

CBSE SAMPLE PAPER - 01

Class 12 - Mathematics

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\int \sqrt{1-9x^2} dx = ?$ [1]
a) $\frac{3x}{2} \sqrt{1-9x^2} + \frac{1}{6} \sin^{-1} 3x + C$ b) $\frac{x}{2} \sqrt{1-9x^2} + \frac{1}{6} \sin^{-1} 3x + C$
c) $\frac{x}{2} \sqrt{1-9x^2} + \frac{1}{18} \sin^{-1} 3x + C$ d) None of these
2. The direction ratios of the line perpendicular to the lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$ are proportional to [1]
a) 4, 5, 7 b) -4, 5, 7
c) 4, -5, -7 d) 4, -5, 7
3. The values of x for which the angle between $\vec{a} = 2x^2 \hat{i} + 4x \hat{j} + \hat{k}$, $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse and the angle between \vec{b} and the z-axis is acute and less than $\frac{\pi}{6}$ are [1]
a) $\frac{1}{2} < x < 15$ b) $x > \frac{1}{2}$ or $x < 0$
c) ϕ d) $0 < x < \frac{1}{2}$
4. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct? [1]
a) None of these b) $P(A|B) \geq P(A)$
c) $P(A|B) = \frac{P(B)}{P(A)}$ d) $P(A|B) < P(A)$
5. $\int \frac{dx}{(4+16x^2)} = ?$ [1]
a) $\frac{1}{32} \tan^{-1} 4x + C$ b) $\frac{1}{4} \tan^{-1} \frac{x}{2} + C$
c) $\frac{1}{16} \tan^{-1} \frac{x}{2} + C$ d) $\frac{1}{8} \tan^{-1} 2x + C$
6. If A and B are two independent events such that $P(A) = 0.3$, $P(A \cup B) = 0.5$, then $P(A/B) - P(B/A) =$ [1]

a) $\frac{2}{7}$

b) $\frac{1}{7}$

c) $\frac{1}{70}$

d) $\frac{3}{35}$

7. The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is [1]

a) $\frac{5}{8}$ sq.units

b) $\frac{9}{8}$ sq.units

c) $\frac{3}{8}$ sq.units

d) $\frac{7}{8}$ sq.units

8. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$. [1]

a) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k})$,
 $\lambda \in R$

b) $\vec{r} = 2\hat{i} + 2\hat{j} + 3\hat{k} + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k})$
 $\lambda \in R$

c) $\vec{r} = 4\hat{i} + 2\hat{j} + 3\hat{k} + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k})$
 $\lambda \in R$

d) $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k} + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k})$
 $\lambda \in R$

9. If \vec{a} and \vec{b} are unit vectors inclined at an angle θ , then the value of $|\vec{a} - \vec{b}|$ is [1]

a) $2 \cos \frac{\theta}{2}$

b) $2 \sin \frac{\theta}{2}$

c) $2 \cos \theta$

d) $2 \sin \theta$

10. What is the equation of a curve passing through (0, 1) and whose differential equation is given by $dy = y \tan x \, dx$? [1]

a) $y = \sec x$

b) $y = \sin x$

c) $y = \operatorname{cosec} x$

d) $y = \cos x$

11. The area bounded by the curves $y = \sin x$ between the ordinates $x = 0$, $x = \pi$ and the x-axis is [1]

a) 2 sq. units

b) 4 sq. units

c) 3 sq. units

d) 1 sq. units

12. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals [1]

a) $\frac{\pi}{4}$

b) $\frac{\pi}{12}$

c) $\frac{\pi}{6}$

d) $\frac{\pi}{24}$

13. The least value of k for which $f(x) = x^2 + kx + 1$ is increasing on (1, 2), is [1]

a) -2

b) 2

c) 1

d) -1

14. If A is any square matrix then which of the following is not symmetric? [1]

a) $A + A^t$

b) $A - A^t$

c) $A^t A$

d) AA^t

15. If A is an invertible matrix, then $\det(A^{-1})$ is equal to [1]

a) $\frac{1}{\det(A)}$

b) 1

c) $\det(A)$

d) none of these

16. The existence of the unique solution of the system of equations: [1]

$$x + y + z = \lambda$$

$$5x - y + \mu z = 10$$

$2x + 3y - z = 6$ depends on

- a) λ and μ both
b) λ only
c) neither λ nor μ
d) μ only

17. Range of $\sec^{-1}x$ is [1]

- a) $[0, \pi]$
b) $[0, \pi] - \{\frac{\pi}{2}\}$
c) None of these
d) $[0, \frac{\pi}{2}]$

18. The equation of the curve satisfying the differential equation $y(x + y^3) dx = x(y^3 - x) dy$ and passing through the point (1, 1) is, [1]

- a) None of these
b) $y^3 + 2x + 3x^2y = 0$
c) $y^3 + 2x - 3x^2y = 0$
d) $y^3 - 2x + 3x^2y = 0$

19. **Assertion (A):** The function $f(x) = \sin x$ decreases on the interval $(0, \frac{\pi}{2})$. [1]

Reason (R): The function $f(x) = \cos x$ decreases on the interval $(0, \frac{\pi}{2})$.

- a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ where, A_{ij} is cofactor of a_{ij} . [1]

Reason (R): $\Delta =$ Sum of the products of elements of any row (or column) with their corresponding cofactors.

- a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the domain of $f(x) = \sin^{-1}(-x^2)$. [2]

22. Find the general solution of the differential equation $(x + 2) \frac{dy}{dx} = x^2 + 5x - 3(x \neq -2)$ [2]

23. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, then show that $A - 3I = 2(I + 3A^{-1})$ [2]

OR

Determine the values of λ for which the following system of equations fail to have a unique solution:

$$\lambda x + 3y - z = 1$$

$$x + 2y + z = 2$$

$$-\lambda x + y + 2z = -1$$

Does it have any solution for this value of λ ?

24. If D, E, F are the mid-points of the sides BC, CA and AB respectively of a triangle ABC, write the value of [2]

$$\vec{AD} + \vec{BE} + \vec{CF}.$$

25. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved. [2]

Section C

26. Evaluate $\int_{-2}^2 x e^{|x|} dx$ [3]

27. Solve : $\frac{dy}{dx} = \cos(x + y) + \sin(x + y)$. [Hint: Substitute $x + y = z$] [3]

OR

Solve $[x\sin^2(\frac{y}{x}) - y] dx + xdy = 0$;

$y = \pi/4$, when $x = 1$

28. Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{i} + \hat{k}$ and other is perpendicular to \vec{b} . [3]

OR

Find λ when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

29. Evaluate the integral: $\int \frac{x^5}{\sqrt{1+x^3}} dx$ [3]

OR

Evaluate: $\int \frac{x}{(1+\sin x)} dx$

30. Discuss the continuity of the $f(x)$ at the indicated point: $f(x) = |x| + |x - 1|$ at $x = 0, 1$ [3]

31. Find the area bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, $X - axis$ and lying in the first quadrant [3]

Section D

32. Solve the following linear programming problem graphically: [5]

Maximize $Z = 50x + 15y$

Subject to

$5x + y \leq 100$

$x + y \leq 60$

$x, y \geq 0$

33. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation. Write all the equivalence classes of R . [5]

OR

Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by n , is an equivalence relation on Z .

34. Show that the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$ intersect. [5]
Also, find their point intersection.

OR

Find the vector equation of the line passing through $(1,2,3)$ and \parallel to the plane $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$

35. Find all the points of discontinuity of f defined by $f(x) = |x| - |x + 1|$. [5]

Section E

36. **Read the text carefully and answer the questions:** [4]

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfectant. The cost of material used to manufacture the tin can is

₹100/m².



- (i) If r cm be the radius and h cm be the height of the cylindrical tin can, then express the surface area as a function of radius (r)
- (ii) Find the radius of the can that will minimize the cost of tin used for making can?
- (iii) Find the height that will minimize the cost of tin used for making can ?

OR

Find the minimum cost of material used to manufacture the tin can.

37. **Read the text carefully and answer the questions:**

[4]

To promote the making of toilets for women, an organization tried to generate awareness through

- i. house calls
- ii. emails and
- iii. announcements.

The cost for each mode per attempt is given below:



1. ₹ 50
2. ₹ 20
3. ₹ 40

The number of attempts made in the villages X, Y and Z are given below:

| | (i) | (ii) | (iii) |
|---|-----|------|-------|
| X | 400 | 300 | 100 |
| Y | 300 | 250 | 75 |
| Z | 500 | 400 | 150 |

Also, the chance of making of toilets corresponding to one attempt of given modes is

1. 2%
2. 4%
3. 20%

- (i) Find total number of toilets that can be expected after the promotion in village X.

- (ii) Find the percentage of toilets that can be expected after the promotion in all the three-villages?
- (iii) Find the cost incurred by the organization on village X.

OR

Find the total cost incurred by the organization on for all the three villages?

38. **Read the text carefully and answer the questions:**

[4]

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.



- (i) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Govind.
- (ii) Find the probability that Priyanka processed the form and committed an error.

Solution

CBSE SAMPLE PAPER - 01

Class 12 - Mathematics

Section A

1. (b) $\frac{x}{2}\sqrt{1-9x^2} + \frac{1}{6}\sin^{-1} 3x + C$

Explanation: $\int \sqrt{1-9x^2} dx = 3 \int \sqrt{\frac{1}{9} - x^2} dx$
 $= 3 \left[\frac{x}{2} \sqrt{\frac{1}{9} - x^2} + \frac{1}{18} \sin^{-1} \frac{x}{(1/3)} \right] + C$
 $= \frac{3x}{2} \sqrt{\frac{1}{9} - x^2} + \frac{1}{6} \sin^{-1} 3x + C$
 $= \frac{x}{2} \sqrt{1-9x^2} + \frac{1}{6} \sin^{-1} 3x + C$

2. (a) 4, 5, 7

Explanation: We have,

$$\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$$

$$\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$$

The direction ratios of the given lines are proportional to 2, -3, 1 and 1, 2, -2.

The vectors parallel to the given lines are $\vec{b}_1 = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$

Vector perpendicular to the vectors b_1 & b_2 is,

$$\vec{b} = \vec{b}_1 \times \vec{b}_2$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= 4\hat{i} + 5\hat{j} + 7\hat{k}$$

Hence, the direction ratios of the line perpendicular to the given two lines are proportional to 4, 5, 7.

3. (d) $0 < x < \frac{1}{2}$

Explanation: $0 < x < \frac{1}{2}$

4. (a) None of these

Explanation: Since, $A \subset B$, $A \cap B = A$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

5. (d) $\frac{1}{8} \tan^{-1} 2x + C$

Explanation: Let $I = \int \frac{dx}{(4x)^2 + 2^2}$

$$4x = t$$

$$4dx = dt$$

$$dx = \frac{dt}{4}$$

$$= \frac{1}{4} \int \frac{dt}{t^2 + 2^2}$$

We know,

$$= \frac{1}{8} \tan^{-1} \frac{t}{2} + c$$

put $t = 4x$

$$= \frac{1}{8} \tan^{-1} \frac{4x}{2} + c$$

$$= \frac{1}{8} \tan^{-1} 2x + c$$

6. (c) $\frac{1}{70}$

Explanation: $P(A) = 0.3$, $P(A \cup B) = 0.5$ (Given)

Since, A and B are two independent events,

$$P(A \cap B) = P(A) P(B)$$

$$P(A \cap B) = 0.3 \times P(B) \dots(i)$$

Also, according to the addition theorem of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.5 = 0.3 + P(B) - 0.3 P(B) \quad \text{From (Given) \& (i)}$$

$$0.7 P(B) = 0.2$$

$$P(B) = \frac{0.2}{0.7} = \frac{2}{7} \dots(\text{ii})$$

Putting value of P(B) in equation (i) we get,

$$P(A \cap B) = 0.3 \times \frac{2}{7} = \frac{3}{10} \times \frac{2}{7}$$

$$P(A \cap B) = \frac{6}{70} \dots(\text{iii})$$

Now,

$$P\left(\frac{A}{B}\right) - P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(B)} - \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{6}{70}}{\frac{2}{7}} - \frac{\frac{6}{70}}{\frac{3}{10}} \quad \text{From (iii) \& (ii) and (Given)}$$

$$= \frac{6}{70} \times \frac{7}{2} - \frac{6}{70} \times \frac{10}{3}$$

$$= \frac{3}{10} - \frac{2}{7}$$

$$= \frac{1}{70}$$

7. (b) $\frac{9}{8}$ sq.units

Explanation: Eliminating y, we get :

$$x^2 - x - 2 = 0 \Rightarrow x = -1, 2$$

Required area :

$$= \int_{-1}^2 \left(\frac{x}{4} + \frac{1}{2} - \frac{x^2}{4} \right) dx = \frac{1}{8}(4 - 1) + \frac{3}{2} - \frac{1}{12}(8 + 1) = \frac{3}{8} + \frac{3}{2} - \frac{3}{4} = \frac{9}{8} \text{ sq.units}$$

8. (a) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k}), \lambda \in R$

Explanation: The equation of the line which passes through the point (1, 2, 3) and is parallel to the vector

$$3\hat{i} + 2\hat{j} - 2\hat{k}, \text{ let vector } \vec{a} = \hat{i} + \hat{j} + \hat{k} \text{ and vector } \vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k},$$

the equation of line is :

$$\vec{a} + \lambda \vec{b} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$$

9. (b) $2 \sin \frac{\theta}{2}$

Explanation: Given \vec{a} and \vec{b} are unit vectors with inclination is θ

$$\text{now, } |\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + |\vec{b}|^2$$

$$= 1 - 2(\vec{a} \cdot \vec{b}) + 1$$

$$= 2 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$= 2 - 2\cos\theta \quad (\text{where vectors are unit vectors})$$

$$= 2(1 - \cos\theta)$$

$$= 4\sin^2 \frac{\theta}{2}$$

$$\text{thus } |\vec{a} - \vec{b}|^2 = 4\sin^2 \frac{\theta}{2}$$

$$\therefore |\vec{a} - \vec{b}| = 2 \sin \frac{\theta}{2}$$

10. (a) $y = \sec x$

Explanation: The given differential equation of the curve is,

$$dy = y \tan x \, dx \Rightarrow \int \frac{dy}{y} = \int \tan x \cdot dx \quad [\text{on integrating}]$$

$$\Rightarrow \log y = \log \sec x + \log C \Rightarrow \log y = \log C \sec x$$

$$\Rightarrow y = C \sec x \dots(\text{i})$$

Since, the curve passes through the origin (0, 1), then

$$1 = C \sec 0 \Rightarrow C = 1$$

\therefore Required equation of curve is, $y = \sec x$

11. (a) 2 sq. units

Explanation: $\int_0^\pi y \, dx = \int_0^\pi \sin x \, dx$

$$\int_0^\pi y \, dx = -[\cos x]_0^\pi$$

$$\int_0^\pi y \, dx = -[-1 - 1]$$

$$\int_0^\pi y \, dx = 2$$

12. (d) $\frac{\pi}{24}$

Explanation: Let $I = \int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$

Taking 9 common from Denominator in I

$$\Rightarrow I = \frac{1}{9} \int_0^{\frac{2}{3}} \frac{dx}{\frac{4}{9}+x^2} = \frac{1}{9} \int_0^{\frac{2}{3}} \frac{dx}{\left(\frac{2}{3}\right)^2+x^2} \quad \left[\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c \right]$$

$$\Rightarrow I = \frac{1}{9} \times \frac{3}{2} \left[\tan^{-1} \frac{x}{\frac{2}{3}} \right]_0^{\frac{2}{3}} = \frac{1}{9} \times \frac{3}{2} \left[\tan^{-1} \frac{3x}{2} \right]_0^{\frac{2}{3}}$$

$$\Rightarrow I = \frac{1}{6} \left[\tan^{-1} \frac{3}{2} \times \frac{2}{3} - \tan^{-1} 0 \right] = \frac{1}{6} \left[\tan^{-1} 1 - \tan^{-1} 0 \right]$$

$$\Rightarrow I = \frac{1}{6} \times \left(\frac{\pi}{4} - 0 \right) = \frac{\pi}{24}$$

$$\therefore \int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \frac{\pi}{24}$$

13. (a) -2

Explanation: Given, $f(x) = x^2 + kx + 1$

For increasing

$$f'(x) = 2x + k$$

$$k \geq -2x$$

thus,

$$k \geq -2x$$

Least value of -2

14. (b) $A - A^t$

Explanation: For every square matrix $(A - A^t)$ is always skew – symmetric.

15. (a) $\frac{1}{\det(A)}$

Explanation: Solution.

Since we know that $|A^{-1}| = \frac{1}{|A|}$

16. (d) μ only

Explanation: The given system of linear equation :-

$$x + y + z = \lambda$$

$$5x - y + \mu z = 10$$

$$2x + 3y - z = 6$$

The matrix equation corresponding to the above system is :

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \lambda \\ 10 \\ 6 \end{bmatrix}$$

Suppose $A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{vmatrix} = 1(1-3\mu) - 1(-5-2\mu) + 1(15 + 2)$$

$$= 1 - 3\mu + 5 + 2\mu + 17 = 23 - \mu$$

For the existence of the unique solution, the value of $|A|$ must not be equal to 0.

Therefore, the existence of the unique solution merely depends on the value of μ . Which is the required solution.

17. (b) $[0, \pi] - \left\{ \frac{\pi}{2} \right\}$

Explanation:

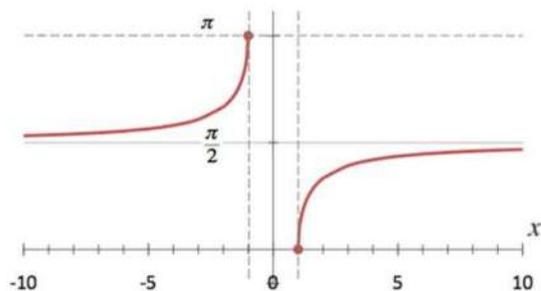
To Find: The range of $\sec^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sec^{-1}(x)$ can be obtained from the graph of

$Y = \sec x$ by interchanging x and y axes. i.e, if (a, b) is a point on $Y = \sec x$ then (b, a) is the point on the function $y = \sec^{-1}(x)$

Below is the Graph of the range of $\sec^{-1}(x)$



From the graph, it is clear that the range of $\sec^{-1}(x)$ is restricted to interval $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

18. (c) $y^3 + 2x - 3x^2y = 0$

Explanation: We have,

$$y(x + y^3) dx = x(y^3 - x) dy$$

$$xy dx + y^4 dx = xy^3 dy - x^2 dy$$

$$xy dx - xy^3 dy + y^4 dx + x^2 dy = 0$$

$$xy dx + x^2 dy + y^4 dx - xy^3 dy = 0$$

$$x(y dx + x^2 dy) + y^3(y dx - x dy) = 0$$

$$x(y dx + x^2 dy) + x^2 y^3 \frac{(y dx - x dy)}{x^2} = 0$$

$$x(y dx + x^2 dy) - x^2 y^3 \frac{(x dy - y dx)}{x^2} = 0$$

$$x(y dx + x^2 dy) - x^2 y^3 d\left(\frac{y}{x}\right) = 0$$

$$x(y dx + x^2 dy) = x^2 y^3 d\left(\frac{y}{x}\right)$$

$$\frac{x(y dx + x^2 dy)}{x^3 y^2} = \frac{y}{x} d\left(\frac{y}{x}\right)$$

$$\int \frac{x(y dx + x^2 dy)}{x^3 y^2} = \int \frac{y}{x} d\left(\frac{y}{x}\right)$$

$$\int \frac{d(xy)}{x^2 y^2} = \int \frac{y}{x} d\left(\frac{y}{x}\right)$$

$$\frac{-1}{xy} = \frac{\left(\frac{y}{x}\right)^2}{2} + c$$

$$\frac{1}{xy} + \frac{\left(\frac{y}{x}\right)^2}{2} + c = 0$$

$$\Rightarrow y^3 + 2x + 2cx^2y = 0$$

Curve passes through (1, 1)

$$1 + 2 + 2c = 0$$

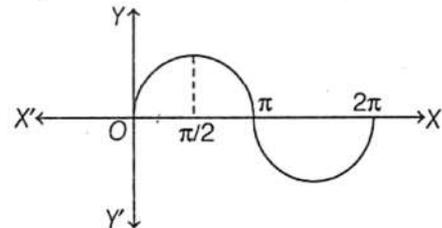
$$1 + 2 + 2c = 0$$

$$c = \frac{-3}{2}$$

$$\Rightarrow y^3 + 2x - 3x^2y = 0$$

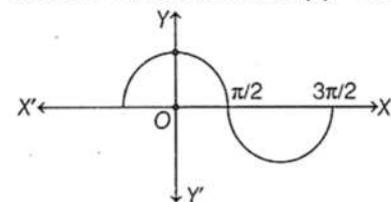
19. (d) A is false but R is true.

Explanation: Assertion: Given, function $f(x) = \sin x$



From the graph of $\sin x$, we observe that $f(x)$ increases on the interval $(0, \frac{\pi}{2})$.

Reason: Given function is $f(x) = \cos x$.



From the graph of $\cos x$, we observe that, $f(x)$ decreases on the interval $(0, \frac{\pi}{2})$.

Hence, Assertion is false and Reason is true.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: By expanding the determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \text{ and } R_1, \text{ we have}$$

$$\Delta = (-1)^{1+1}a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3}a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}, \text{ where } A_{ij} \text{ is cofactor of } a_{ij}$$

$$= \text{Sum of products of elements of } R_1 \text{ with their corresponding cofactors.}$$

Section B

21. The domain of $\sin^{-1} x$ is $[-1, 1]$. Therefore, $f(x) = \sin^{-1}(-x^2)$ is defined for all x satisfying $-1 \leq -x^2 \leq 1$

$$\Rightarrow 1 \geq x^2 \geq -1$$

$$\Rightarrow 0 \leq x^2 \leq 1$$

$$\Rightarrow x^2 \leq 1$$

$$\Rightarrow x^2 - 1 \leq 0$$

$$\Rightarrow (x-1)(x+1) \leq 0$$

$$\Rightarrow -1 \leq x \leq 1$$

Hence, the domain of $f(x) = \sin^{-1}(-x^2)$ is $[-1, 1]$.

22. The given differential equation may be written as

$$\frac{dy}{dx} = \frac{x^2+5x-3}{x+2}$$

$$\Rightarrow dy = \left(\frac{x^2+5x-3}{x+2} \right) dx \text{ [separating the variables]}$$

$$\Rightarrow \int dy = \int \left(\frac{x^2+5x-3}{x+2} \right) dx$$

$$\Rightarrow y = \int \left\{ x + 3 - \frac{9}{(x+2)} \right\} dx + C, \text{ where } C \text{ is an arbitrary constant}$$

[on dividing $(x^2 + 5x - 3)$ by $(x + 2)$]

$$\Rightarrow y = \frac{x^2}{2} + 3x - 9 \log|x+2| + C$$

Therefore, $y = \frac{x^2}{2} + 3x - 9 \log|x+2| + C$ is the required general solution of the given differential equation.

23. We have

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$

$$|A| = 4 - 10 = -6 \text{ adj } A = \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-6} \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$$

To Show: $A - 3I = 2(I + 3A^{-1})$

$$\text{LHS } A - 3I = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$$

$$\text{R.H.S } 2(I + 3A^{-1}) = 2I + 6A^{-1} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 6 \cdot \frac{1}{-6} \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$$

Hence, $A - 3I = 2(I + 3A^{-1})$

OR

Given system of equations is

$$\lambda x + 3y - z = 1$$

$$x + 2y + z = 2$$

$$-\lambda x + y + 2z = -1$$

The given system of equations will fail to have unique solution, if $D = 0$

$$\text{i.e. } \begin{vmatrix} \lambda & 3 & -1 \\ 1 & 2 & 1 \\ -\lambda & 1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \lambda(4 - 1) - 3(2 + \lambda) - (1 + 2\lambda) = 0$$

$$\Rightarrow 3\lambda - 6 - 3\lambda - 1 - 2\lambda = 0$$

$$\Rightarrow -2\lambda - 7 = 0$$

$$\Rightarrow \lambda = -\frac{7}{2}$$

For $\lambda = -\frac{7}{2}$, we obtain

$$D_1 = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix} = -16 \neq 0$$

Thus, for $\lambda = -\frac{7}{2}$, we have $D = 0$ and $D_1 \neq 0$

Hence, the given system of equations has no solution for $\lambda = -\frac{7}{2}$

24. Here, it is given that DD, E, F are the midpoints of the sides BC, CA, AB respectively.

Then, the position vectors of the midpoints D, E, F are given by $\frac{\vec{b}+\vec{c}}{2}$, $\frac{\vec{c}+\vec{a}}{2}$, $\frac{\vec{a}+\vec{b}}{2}$

$$\text{Now we have, } \vec{AD} + \vec{BE} + \vec{CF} = \left(\frac{\vec{b}+\vec{c}}{2}\right) - \vec{a} + \left(\frac{\vec{c}+\vec{a}}{2}\right) - \vec{b} + \left(\frac{\vec{a}+\vec{b}}{2}\right) - \vec{c}$$

$$= 2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right) - (\vec{a} + \vec{b} + \vec{c})$$

$$= (\vec{a} + \vec{b} + \vec{c}) - (\vec{a} + \vec{b} + \vec{c})$$

$$= \vec{0}$$

25. Given:

$$P(A) = \text{Probability of solving the problem by A} = \frac{1}{2}$$

$$P(B) = \text{Probability of solving the problem by B} = \frac{1}{3}$$

Since, A and B both are independent.

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

$$\Rightarrow P(A \cap B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

The problem is solved, i.e. it is either solved by A or it is solved by B.

$$= P(A \cup B)$$

$$\text{As we know, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6}$$

$$\Rightarrow P(A \cup B) = \frac{2}{3}$$

Section C

26. The given integral can be written as

$$I = \int_{-2}^0 x e^{-x} dx + \int_0^2 x e^x dx$$

For

$$\int_{-2}^0 x e^{-x} dx$$

Using Integration By parts

$$\int fg = fg - \int fg'$$

$$f' = e^{-x}, g = x$$

$$f = -e^{-x}, g' = 1$$

$$\int_{-2}^0 x e^{-x} dx = \{-x e^{-x}\}_{-2}^0 + \int_{-2}^0 e^{-x} dx$$

$$\int_{-2}^0 x e^{-x} dx = \{-x e^{-x} - e^{-x}\}_{-2}^0$$

$$\int_{-2}^0 x e^{-x} dx = \{(-1) - (2e^2 - e^2)\}$$

$$\int_{-2}^0 x e^{-x} dx = \{-1 - e^2\}$$

For

$$\int_0^2 x e^x dx$$

Using Integration By parts

$$\int fg = fg - \int fg'$$

$$f' = e^x, g = x$$

$$f = e^x, g' = 1$$

$$\int_0^2 x e^x dx = \{x e^x\}_0^2 - \int_0^2 e^x dx$$

$$\int_0^2 x e^x dx = \{x e^x - e^x\}_0^2$$

$$\int_0^2 x e^x dx = 2e^2 - e^2 + 1$$

$$\int_0^2 x e^x dx = e^2 + 1$$

Hence answer is,

$$\int_{-2}^2 x e^H dx = -1 - e^2 + e^2 + 1 = 0$$

27. We have, $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$

Using the hint given and substituting $x+y = z$

$$\Rightarrow \frac{d(z-x)}{dx} = \cos z + \sin z$$

Differentiating $z-x$ with respect to x

$$\Rightarrow \frac{dz}{dx} - 1 = \cos z + \sin z$$

$$\Rightarrow \frac{dz}{dx} = 1 + \cos z + \sin z$$

$$\Rightarrow \frac{dz}{1 + \cos z + \sin z} = dx$$

Integrating both sides, we have

$$\Rightarrow \int \frac{dz}{1 + \cos z + \sin z} = \int dx$$

We know that $\cos 2z = 2 \cos^2 z - 1$ and $\sin 2z = 2 \sin z \cos z$

$$\Rightarrow \int \frac{dz}{1 + 2 \cos^2 \frac{z}{2} - 1 + 2 \sin \frac{z}{2} \cos \frac{z}{2}} = x$$

$$\Rightarrow \int \frac{dz}{2 \cos^2 \frac{z}{2} + 2 \sin \frac{z}{2} \cos \frac{z}{2}} = x$$

$$\Rightarrow \int \frac{dz}{2 \cos \frac{z}{2} (\cos \frac{z}{2} + \sin \frac{z}{2})} = x$$

$$\Rightarrow \int \frac{dz}{2 \cos^2 \frac{z}{2} \left(1 + \frac{\sin \frac{z}{2}}{\cos \frac{z}{2}}\right)} = x$$

$$\Rightarrow \int \frac{\sec^2 \frac{z}{2} dz}{2(1 + \tan \frac{z}{2})} = x$$

Let $1 + \tan \frac{z}{2} = t$

Differentiating with respect to z , we get

$$\frac{dt}{dz} = \frac{\sec^2 \frac{z}{2}}{2}$$

$$\text{Hence } \frac{\sec^2 \frac{z}{2} dz}{2} = dt$$

$$\Rightarrow \int \frac{dt}{t} = x$$

$$\Rightarrow \log t + c = x$$

Resubstituting t

$$\Rightarrow \log(1 + \tan \frac{z}{2}) + c = x$$

Resubstitute z

$$\Rightarrow \log\left(1 + \tan \frac{x+y}{2}\right) + c = x$$

OR

$$\{x \sin^2\left(\frac{y}{x}\right) - y\} dx + x dy = 0$$

$$\sin^2\left(\frac{y}{x}\right) - \frac{y}{x} + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right) \dots (i)$$

let $y = vx$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

put $\frac{dy}{dx}$ in eq (i)

$$v + x \frac{dv}{dx} = v - \sin^2 v$$

$$\int \cos^2 v dv = \int -\frac{dx}{x}$$

$$-\cot v = -\log x + c$$

$$\log x - \cot v = c$$

$$\log x - \cot\left(\frac{y}{x}\right) = c$$

$$\text{When } x = 1, y = \frac{\pi}{4}$$

$$c = -1$$

$$\log x - \cot\left(\frac{y}{x}\right) = -1$$

$$\log x - \cot\left(\frac{y}{x}\right) = -\log e$$

$$\log ex = \cot\left(\frac{y}{x}\right)$$

28. Let, $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$

Also, given $\vec{b} = 3\hat{i} + \hat{k}$

Also, let

$$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v} \dots (i)$$

Where \vec{u} is parallel to \vec{b} and \vec{v} is perpendicular to \vec{b} .

now \vec{u} is parallel to \vec{b} .

$$\vec{u} = \lambda\vec{b}$$

$$= \lambda(3\hat{i} + \hat{k})$$

$$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k} \dots (ii)$$

put value of u in equation (i)

$$5\hat{i} - 2\hat{j} + 5\hat{k} = (3\lambda\hat{i} + \lambda\hat{k}) + \vec{v}$$

$$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$$

$$\vec{v} = (5 - 3\lambda)\hat{i} - 2\hat{j} + (5 - \lambda)\hat{k}$$

Since, \vec{v} is perpendicular to \vec{b}

Then $\vec{v} \cdot \vec{b} = 0$

$$[(5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}] \cdot (3\hat{i} + 0\hat{j} + \hat{k}) = 0$$

$$(5 - 3\lambda)(3) + (-2)(0) + (5 - \lambda)(1) = 0$$

$$15 - 9\lambda + 0 + 5 - \lambda = 0$$

$$20 - 10\lambda = 0$$

$$\Rightarrow -10\lambda = -20$$

$$\Rightarrow \lambda = 2$$

putting value of λ in equation (ii)

$$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$$

$$= 3(2)\hat{i} + (2)\hat{k}$$

$$\vec{u} = 6\hat{i} + 2\hat{k}$$

put the value of u in equation (i)

$$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$$

$$5\hat{i} - 2\hat{j} + 5\hat{k} = (6\hat{i} + 2\hat{k}) + \vec{v}$$

$$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$$

$$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{a} = (6\hat{i} + 2\hat{k}) + (-\hat{i} - 2\hat{j} + 3\hat{k})$$

OR

Given vectors are, $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$, $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

The projection of \vec{a} along \vec{b}

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\vec{a} \cdot \vec{b} = (\lambda\hat{i} + \hat{j} + 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})$$

$$= 2\lambda + 6 + 12$$

$$= 2\lambda + 18$$

$$|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2}$$

$$= \sqrt{49} = 7$$

Given, the projection of vector a along vector b is 4.

$$\therefore \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 4$$

$$\Rightarrow \frac{2\lambda + 18}{7} = 4$$

$$\Rightarrow 2\lambda + 18 = 28$$

$$\Rightarrow 2\lambda = 10$$

$$\Rightarrow \lambda = 5$$

29. Let the given integral be,

$$I = \int \frac{x^5 dx}{\sqrt{1+x^3}}$$

$$= \int \frac{x^3 x^2 dx}{\sqrt{1+x^3}}$$

Put $x^3 = t$

$$\Rightarrow 3x^2 dx = dt$$

$$\Rightarrow x^2 dx = \frac{dt}{3}$$

$$\therefore I = \frac{1}{3} \int \frac{t dt}{\sqrt{1+t}}$$

$$= \frac{1}{3} \int \left(\frac{1+t-1}{\sqrt{1+t}} \right) dt$$

$$= \frac{1}{3} \int \left(\sqrt{1+t} - \frac{1}{\sqrt{1+t}} \right) dt$$

$$= \frac{1}{3} \left[\frac{(1+t)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(1+t)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right] + C$$

$$= \frac{2}{9} (1+t)^{\frac{3}{2}} - \frac{2}{3} (1+t)^{\frac{1}{2}} + C$$

$$= \frac{2}{9} (1+x^3)^{\frac{3}{2}} - \frac{2}{3} (1+x^3)^{\frac{1}{2}} + C$$

$$= \frac{2}{9} (1+x^3)^{\frac{1}{2}} (1+x^3-3) + C$$

$$= \frac{2}{9} \sqrt{1+x^3} (x^3-2) + C$$

OR

$$\text{We can write it as } \int \frac{x}{1+\sin x} dx = \int \frac{x(1-\sin x)}{(1+\sin x)(1-\sin x)} dx$$

$$= \int \frac{x(1-\sin x)}{1-\sin^2 x} dx$$

$$= \int \frac{x(1-\sin x)}{\cos^2 x} dx$$

$$= \int x \sec^2 x dx - \int x \tan x \sec x dx$$

Using by parts we have,

$$\int x \sec^2 x dx - \int x \sec x \tan x dx = \left(x \int \sec^2 x dx - \int \left(\frac{dx}{dx} \cdot \int \sec^2 x dx \right) dx \right)$$

$$- \left(x \int \sec x \tan x dx - \int \left(\frac{dx}{dx} \cdot \int \sec x \tan x dx \right) dx \right)$$

$$= (x \tan x - \int 1 \cdot \tan x dx) - (x \cdot \sec x - \int 1 \cdot \sec x dx)$$

$$= x \tan x - \ln|\sec x| - x \sec x + \ln|\sec x + \tan x| + c$$

$$= x(\tan x - \sec x) + \ln \left| \frac{\sec x + \tan x}{\sec x} \right| + c$$

$$= x(\tan x - \sec x) + \ln|1+\sin x| + c$$

30. Given function is: $f(x) = |x| + |x-1|$

We have,

$$(\text{LHL at } x=0) = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h)$$

$$= \lim_{h \rightarrow 0} [(0-h) + |0-h-1|] = 1$$

$$(\text{RHL at } x=0) = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h)$$

$$= \lim_{h \rightarrow 0} [(0+h) + |0+h-1|] = 1$$

Also,

$$f(0) = |0| + |0-1| = 0+1=1$$

Now,

$$(\text{LHL at } x=1) = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h)$$

$$= \lim_{h \rightarrow 0} (|1-h| + |1-h-1|) = 1+0=1$$

$$(\text{RHL at } x=1) = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h)$$

$$= \lim_{h \rightarrow 0} (|1+h| + |1+h-1|) = 1+0=1$$

Also,

$$f(1) = |1| + |1-1| = 1+0=1$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0) \text{ and}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

Hence, $f(x)$ is continuous at $x=0, 1$

31. Given curves are $y = \sqrt{x}$(1)

and $2y-x+3=0$(2)

on solving (1) and (2), we get

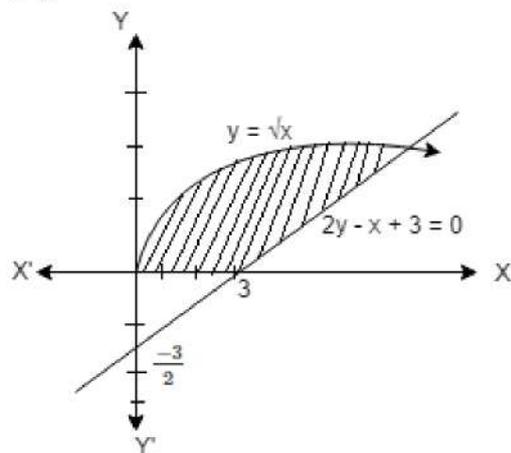
$$2\sqrt{x} - (\sqrt{x})^2 + 3 = 0$$

$$(\sqrt{x})^2 - 2\sqrt{x} - 3 = 0$$

$$(\sqrt{x} - 3)(\sqrt{x} + 1) = 0$$

$$\sqrt{x} = 3 \text{ [as } \sqrt{x} = -1 \text{ is not possible]}$$

$$\therefore y = 3$$



Hence, required area

$$= \int_0^3 (x_2 - x_1) dy$$

$$= \int_0^3 \{ (2y + 3) - y^2 \} dy$$

$$= \left[y^2 + 3y - \frac{y^3}{3} \right]_0^3$$

$$= 9 + 9 - 9 = 9 \text{ sq. units}$$

Section D

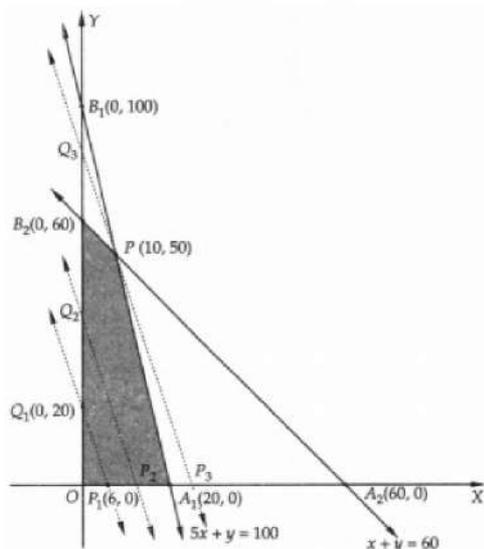
32. We first convert the inequations into equations to obtain the lines $5x + y = 100$, $x + y = 60$, $x = 0$ and $y = 0$.

The line $5x + y = 100$ meets the coordinate axes at $A_1(20, 0)$ and $B_1(0, 100)$. Join these points to obtain the line $5x + y = 100$.

The line $x + y = 60$ meets the coordinate axes at $A_2(60, 0)$ and $B_2(0, 60)$. Join these points to obtain the line $x + y = 60$.

Also, $x = 0$ is the y-axis and $y = 0$ is the x-axis.

The feasible region of the LPP is shaded in a figure. The coordinates of the corner-points of the feasible region OA_1PB_2 are $O(0, 0)$, $A_1(20, 0)$, $P(10, 50)$ and $B_2(0, 60)$.



Now, we take a constant value, say 300 (i.e. 2 times the l.c.m. of 50 and 15) for Z. Then,

$$300 = 50x + 15y$$

This line meets the coordinate axes at $P_1(6, 0)$ and $Q_1(0, 20)$. Join these points by a dotted line. Now, move this line parallel to itself in the increasing direction i.e. away from the origin. P_2Q_2 and P_3Q_3 are such lines. Out of these lines locate a line that is farthest from the origin and has at least one point common to the feasible region.

Clearly, P_3Q_3 is such line and it passes through the vertex P (10, 50) the convex polygon OA_1PB_2 . Hence, $x = 10$ and $y = 50$ will give the maximum value of Z.

The maximum value of Z is given by

$$Z = 50 \times 10 + 15 \times 50 = 1250.$$

33. $R = \{(a,b) = |a.b| \text{ is divisible by } 2\}$

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivity

For any $a \in A, |a-a|=0$ Which is divisible by 2.

$\therefore (a, a) \in r$ for all $a \in A$

So, R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all $a, b \in R$

$|a-b|$ is divisible by 2

$|b-a|$ is divisible by 2

$(a, b) \in r \Rightarrow (b, a) \in R$

So, R is symmetric .

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$

$|a-b|$ is divisible by 2

$|b-c|$ is divisible by 2

Two cases :

Case 1:

When b is even

$(a, b) \in R$ and $(b, c) \in R$

$|a-c|$ is divisible by 2

$|b-c|$ is divisible by 2

$|a-c|$ is divisible by 2

$\therefore (a, c) \in R$

Case 2:

When b is odd

$(a, b) \in R$ and $(b, c) \in R$

$|a-c|$ is divisible by 2

$|b-c|$ is divisible by 2

$|a-c|$ is divisible by 2

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

So R is transitive.

Hence, R is an equivalence relation

OR

We observe the following properties of relation R.

Reflexivity: For any $a \in \mathbb{N}$

$$a - a = 0 = 0 \times n$$

$\Rightarrow a - a$ is divisible by n

$\Rightarrow (a, a) \in R$

Thus, $(a, a) \in R$ for all $a \in \mathbb{Z}$. So, R is reflexive on Z

Symmetry: Let $(a, b) \in R$. Then,

$(a, b) \in R$

$\Rightarrow (a - b)$ is divisible by n

$\Rightarrow (a - b) = np$ for some $p \in \mathbb{Z}$

$\Rightarrow b - a = n(-p)$

$\Rightarrow b - a$ is divisible by n [$\because p \in \mathbb{Z} \Rightarrow -p \in \mathbb{Z}$]

$\Rightarrow (b, a) \in R$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in \mathbb{Z}$.

So, R is symmetric on Z.

Transitivity: Let a, b, c ∈ Z such that (a, b) ∈ R and (b, c) ∈ R. Then,

(a, b) ∈ R

⇒ (a - b) is divisible by n

⇒ a - b = np for some p ∈ Z

and, (b, c) ∈ R

⇒ (b - c) is divisible by n

⇒ b - c = nq for some q ∈ Z

∴ (a, b) ∈ R and (b, c) ∈ R

⇒ a - b = np and b - c = nq

⇒ (a - b) + (b - c) = np + nq

⇒ a - c = n(p + q)

⇒ a - c is divisible by n [∵ p, q ∈ Z ⇒ p + q ∈ Z]

⇒ (a, c) ∈ R

Thus, (a, b) ∈ R and (b, c) ∈ R ⇒ (a, c) ∈ R for all a, b, c ∈ Z.

34. Here, it is given that

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

Here,

$$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a}_2 = 4\hat{i} + \hat{j}$$

$$\vec{b}_2 = 5\hat{i} + 2\hat{j} + \hat{k}$$

Thus,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(3 - 8) - \hat{j}(2 - 20) + \hat{k}(4 - 15)$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = -5\hat{i} + 18\hat{j} - 11\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$$

$$= \sqrt{25 + 324 + 121}$$

$$= \sqrt{470}$$

$$\vec{a}_2 - \vec{a}_1 = (4 - 1)\hat{i} + (1 - 2)\hat{j} + (0 - 3)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} - \hat{j} - 3\hat{k}$$

Now, we have

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k})$$

$$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$$

$$= -15 - 18 + 33$$

$$= 0$$

Thus, the distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{0}{\sqrt{470}} \right|$$

$$\therefore d = 0 \text{ units}$$

As d = 0

Thus, the given lines intersect each other.

Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.

For that putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$$\Rightarrow \vec{L}_1 : x\hat{i} + y\hat{j} + z\hat{k} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\Rightarrow \vec{L}_2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

$$\Rightarrow \vec{L}_1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$$

$$\Rightarrow \vec{L}_2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$$

$$\Rightarrow \vec{L}_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\therefore \vec{L}_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$$

General point on L1 is

$$x_1 = 2\lambda + 1, y_1 = 3\lambda + 2, z_1 = 4\lambda + 3$$

Suppose, P(x₁, y₁, z₁) be point of intersection of two given lines.

Thus, point P satisfies the equation of line \vec{L}_2 .

$$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3-0}{1}$$

$$\therefore \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2}$$

$$\Rightarrow 4\lambda - 6 = 15\lambda + 5$$

$$\Rightarrow 11\lambda = -11$$

$$\Rightarrow \lambda = -1$$

Thus, x₁ = 2(-1) + 1, y₁ = 3(-1) + 2, z₁ = 4(-1) + 3

$$\Rightarrow x_1 = -1, y_1 = -1, z_1 = -1$$

Therefore, point of intersection of given lines is (-1, -1, -1).

OR

Line passing through (1, 2, 3)

ie $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and parallel to the given planes is perpendicular to the vectors

$$\vec{b}_1 = \hat{i} - \hat{j} + 2\hat{k} \text{ and}$$

$$\vec{b}_2 = 3\hat{i} + \hat{j} + \hat{k}$$

Required line is parallel to $\vec{b}_1 \times \vec{b}_2$

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = \hat{i}(-1-2) - \hat{j}(1-6) + \hat{k}(1+3) = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Required equation of line is :-

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$$

35. It is given that $f(x) = |x| - |x+1|$

The given function f is defined for real number and f can be written as the composition of two functions, as

$$f = g \circ h, \text{ where } g(x) = |x| \text{ and } h(x) = |x+1|$$

Then, $f = g \circ h$

First we have to prove that $g(x) = |x|$ and $h(x) = |x+1|$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Now, g is defined for all real number.

Let k be a real number.

Case I: If $k < 0$,

Then $g(k) = -k$

$$\text{And } \lim_{x \rightarrow k} g(x) = \lim_{x \rightarrow k} (-x) = -k$$

$$\text{Thus, } \lim_{x \rightarrow k} g(x) = g(k)$$

Therefore, g is continuous at all points x, i.e., $x > 0$

Case II: If $k > 0$,

Then $g(k) = k$ and

$$\lim_{x \rightarrow k} g(x) = \lim_{x \rightarrow k} x = k$$

$$\text{Thus } \lim_{x \rightarrow k} g(x) = g(k)$$

Therefore, g is continuous at all points x, i.e., $x < 0$.

Case III: If $k = 0$,

Then, $g(k) = g(0) = 0$

$$\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} (-x) = 0$$

$$\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} (x) = 0$$

$$\therefore \lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^+} g(x) = g(0)$$

Therefore, g is continuous at $x = 0$

From the above 3 cases, we get that g is continuous at all points.

$g(x) = |x + 1|$ can be written as

$$g(x) = \begin{cases} -(x + 1), & \text{if } x < -1 \\ x + 1, & \text{if } x \geq -1 \end{cases}$$

Now, h is defined for all real number.

Let k be a real number.

Case I: If $k < -1$,

Then $h(k) = -(k + 1)$

$$\text{And } \lim_{x \rightarrow k} h(x) = \lim_{x \rightarrow k} [-(x + 1)] = -(k + 1)$$

$$\text{Thus, } \lim_{x \rightarrow k} h(x) = h(k)$$

Therefore, h is continuous at all points x, i.e., $x < -1$

Case II: If $k > -1$,

Then $h(k) = k + 1$ and

$$\lim_{x \rightarrow k} h(x) = \lim_{x \rightarrow k} (x + 1) = k + 1$$

$$\text{Thus, } \lim_{x \rightarrow k} h(x) = h(k)$$

Therefore, h is continuous at all points x, i.e., $x > -1$.

Case III: If $k = -1$,

Then, $h(k) = h(-1) = -1 + 1 = 0$

$$\lim_{x \rightarrow -1^-} h(x) = \lim_{x \rightarrow -1^-} [-(x + 1)] = -(-1 + 1) = 0$$

$$\lim_{x \rightarrow -1^+} h(x) = \lim_{x \rightarrow -1^+} (x + 1) = -(-1 + 1) = 0$$

$$\therefore \lim_{x \rightarrow -1^-} h(x) = \lim_{x \rightarrow -1^+} h(x) = h(-1)$$

Therefore, g is continuous at $x = -1$

From the above 3 cases, we get that h is continuous at all points.

Hence, g and h are continuous function.

Therefore, $f = g - h$ is also a continuous function.

Section E

36. Read the text carefully and answer the questions:

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfectant. The cost of material used to manufacture the tin can is ₹100/m².



(i) Given, r cm is the radius and h cm is the height of required cylindrical can.

Given that, volume of cylinder = 3l = 3000 cm³ (\because 1l = 1000 cm³)

$$\Rightarrow \pi r^2 h = 3000 \Rightarrow h = \frac{3000}{\pi r^2}$$

Now, the surface area, as a function of r is given by

$$S(r) = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \left(\frac{3000}{\pi r^2} \right)$$

$$= 2\pi r^2 + \frac{6000}{r}$$

$$(ii) \text{ Now, } S(r) = 2\pi r^2 + \frac{6000}{r}$$

$$\Rightarrow S'(r) = 4\pi r - \frac{6000}{r^2}$$

To find critical points, put $S'(r) = 0$

$$\Rightarrow \frac{4\pi r^3 - 6000}{r^2} = 0$$

$$\Rightarrow r^3 = \frac{6000}{4\pi} \Rightarrow r = \left(\frac{1500}{\pi}\right)^{1/3}$$

$$\text{Also, } S''(r)|_r = \sqrt[3]{\frac{1500}{\pi}} = 4\pi + \frac{12000 \times \pi}{1500}$$

$$= 4\pi + 8\pi = 12\pi > 0$$

Thus, the critical point is the point of minima.

(iii) The cost of material for the tin can is minimized when $r = \sqrt[3]{\frac{1500}{\pi}}$ cm and the height is $\frac{3000}{\pi \left(\sqrt[3]{\frac{1500}{\pi}}\right)^2} = 2\sqrt[3]{\frac{1500}{\pi}}$ cm.

OR

$$\text{We have, minimum surface area} = \frac{2\pi r^3 + 6000}{r}$$

$$= \frac{2\pi \cdot \frac{1500}{\pi} + 6000}{\sqrt[3]{\frac{1500}{\pi}}} = \frac{9000}{7.8} = 1153.84 \text{ cm}^2$$

Cost of 1 m² material = ₹100

∴ Cost of 1 cm² material = ₹ $\frac{1}{100}$

∴ Minimum cost = ₹ $\frac{1153.84}{100} = ₹11.538$

37. Read the text carefully and answer the questions:

To promote the making of toilets for women, an organization tried to generate awareness through

- i. house calls
- ii. emails and
- iii. announcements.

The cost for each mode per attempt is given below:



1. ₹ 50
2. ₹ 20
3. ₹ 40

The number of attempts made in the villages X, Y and Z are given below:

| | (i) | (ii) | (iii) |
|---|-----|------|-------|
| X | 400 | 300 | 100 |
| Y | 300 | 250 | 75 |
| Z | 500 | 400 | 150 |

Also, the chance of making of toilets corresponding to one attempt of given modes is

1. 2%
2. 4%
3. 20%

(i) Total number of toilets that can be expected in each village is given by the following matrix.

$$\begin{matrix} X \\ Y \\ Z \end{matrix} \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$$

\therefore Required probability = $P\left(\frac{\bar{E}_1}{A}\right)$

$$= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$$

(ii) Let A be the event of committing an error and E_1 , E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$$

$$P\left(\frac{A}{E_1}\right) = 0.06, P\left(\frac{A}{E_2}\right) = 0.04, P\left(\frac{A}{E_3}\right) = 0.03$$

$$P(A \cap E_2) = P\left(\frac{A}{E_2}\right) \cdot P(E_2)$$

$$\Rightarrow 0.04 \times 0.2 = 0.008$$