CBSE Test Paper 05

Chapter 11 Three Dimensional Geometry

1. Find the angle between the following pairs of lines: $\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k}$ + $\lambda\left(\hat{i} - \hat{j} - 2\hat{k}.\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - 56\hat{k}$ + $\mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}.\right)$,$\lambda, \mu \in R$.

   a. $\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$
   b. $\theta = \cot^{-1}\left(\frac{8}{5\sqrt{3}}\right)$
   c. $\theta = \sin^{-1}\left(\frac{8}{5\sqrt{3}}\right)$
   d. $\theta = \tan^{-1}\left(\frac{8}{5\sqrt{3}}\right)$

2. $\overrightarrow{PQ}$ is a vector joining two points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2).If $\left|\overrightarrow{PQ}\right| = d$, Direction cosines of $\overrightarrow{PQ}$ are

   a. $\frac{x_2 - x_1}{d}, \frac{y_2 - y_1}{d}, \frac{z_2 + z_1}{d}$
   b. $\frac{x_2 - x_1}{d}, \frac{y_2 + y_1}{d}, \frac{z_2 - z_1}{d}$
   c. $\frac{x_2 - x_1}{d}, \frac{y_2 - y_1}{d}, \frac{z_2 - z_1}{d}$
   d. $\frac{x_2 + x_1}{d}, \frac{y_2 - y_1}{d}, \frac{z_2 - z_1}{d}$

3. Find the shortest distance between the lines : $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$ +$\lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}.\right)$ + $\mu\left(2\hat{i} + 3\hat{j} + \hat{k}.\right)$

   a. $\frac{3}{\sqrt{17}}$
   b. $\frac{3}{\sqrt{23}}$
   c. $\frac{3}{\sqrt{27}}$
   d. $\frac{3}{\sqrt{19}}$

4. In the following case, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them. 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z – 1 = 0.

a. The planes are at 45°
b. The planes are parallel
c. The planes are at 55°
d. The planes are perpendicular

5. Find the angle between the planes whose vector equations are $\vec{r}.\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5$, and $\vec{r}.\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=3$.

   a. $\tan^{-1}\left(\frac{15}{\sqrt{731}}\right)$
   b. $\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$
   c. $\sin^{-1}\left(\frac{15}{\sqrt{731}}\right)$
   d. $\cot^{-1}\left(\frac{15}{\sqrt{731}}\right)$

6. The direction cosines of the vector $(2\hat{i}+2\hat{j}-\hat{k})$ are ________.
7. The vector equation of a plane which is at a distance p from the origin, where $\hat{n}$ is the unit vector normal to the plane is ________.
8. The vector equation of the line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ is ________.

9. If a line makes angles α, β, γ with the position direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

10. The Cartesian equation of a line is $\frac{x+3}{2}=\frac{y-5}{4}=\frac{z+6}{2}$ Find the vector equation for the line.

11. Find the direction cosines of the line passing through the two points(-2, 4, -5) and (1, 2, 3).

12. Find the Cartesian equation of the line which passes through the point $(-2,4,-5)$ and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

13. Find the vector equation of the line passing through the point A (1, 2, -1) and parallel to the line 5x - 25 = 14 -7y = 35 z.

14. Show that the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular to each other.

15. Find the angle between the lines
$\vec{r} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$
$\vec{r} = \left(2\hat{i} - \hat{j} - 56\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

16. Find the equation of the plane through the intersection of the planes 3x - y + 2z - 4 = 0 and x + y + z - 2 = 0 and the point (2,2,1).

17. Find the equation of the perpendicular from point (3, -1,11) to line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the coordinates of foot of perpendicular and the length of perpendicular.

18. Find the distance of the point P(3, 4, 4) from the point, where the line joining the points A(3, -4, -5) and B(2, -3, 1) intersects the plane 2x + y + z = 7.

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Solution

1. a. $\theta=\cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$

   Explanation: If θ is the acute angle between $\overrightarrow{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ and $\overrightarrow{r}=\overrightarrow{a_2}+\lambda\overrightarrow{b_2}$ then cosine of the angle between these two lines is given by:

   Here, $\overrightarrow{b_1}=\hat{i}-\hat{j}-2\hat{k}, \overrightarrow{b_2}=3\hat{i}-5\hat{j}-4\hat{k}$

   Then,

   $\cos\theta=\left|\frac{(\hat{i}-\hat{j}-2\hat{k}).(3\hat{i}-5\hat{j}-4\hat{k})}{\left|(\hat{i}-\hat{j}-2\hat{k})\right|\left|(3\hat{i}-5\hat{j}-4\hat{k})\right|}\right|$

   $\cos\theta=\left|\frac{16}{\sqrt{6}\sqrt{50}}\right|=\left|\frac{16}{\sqrt{300}}\right|$

   $\Rightarrow\theta=\cos^{-1}\left(\left|\frac{16}{10\sqrt{3}}\right|\right)$

   $=\cos^{-1}\left(\left|\frac{8}{5\sqrt{3}}\right|\right)$

2. c. $\frac{x_2-x_1}{d}, \frac{y_2-y_1}{d}, \frac{z_2-z_1}{d}$

   Explanation: since we know Direction cosines of a line are coefficient of i, j, k of a unit vector along that line,first find a vector $\overrightarrow{PQ}=(x2-x1)\hat{i}+(y2-y1)\hat{j}+(z2-z1)\hat{k}$ then to convert it unit vector divide by its magnitute $|\overrightarrow{PQ}|$ the coefficient of this unit vector will be

3. d. $\frac{3}{\sqrt{19}}$

   Explanation: On comparing the given equations with: $\overrightarrow{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ **and** $\overrightarrow{r}=\overrightarrow{a_2}+\mu\overrightarrow{b_2}$, we get

   $\overrightarrow{a_1}=\hat{i}+2\hat{j}+3\hat{k}, \overrightarrow{b_1}=\hat{i}-3\hat{j}+2\hat{k},$

   and $\overrightarrow{a_2}=4\hat{i}+5\hat{j}+6\hat{k}, \overrightarrow{b_2}=2\hat{i}+3\hat{j}+\hat{k}$

   $\therefore S.D=\left|\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_2})}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right|$

$$= \left| \frac{(-9\hat{i}+3\hat{j}+9\hat{k}).(3\hat{i}+3\hat{j}+3\hat{k})}{\sqrt{171}} \right|$$
$$= \left| \frac{-27+9+27}{3\sqrt{19}} \right| = \left| \frac{9}{3\sqrt{19}} \right| = \frac{3}{\sqrt{19}}$$

4. b. The planes are parallel

Explanation: We have, 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z = 0. Here ,
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{2}{3}$
Therefore , the given planes are parallel.

5. b. $\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$

Explanation: We have,
$$\overrightarrow{n_1} = (2\hat{i} + 2\hat{j} - 3\hat{k});$$
$$\overrightarrow{n_2} = (3\hat{i} - 3\hat{j} + 5\hat{k});$$
$$\therefore \cos\theta = \left| \frac{\overrightarrow{n_1}\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|} \right|$$
$$\Rightarrow \left| \frac{(2\hat{i}+2\hat{j}-3\hat{k}).(3\hat{i}-3\hat{j}+5\hat{k})}{\sqrt{4+4+9}\sqrt{9+9+25}} \right| = \left| \frac{-15}{\sqrt{17}\sqrt{43}} \right|$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$$

6. $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$
7. $\vec{r}.\hat{n} = p$
8. $(x-5)\hat{i} + (y+4)\hat{j} + (z-6)\hat{k} = \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

9. If a line makes angles α, β, γ with the coordinate axes.

   Then, direction cosines of the line are (cos α, cos β ,cos γ)

   ∴$\sin^2\alpha+\sin^2\beta+\sin^2\gamma$
$$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$$
$$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$
$$= 3 - 1 = 2 \quad [\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$$

10. Comparing the given equation with the standard equation form
$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
We get a =2, b = 4, c = 2 and x_1 = -3, y_1 = 5, z_1 = -6

    Then, $\vec{a} = -3\hat{i} + 5\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

    Therefore, required equation is,

$\vec{r} = -3\hat{i} + 5\hat{j} - 6\hat{k} + \lambda\left(2\hat{i} + 4\hat{j} + 2\hat{k}\right)$

11. Let the given points be P(-2, 4, -5) and Q(1, 2, 3). Then,
$|PQ| = \sqrt{(1+2)^2 + (2-4)^2 + (3+5)^2}$
$= \sqrt{9+4+64}$
$= \sqrt{77}$
The direction cosines of the line joining two points is
$\left(\frac{1+2}{\sqrt{77}}, \frac{2-4}{\sqrt{77}}, \frac{3+5}{\sqrt{77}}\right)$
$\left(\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right)$

12. Given: A point on the line is $(-2, 4, -5) = (x_1, y_1, z_1)$
Equation of the given line in Cartesian form is $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$
∴ Direction ratios of the given line are its denominators 3, 5, 6 = a, b, c
∴Equation of the required line is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
$\Rightarrow \frac{x-(-2)}{3} = \frac{y-4}{5} = \frac{z-(-5)}{6} = \frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

13. According to the question,
Line is $5x - 25 = 14 - 7y = 35z$.
$\Rightarrow \frac{x-5}{1/5} = \frac{2-y}{1/7} = \frac{z}{1/35} \Rightarrow \frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$
$\Rightarrow$ Direction ratio of the given line = $\frac{1}{5}, -\frac{1}{7}, \frac{1}{35}$
$\Rightarrow$ Direction ratio of a line parallel to the given line= $\frac{1}{5}, -\frac{1}{7}, \frac{1}{35}$
:. The required equation of a line passing through the point $A(1, 2, -1)$ and parallel to the given line is
$\frac{x-1}{1/5} = \frac{y-2}{-1/7} = \frac{z+1}{1/35}$

14. Equation of one line $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$
∴ Direction ratios of this line are $7, -5, 1 = a_1, b_1, c_1$
$\Rightarrow \vec{b}_1 = 7\hat{i} - 5\hat{j} + \hat{k}$
Again equation of another line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$
∴ Direction ratios of this line are 1, 2, 3 $= a_2, b_2, c_2$
$\Rightarrow \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$
Now $\vec{b}_1 . \vec{b}_2 = a_1a_2 + b_1b_2 = c_1c_2 = 7 \times 1 + (-5) \times 2 + 1 \times 3 = 7 - 10 + 3 = 0$

Hence, the given two lines are perpendicular to each other.

15. Let θ is the angle between the given lines

$\vec{b}_1 = \hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b}_2 = 3\hat{i} - 5\hat{j} - 4\hat{k}$

$\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

= $\left|\frac{(\hat{i}-\hat{j}-2\hat{k}).(3\hat{i}-5\hat{j}-4\hat{k})}{|\hat{i}-\hat{j}-2\hat{k}||3\hat{i}-5\hat{j}-4\hat{k}|}\right|$

$= \left|\frac{3+5+8}{\sqrt{6}\sqrt{50}}\right| = \frac{16}{\sqrt{50}}$

= $\frac{16}{\sqrt{6}\ 5\sqrt{2}}$

= $\frac{16}{\sqrt{2}\times\sqrt{3}\times\sqrt{5}\times\sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{3}}$

= $\frac{16\sqrt{3}}{2\times3\times5}$

$\cos\theta = \frac{8\sqrt{3}}{15}$

$\theta = \cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

16. Equation of any plane through the intersection of given planes can be taken as

$(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$...... (i)

The point (2,2,1) lies in this plane,therefore

3(2) - 2 + 2(1) - 4 + λ(2+2+1-2) = 0

2 + λ(3) = 0

$\lambda = \frac{-2}{3}$

put value of λ in eq(i), we get

$(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$

7x - 5y + 4z - 8 = 0

17. Firstly, determine any point P on the given line and DR's between given point Q and P, using the relation $a_1a_2+b_1 b_2+c_1 c_2=0$, where (a_1, b_1, c_1) and (a_2, b_2, c_2) and DR's of PQ and given line.

Given equation of line AB is

$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda(\text{ (say)}$

$\Rightarrow \quad \frac{x}{2} = \lambda, \frac{y-2}{3} = \lambda$

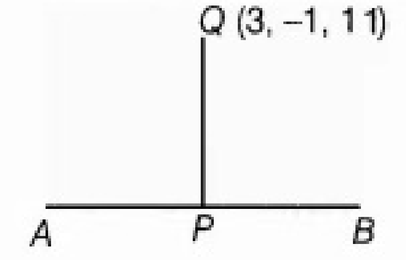
and $\quad \frac{z-3}{4} = \lambda$

$\Rightarrow \quad x=2\lambda, y=3\lambda+2$

and $z=4\lambda+3$

:. Any point P on the given line

$=(2\lambda, 3\lambda+2, 4\lambda+3)$



Let P be the foot of perpendicular drawn from point Q(3, -1,11) on line AB Now, DR's of line

$QP=(2\lambda-3, 3\lambda+2+1, 4\lambda+3-11)$

$=(2\lambda-3, 3\lambda+3, 4\lambda-8)$

Here, $a_1=2\lambda-3, b_1=3\lambda+3, c_1=4\lambda-8$

and $a_2=2, b_2=3, c_2=4$

Since, $QP \perp AB$

$\therefore$ We have, $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ....(i)

$\Rightarrow 2(2\lambda-3)+3(3\lambda+3)+4(4\lambda-8)=0$

$\Rightarrow \quad 4\lambda-6+9\lambda+9+16\lambda-32=0$

$\Rightarrow \quad 29\lambda-29=0 \Rightarrow 29\lambda=29 \Rightarrow \lambda=1$

$\therefore$ Foot of perpendicular is $P=(2,3+2,4+3)$

$=(2,5,7)$

Now, equation of perpendicular QP, where Q(3, -1, 11) and P(2, 5, 7), is

$\frac{x-3}{2-3}=\frac{y+1}{5+1}=\frac{z-11}{7-11}$

$\left[\begin{array}{c}\text{using two point form of equation of line,} \\ \text{i.e. } \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}\end{array}\right]$

$\Rightarrow \quad \frac{x-3}{-1}=\frac{y+1}{6}=\frac{z-11}{-4}$

Now, length of perpendicular QP = distance between points Q(3, -1, 11) and P (2, 5, 7)

$=\sqrt{(2-3)^2+(5+1)^2+(7-11)^2}$

$\left[\because \text{ distance }=\sqrt{(x_2-x_1)_2+(y_2-y_1)_2+(z_2-z_1)_2}\right]$

$=\sqrt{1+36+16}=\sqrt{53}$

Hence, length of perpendicular is $\sqrt{53}$.

18. The direction ratios of line joining $A(3,-4,-5)\ and\ B(2,-3,1)$ are
$[(2-3),(-3+4),(1+5)]=(-1,1,6)$
The equation of line passing through (3, -4, -5) and having Direction ratios (-1, 1, 6) is given by
$\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}\left[\because \frac{x-x_1}{a}-\frac{y-y}{b}-\frac{z-z_1}{c}\right]$
Suppose $\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=\lambda(\text{ say })$
$\Rightarrow x=-\lambda+3, y=\lambda-4$ and $z=6\lambda-5$
The general point on the line is given by
$(3-\lambda, \lambda-4, 6\lambda-5)$
Line intersect the plane $2x+y+z=7$.
So, General point on the line $(3-\lambda, \lambda-4, 6\lambda-5)$ satisfy the equation of plane.
$\therefore 2(3-\lambda)+\lambda-4+6\lambda-5=7$
$\Rightarrow 6-2\lambda+\lambda-4+6\lambda-5=7 \Rightarrow 5\lambda=10$
$\lambda=2$
The point of intersection of line and plane is
$(3-2,2-4,6\times 2-5)=(1,-2,7)$.
Distance between (3, 4, 4) and (1, -2, 7)
$=\sqrt{(3-1)^2+(4+2)^2+(4-7)^2}$
$=\sqrt{4+36+9}=\sqrt{49}=7units$