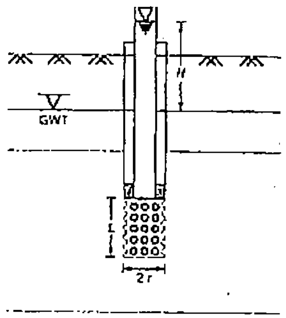


Fig. 6.30

6.9.3 Indirect Methods

(a) Kozney-Carman Equation:

$$\text{Permeability, } k = \frac{1}{k_k}\cdot\frac{\gamma_w}{\mu}\cdot\frac{e^3}{1+e}\cdot D_{10}^2$$

k_K = Kozney-Karman constant
e = void ratio
D_{10} = Effective diameter of soil
μ = Dynamic viscosity coefficient
γ_w = Unit weight of water

(b) Allen Hazen's Equation: This equation is valid for granular soils with effective size of 0.1 mm to 3 mm i.e sands

$$k = C.D^2_{10}\,(\text{cm/sec})$$

where, C = Hazen's constant
= 100 to 150
D_{10} = effective size in cm

(c) Loudan's Equation:

$$\log(kS^2) = a + bn$$

where, a and b are constant, their values being 1.365 and 5.15 respectively at 10°C
n = porosity of soil
S = specific surface area
= surface area/volume

Do You Know?

If soil particles are not spherical and are of variable size then, if these particles pass through sieve size 'a' and retain on sieve size 'b' then the specific surface area,

$$S = \frac{6}{\sqrt{ab}}$$

(d) Terzaghi's equation:

$$k = 200\,e^2 D^2_{10}\,(\text{cm/sec})$$

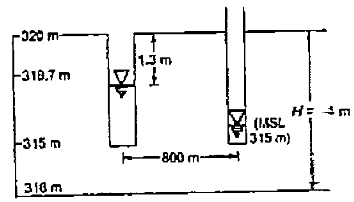
(e) Consolidation equation:

$$k = C_v \gamma_\omega m_v\,(\text{cm/sec})$$

where, γ_ω = unit weight of water (N/cm³)
C_v = coefficient of consolidation (cm²/sec)
m_v = Volume compressibility / modulus of volume change (cm²/N)

Example 6.22 A test boring was done at an elevation 320 m MSL, and it was found that the phreatic surface (water table) is 1.3 m below the ground surface. An aquifer was identified and sample of soil was having effective size of 16 mm. A piezometer was installed 800m downstream from the boring and showed water level at elevation 315 m MSL. If thickness of aquifer was 4 m between both points estimate.

(a) The permeability using Hazen's formula (C = 12)
(b) Quantity of flow per m width.



Solution:

Given, C = 12, D = 16 mm

(a) Using Allen Hazen formula,

$$k = C(D_{10})^2$$
$$= 12 \times (0.16)^2$$
$$= 0.31 \text{ mm/s}$$

(b) The head loss between two points,

$$H_L = (320 - 1.3) - 315 = 3.7 \text{ m}$$

$\therefore$ Hydraulic gradient, $i = \frac{H_L}{L} = \frac{3.7}{800}$

Using Darcy's Formula,

$$Q = k\,.\,i\,.\,A = 0.31 \times \frac{3.7}{800} \times (4 \times 1) = 1.433 \times 10^{-3} \text{ m}^3/\text{s}$$

Example 6.23 The coefficient of permeability of fine sand is 0.022 cm/s at a void ratio of 0.72. Estimate permeability of soil if sand is compacted to a void ratio of 0.57. Use Kozeny-Carman formula.

Solution:

From Kozeny-Carman Formula, $$k = \frac{1}{k_k}\cdot\frac{\gamma_\omega}{\mu}\cdot\frac{e^3}{1+e}\cdot D_{10}^2$$

For a given soil the term '$\frac{1}{k_k}\frac{\gamma_\omega}{\mu}D_{10}^2$' would remain constant

Hence, $$k = C\frac{e^3}{1+e}$$

$$\therefore \quad \frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}}$$

$$\frac{0.022}{k_2} = \frac{(0.72)^3}{1+0.72} \times \frac{1+0.57}{(0.57)^3} = 1.84$$

$$k_2 = \frac{0.022}{1.84} = 0.012 \text{ cm/s}$$

6.10 Factors Affecting Permeability

The permeability depends on the soil properties and fluid properties both.

The Kozney-Carman equation is quite useful, it reflects the effect of factors that affect permeability.

$$k = \frac{1}{K_k} \cdot \frac{\gamma_w}{\mu} \cdot \frac{e^3}{1+e} D_{10}^2$$

(a) Grain Size: The coefficient of permeability (k) includes D^2_{10}, where D_{10} is a measure of grain size.

i.e., $$k \ \alpha \ D_{10}^2$$

If the void ratio is same, then permeability is more in coarse soil than in fine soil.

$$k_{gravel} > k_{sand} > k_{silt} > k_{clay}$$

(b) Void ratio:

- As per Kozeny-Carman equation, coefficient of permeability is directly proportional to $\frac{e^3}{1+e}$.

  i.e $$k \alpha \frac{e^3}{1+e} \quad \text{o} \quad k \alpha e^2$$

- If particle size is same, then loose soils are more permeable than dense soils.
- The plot of void ratio (e) against permeability k (at log scale) is approximately a straight line for all soils.

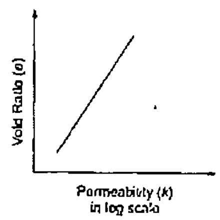


Fig.6.31

(c) Particle shape:

- It is expressed in terms of specific surface area and permeability relates to specific surface area as $k \alpha \frac{1}{S^2}$
- The angular particles have greater specific surface than the rounded particles.
- For the same void ratio, the soils with angular particles are less permeable than those with rounded particles.

(d) Degree of Saturation: Permeability is directly proportional to degree of saturation i.e $k \alpha$ Degree of saturation (S).

In partially saturated soils, entrapped air causes blockage in the flow of water. Hence, the permeability of partially saturated soil is lesser than that of a fully saturated soil.

(e) Entrapped air and gases: Entrapped air and gases in the voids obstruct flow and cause the reduction of permeability.

As far as possible, sample should be fully saturated before the permeability test.

(f) Structure of Soil particles: For stratified soils, permeability is more in horizontal direction as compared to the vertical direction.

(g) Properties of pore fluid: The coefficient of permeability also includes $\frac{\gamma_w}{\mu}$

i.e $$k \ \alpha \ \frac{\gamma_w}{\mu}$$

For water, relatively the unit weight remain constant but viscosity (μ) decreases with increasing temperature.

i.e $$\mu \ \alpha \ \frac{1}{T}$$

Hence, $k \alpha T$

Do You Know?

Usually, permeability is represented at 20°C, test is conducted at T° C, then k_{20} is given by

$$k_{20} = k_T \cdot \frac{\mu_T}{\mu_{20}}$$

$$k_T \cdot \frac{\mu_T}{\mu_{20}} = R_T = \text{Temperature correction factor}$$

$$= 2.42 - 0.475 \log_e(T), \text{ where } T \text{ in } ^\circ C$$

(h) Adsorbed water: Adsorbed water or water film which is strongly attached to the soil solids, reduces flow area available for passage of water, which reduces permeability to some extent.

(i) Impurities: Due to presence of impurities, voids are blocked and the permeability is reduced.

(j) Presence of minerals in water:

- The permeability of clay, depends upon the cation absorb on the surface of mineral.
- If void ratio is constant, the permeability is increased in the following order.

  $k < Na < H < Ca$ for montmorillonite

  $Na < k < Ca < H$ for kaolinite

- For the construction of core of earthen dams, the clay being treated with salt water (Na-water) so that seepage can reduced.

6.11 Coefficient of Absolute Permeability

It was defined by Darcy and this parameter is independent of fluid properties like γ_w and μ. It is only depends on the soil properties.

It is defined as

$$k_0 = k \cdot \frac{\mu}{\gamma_w}$$

Unit: m^2, cm^2 or Darcy

$$1 \text{ Darcy} = 0.987 \times 10^{-8} \text{ cm}^2$$

It is also known as intrinsic permeability or specific permeability.

Example 6.24 What is the intrinsic permeability of saturated soil which have hydraulic conductivity of 15.24 m/day. Assuming ground water is at atmospheric pressure at 20°C having density of 998.2 kg/m³ and viscosity of 1.002×10^{-3} kg/m-sec.

Solution:

Given,

$$\gamma_w = 998.2 \text{ kg/m}^3$$

$$\mu = 1.002 \times 10^{-3} \text{ kg/m-s}$$

$$k = 15.24 \text{ m/day} = \frac{15.24}{24 \times 60 \times 60} = 1.76 \times 10^{-4} \text{ m/sec}$$

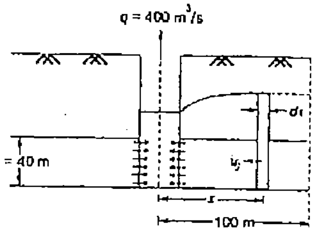
Using,

$$k_0 = k\frac{\mu}{\gamma_w} = \frac{1.76 \times 10^{-4} \times 1.002 \times 10^{-3}}{998.2} = 1.77 \times 10^{-4} \text{ m}^2$$

Example 6.25 A fully penetrating well of radius 0.2 m pumps at a constant rate of 40 m³/hr from a confined sandy aquifer of thickness 40 m and porosity 0.25. Consider only microscopic flow velocity. How long will it take for a pollutant to reach the well if introduced into the aquifer at 100 m from it?

Solution:

Let at any instant of time 't', pollutant is at a distance 'x' from centre of well. Let it moves by dx in time dt velocity of flow at that time,



$$v_s = \frac{-dx}{dt}$$

Here -ve sign indicate that x is decreasing with time.

$$v_s = \frac{v}{n} = \frac{-dx}{dt}$$

or

$$\frac{(q/A)}{n} = \frac{-dx}{dt}$$

$$\frac{q}{2\pi x H} \times \left(\frac{1}{n}\right) = \frac{-dx}{dt}$$

Microscopic flow means, flow is only through confined layer, Hence flow contribution apart from confined aquifer is neglect

$$\therefore \quad \int_0^t dt = \frac{2\pi n H}{q}\int_R^r -x\,dx$$

$$t = \frac{2\pi n H}{q}\left(\frac{R^2 - r^2}{2}\right) = \frac{\pi \times 0.25 \times 40}{400}(100^2 - 0.2^2)$$

$$= 785.395 \text{ hrs or } 32.725 \text{ days}$$

Summary

- Permeability of soil is defined as the property to permit water to percolate through continuously interconnected pores.
- Darcy's Law is valid when flow is laminar, the soil is fully saturated, continuity condition should be valid.
- The apparatus used for the determination of coefficient of permeability of soil in laboratory are called permeameter.
- Constant head permeability test is adopted for coarse-grained soils, where as falling head permeability test is suitable for fine-grained soils.
- Pumping out test is suitable for test below the water table whereas pumping in test can be conducted irrespective of the position of the water table.
- In stratified soils, the average coefficient of permeability in the horizontal direction is more than the average coefficient of permeability in the vertical direction.
- The factors affecting permeability are: Grain size, Void ratio, Particle shape, degree of saturation, entrapped air, structure of soil particle, property of pore fluid, absorbed water etc.
- Absolute permeability of soil is independent of fluid properties, it depends only on soil properties.

Objective Brain Teasers

Q.1 In a falling head permeability test drop in head occurs from 60 m to 40 m in 10 minutes. The cross-sectional area of soil sample and stand pipe is 20 cm² and 2 cm² respectively. If the length of the soil sample is 15 cm then the permeability of soil sample in mm/sec is

(a) 1.01 (b) 0.12
(c) 0.01 (d) 0.001

Q.2 The saturated unit weight of sand in the bed of a pond 20 m deep is 20 kN/m³. Unit weight of water is 10 kN/m³. The effective stress at 4 m below bed level of pond is

(a) 40 kN/m² (b) 20 kN/m²
(c) 60 kN/m² (d) 80 kN/m²

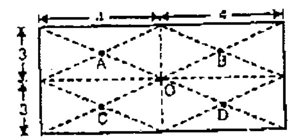
Q.3 Assertion (A) : Constant head permeability test is not used for fine grained soil.
Reason (R) : Lesser the permeability of soil, lesser is the discharge

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.4 In a falling head permeability test, the time taken for the head to fall from 27 cm to 3 cm is 10 minutes. If the test is repeated with same initial head, i.e. 27 cm, what time would it take for the head to fall 9 cm

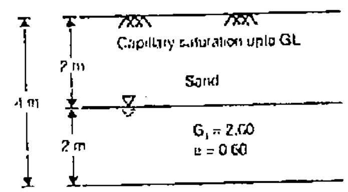
(a) 3 minutes (b) 5 minutes
(c) 6 minutes (d) 7.5 minutes

Q.5 The raft footing of carries a uniform load of 200 kN/m². With the load approximation as shown in the figure, the incremental stress at 5 m below the centre as per Bousinesq's theory is



(a) 13.2 kN/m² (b) 52 kN/m²
(c) 105 kN/m² (d) 26.2 kN/m²

Q.6 The value of effective stress at 4 m depth for the subsoil condition as shown in figure is (Assume γ_w = 10 kN/m³)



(a) 80 kN/m² (b) 20 kN/m²
(c) 60 kN/m² (d) 30 kN/m²

Q.7 A sample of clay and a sample of sand have the same specific gravity and void ratio. Their permeabilities would differ because
(a) their porosities would be different
(b) their degree of saturation would be different
(c) their densities would be different
(d) the size range of their void would be different

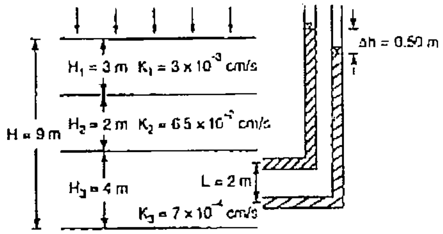
Q.8 Consider the following statements:
1. Soil with high void ratio has always more coefficient of permeability soil with lower void ratio.
2. Constant head permeability test is used for fine grained soil.
3. As temperature increases, the coefficient of permeability of soil also increases.
4. As the specific surface area of soil particle increases the coefficient of permeability decreases.

Which of these statements are correct?
(a) 1, 3 and 4 (b) 2 and 3
(c) 3 and 4 (d) 2, 3 and 4

Q.9 A 30 cm well completely penetrates an unconfined aquifer of depth 40 m. After a long period of pumping at a steady rate of 1500 lpm, the drawdown in two observation wells which are 25 m and 75 m from the pumping well were found to be 3.5 m and 2.0 m respectively. The draw down at the pumping well is
(a) 28.49 m (b) 11.51 m
(c) 10.62 m (d) 12.34 m

Q.10 A capillary permeability test was conducted in two stages under a head of 60 cm and 180 cm respectively at the entry end. In the first stage, the wetted surface moved from 1.5 cm to 7 cm in 7 minutes. In the second stage, it advanced from 7 cm to 18.5 cm in 24 minutes. The degree of saturation at the end of the test was 85% and the porosity was 35%. The capillary head and the coefficient of permeability are respectively.
(a) 84.65 cm and 4.85×10^{-3} cm/min
(b) 92.43 cm and 4.85×10^{-3} cm/min
(c) 92.43 cm and 6.87×10^{-3} cm/min
(d) 84.65 cm and 6.87×10^{-3} cm/min

Q.11 Consider the three layered soil strata shown in the figure below. The thickness and coefficient of vertical permeability of each layer is mentioned in the figure.



The total head loss in three layers is
(a) 1.52 m (b) 1.23 m
(c) 1.18 m (d) 0.87 m

Q.12 Specific capacity of a confined well under equilibrium conditions and within the working limits of drawdown is
(a) is constant at all drawdowns.
(b) decreases as the drawdown increases.
(c) increases as the drawdown increases.
(d) None of the above.

Q.13 In a falling head permeability test on a soil, the time taken for the head to fall from h_0 to h_1 is t. The test is repeated with same initial head h_0, the final head h' is noted in time t/2. Which one of the following equation gives the relation between h', h_0 and h_1?
(a) $h' = h_0/h_1$ (b) $h' = \sqrt{h_0/h_1}$
(c) $h' = h_0 h_1$ (d) $h' = \sqrt{h_0 h_1}$

Q.14 Which one of the following factors are associated with the behaviour of sand mass during earthquake to cause liquefaction?
1. Number of stress cycle
2. The frequency and amplitude of vibration of waves generated by an earthquake
3. Characteristics of sand
4. Relative density

Select the correct answer using the codes given below:
(a) 1, 2, 3 and 4 (b) 1, 2, and 3
(c) 2 and 4 (d) 3 and 4

Q.15 A dry soil with suction capacity '0' rests over a saturated soil. The height of capillary rise is
(a) 0 (b) 1 mm
(c) 1 cm (d) 1 m

Q.16 Consider the following statements:
1. In a sedimentary soil deposit, permeability in the horizontal direction is greater than that in the vertical direction.
2. Permeability of two sand specimens having equal void ratio will be always same

Which of these statement(s) is (are) correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Q.17 Consider the following statements regarding coefficient of permeability 'k' of the soil.
1. It is proportional to square of diameter of soil particle.
2. It decreases with increase in temperature due to reduction in viscosity.
3. It is inversely proportional to square of specific surface.

Which of these statements are correct?
(a) 1 and 3 (b) 2 and 3
(c) 1 and 2 (d) All of these

Q.18 Approximate ratio of the permeabilities of two clay soils, having D_{10} values of 0.8 mm and 0.4 mm respectively, is
(a) 0.25 (b) 2.05
(c) 2 (d) 4

Direction: The following items consists of two statements, one labeled as 'Assertion A' and the other as 'Reason R'. Select your answer to these items using the codes given below.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.19 Assertion (A): Mohr's circle for unconfined compression test passes through the origin.
Reason (R): In an unconfined compression test, the axial stress is equal to confining stress.

Q.20 Assertion (A): Rankine's earth pressure theory is a simplified form of Coulomb's earth pressure theory.
Reason (R): Coulomb's theory consider effect of pore pressures.

Answers

1. (c)	2. (a)	3. (a)	4. (b)	5. (c)
6. (c)	7. (d)	8. (c)	9. (b)	10. (d)
11. (c)	12. (b)	13. (d)	14. (a)	15. (c)
16. (a)	17. (a)	18. (d)	19. (c)	20. (b)

Hints and Explanations:

1. (c)

$$K = \frac{2.3\,aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right)$$
$$= \frac{2.3 \times 2 \times 150}{20 \times (10 \times 60)}\log_{10}\left(\frac{60}{40}\right)$$
$$= 1.012 \times 10^{-3} \text{ cm/sec}$$
$$= 0.01 \text{ mm/sec}$$

2. (a)

$$\bar{\sigma} = \sigma_T - u$$
$$\sigma_T = (\gamma_w \times 20) + \gamma_{sat} \times 4$$
$$= (10 \times 20) + (20 \times 4) = 280 \text{ kN/m}^2$$
$$u = \gamma_w(20 + 4) = 240 \text{ kN/m}^2$$
$$\bar{\sigma} = 280 - 240 = 40 \text{ kN/m}^2$$

4. (b)

For first test

$$t_1 = \frac{2.303\,aL}{Ak}\log_{10}\left(\frac{h_1}{h_2}\right)$$
$$\Rightarrow \quad 10 = \frac{2.303\,aL}{Ak}\log_{10}\left(\frac{27}{3}\right) \quad \text{...(i)}$$

For second test

$$t_2 = \frac{2.303\,aL}{Ak}\log_{10}\left(\frac{27}{9}\right) \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\frac{t_2}{10} = \frac{\log_{10}\left(\frac{27}{9}\right)}{\log_{10}\left(\frac{27}{3}\right)} = \frac{\log_{10}3}{\log_{10}9}$$

$$\Rightarrow \frac{t_2}{10} = \frac{\log_{10}3}{\log_{10}3^2} = \frac{\log_{10}3}{2\log_{10}3}$$

$$\Rightarrow t_2 = \frac{10}{2} = 5 \text{ minutes}$$

5. (c)

The area for each point = 3 × 4 = 12 m

The load at any point = 12 × 200 = 2400 kN

Due to load at A, the incremental stress below centre O

$$= \frac{3Q\,z^3}{2\pi R^5} = \frac{3\times 2400}{2\pi}\times\frac{5^3}{(25^2+5^2)^{5/2}}$$

$$= 26.2 \text{ kN/m}^2$$

Thus the total increment

$$= 4 \times 26.2 = 104.9 = 105 \text{ kN/m}^2$$

6. (c)

$$\gamma_{sat(sand)} = \frac{G+e}{1+e}\gamma_w = \frac{2.6+0.6}{1+0.6}\times 10$$

$$= 20 \text{ kN/m}^3$$

The sand is saturated by gravity flow below water table and by capillary flow upto a height of 2 m above water table.

$\therefore \quad \sigma_{at\,4\,m} = \gamma_{sat} \times 4 = 20 \times 4 = 80$ kN/m²

$U = 2\gamma_w = 20$ kN/m²

$\therefore \quad \bar{\sigma} = 80 - 20 = 60$ kN/m²

9. (b)

q = 1500 litres/min

= 1.5 m³/min = 0.025 m³/sec

$h_2 = H - s_2 = 40 - 2 = 38$ m

$h_1 = H - s_1 = 40 - 3.5 = 36.5$ m

We know that,

$$q = \frac{1.36k(h_2^2 - h_1^2)}{\log_{10} r_2/r_1}$$

$$\text{or} \quad 0.025 = \frac{1.36k(38^2 - 36.5^2)}{\log_{10} 75/25}$$

From which,

$k = 7.848 \times 10^{-5}$ m/sec

At the well face,

$$q = \frac{1.36k(h_1^2 - h_w^2)}{\log_{10}(r_1/r_w)}$$

$$\therefore \quad 0.025 = \frac{1.36\times 7.848\times 10^{-5}(36.5^2 - h_w^2)}{\log_{10}\left(\frac{25}{0.15}\right)}$$

or $\quad h_w^2 = 36.5^2 - 520.42 = 811.83$

$h_w = 28.49$ m

∴ Draw down at well face,

$s_w = H - h_w = 40 - 28.49 = 11.51$ m

10. (d)

First stage:

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}\times(h_{01} + h_c)$$

$$\frac{7^2 - 1.5^2}{7} = \frac{2k}{0.85\times 0.35}(60 + h_c)$$

or $\quad k(60 + h_c) = 0.9934$ ...(i)

Second stage:

$$\frac{18.5^2 - 7^2}{24} = \frac{2k}{0.85\times 0.35}(180 + h_c)$$

or $k(180 + h_c) = 1.8175$ ...(ii)

From equation (i) and (ii)

$$\frac{180 + h_c}{60 + h_c} = \frac{1.8175}{0.9934} = 1.8296$$

from which h_c = 84.65 cm

Hence from equation (i),

$k = 6.87 \times 10^{-3}$ cm/min.

11. (c)

$$K_v = \frac{H}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}}$$

$$= \frac{9}{\frac{3}{3\times 10^{-3}} + \frac{2}{6.5\times 10^{-2}} + \frac{4}{7\times 10^{-4}}}$$

$= 1.33 \times 10^{-3}$ cm/s

For continuity of flow, velocity is the same

$$K_3 \cdot \frac{\Delta h}{L} = K_v \cdot \frac{\Delta h_{total}}{H}.$$

where Δh_{total} = total head loss in three layers

$$\therefore \Delta h_{total} = K_3 \cdot \frac{\Delta h}{L} \cdot \frac{H}{K_v}$$

$$= 7\times 10^{-4} \times \frac{0.50}{2} \times \frac{9}{1.33\times 10^{-3}}$$

$= 1.184$ m

12. (b)

$$\text{Specific capacity} = \frac{1}{C_1 + C_2 Q}$$

(C_1 and C_2 are constant)

This equation shows that specific capacity decreases as Q increases and Q increases only when drawdown is increased by heavier pumping and hence, specific capacity decreases as drawdown increases.

13. (d)

In falling head permeability test

$$k = 2.3\frac{aL}{At}\log_{10}\left(\frac{h_0}{h_1}\right)$$

$$= 2.3\frac{aL}{A(t/2)}\log_{10}\left(\frac{h_0}{h'}\right)$$

$$\therefore \log_{10}\left(\frac{h_0}{h_1}\right) = 2\log\left(\frac{h_0}{h'}\right)$$

$$\Rightarrow \frac{h_0}{h_1} = \left(\frac{h_0}{h'}\right)^2$$

$$\Rightarrow h' = \sqrt{h_0 h_1}$$

14. (a)

Liquefaction of soil during earthquake depends on stress cycle, type of soil characteristics, vibration and relative density of sand.

15. (c)

Suction capacity

$= \log_{10} h_c = 0$

$\Rightarrow \quad h_c = 1$ cm

17. (a)

1. $k \propto (\text{size})^2$

$\therefore \quad k = C D_{10}^2$

2. $k \propto$ temperature
3. $k \propto$ specific surface
4. $k \propto \left(\frac{1}{\text{viscosity}}\right)$

$$k = \frac{1}{C}\times\frac{1}{S^2}\times\frac{\gamma_w}{\mu}\times\frac{e^3}{1+e}$$

18. (d)

$k_1 = C D_{10}^2$

$$\therefore \quad \frac{k_1}{k_2} = \left(\frac{D_{1_{10}}}{D_{2_{10}}}\right)^2$$

$$\frac{k_1}{k_2} = \left(\frac{0.8}{0.4}\right)^2$$

$$\Rightarrow \quad \frac{k_1}{k_2} = 4.0$$

19. (c)

The all round pressure (confining pressure) is zero in unconfined compression test and the Mohr's circle passes through origin.

Student's Assignments

Q 1. A sand sample of 25 cm length was subjected to a constant head permeability test in a permeameter having an area of 30 m². A discharge of 100 cm³ was obtained in a period of 1 minute under a head of 39 cm. Mass of dry sand in the sample was 1350 g. The specific gravity of the sand particles was 2.67. Determine:

(i) the coefficient of permeability

(ii) the superficial velocity and

(iii) the seepage velocity

[Ans. 0.0356 cm/s, 0.056 cm/s, 0.0183 cm/s]

Q 2. Calculate the coefficient of permeability of a soil sample 8 cm in height and cross sectional area 60 cm^2. It is observed that in 12 minutes, 600 ml of water passed down under as effective constant head of 50 cm. On oven drying, the test specimen weight 750 gm. Taking G_s = 2.70, calculate the seepage velocity of water during the test.

[Ans. 2.22 × 10^{-3} cm/sec, 0.33 cm/sec.]

Q 3. A falling head permeability test was performed on a sand sample and the following data were recorded:

Cross-sectional area of permeameter = 100 cm^2; length of the soil sample = 15 cm; area of the stand pipe = 1cm^2; time taken for the head to fall from 150 cm to 50 cm = 8 min, temperature of water was 25°C. Dry mass of soil specimen = 2.2 kg and G_S = 2.66. Calculate the coefficient of permeability of the soil for a void ratio of 0.70 and standard temperature 20°C.

[Ans. 2 × 10^{-4} cm/s]

Q 4. The following data were recorded in a constant head permeability test:

Head lost over a sample length of 18 cm = 24.7 cm

Quantity of water collected in 60 seconds = 626 ml

Internal diameter of permeability = 7.5 cm

Porosity of the soil sample = 44%

Calculate the coefficient of permeability of the soil. Also determine the discharge velocity and the seepage velocity during the test.

[Ans. [0.172 cm,s 0.236 cm/s, 0.537 cm/s]

Q 5. In a capillary permeability test conducted in two stages under a head of 50 cm and 200 cm, in the first stage the wetted surface rose from 20 mm to 80 mm in 6 minutes. In the second stage it rose from 80 mm to 200 mm in 20 minutes. If the degree of saturation is 90% and the porosity is 30%, determine the capillary head and the coefficient of permeability.

[Ans. 170.49 cm, 1.02 × 10^4 cm/sec]

Q 6. A sand deposit of 12 m thickness overlies a clay layer. The water table is 3 m below the ground surface. In a pump out test, the water is pumped out at rate of 540 litres/min when steady-state conditions are reached. Two observation wells are located at 18 m and 36 m from the centre of the test well. The depth of the drawdown curve are 1.8 m and 1.5 m respectively, for these two wells. Determine the coefficient of permeability.

[Ans. 17.96 cm/min]

Q 7. A pumping out test was carried out in the field, in order to determine the average coefficient of permeability of 18 m thick sand layer. The ground water table is located at a depth of 2.2 m below the ground level. A steady state was reached when the discharge from the well was 21.5 lit/sec. At this stage, the drawdown in the test well was 2.54 m, while the drawdown in two observation well situated at 8 m and 20 m from the test well were found to be 1.76 m and 1.27 m respectively.

Determine:

(i) Coefficient of permeability of the sand layer in m/day

(ii) Radius of influence of test well

(iii) Effective size of the sand.

[Ans. 38.70 m/day, 161.29 m, 0.212 mm]

Q 8. A soil has the coefficient of permeability of 0.4 × 10^{-4} cm/sec at a void ratio of 0.65 and temperature of 30° C. Determine the coefficient of permeability at the same void ratio and a temperature of 20° C. At 20°C, γ_w = 0.998 g/ml and μ = 0.0101 poise and at 30°C, γ_w = 0.996 gm/ml and μ = 0.008 poise. What would be the coefficient of permeability at a void ratio of 0.75 and temperature of 20° C?

[Ans. 0.317 × 10^{-4} cm/sec;
0.422 × 10^{-4} cm/sec]

■■■■