Chapter 4

Factorization

Exercise 4.1

Factorise the following (1 to 9)

1. (i) $8xy^3 + 12x^2y^2$

Solution:

$8xy^3 + 12x^2y^2$

Take out common in both terms,

Then, $4xy^2(2y + 3x)$

Therefore, HCF of $8xy^3$ and $12x^2y^2$ is $4xy^2$

(ii) $15x^3 – 9ax^2$

Solution:

$15x^3 – 9ax^2$

Take out common in both terms,

Then , $3ax^2 (5x – 3)$

Therefore, HCF of $15ax^3$ and $9a^2$ is $3ax^2$.

Question 2.

(i) $21py^2 – 56py$

Solution:

$21py^2 – 56py$

Take out common in both terms,

Then , $7py (3y – 8)$

Therefore , HCL of $21py^2$ and 56 py is 7py.

(ii) $4x^3 – 6x^2$

Solution:

$4x^3 – 6x^2$

Take out common in both terms,

Then , $2x^2 (2x – 3)$

Therefore, HCF of $4x^3$ and $6x^2$ is $2x^2$.

Question 3.

(i) $2\pi r^2 - 4\pi r$

Solution:

Take out common in both terms,

Then , $2\pi r(r -2)$

Therefore , HCL of $2\pi r^2$ and $4\pi r$ is $2\pi r$.

(ii) $18m + 16n$

Solution :

$18m + 16n$

Take out common in both terms ,

Then , $2 (9m – 8n)$

Therefore , HCF of 18m and 16n is 2.

Question 4.

(i) $25abc^2 – 15a^2b^2c$

Solution:

$25abc^2 – 15a^2b^2c$

Take out common in both terms,

Then , $5abc (5c – 3ab)$

Therefore, HCF of $25abc^2$ and $15a^2b^2c$ is 5abc

(ii) $28p^2q^2r – 42pq^2r^2$

Solution:

$28p^2q^2r – 42pq^2r^2$

Take out common in both terms,

Then , $14pq^2r (2p – 3r)$

Therefore , HCF or $28p^2q^2r$ and $42pq^2r^2$ is $14pq^2r$.

Question 5.

(i) $8x^3 – 6x^2 + 10x$

Solution:

$8x^3 – 6x^2 + 10x$

Take out common in both terms,

Then , $2x (4x^2 – 3x + 5)$

Therefore, HCF of $8x^3$, $6x^2$ and $10x$ is $2x$.

(ii) $14mn + 22m – 62p$

Solution:

$14mn + 22m – 62p$

Take out common in both terms,

Then , $2 (7mn + 11m – 31p)$

Therefore, HCF of $14mn$, $22m$ and $62p$ is 2.

Question 6.

(i) $18p^2q^2 – 24pq^2 + 30p^2q$

Solution:

$18p^2q^2 – 24pq^2 + 30p^2q$

Take out common in both terms,

Then, $6pq (3pq – 4q + 5q)$

Therefore, HCL of $18p^2q^2$, $24pq^2$ and $30p^2q$ is 6pq.

(ii) $27a^3b^3 – 18a^2b^3 + 75a^3b^2$

Solution:

$27a^3b^3 – 18a^2b^3 + 75a^3b^2$

Take out common in both terms,

Then , $3a^2b^2$ (9a – 6b + 25a)

Therefore , HCL of $27a^3b^3$, $18a^2b^3$ and $75a^3b^2$ is 3 a^2b^2

Question 7.

(i) 15a (2p – 3q) – 10b (2p – 3q)

Solution:

15a (2p – 3q) – 10b (2p – 3q)

Take out common in both terms,

Then 5 (2p – 3q) [3a – 2b]

Therefore, HCF of 15a (2p – 3q) and 10b (2p – 3q) is 5 (2p – 3q) .

Question 8.

(i) $6(x + 2y)^3 + 8 (x + 2y)^2$

Solution:

$6(x + 2y)^3 + 8 (x + 2y)^2$

Take out common in all terms,

Then, $2(x + 2y)^2$ [3 (x + 2y) + 4]

Therefore, HCF of $6(x + 2y)^3$ and $8 (x + 2y)^2$ is $2(x + 2y)^2$

(ii) $14 (a – 3b)^3 – 21p (a – 3b)$

Solution:

$14 (a – 3b)^3 – 21p (a – 3b)$

Take out common in all terms,

Then , $7(a – 3b) [2(a – 3b)^2 – 3p]$

Therefore, HCF of $14 (a – 3b)^3$ and $21p (a – 3b)$ is $7 (a – 3b)$.

Question 9.

(i) $10a(2p + q)^3 – 15b (2p + q)^2 + 35 (2p + q)$

Solution:

$10a(2p + q)^3 – 15b (2p + q)^2 + 35 (2p + q)$

Take out common in all terms,

Then , $5 (2p + q) [2a (2p + q)^2 – 3b (2p + q) + 7]$

Therefore , HCF of $10a(2p + q)^3$, $15b (2p + q)^2$ and $35 (2p + q)$ is $5(2p + q)$.

(ii) $x (x^2 + y^2 – z^2) +y(-x^2 – y^2 + z^2) –z(x^2 + y – z^2)$

Solution:

$x (x^2 + y^2 – z^2) +y(-x^2 – y^2 + z^2) –z(x^2 + y – z^2)$

take out common in all terms,

then , $(x^2 + y^2 – z^2)$ [x – y – z]

Therefore HCL of x ($x^2 + y^2 – z^2$) , y($-x^2 – y^2 + z^2$) and ,z($x^2 + y – z^2$) is ($x^2 + y^2 – z^2$)

Exercise 4.2

Factorise the following (1 to 13) :

Question 1.

(i) $x^2 + xy – x – y$

Solution :

$x^2 + xy – x – y$

take out common in all terms,

$x(x + y) – 1 (x + y)$

$(x + y) (x -1)$

(ii) $y^2 – yz – 5y + 5z$

Solution:

$y^2 – yz – 5y + 5z$

take out common in all terms,

$x(5y – 7) –y (5y – 7)$

$(5y – 7) (x – y)$

(ii) $5p^2 – 8pq – 10p + 16q$

Solution:

$5p^2 – 8pq – 10p + 16q$

take out common in all terms,

$p (5p – 8q) -2 (5p – 8q)$

$(5p – 8q) (p – 2)$

Question 3.

(i) $a^2b – ab^2 + 3a – 3b$

Solution:

$a^2b – ab^2 + 3a – 3b$

take out common in all terms,

$ab(a – b) + 3 (a – b)$

$(a – b) (ab + 3)$

(ii) $x^3 – 3x^2 + x - 3$

Solution:

$x^3 – 3x^2 + x - 3$

take out common in all terms,

$x^2(x – 3) + 1 (x – 3)$

$(x – 3) (x^2 + 1)$

Question 4.

(i) $6xy^2 – 3xy – 10y + 5$

Solution:

$6xy^2 – 3xy – 10y + 5$

take out common in all terms,

$3xy (2y – 1) – 5 (2y – 1)$

$(2y – 1) (3xy – 5)$

(ii) $3ax – 6ay – 8by + 4bx$

Solution:

$3ax – 6ay – 8by + 4bx$

take out common in all terms,

$3a (x – 2y) + 4b (x – 2y)$

$(x – 2y) (3a + 4b)$

Question 5.

(i) $1 – a – b + ab$

Solution:

$1 –a – b + ab$

Take out common in all terms,

$1(1 –a) –b(1 –a)$

$(1 – a) (1 – b)$

(ii) $a (a – 2b – c) + 2bc$

Solution:

$a (a – 2b – c) + 2bc$

above question can be written as,

$a^2 – 2ab – ac + 2bc$

Take out common in all terms,

$a (a – 2b) –c (a + 2b)$

$(a – 2b) (a –c)$

Question 6.

(i) $x^2 + xy (1 + y) + y^3$

Solution:

$x^2 + xy (1 + y) + y^3$

Above question can be written as,

$X^2 + xy + xy^2 + y^3$

Take out common in all terms,

$X(x + y) + y^2(x +y)$

$(x + y) (x + y^2)$

(ii) $y^2 – xy (1 –x) –x^3$

Solution:

$y^2 – xy (1 –x) –x^3$

Above question can be written as,

$Y^2 – xy + x^2y – x^3$

Take out common in all terms,

$Y(y – x) + x^2 (y – x)$

$(y – x) (y + x^2)$

Question 7.

(i) $ab^2 + (a – 1) b – 1$

Solution:

$ab^2 + (a – 1) b – 1$

Above question can be written as,

$Ab^2 + ab – b – 1$

Take out common in all terms,

$Ab (b + 1) – 1 (b + 1)$

$(b + 1) (ab – 1)$

(ii) $2a – 4b – xa + 2bx$

Solution:

$2a – 4b – xa + 2bx$

Take out common in all terms,

$2(a – 2b) –x (a – 2b)$

$(a – 2b) (2 – x)$

Question 8.

(i) 5ph – 10qk + 2rph – 4qrk

Solution:

5ph – 10qk + 2rph – 4qrk

Re – arranging the given question we get,

5ph + 2rph – 10qk – 4qrk

Take out common in all terms,

Ph(5 + 2r) – 2qk (5 + 2r)

(5 + 2r) (ph – 2qk)

(ii) x^2 – x (a + 2b) + 2ab

Solution:

x^2 – x (a + 2b) + 2ab

above question can be written as,

x^2 – xa – 2xb + 2ab

take out common in all terms,

x(x – a) – 2b(x – a)

(x – a) (x – 2b)

Question 9.

(i) $ab (x^2 + y^2) – xy (a^2 + b^2)$

Solution:

$ab (x^2 + y^2) – xy (a^2 + b^2)$

above question can be written as,

$abx^2 + aby^2 – xya^2 – xyb^2$

Re – arranging the above we get,

$Abx^2 – xyb^2 + aby^2 – xya^2$

Take out common in all terms,

$bx (ax – by) + ay (by – xy)$

$bx (ax – by) –ay (ax – by)$

$(ax – by) (bx – ay)$

(ii) $(ax + by)^2 + (bx – ay)^2$

Solution:

By expanding the give question, we get,

$(ax)^2 + (by)^2 + 2axby + (bx)^2 + (ay)^2 – 2bxay$

$A^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$

Re – arranging the above we get,

$A^2x^2 + a^2y^2 + b^2y^2 + b^2x^2$

Take out common in all terms,

$A^2 (x^2 + y^2) + b^2 (x^2 + y^2)$

$(x^2 + y^2) (a^2 + b^2)$

Question 10.

(i) $a^3 + ab (1 – 2a) – 2b^2$

Solution:

$a^3 + ab (1 – 2a) – 2b^2$

above question can be written as ,

$a^3 + ab – 2a^2b – 2b^2$

Re- arranging the above we get,

$a^3 – 2a^2b + ab – 2b^2$

take out common in all terms,

$a^2 (a – 2b) + b (a – 2b)$

$(a – 2b) (a^2 + b)$

(ii) $3x^2y – 3xy + 12x – 12$

Solution:

$3x^2y – 3xy + 12x – 12$

Take out common in all terms,

$3xy (x – 1) + 12 (x – 1)$

$(x – 1) (3xy + 12)$

Question 11.

$A^2b + ab^2 – abc – b^2c + axy + bxy$

Solution:

$A^2b + ab^2 – abc – b^2c + axy + bxy$

Re – arranging the above we get,

$A^2b – abc + axy + ab^2 – b^2c + bxy$

Take out common in all terms.

$A(ab – bc + xy) + b (ab – bc + xy)$

$(a + b) (ab – bc + xy)$

Question 12.

$Ax^2 – bx^2 + ay^2 – by^2 + az^2 – bz^2$

Solution:

$Ax^2 – bx^2 + ay^2 – by^2 + az^2 – bz^2$

Re – arranging the above we get,

$Ax^2 + ay^2 + az^2 – bx^2 – by^2 – bz^2$

Take out common in all terms,

$A(x^2 + y^2 + z^2) –b(x^2 + y^2 + z^2)$

$(x^2 + y^2 + z^2) (a – b)$

Question 13

$X - 1 - 1 (x - 1)^2 + ax - a$

Solution:

$X - 1 - 1 (x - 1)^2 + ax - a$

By expanding the above we get,

$X - 1 - 1 (x^2 + 1 - 2x) + ax - a$

$X - 1 - 1 x^2 - 1 + 2x + ax - a$

$2x - x^2 + ax - 2 + x - a$

Take out common in all terms,

$X (2 - x + a) - 1 (2 - x + a)$

$(2 - x + a) (x - 1)$

Exercise 4.3

Factorise the following (1 to 17) :

Question 1.

$4x^2 – 25y^2$

Solution :

We know that, $a^2 – b^2 = (a + b) (a – b)$

So, $(2x)^2 – (5y)^2$

Then, $(2x + y) (2x – 5y)$

(ii) 9x2 – 1

Solution:

We know that, $a^2 – b^2 = (a + b) (a – b)$

So, $(3x)^2 – 1^2$

Then, (3x + 1) (3x -1)

Question 2.

(i) $150 – 6a^2$

Solution:

$150 – 6a^2$

Take out common in all terms,

$6(25 – a^2)$

$6(5^2 – a^2)$

We know that, $a^2 – b^2 = (a + b) (a –b)$

So, $6 (5 + a) (5 – a)$

(ii) $32x^2 – 18y^2$

Solution:

$32x^2 – 18y^2$

Take out common in all terms,

$2(16x^2 – 9y^2)$

$2((4x)^2 – (3y)^2)$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$2(4x + 3y) (4x – 3y)$

Question 3.

(i) $(x – y)^2 – 9$

Solution:

$(x – y)^2 – 9$

$(x – y)^2 – 3^2$

We know that, $a^2 – b^2 = (a + b) (a –b)$

$(x – y + 3) (x – y – 3)$

(ii) $9(x + y)^2 – x^2$

Solution:

$9[(x + y)^2 – x^2]$

We know that, $a^2 – b^2 = (a + b) (a –b)$

$9[(x + y + x) (x + y – x)]$

So, $9(2x + y) y$

$9y (2x + y)$

Question 4.

(i) $20x^2 – 45y^2$

Solution;

$20x^2 – 45y^2$

Take out common in all terms,

$5(4x^2 – 9y^2)$

$5((2x)^2 – (3y)^2$

We know that , $a^2 – b^2 = (a + b) (a – b)$

$5(2x + 3y) (2x – 3y)$

(ii) $9x^2 – 4(y + 2x)^2$

Solution:

$9x^2 – 4(y + 2x)^2$

Above question can be written as,

$(3x)^2 – [2(y + 2x)]^2$

$(3x)^2 – (2y + 4x)^2$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$(3x + 2y + 4x) (3x – 2y – 4x)$

$(7x + 2y) (-x- 2y)$

Question 5.

(i) $2(x – 2y)^2 – 50y^2$

Solution:

$2(x – 2y)^2 – 50y^2$

Take out common in all terms,

$2[(x – 2y)^2 – 25y^2]$

$2[(x – 2y)^2 – (5y)^2]$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$2[(x – 2y + 5y) (x – 2y – 5y)]$

$2[(x + 3y) (x – 7y)]$

$2(x + 3y) (x – 7y)$

(ii) $32 – 2 (x – 4)^2$

Solution:

$32 – 2 (x – 4)^2$

Take out common in all terms,

$2[16 - (x - 4)^2]$

$2[42 - (x - 4)^2]$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$2[(4 + x - 4)(4 - x + 4)$

$2[(x)(8 - x)]$

$2x(8 - x)$

Question 6.

(i) $108a^2 - 3(b - c)^2$

Solution:

$108a^2 - 3(b - c)^2$

Take out common in all terms,

$3[36a^2 - (b - c)^2]$

$3[(6a)^2 - (b - c)^2]$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$3[(6a + b - c)(6a - b + c)$

(ii) $\pi a^5 - \pi^3 ab^2$

Solution:

$\pi a^5 - \pi^3 ab^2$

Take out common in all terms,

$\pi a(a^4 - \pi^2 b^2)$

$\pi a((a^2)^2 - (\pi b)^2)$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$\pi a(a^2 + \pi b)(a^2 - \pi b)$

Question 7.

(i) $50x^2 - 2(x - 2)^2$

Solution'

$50x^2 - 2(x - 2)^2$

Take out common in all terms,

$2[25x^2 - (x - 2)^2]$

$2[(5x)^2 - (x - 2)^2]$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$2[(5x + x - 2)(5x - x + 2)]$

$2[(6x - 2)(4x + 2)]$

$2(6x - 2)(4x + 2)$

(ii) $(x - 2)(x + 2) + 3$

Solution:

We know that, $a^2 - b^2 = (a + b)(a - b)$

$(x^2 - 2^2) + 3$

$X^2 - 4 + 3$

$X^2 - 1$

Then,

$(x + 1) (x – 1)$

Question 8.

(i) $x – 2y – x^2 + 4y^2$

Solution:

$X – 2y – x^2 + 4y^2$

$X – 2y – (x^2 + (2y)^2)$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$X – 2y – [(x + 2y) (x – 2y)]$

Take out common in all terms,

$(x – 2y) (1 – (x + 2y)$

$(x – 2y) (1 – x – 2y)$

(ii) $4a^2 – b^2 + 2a + b$

Solution:

$4a^2 – b^2 + 2a + b$

$(2a)^2 – b^2 + 2a + b$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$((2a + b) (2a – b)) + 1 (2a + b)$

Take out common in all terms,

(2a + b) (2a – b + 1)

Question 9.

(i) a (a – 2) –b (b – 2)

Solution:

a (a – 2) –b (b – 2)

above question can be written as,

$a^2 – 2a – b^2 – 2b$

re – arranging the above terms, we get,

$a^2 – b^2 – 2a – 2b$

we know that, $a^2 – b^2 = (a + b) (a –b)$

[(a + b) (a – b)] -2 (a – b)

Take out common in all terms,

(a – b) (a + b – 2)

(ii) a (a – 1) –b (b -1)

Solution:

a (a – 1) –b (b -1)

above question can be written as,

$a^2 – a – b^2 + b$

Re – arranging the above terms, we get,

$A^2 – b^2 – a + b$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$[(a + b) (a – b)] -1 (a – b)$

Take out common in all terms,

$(a – b) (a + b – 1)$

Question 10.

(i) $9 – x^2 + 2xy – y^2$

Solution:

$9 – x^2 + 2xy – y^2$

Above terms can be written as,

$9 – x^2 + xy + xy – y^2$

Now,

$9 – x^2 + xy + 3x – 3x + 3y – 3y + xy – y^2$

Re – arranging the above terms, we get,

$9 – 3x + 3y + 3x – x^2 + xy + xy – 3y – y^2$

Take out common in all terms,

$3(3 –x +y) +x (3 – x + y) +y(-3 – y + x)$

$3(3 – x + y) +x (3 – x + y) –y (3 – x + y)$

$(3 – x + y) (3 + x –y)$

(ii) $9x^4 – (x^2 + 2x + 1)$

Solution:

$9x^4 – (x^2 + 2x + 1)$

Above terms can be written as,

$(3x^2)^2 – (x + 1)^2$ [because $(a + b)^2 = a^2 + 2ab + b^2$]

We know that, $a^2 – b^2 = (a + b) (a – b)$

So , $(3x^2 + x + 1) (3x2 – x – 1)$

Question 11.

(i) $9x^4 - x^2 – 12x – 36$

Solution:

$9x^4 - x^2 – 12x – 36$

Above terms can be written as,

$9x^4 – (x^2 + 12x + 36)$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$(3x^2)^2 – (x^2 + (2 \times 6 \times x) + 6^2$

So, $(3x^2)^2 – (x + 6)^2$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$(3x^2 + x + 6) (3x^2 – x – 6)$

(ii) $x^3 – 5x^2 – x + 5$

Solution :

$x^3 – 5x^2 – x + 5$

take out common in all terms,

$x^2 (x – 5) -1 (x – 5)$

$(x -5) (x^2 – 1)$

$(x – 5) (x^2 – 12)$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$(x – 5) (x + 1) (x – 1)$

Question 12.

(i) $a^4 – b^4 + 2b^2 – 1$

Solution:

$a^4 – b^4 + 2b^2 – 1$

above terms can be written as,

$a^4 –(b^4 - 2b^2 + 1)$

we know that, $(a – b)^2 = a^2 – 2ab + b^2$

$a^4 – ((b^2)^2) – (2 \times b^2 \times 1) + 1^2)$

$(a^2)^2 – (b^2 – 1)^2$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$(a^2 + b^2 -1) (a^2 – b^2 + 1)$

(ii) $x^3 – 25x$

Solution;

$X^3 – 25x$

Take out common in all terms,

$X (x^2 – 25)$

Above terms can be written as,

$X (x^2 – 5^2)$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$X (x + 5) (x – 5)$

Question 13.

(i) $2x^4 – 32$

Solution:

$2x^4 – 32$

Take out common in all terms,

$2(x^4 – 16)$

Above terms can be written as,

$2((x^2)^2 – 4^2)$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$2(x^2 + 4) (x^2 – 4)$

$2(x^2 + 4) (x^2 – 2^2)$

$2(x^2 + 4) (x + 2) (x – 2)$

(ii) $a^2 (b + c) – (b + c)^3$

Solution:

$a^2 (b + c) – (b + c)^3$

take out common in all terms,

$(b + c) (a^2 – (b + c)^3$

We know that , $a^2 – b^2 = (a + b) (a – b)$

$(b + c) (a + b + c) (a – b – c)$

Question 14.

(i) $(a + b)3 – a – b$

Solution:

$(a + b)3 – a – b$

Above terms can be written as,

$(a + b)3 – (a + b)$

Take out common in all terms,

$(a + b) [(a + b)2 – 1)]$

$(a + b) [(a + b)2 -12]$

We know that, $a2 – b2 = (a + b) (a – b)$

$(a + b) (a + b + 1) (a + b – 1)$

(ii) $X^2 – 2xy + y^2 – a^2 – 2ab – b^2$

Solution:

$X^2 – 2xy + y^2 – a^2 – 2ab – b^2$

Above terms can be written as,

$(x^2 – 2xy + y^2) – (a^2 + 2ab + b^2)$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a – b)^2 = a^2 – 2ab + b^2$

$(x^2 – (2 \times x \times y) + y^2) – (a^2 + (2 \times a \times b) + b^2)$

$(x – y)^2 – (a + b)^2$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$[(x – y) + (a + b)][(x –y) – (a + b)]$

$(x – y + a + b) (x – y – a – b)$

Question 15.

(i) $(a^2 – b^2) (c^2 – d^2) – 4abcd$

Solution:

$(a^2 – b^2) (c^2 – d^2) – 4abcd$

$A^2 (c^2 – d^2) –b^2 (c^2 – d^2) – 4abcd$

$A^2c^2 – a^2d^2 – b^2c^2 + b^2d^2 – 4abcd$

$A^2c^2 + b^2d^2 – a^2d^2 – b^2c^2 – 2abcd – 2abcd$

Re arranging the above terms, we get,

$A^2c^2 + b^2d^2 - 2abcd – a^2d^2 – b^2c^2 – 2abcd$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a – b)^2 = a^2 – 2ab + b^2$

$(ac – bd)^2 – (ad – bc)^2$

(ac – bd + ad – bc) (ac – bd – ad + bc)

(ii) $4x^2 – y^2 – 3xy + 2x – 2y$

Solution:

$4x^2 – y^2 – 3xy + 2x – 2y$

Above terms can be written as,

$X^2 + 3x^2 – y^2 – 3xy + 2x - 2y$

Rearranging the above terms, we get,

$(x^2 – y^2) + (3x^2 – 3xy) + (2x – 2y)$

We know that, $a^2 – b^2 = (a + b) (a – b)$ and take out common terms,

(x + y) (x –y) + 3x (x –y) +2 (x –y)

(x – y) [(x + y) + 3x + 2]

(x – y) (x + y + 3x + 2)

(x – y) (4x + y + 2)

Question 16.

(i) $x^2 + \frac{1}{x^2} - 11$

Solution:

$x^2 + \frac{1}{x^2} - 11$

above terms can be written as,

$x^2 + \frac{1}{x^2} - 2 - 9$

then , $(x^2 + \frac{1}{x^2}) - 2) - 3^2$

we know that, $(a - b)^2 = a^2 - 2ab + b^2$,

$(x^2 - (2 \times x^2 \times \frac{1}{x^2}) + \left(\frac{1}{x}\right)^2$

$\left(x - \frac{1}{x}\right)^2 - 3^2$

We know that, $a^2 - b^2 = (a + b) (a - b)$

$(x - \frac{1}{x} + 3) (x - \frac{1}{x} - 3)$

(ii) $x^4 + 5x^2 + 9$

Solution:

$x^4 + 5x^2 + 9$

$x^4 + 6x^2 - x^2 + 9$

$(x^4 + 6x^2 + 9) - x^2$

$((x^2)^2 + (2 \times x^2 \times 3) + 3^2$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$,

$((x^2)^2 + (2 \times x^2 \times 3) + 3^2)$

So, $(x^2 + 3)^2 - x^2$

We know that , $a^2 - b^2 = (a + b) (a - b)$

$(x^2 + 3 + x) (x^2 + 3 - x)$

Question 17.

(i) $a^4 + b^4 – 7a^2b^2$

Solution:

$A^4 + b^4 – 7a^2b^2$

Above terms can be written as,

$A^4 + b^4 + 2a^2b^2 – 9a^2b^2$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$[(a^2)^2 + (b^2)^2 + (2 \times a^2 \times b^2)] – (3ab)^2$

$(a^2 + b^2)^2 – (3ab)^2$

We know that, $a^2 – b^2 = (a + b) (a –b)$

$(a^2 + b^2 + 3ab) (a^2 + b^2 – 3ab)$

(ii) $x^4 – 14x^2 + 1$

Solution:

$X^4 – 14x^2 + 1$

Above terms can be written as,

$X^4 + 2x^2 + 1 – 16x^2$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$,

So, $[(x^2)^2 + (2 \times x^2 \times 1) + 12] – 16x^2$

$(x^2 + 1)^2 – (4x)^2$

We know that, $a^2 – b^2 = (a + b) (a – b)$

$(x^2 + 1 + 4x) (x^2 + 1 – 4x)$

Question 18.

Express each of the following as the difference of two squares:

(i) $(x^2 – 5x + 7) (x^2 + 5x + 7)$

Solution:

$(x^2 – 5x + 7) (x^2 + 5x + 7)$

Rearranging the above terms, we get,

$((x^2 + 7) – 5x) ((x^2 + 7) + 5x)$

As, we know that , $a^2 – b^2 = (a + b) (a –b)$

So,$(x^2 + 7)^2 – (5x)^2$

$(x^2 + 7) – 25x^2$

(ii) $(x^2 – 5x + 7) (x^2 – 5x – 7)$

Solution :

$(x^2 – 5x + 7) (x^2 – 5x – 7)$

$[(x^2 – 5x + 7) ((x^2 – 5x) – 7)$

As, we know that, $a^2 – b^2 = (a + b) (a – b)$

$(x^2 – 5x)^2 – 7^2$

$(x^2 – 5x)^2 – 49$

(iii) $(x^2 + 5x – 7) (x^2 – 5x + 7)$

Solution:

$(x^2 + 5x – 7) (x^2 – 5x + 7)$

$[x^2 + (5x – 7) [x^2 – (5x – 7)]$

As, we know that, $a^2 – b^2 = (a + b) (a – b)$

$X^2 – (5x – 7)^2$

We know that, $(a – b)^2 = a^2 – 2ab + b^2$.

$X^2 – [(5x)^2 – (2\times 5x \times 7) + 7^2]$

$X^2 – (25x^2 – 70x + 49)$

$X^2 – 25x^2 + 70x – 49$

$-24x^2 + 70x – 49$

Question 19.

Evaluate the following by using factors:

(i) $(979)^2 – (21)^2$

(ii) $(99.9) – (0.1)^2$

Solution:

(i) $(979)^2 – (21)^2$

We know that

$= (979 + 21) (979 – 21)$

So we get

$= 1000 \times 958$

$= 958000$

(ii) $(99.9) – (0.1)^2$

= we know that

$= (99.9 + 0.1) (99.9 – 0.1)$

So we get

$= 100 \times 99.8$

$= 9980$

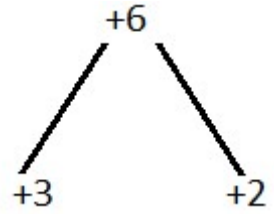
Exercise 4.4

Factorise the following (1 to 18) :

Question 1.

(i) $x^2 + 5x + 6$

Solution:



$x^2 + 3x + 2x + 6$

take out common in all terms we get,

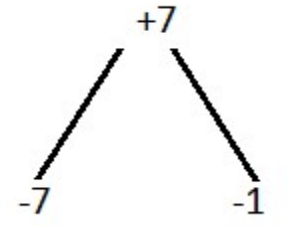
$x(x + 3) + 2(x + 3)$

$(x + 3) (x + 2)$

(ii) $x^2 – 8x + 7$

Solution :

$x^2 – 8x + 7$



$X^2 – 7x – x + 7$

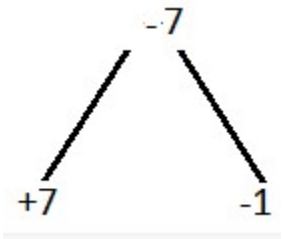
Take out common in all terms we get,

$X(x – 7) -1(x – 7)$

(x – 7) (x – 1)

Question 2.

(i) $x^2 + 6x – 7$



$x^2 + 6x – 7$

$x^2 + 7x – x – 7$

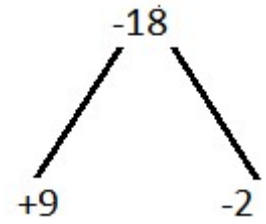
take out common in all terms we get,

x(x + 7) -1(x + 7)

(x + 7) (x -1)

(ii) $y^2 + 7y – 18$

Solution :



$y^2 + 7y – 18$

$y^2 + 9y – 2y – 18$

take out common in all terms we get,

y(y + 9) -2 (y + 9)

(y + 9) (y – 2)

Question 3.

(i) $y^2 – 7y – 18$

Solution:

$Y^2 – 7y – 18$

$Y^2 + 2y – 9y – 18$

Take out common in all terms we get,

Y(y + 2) – 9(y + 2)

(y + 2) (y -2)

(ii) $a^2 – 3a – 54$

Solution:

$a^2 – 3a – 54$

$a^2 + 6a – 9a – 54$

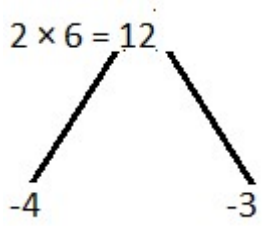
take out common in all terms we get,

a(a + 6) -9(a + 6)

so, (a + 6) (a – 9)

Question 4.

(i) $2x^2 - 7x + 6$



$2x^2 - 7x + 6$

$2x^2 - 4x - 3x + 6$

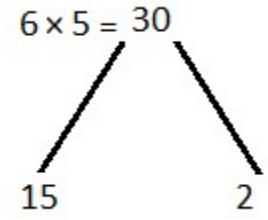
Take out common in all terms we get,

$2x(x - 2) -3(x - 2)$

$(x - 2) (2x - 3)$

(ii) $6x^2 + 13x - 5$

Solution:



$6x^2 + 15x - 2x - 5$

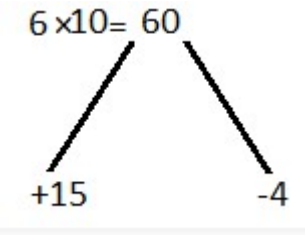
Take out common in all terms we get,

$3x (2x + 5) -1(2x + 5)$

$(2x + 5) (3x - 1)$

Question 5.

$6x^2 + 11x – 10$



Solution:

$6x^2 + 11x – 10$

$6x^2 + 15x – 4x – 10$

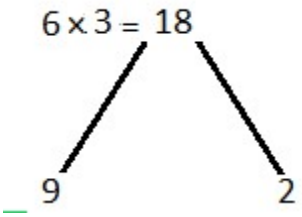
Take out common in all terms we get,

$3x (2x + 5) -2 (2x + 5)$

$(2x + 5) (3x – 2)$

(ii) $6x^2 – 7x -3$

Solution:



$6x^2 – 9x + 2x – 3$

Take out common in all terms we get,

$3x (2x – 3) +1(2x – 3)$

$(2x – 3) (3x + 1)$

Question 6.

(i) $2x^2 – x – 6$

Solution: $2x^2 – x – 6$

$2x^2 – 4x + 3x – 6$

Take out common in all terms we get,

$2x (x – 2) +3(x -2)$

$(x – 2) (2x + 3)$

(ii) $1 – 18y – 63y^2$

Solution:

$1 – 18y – 63y^2$

$1 – 21y + 3y – 63y^2$

Take out common in all terms we get,

$1(1 – 21y) +3y(1 – 21y)$

$(1 – 21y) (1 + 3y)$

Question 7

(i) $2y^2 + y – 45$

Solution:

$2y^2 + y – 45$

$2y^2 + 10y – 9y – 45$

Take out common in all terms we get,

$2y (y + 5) – 9 (y + 5)$

$(y + 5) (2y – 9)$

(ii) $5 – 4x – 12x^2$

Solution :

$5 – 4x – 12x^2$

$5 – 10x + 6x – 12x^2$

Take out common in all terms we get,

$5(1-2x) + 6x (1 – 2x)$

$(1 – 2x) (5 + 6x)$

Question 8.

(i) $x(12x + 7) – 10$

Solution:

$x(12x + 7) – 10$

above terms can be written as,

$12x^2 + 7x – 10$

$12x^2 + 15x – 8x – 10$

Take out common in all terms we get,

$3x (4x + 5) -2(4x + 5)$

$(4x + 5) (3x – 2)$

(ii) $(4 – x)^2 – 2x$

Solution:

$(4 – x)^2 – 2x$

We know that, $(a – b)^2 = a^2 – 2ab + b^2$

So, $(4^2 – (2 \times 4 \times x) + x^2) – 2x$

$16 – 8x + x^2 – 2x$

$X^2 – 10x + 16$

$X^2 – 8x – 2x + 16$

Take out common in all terms we get,

$X(x – 8) -2 (x -8)$

$(x – 8) (x – 2)$

Question 9.

(i) $60x2 – 70x – 30$

Solution:

$60x2 – 70x – 30$

Take out common in all terms we get,

$10(6x2 – 7x – 3)$

$10 (6x2 – 9x + 2x – 3)$

Again, take out common in all terms we get,

$10(3x (2x – 3) + 1(2x – 3)$

$10(2x – 3) (3x + 1)$

(ii) $x^2 – 6xy – 7y^2$

Solution:

$X^2 – 6xy – 7y^2$

$X^2 – 7xy + xy – 7y^2$

Take out common in all terms we get,

$X(x – 7y) + y(x – 7y)$

$(x – 7y) (x + y)$

Question 10.

(i) $2x^2 + 13xy – 24y^2$

Solution:

$2x^2 + 13xy – 24y^2$

$2x^2 + 16xy – 3xy – 24y^2$

Take out common in all terms we get,

$2x (x + 8y) -3y(x + 8y)$

$(x + 8y) (2x – 3y)$

(ii) $6x^2 – 5xy – 6y^2$

Solution:

$6x^2 – 5xy – 6y^2$

$6x^2 – 9xy + 4xy – 6y^2$

Take out common in all terms we get,

$3x (2x – 3y) + 2y(2x – 3y)$

$(2x – 3y) (3x + 2y)$

Question 11.

(i) $5x^2 + 17xy – 12y^2$

Solution:

$5x^2 + 17xy – 12y^2$

$5x^2 + 20xy – 3xy – 12y^2$

Take out common in all terms we get,

$5x (x + 4y) – 3y(x + 4y)$

$(x + 4y) (5x – 3y)$

$(x + 4y) (5x – 3y)$

(ii) $x^2y^2 – 8xy – 48$

Solution:

$X^2y^2 – 8xy – 48$

$X^2y^2 – 12xy + 4xy – 48$

Take out common in all terms we get,

$Xy (xy – 12) +4 (xy – 12)$

$(xy – 12) (xy + 4)$

Question 12.

(i) $2a^2b^2 – 7ab – 30$

Solution:

$2a^2b^2 – 7ab – 30$

$2a^2b^2 – 12ab + 5ab – 30$

Take out common in all terms we get,

$2ab (ab – 6) +5(ab – 6)$

$(ab – 6) (2ab + 5)$

(ii) $a (2a – b) – b^2$

Solution:

$(2a – b) – b^2$

Above terms can be written as,

$2a^2 – ab – b^2$

$2a^2 – 2ab + ab – b^2$

Take out common in all terms we gets,

$2a (a – b) +b(a – b)$

$(a – b) (2a + b)$

Question 13.

(i) $(x –y)^2 – 6(x – y) +5$

Solution:

$(x –y)^2 – 6(x – y) +5$

Above terms can be written as,

$(x – y)^2 -5 (x – y) – (x – y) + 5$

$(x – y) (x – y – 5) -1 (x – y – 5)$

Then ,

$(x – y – 5) (x – y – 1)$

(ii) $(2x – y)^2 – 11 (2x – y) + 28$

Solution:

$(2x – y)^2 – 11 (2x – y) + 28$

Above terms can be written as,

$(2x –y)^2 – 7 (2x – y) -4(2x – y) + 28$

$(2x – y) (2x – y – 7) -4(2x – y – 7)$

$(2x – y – 7) (2x – y – 4)$

Question 14.

(i) $4(a – 1)^2 -4 (a – 1) – 3$

Solution:

$4(a – 1)^2 -4 (a – 1) – 3$

Above terms can be written as,

$4(a – 1)^2 - 6(a – 1) +2(a – 1) -3$

Take out common in all terms we get,

$2(a – 1) [2(a -1) – 3] +1 [2 (a – 1) – 3)]$

$(2 (a – 1) – 3) (2 (a – 1) +1)$

$(2a – 2 – 3) (2a – 2 + 1)$

$(2a – 5) (2a – 1)$

(ii) $1 – 2a – 2b – 3 (a + b)^2$

Solution:

$1 – 2a – 2b – 3 (a + b)^2$

Above terms can be written as,

$1 – 2 (a + b) – 3 (a + b)^2$

$1 – 3 (a + b) + (a + b) – 3 (a + b)^2$

Take out common in all terms we get,

$1 (1 – 3 (a + b)) + (a + b) (1 – (a + b))$

$(1 – 3 (a + b) (1 + (a + b))$

$(1 – 3a + 3b) (1 + a + b)$

Question 15.

(i) $3 – 5a – 12(a + b)^2$

Solution:

Above terms can be written as,

$3 – 5 (a + b) – 12 (a + b) ^ 2$

$3 – 9 (a + b) +4 (a + b) -12(a +b)^2$

Take out common in all terms we get,

$3(1 – 3 (a + b)) +4 (a +b) (1 – 3 (a +b))$

$(1 – 3 (a + b)) (3 + 4(a + b))$

$(1 – 3a – 3b) (3 + 4a + 4b)$

(ii) $a^4 – 11a^2 + 10$

Solution:

$A^4 – 10a^2 – a^2 + 10$

Take out common in all terms we get,

$A^2 (a^2 – 10) – 1 (a^2 – 10)$

$(a^2 – 10) (a^2 – 1)$

Question 16.

(i) $(x + 4)^2 – 5xy – 20y – 6y^2$

Solution :

$(x + 4)^2 – 5xy – 20y – 6y^2$

Above terms can be written as,

$(x + 4)2 – 5y (x + 4) – 6y2$

$(x + 4)2 – 6y(x + 4) +y (x + 4) – 6y2$

Take out common in all terms we get ,

$(x + 4) (x + 4 – 6y) +y(x + 4 – 6y)$

$(x – 6y + 4) (x + 4 + y)$

(ii) $(x^2 – 2x^2) – 23 (x^2 – 2x) + 120$

Solution:

$(x^2 – 2x^2) – 23 (x^2 – 2x) + 120$

Above terms can be written as,

$(x^2 – 2x)^2 – 15 (x^2 – 2x) -8(x^2 – 2x) + 120$

Take out common in all terms we get,

$(x^2 – 2x) (x^2 – 2x – 15) -8 (x^2 – 2x – 15)$

$(x^2 – 2x – 15) (x^2 – 2x – 8)$

Question 17.

$4(2a – 3)^2 – 3(2a – 3) (a -1) -7 (a – 1)^2$

Solution:

$4(2a – 3)^2 – 3(2a – 3) (a -1) -7 (a – 1)^2$

Let us assume, $2a – 3 = p$ and $a – 1 = q$

So $4p^2 – 3pq – 7q^2$

Then,

$4p^2 – 7pq + 4pq – 7q^2$

out common in all terms we get,

P(4p – 7q) + q (4p – 7q)

(4p – 7q) (p + q)

Now, substitute the value of p and q we get,

(4(2a – 3) -7 (a – 1)) (2a – 3 + a – 1)

(8a – 12 – 7a + 7) (3a – 4)

(a – 5) (3a – 4)

Question 18.

$(2x^2 + 5x) (2x^2 + 5x – 19) + 84$

Solution:

$(2x^2 + 5x) (2x^2 + 5x – 19) + 84$

Let us assume, $2x^2 + 5x = p$

So, (P) (P – 19) + 84

$P^2 – 19p + 84$

$P^2 – 12p – 7p + 84$

P(p – 12) – 7(p – 12)

(p – 12) (p – 7)

Now, substitute the value of p we get,

$(2x^2 + 5x – 12) (2x^2 + 5x – 7)$

Exercise 4.5

Factorise the following (1 to 13) :

Question 1.

(i) $8x^3 + y^3$

Solution:

$8x^3 + y^3$

Above terms can be written as,

$(2x)^3 + y^3$

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

Where, a = 2x , b = y

Then , $(2x)^2 + y^3 = (2x + y) (2x)^2 – (2x \times y) + y^2$

$= (2x + y) (4x^2 – 2xy + y^2)$

(ii) $64x^3 – 125y^3$

Solution:

$64x^3 – 125y^3$

Above terms can be written as,

$(4x)^3 – (5y)^3$

We know that, $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$

Where, a = 4x b = 5y

Then , $(4x)^3 – (5y)^3 = (4x – 5y) (4x)^2 + (4x \times 5y) + 5y^2)$

$= (4x – 5y) (16x^2 + 20xy + 25y^2)$

Question 2.

(i) $64x^3 + 1$

Solution:

$64x^3 + 1$

Above terms can be written as,

$(4x)^3 + 1^3$

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

Where $a = 4x , b = 1$

Then , $(4x)^3 + 1^3 = (4x + 1) (4x)^2 – (4x \times 1) + 12$

$= (4x + 1) (16x^2 – 4x \ \ 1)$

(ii) $7a^3 + 56b^3$

Solution:

$7a^3 + 56b^3$

Take out common in all terms we get,

$7(a^3 + 8b^3)$

Above terms can be written as,

$7(a^3 + (2b)^3$

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

Where , $a = a , b = 2b$

Then , $7[(a)^3 + (2b)^3] = 7[(a + 2b)\ (a)^2 – (a \times 2b) + (2b)^2)]$

$= 7(\ a + 2b)\ (\ a^2 – 2ab + 4b^2)$

Question 3.

(i) $\left(\frac{x^6}{343}\right) + \left(\frac{343}{x^6}\right)$

Solution:

$\left(\frac{x^6}{343}\right) + \left(\frac{343}{x^6}\right)$

Above terms can be written as,

$\left(\frac{x^2}{7}\right)^3 + \left(\frac{7}{x^2}\right)^3$

We know that, a3 + b3 = (a + b) (a2 – ab + b2)

Where , a =

$a = \left(\frac{x^2}{7}\right) \quad b = \left(\frac{7}{x^2}\right)$

Then

$\left(\frac{x^2}{7}\right)^3 + \left(\frac{7}{x^2}\right)^3 = [\left(\frac{x^2}{7}\right) + \left(\frac{7}{x^2}\right)][\ \left(\frac{x^2}{7}\right)^2 - \left(\frac{x^2}{7}\right) \times \left(\frac{7}{x^2}\right) + \left(\frac{7}{x^2}\right)^2$

$= [\left(\frac{x^2}{7}\right) + \left(\frac{7}{x^2}\right)\ ]\ [\left(\frac{x^4}{49}\right) - 1\left(\frac{49}{x^4}\right)$

(ii) $8x^3 - \frac{1}{27}y^3$

Solution:

$8x^3 - \frac{1}{27}y^3$

Above terms can be written as,

$(2x)^3 - (\frac{1}{3y})^3$

We know that , $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

Where, a = 2x, b = $\frac{1}{3y}$

Then , $(2x)^3 - (\frac{1}{3y})^3 = (2x - \frac{1}{3y}) ((2x)2 + (2x \times \frac{1}{3y}) + (3y)^2$

$= (2x - \frac{1}{3y}) (4x^2 + \frac{2x}{3y}) + 9y^2$

Question 4.

(i) $x^2 + x^5$

Solution:

$X^2 + x^5$

Take out common in all terms we get,

$X^2 (1 + x^3)$

$X^2 (1^3 + x^3)$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

Where , a = 1 , b = x

$= x^2 [(1 +x) (1^2 - (1 \times x) + x^2]$

$= x^2 (1 + x) (1 - x + x^2)$

(ii) $32x^4 - 500$

Solution:

$32x^4 – 500x$

Take out common in all terms we get,

$4x (8x^3 – 125)$

Above terms can be written as,

$4x (2x)^3 – 5^3$

We know that, $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$

Where, $a = 2x , b = 5$

$= 4x (2x – 5) (2x)^2 + (2x \times 5) + 5^2$

$= 4x (2x – 5) (4x^2 + 10x + 25)$

Question 5.

(i) $27x^3y^3 – 8$

Solution:

$27x^3y^3 – 8$

Above terms can be written as,

$(3xy)^3 – 2^3$

We know that, $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$

Where , $a = 3xy , b = 2$

$(3xy – 2) (3xy)^2 + (3xy \times 2) + 2^2)$

$= (3xy – 2)(9x^2y^2 + 6xy + 4)$

(ii) $27(x + y)^3 + 8(2x – y)^3$

Solution:

$27(x + y)^3 + 8 (2x – y)^3$

Above terms can be written as,

$3^3 (x + y)^3 + 2^3 (2x – y)^3$

$(3(x +y)^3 + (2 (x –y))^3$

We know that, $a^3 + b^3 = (a +b) (a^2 – ab + b^2)$

Where , $a = 3 (x + y)$, $b = 2(x – y)$

$= [3(x + y) + 2(2x – y)][(3 (x +y)^3 – (3(x + y) \times 2(2x – y)) + (2(2x – y))^{2]}$

$= [3x + 3y + 4x – 2y][9(x + y)2 – 6 (x + y) (2x – y) +4(2x – y)]$

$= (7x – y) [9 (x^2 + y^2 + 2xy) – 6(2x^2 – xy + 2xy – y^2) +4 (4x^2 + y^2 – 4xy)]$

$= (7x – y) [9x^2 + 9y^2 + 18xy – 12x^2 – 6xy – 6y^2 + 16x^2 + 4y^2 – 16xy]$

$= (7x – y) [13x^2 – 4xy + 19y^2]$

Question 6.

(i) $a^3 + b^3 + a + b$

Solution :

$A^3 + b^3 + a + b$

$(a^3 + b^3) + (a + b)$

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

$[(a + b) (a^2 – ab + b^2)] + (a + b)$

$(a + b) (a^2 – ab + b^2 + 1)$

(ii) $a^3 – b^3 – a + b$

Solution:

$A^3 – b^3 – a + b$

$(a^3 – b^3) – (a – b)$

We know that, $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$

$[(a – b) (a^2 + ab + b^2)]-(a – b)$

$(a – b) (a^2 + ab + b^2 – 1)$

Question 7.

(i) $x^3 + x + 2$

Solution:

$x^3 + x + 2$

above terms can be written as,

$x^3 + x + 1+1$

rearranging the above terms, we get

$(x^3 + 1) (x +1)$

$(x^3 + 1^3) (x +1)$

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

$[(x + 1) (x^2 – x + 1)]+ (x + 1)$

$(x + 1) (x^2 – x + 1 + 1)$

$(x + 1) (x^2 – x + 2)$

(ii) $a^3 – a – 120$

Solution:

$A^3 – a – 120$

Above terms can be written as,

$A^3 – a – 125 + 5$

Rearranging the above terms, we get

$A^3 – 125 – a + 5$

$(a^3 – 125) – (a – 5)$

$(a^3 – 5^3) – (a – 5)$

We know that , $a3 – b3 = (a – b) (a^2 + ab + b^2)$

$[(a – 5) (a^2 + 5a + 5^2)] – (a – 5)$

$(a – 5) (a^2 + 5a + 25) – (a – 5)$

$(a – 5) (a^2 + 5a + 25 -1)$

$(a – 5) (a^2 + 5a + 24)$

Question 8.

(i) $x^3+ 6x^2 + 12x + 16$

Solution:

$x^3+ 6x^2 + 12x + 16$

$x^3 + 6x^2 + 12x + 8 + 8$

above terms can be written as,

$(x^3 + (3 \times 2 \times x^2) + (3 \times 2^2 \times x) +2^3) +8$

We know that, $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Now $a = x$ and $b = 2$

So, $(x + 2)^3 +2^3$

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

$(x + 2 + 2) ((x + 2)^2 – (2 \times (x + 2) + 2^2)$

$(x + 4)(x^2 + 4 + 4x – 2x – 4 +4)$

$(x + 4) (x^2 + 2x + 4)$

(ii) $a^3 – 3a^2b + 3ab^2 – 2b^3$

Solution :

$A^3 – 3a^2b + 3ab^2 – 2b^3$

Above terms can be written as,

$A^3 – 3a^2b + 3ab^2 – b^3 – b^3$

We know that, $(a – b)^3 = a^3 – b^3 – 3a^2b + 3ab^2$

So, $(a – b)^3 + b^3$

We also know that, a3 – b3 = (a – b) (a2 + ab + b2)

Where, a = a – b , b = b

(a – b – b) (a – b)2 + (a – b) b + b2

(a – 2b) (a2 + b2 – 2ab + ab – b2 + b2)

(a – 2b) (a2 + b2 – ab)

Question 9.

(i) $2a^3 + 16b^3 – 5a – 10b$

Solution:

$2a^3 + 16b^3 – 5a – 10b$

Above terms can be written as,

$2(a^3 + 8b^3)$ -5 (a + 2b)

$2(a^3 + (2b)^3) – 5$ (a + 2b)

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

$2[(a + 2b) (a^2 – 2ab + 4b^2)]$ -5 (a + 2b)

(a + 2b) ($2a^2 – 4ab + 8b^2 – 5$)

(ii) $a^3 – \frac{1}{a^3}$ -2a + $\frac{2}{a}$

Solution:

$a^3 – \frac{1}{a^3}$ -2a + $\frac{2}{a}$

$= a^3 - \left(\frac{1}{a}\right)^3 - 2a + \frac{2}{a}$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$[(a - \frac{1}{a}) - (a^2 + (a \times \frac{1}{a}) + \left(\frac{1}{a}\right)^2] - 2(a - \frac{1}{a})$

$(a - \frac{1}{a})(a^2 + 1 + \frac{1}{a^2}) - 2(a - \frac{1}{a})$

$(a - \frac{1}{a})(a^2 + 1 + \frac{1}{a^2} - 2)$

$(a - \frac{1}{a})(a^2 + \frac{1}{a^2} - 1)$

Question 10.

(i) $a^6 - b^6$

Solution:

$A^6 - b^6$

Above terms can be written as,

$(a^2)^3 - (b^2)^3$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So, $a = a^2$, $b = b^2$

$(a^2 - b^2)((a^2)^2 + a^2b^2 + (b^2)^2)$

$(a^2 - b^2)(a^4 + a^2b^2 + b^4)$

(ii) $x^6 – 1$

Solution:

$X^6 – 1$

Above terms can be written as,

$(x^2)^3 – 1^3$

We know that, $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$

So, $a = x^2$, $b = 1$

$(x^2- 1) ((x^2)^2 + (x^2 \times 1) + 1^2)$

$(x^2 -1) (x^4 + x^2 + 1)$

Question 11.

(i)$64x^6 – 729y^6$

Solution:

$64x^6 – 729y^6$

Above terms can be written as,

$(2x)^6 – (3y)^6$

$[(2x)^2]^3 – [(3y)^2]^3$

We know that, $a^3 – b^3 = (a – b) (a2 + ab + b2)$

So, $a = (2x)^2$, $b = (3y)^2$

$[(2x)^2 – (3y)^2] [(2x)^2)^2 + ((2x)^2 \times (3y)^2 + ((3y)^2)^2]$

$(4x^2 – 9y^2) [16x^4 + (4x^2 \times 9y^2) + (9y^2)^2]$

$(4x^2 – 9y^2) [16x^4 + 36x^2y^2 + 81y^4] [(2x)^2 – (3y)^2] [16x^4 + 36x^2y^2 + 81y^4]$

$(2x + 3y)\ (\ 2x - 3y)\ (\ 16x4 + 36x^2y^2 + 81y^4)$

(ii) $x^3 - \frac{8}{4}$

Solution:

$X^3 - \frac{8}{x}$

Above terms be written as,

$\frac{1}{x}\ (x3 - 8\ )$

$\frac{1}{x}\ [x^3 - 2^3]$

We know that, $a^3 - b^3 = (\ a - b)\ (\ a^2 + ab + b^2)$

So, a = x , b = 2

$\frac{1}{x}\ (\ x - 2)(\ x^2 + 2x + 4)$

Question 12.

(i) 250 (a – b)3 + 2

Solution:

250 (a – b)3 + 2

Take out common in all terms we get,

2(125 (a – b)3 + 1)

2[(5 (a – b) 3 + 13

We know that, a3 + b3 = (a + b) (a2 – ab + b2)

= 2[(5a – 5b + 1) (5a – 5b)2 – (5a – 5b) 1 + 12)]

= 2 (5a – 5b + 1) (25a2 + 25b2 – 50ab – 5a + 5b + 1)

(ii) $32\ a^2x^2 – 8b^2x^3 – 4a^2y^3 + b^2y^3$

Solution:

$32\ a^2x^2 – 8b^2x^3 – 4a^2y3 + b^2y^3$

Take out common in all terms we get,

$8x^3\ (4a^2 – b^2) –y^3(4a^2 – b^2)$

$(4a^2 – b^2) (8x^3 – y^3)$

Above terms can be written as,

$((2a)^2 – b^2\ ) ((2x)^3 – y^3\ )$

We know that, $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$ and $(a^2 – b^2) = (a + b) (a – b)$

$(2a + b) (2a – b) [(2x – y\ ) ((2x)^2 + 2xy + y^2)]$

$(2a + b) (2a – b) (2x – y) (4x^2 + 2xy + y^2)$

Question 13.

(i) $x^9 + y^9$

Solution:

$X^9 +y^9$

Above terms can be written as,

$(x^3)^3 + (y^3)^3$

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

Where, $a = x^3$, $b = y^3$

$(x^3 + y^3) ((x^3)^2 – x^3y^3 + (y^3)^2)$

$(x^3 + y^3) (x^6 – x^3y^3 + y^6)$

Then , $(x^3 + y^3)$ in the form of $(a^3 + b^3)$

$(x + y) (x^2 – xy + y^2) (x^6 – x^3y^3 + y^6)$

(ii) $x^6 – 7x^3 – 8$

Solution:

$X^6 – 7x^3 – 8$

Above terms can be written as,

$(x^2)^3 – 7x^3 – x^3 + x^3 – 8$

$(x2)^3 – 8x^3 + x^3 – 2^3$

$(((x^2)^3 – (2x)^3)+ (x^3 – 2^3)$

We know that, $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$

$(x^2 – 2x) ((x^2)^2 + (x^2 \times 2x) + (2x)^{2)} + (x – 2) (x^2 + 2x + 2^2)$

$X(x – 2) x^2 (x^2 + 2x + 4) + (x – 2) (x^2 + 2x + 4)$

Take out common in all terms we get,

$(x – 2) (x^2 + 2x + 4) ((x \times x^2) +1)$

$(x – 2) (x^2 + 2x + 4) (x^3 + 1)$

So, above terms are in the form of $a^3 + b^3$

Therefore, $(x -2) (x^2 + 2x + 4) (x + 1) (x^2 – x + 1)$

Chapter test

Factorise the following (1 to 12) :

Question 1.

(i) $15 (2x – 3)^3 – 10 (2x – 3)$

Solution :

$15 (2x – 3)3 – 10 (2x – 3)$

Take out common in both terms,

Then, $5(2x – 3) [3(3(2x – 3)^2 -2]$

(ii) $a(a – c) (b + c) – d (c – d)$

Solution:

$a (b –c) (b + c) – d (c – b)$

above terms can be written as,

$a(b – c) (b + c) +d (b – c)$

take out common in both terms,

$(b – c) [a (b + c) + d]$

$(b – c) (ab + ac + d)$

Question 2.

(i) $2a^2x – bx + 2a^2 – b$

Solution:

$2a^2x – bx + 2a^2 – b$

Rearrange the above terms we get,

$2a^2x + 2a – bx – b$

Take out common in both terms,

$2a^2 (x + 1) – b (x + 1)$

$(x + 1) (2a^2 – b)$

(ii) $p^2 – a (a + 2b) p + 2ab$

Solution:

$p^2 – a (a + 2b) p + 2ab$

above terms can be written as,

$p^2 – ap – 2bp + 2ab$

take out common in both terms,

$p(p – a) -2b(p – a)$

$(p – a) (p – 2b)$

Question 3.

(i) $(x^2 – y^2) z+(y^2 – z^2) x$

Solution:

$(x^2 – y^2) z+(y^2 – z^2) x$

Above terms can be written as,

$Zx^2 – zy^2 + xy^2 – xz^2$

Re arranging the above terms we get,

$Zx^2 – xz^2 + xy^2 – zy^2$

Take out common in both terms,

$Zx (x – z) + y^2(x – z)$

$(x – z) (zx + y^2)$

(ii) $5a^4 – 5a^3 + 30a^2 – 30a$

Solution:

$5a^4 – 5a^3 + 30a^2 – 30a$

Take out common in both terms,

$5a (a^3 – a^2 + 6a – 6)$

$5a[a^2 (a – 1) +6(a – 1)]$

$5a (a – 1) (a^2 + 6)$

Question 4.

(i) $b(c – d)^2 + a(d – c) + 3c – 3d$

Solution:

$b (c –d)^2 + a (d – c) + 3c – 3d$

above terms can be written as,

$b (c – d)^2 – a (c – d) + 3c – 3d$

$b (c – d)^2 – a (c – d) + 3(c – d)$

take out common in both terms,

$(c - d)\ [\ b(\ c - d) - a + 3]$

$(\ c - d)\ (\ bc - bd - a + 3)$

(ii) $x^3 - x^2 - xy + x + y - 1$

Solution:

$x^3 - x^2 - xy + x + y - 1$

re arrange the above terms we get,

$x^3 - x^2 - xy + y + x - 1$

take out common in both terms,

$x^2(\ x - 1) - y\ (\ x - 1) + 1(\ x - 1)$

$(x - 1)\ (\ x^2 - y + 1)$

Question 5.

(i) $x\ (\ x + z) - y(\ y + z)$

Solution:

$x\ (\ x + z) - y(\ y + z)$

$x^2 + xz - y^2 - yz$

re arrange the above terms we get,

$x^2 - y^2 + xz - yz$

we know that, $(a^2 - b^2\ ) = (\ a + b)\ (\ a - b)$

so , $(\ x + y)\ (x - y) + z(\ x - y)$

$(\ x - y)\ (\ x + y + z)$

(ii) $a^{12}x^4 – a^4x^{12}$

Solution:

$a^{12}x^4 – a^4x^{12}$

take out common in both terms,

$a^4x^4 (a^8 – x^8)$

$a^4x^4((a^4)^2 – (x^4)^2)$

we know that, $(a^2 – b^2) = (a + b) (a – b)$

$a^4x^4(a^4 + x^4) (a^4 – x^4)$

$a^4x^4 (a^4 + x^4) ((a^2)^2 – (x^2)^2)$

$a^4x^4(a^4 + x^4) (a^2 + x^2) (a^2 – x^2)$

$a^4x^4 (a^4 + x^4) (a^2 + x^2) (a + x) (a – x)$

Question 6.

(i) $9x^2 + 12x + 4 – 16y^2$

Solution:

$9x^2 + 12x + 4 – 16y^2$

Above terms can be written as,

$(3x)^2 + (2 \times 3x \times 2) + 22 – 16y^2$

Then, $(3x + 2)^2 + (4y)^2$

$(3x + 2 + 4y) (3x + 2 – 4y)$

(ii) $x^4 + 3x^2 + 4$

Solution:

$X^4 + 3x^2 + 4$

Above terms can be written as,

$(x^2)^2 + 3(x^2) + 4$

$(x^2)^2 + (2)^2 + 4x^2 – x^2$

$(x^2 + 2)^2 – (x^2)$

We know that,

$(a^2 – b^2) = (a + b) (a – b)$

$(x^2 + 2 + x) (x^2 + 2 – x)$

$(x^2 + x + 2) (x^2 – x + 2)$

Question 7.

(i) $21x^2 – 59xy + 40y^2$

Solution:

$21x^2 – 59xy + 40y^2$

By multiplying first and last term we get, $12 \times 40 = 840$

Then , $(-35) \times (-24) = 840$

So, $21x^2 – 35xy – 24xy + 40y^2$

$7x (3x – 5y) -8y (3x – 5y)$

$(3x – 5y) (7x – 8y)$

(ii) $4x^3y - 44x^2y + 112xy$

solution:

take out common in all terms,

$4xy(x^2 - 11x + 28)$

Then $4xy (x^2 - 7x - 4x + 28)$

$4xy [x(x - 7) -4(x + 7)]$

$4xy (x -7) (x - 4)$

Question 8.

(i) $x^2y^2 - xy - 72$

Solution:

$X^2y^2 - xy - 72$

$X^2y^2 - 9xy + 8xy - 72$

Take out common in all terms,

$Xy (xy - 9) +8(xy - 9)$

$(xy - 9) (xy + 8)$

(ii) $9x^3y + 41x^2y^2 + 20xy^3$

Solution:

$9x^3y + 41x^2y^2 + 20xy^3$

Take out common in all terms,

$Xy(9x^2 + 41xy + y^2)$

Above terms can be written as,

$Xy(9x^2 + 36xy + 5xy + 20y^2)$

$Xy\ [9x(x + 4y) + 5y(x+ 4y)\ ]$

$Xy\ (x + 4y)\ (\ 9x + 5y)$

Question 9.

(i) $(\ 3a – 2b\ )^2 + 3(\ 3a – 2b) – 10$

Solution:

$(3a – 2b)^2 + 3(\ 3a – 2b) – 10$

Let us assume, $(\ 3a – 2b) = p$

$P^2 + 3p – 10$

$P^2 + 5p – 2p – 10$

Take out common in all terms,

$P(p + 5) – 2(\ p + 5)$

$(\ p + 5)\ (\ p – 2)$

Now, substitute the value of p

$(\ 3a – 2b + 5)\ (\ 3a – 2b – 2)$

(ii) $(\ x^2 – 3x)\ (\ x^2 – 3x + 7) + 10$

Solution:

$(\ x^2 – 3x)\ (\ x^2 – 3x + 7) + 10$

Let us assume, $(x^2 – 3x\ ) = q$

$q (q + 7) + 10$

$q^2 + 7q + 10$

$q^2 + 5q + 2q + 10$

$q(q + 5) +2(q + 5)$

$(q + 5) (q + 2)$

Now, substitute the value of q

$(x^2 – 3x + 5) (x^2 – 3x + 2)$

Question 10.

(i) (x2 –x) (4x2 – 4x – 5) – 6

Solution:

$(x^2 –x) (4x^2 – 4x – 5) – 6$

$(x^2 – x) [(4x^2 – 4x) – 5] – 6$

$(x^2 – x) [4(x^2 – x) -5]-6$

Let us assume $x^2 – x = q$

So, $q [4q – 5] – 6$

$4q^2 – 5q – 6$

$4q^2 – 5q – 6$

$4q^2 – 8q + 3q – 6$

$4q (q – 2) + 3 (q – 2)$

$(q – 2) (4q + 3)$

Now, substitute the value of q

$(x^2 – x – 2) (4(x^2 – x) + 3)$

$(x^2 – x – 2) (4x^2 – 4x + 3)$

$(x^2 – 2x + x – 2) (4x^2 – 4x + 3)$

$[x(x -2) + 1(x-2)](4x^2 – 4x^2 – 4x + 3)$

$(x – 2) (x + 1) (4x^2 – 4x + 3)$

(ii) $x^4 + 9x^2y^2 + 81y^4$

Solution :

$x^4 + 9x^2y^2 + 81y^4$

above terms can be written as,

$((x^2)^2 + (2 \times x^2 \times 9y^2) + (9y^2)^2 – 9x^2y^2$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$(x^2 + 9y^2)^2 – (3xy)^2$

$(x^2 + 9y^2 + 3xy) (x^2 + 9y^2 – 3xy)$

Question 11.

(i) $\left(\frac{8}{27}\right)x^3 - \left(\frac{8}{27}\right)y^3$

Solution:

$\left(\frac{8}{27}\right)x^3 - \left(\frac{8}{27}\right)y^3$

Above terms can be written as,

$\left(\frac{2}{3}x\right)^3 \left(\frac{1}{2}y\right)^3$

We know that, a3 – b3 = (a – b) (a^2 + ab + b^2)

$\frac{2}{3}x - \frac{1}{2}y [\frac{2}{3}x + \frac{2}{3}x \times \frac{1}{2}y + \left(\frac{1}{2}y\right)^2$

$\frac{2}{3}x - \frac{1}{2}y [\frac{4}{9}x^2 + \frac{xy}{3} + \frac{y^2}{4}]$

(ii) x6 + 63x3 – 64

Solution:

X6 + 63x3 – 64

Above terms can be written as,

X6 + 64x3 – 64

Take out common in all terms,

X3 (x3 + 64) -1 (x3 + 64)

(x3 + 64) (x3 – 1)

(x3 + 43) (x3 – 13)

We know that, a3 – b3 = (a –b) (a2 + ab + b2) and a3 + b3 = (a + b)

(a2 – ab + b2)

So, (x + 4) [x2 – 4x + 42] (x – 1) [x2 + x + 12]

(x + 4) (x2 – 4x + 16) (x -1) (x2 + x + 1)

Question 12.

(i) x3 + x2 - $\frac{1}{x^2} + \frac{1}{x^3}$

Re arranging the above terms, we get

$X^3 + \frac{1}{x^3} + x^2 - \frac{1}{x^2}$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$ and $(a^2 - b^2) = (a + b) (a - b)$

$(x + \frac{1}{x}) (x^2 - 1 + \frac{1}{x^2}) + (x + \frac{1}{x}) (x - \frac{1}{x})$

$(x + \frac{1}{x}) [x^2 - 1 + \frac{1}{x^2} + x - \frac{1}{x}]$

(ii) $(x + 1)^6 - (x - 1)^6$

Solution:

$(x + 1)^6 - (x - 1)^6$

Above terms can be written as,

$((x + 1)^3)^2 - ((x - 1)^3)^2$

We know that, $(a^2 - b^2) = (a + b) (a - b)$

$[(x + 1)^3 + (x - 1)^3] [(x + 1)^3 - (x - 1)^3] [(x + 1) + (x - 1)][(x + 1)^2 - (x - 1) (x + 1) + (x - 1)^2] [(x - 1)][(x + 1)^2 + (x - 1) (x + 1) + (x - 1)^2]$

$(x + 1 + x - 1) [x^2 + 2x + 1 - x^2 + 1 + x^2 + 1 - 2x (x + 1) - x + 1] [x^2 + 2x + 1 + x^2 - 1 + x^2 - 2x + 1]$

By simplifying we get,

$2x (x^2 + 3)^2 (3x^2 + 1)$

$4x (x^2 + 3) (3x^2 + 1)$

Question 13.

Show that $(97)^3 + (14)^3$ is divisible by 111

Solution:

From the question,

$(97)^3 + (14)^3$

We know that $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

So

$(97 + 14) [(97)^2 – (97\times 14) + (14)^2]$

$111 [(97)^2 – (97\times 14) + (14)^2]$

Therefore, it is clear that the given expression is divisible by 111

Question 14.

If $a + b = 8$ and $ab = 15$, find the value of $a^4 + a^2b^2 + b^4$.

Solution:

$A^4 + a^2b^2 + b^4$

Above terms can be written as,

$A4 + 2a^2b^2 + b^4 – a^2b^2$

$(a^2)^2 + 2a^2b^2 + (b^2)^2 – (ab)^2$

$(a^2 + b^2)^2 – (ab)^2$

$(a^2 + b^2 + ab) (a^2 + b – ab)$

$A + b = 8 , ab = 15$

So, $(a + b)^2 = 8^2$

$A^2 + 2ab + b^2 = 64$

$A^2 + 2(15) + b^2 = 64$

$A^2 + b^2 + 30 = 64$

By transposing ,

$A^2 + b^2 = 64 - 30$

$A^2 + b^2 = 34$

Then , $a^4 + a^2b^2 + b^4$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

$= (34 + 15)(34 - 15)$

$= 49 \times 19$

$= 931$