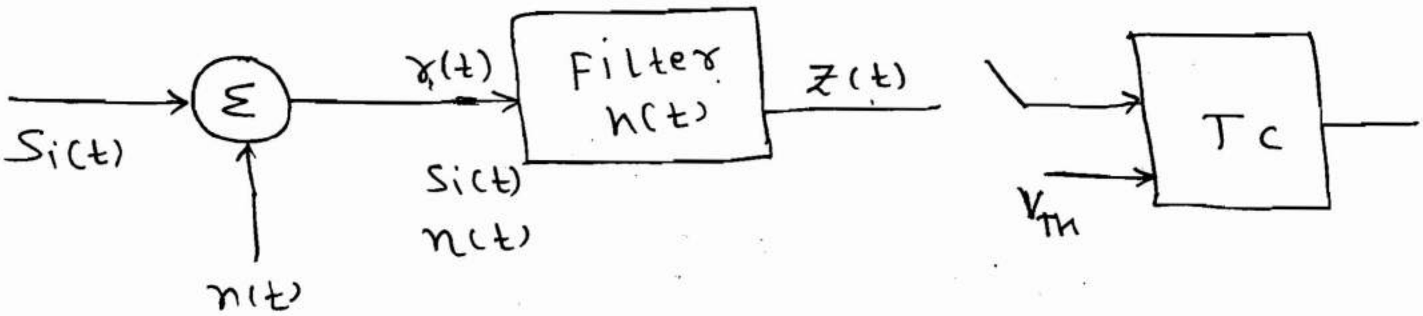
☆ Probability of Error Calculation:

(IMP)

⇒



⇒ $r(t) = S_i(t) + n(t).$

$z(t) = a_i(t) + n_o(t).$

→ assume that the signal is sampled at $t = T_b$.

$\therefore \; Z(T_b) = a_i(T_b) + n_o(T_b).$

→ The input to the Threshold Comparator

$z = a_i + n_o.$

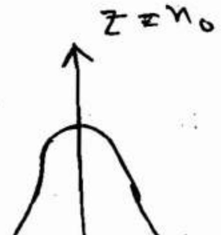
Case - I:

⇒ Assume that the signal is not present at the input to the receiver.

$z = n_o$

⇒ n_o is assumed as a gaussian random variable with zero mean.

$E[z] = E[n_o] = 0.$



Case- II:

$\Rightarrow$ Assume that the binary symbol 1 is present at the input of the receiver.

$\Rightarrow$ $$Z = a_1 + n_0.$$

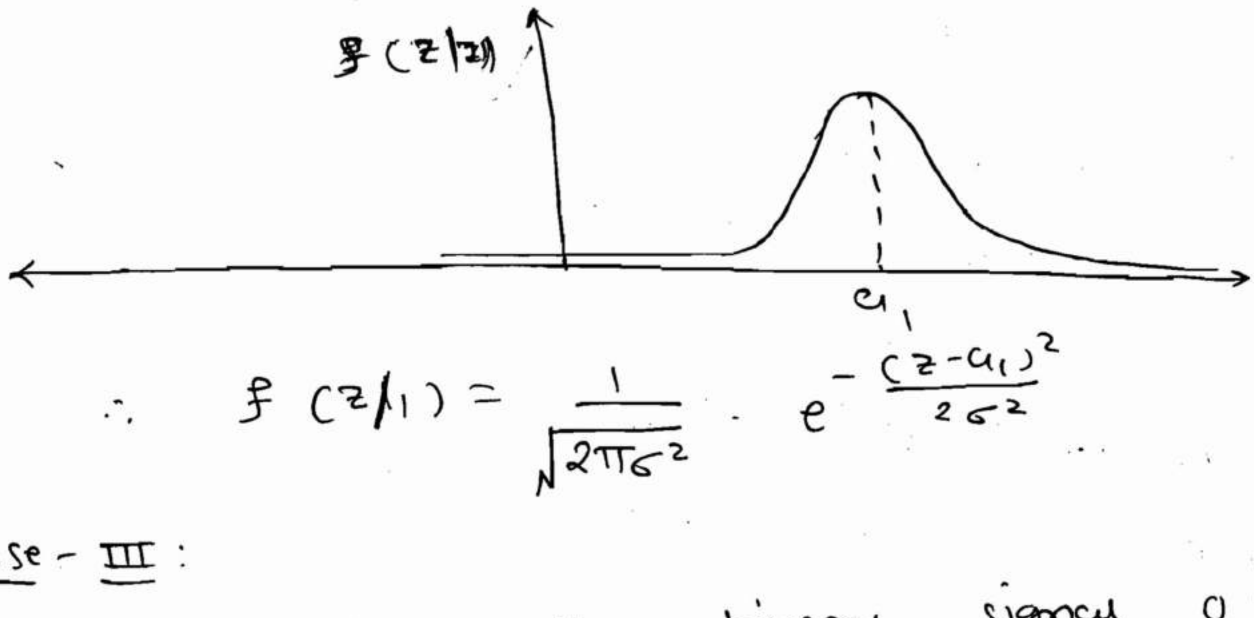
$$E[z] = E[a_1 + n_0]$$

$$= E[a_1] + E[n_0]$$

$$E[z] = a_1 + 0$$

$$\therefore \boxed{E[z] = a_1}$$

$\Rightarrow$ In this case also z is Gaussian random variable with mean $= a_1$.



$$\therefore \quad f(z/1) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(z-a_1)^2}{2\sigma^2}}$$

Case - III :

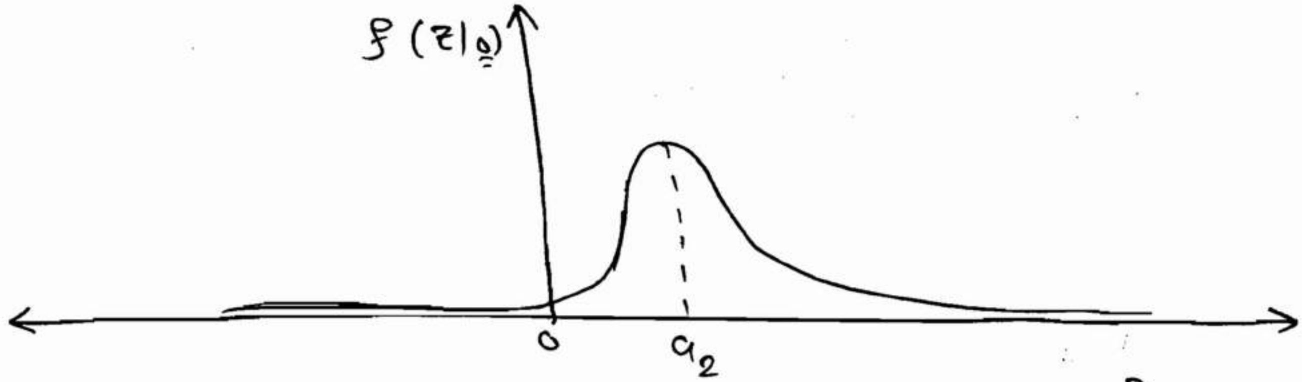
$\Rightarrow$ Assume that the binary signal 0 is present at the input to the receiver.

$\Rightarrow$ $$Z = a_2 + n_0.$$

$$\therefore E[Z] = E[a_2] + E[n_0]$$

$$\boxed{E[z] = a_2}$$

$\Rightarrow$



$\Rightarrow \quad f(z|0) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(z-a_2)^2}{2\sigma^2}}$.

$\Rightarrow$ The i/p to the Threshold comparator is the avg. value of a_1+n_0 & a_2+n_0.

So, $z = \frac{a_1+a_2}{2} = V_{th}$.

$\Rightarrow$ Assume that the binary symbol 1 is transmitted through the channel.

$\Rightarrow$ The i/p to the threshold comparator $z = a_1+n_0$. The o/p of the comparator will be '0' (or) error occurs if the following condition is satisfied:

$\Rightarrow \quad 1 \longrightarrow$ error $[z < V_{th}]$.

$$\boxed{P_{e_1} = P[z < V_{th}].} \leftarrow \text{H.B.}$$

similarly.

$0 \longrightarrow$ error occurs $[z > V_{th}]$.

$$\Rightarrow \boxed{Pe_1 = \int_{-\infty}^{V_{TH}} f(z|1)\, dz} \quad -① \leftarrow \underline{H.B.}$$

$$\boxed{Pe_0 = \int_{V_{TH}}^{\infty} f(z|0)\, dz} \quad -② \leftarrow \underline{H.B.}$$

⇒ Practically the effect of noise on the binary symbol '0' and '1' will be same. So, the prob. of error will also be same. i.e. $Pe_1 = Pe_0$.

$$\Rightarrow \quad P_e = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\frac{a_1+a_2}{2}}^{\infty} \cdot e^{\frac{(z-a_2)^2}{2\sigma^2}} \, dz.$$

⇒ The above integral can be represent in a function.

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-y^2/2} \, dy.$$

~~let,~~ Property: $Q(x) \downarrow$ when $x \uparrow$

let, $y = \frac{z-a_2}{\sigma}$ ⇒ $\frac{dy}{dz} = \frac{1}{\sigma}$ ⇒ $\boxed{dz = \sigma\, dy}$

$$\therefore \quad P_e = \frac{\sigma}{\sqrt{2\pi\sigma^2}} \int_{\frac{a_1 - a_2}{2\sigma}}^{\infty} e^{-y^2/2} \, dy$$

$$\therefore \quad \boxed{P_e = Q\left[\frac{a_1 - a_2}{2\sigma}\right].}$$

$$\therefore \quad P_e = Q\left[\sqrt{\frac{(a_1 - a_2)^2}{4\sigma^2}}\right].$$

mean square value = power = σ^2.
= Noise power.

$$\therefore \quad P_e = Q\left[\sqrt{\frac{\text{Difference signal power}}{4 \times \text{Noise power}}}\right].$$

$$\therefore \quad P_e = Q\left[\sqrt{\frac{2E_d}{4N_0}}\right].$$

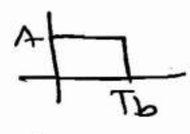
$$\therefore \quad \boxed{P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right].}$$

H.B.
MIMP.

$$\therefore \quad \boxed{E_d = \int_0^{T_b} [S_1(t) - S_2(t)]^2 \, dt}$$

* Probability of Error in PCM System:

$\Rightarrow$ ON- OFF

$s_1(t) = A$ 

NRZ

$S_1(t) = A$

$S_2(t) = -A$.

$\therefore S_1(t) - S_2(t) = A$

$$\therefore E_D = \int_0^{T_b} A^2 \cdot dt$$

$$\therefore \boxed{E_D = A^2 \cdot T_b}$$

$$\therefore \boxed{P_e = Q\left[\underbrace{\sqrt{\frac{A^2 T_b}{2N_0}}}_{x_1}\right]} \leftarrow \text{H.B.}$$

$S_1(t) - S_2(t)$
$= A - (-A) = 2A$

$$\therefore E_D = \int_0^{T_b} 4A^2 \cdot dt$$

$$\therefore \boxed{E_D = 4A^2 T_b}$$

$$P_e = Q\left[\sqrt{\frac{4A^2 T_b}{2N_0}}\right]$$

$$\boxed{P_e = Q\left[\underbrace{\sqrt{\frac{2A^2 T_b}{N_0}}}_{x_2}\right]}$$

H.B.

$x_2 > x_1$

$\therefore P_e(x_2) < P(x_1)$.

$\therefore \boxed{P_e(\text{NRZ}) < P_e[\text{ON-OFF}]}$.

Q-1 A received NRZ signal assume the voltage level 500 mV and -500mV respectively for '1' and '0'. The received signal is affected by white noise having a two sided spectrum density of 10^{-10} W/Hz. Determine the bit rate so that $P_e = 10^{-5}$.

$Q(x) = 10^{-5}$, $x = 4.27$.

Sol^n: $A = 500$ mV

$N_0 = 10^{-10}$ ... $N_0 = 2\times10^{-10}$ W/Hz

$$\therefore \quad P_e = Q\left[\sqrt{\frac{2A^2T_b}{N_o}}\right] = 10^{-5}.$$

$$\therefore \quad \sqrt{\frac{2A^2T_b}{N_o}} = (4.27)$$

$$\therefore \quad \frac{2A^2T_b}{N_o} = (4.27)^2$$

$$\therefore \quad T_b = \frac{(4.27)^2 \times \cancel{2} \times 10^{-10}}{0.25 \times \cancel{2}}$$

$$\therefore \quad T_b = \underset{72.93}{\cancel{145.8632}} \times 10^{-10}.$$

$$\therefore \quad R_b = \frac{1}{T_b}$$

$$\therefore \quad R_b = \overset{13.71}{\cancel{6.856}} \times 10^{7}$$

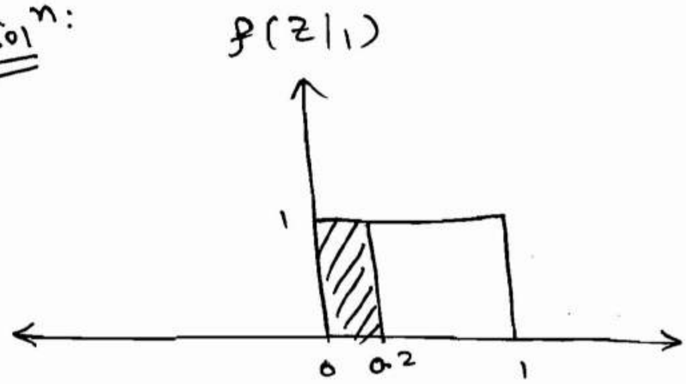
$$\therefore \quad \boxed{R_b = \underset{137}{\cancel{68}} \text{ Mbps.}}$$

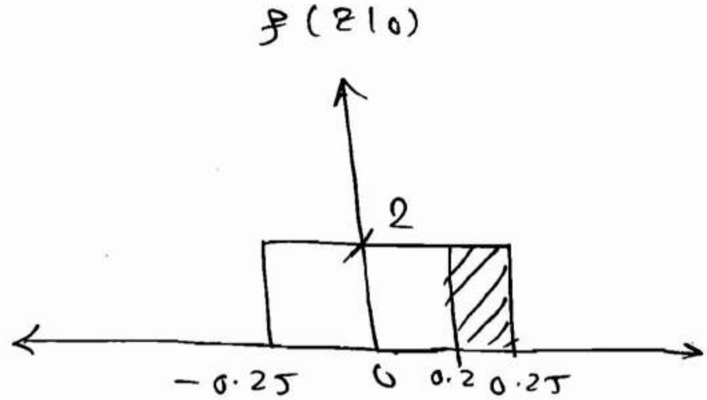
=> Probability of error should be minimum only either A (or) T_b should be as maximum (or) very high. So, the bit rate should be as minimum as possible.

$\boxed{Q-2}$ Consider a digital Communication System when the binary symbol 1 is transmitted the input to the Threshold Comparator can be any value betn 0 to 1 volt. with equal prob.. when binary symbol 0 is transmitted the input to the Threshold

$-0.25V$ to $+0.25V$ with equal probability. The Threshold value used at the Comparator is $0.2V$. Determine the prob. of 0 & 1. Also Calculate the avg. prob. of errors.

Soln:





$\therefore$ 1 $\rightarrow$ error occurs $[z < 0.2]$.

$$P_{e_1} = P[z < 0.2] = \int_{-\infty}^{0.2} f(z|1)\, dz$$

$$\therefore P_{e_1} = \int_{0}^{0.2} 1 \cdot dz$$

$$\boxed{P_{e_1} = 0.2}$$

$\therefore$ 0 $\longrightarrow$ error occurs $[z > 0.2]$.

$$P_{e_0} = P[z > 0.2] = \int_{0.2}^{\infty} f(z|0)\, dz$$

$$= \int_{0.2}^{0.25} 2 \cdot dz$$

$$= 2\,[0.05].$$

$$\therefore P_{avg} = \frac{P_{e_0} + P_{e_1}}{2}$$

$$\therefore P_{avg} = \frac{0.1 + 0.2}{2}$$

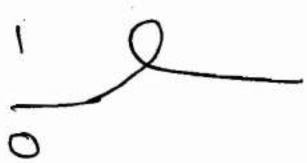
⇒ Prob. of error (Bit error rate) is depends on the threshold value.

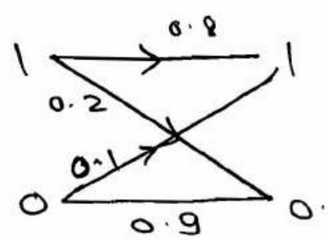
⇒ The threshold value should be selected so that the prob. of error is minimum.

⇒ The optimum Threshold value will be equal to the intersection of the two pdf of $\underline{f(z|1)}$ and $\underline{f(z|0)}$. ↑ H.B.

* To determine the avg. Probability of error the following procedure is used:

⇒





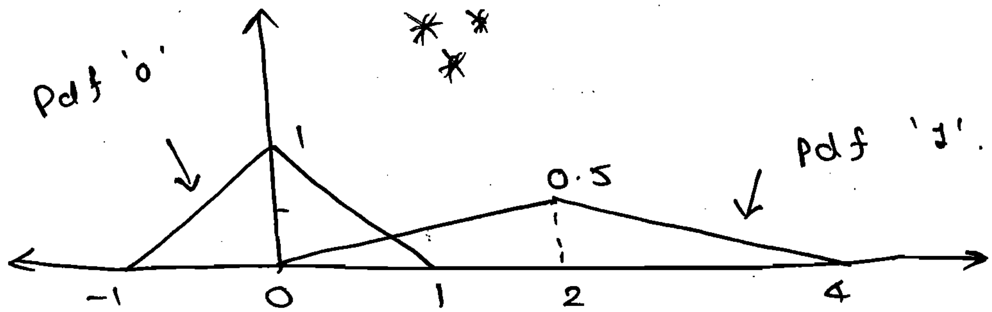
$$\Rightarrow \boxed{P_e = P(1) \cdot P_{e1} + P(0) \cdot P_{e0}.}$$

$\therefore$ if $P(1) = P(0)$.

$$\therefore P_e = \frac{1}{2}(0.2) + \frac{1}{2}(0.1)$$

$$\boxed{P_e = 0.15}$$

$\boxed{Q}$ Bits 1 and 0 are transmitted with equal Probability at the receiver the pdf of the respective received signals are as shown in the figure.



Q-1 If the decision threshold is 1 the bit error rate will be

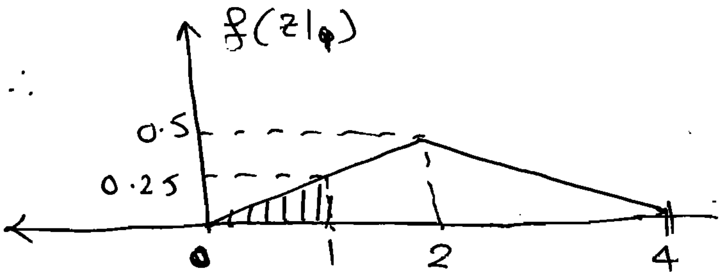
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$.

Sol^n:

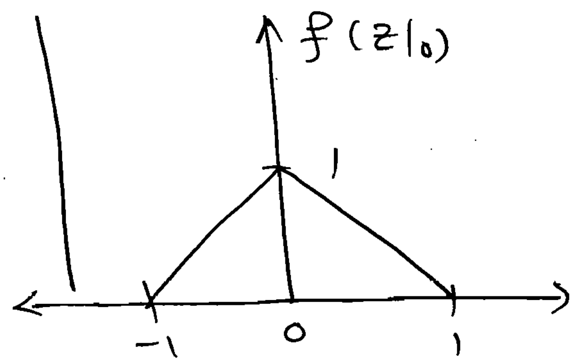
$$P_e = P(1) \cdot P_{e1} + P(0) \cdot P_{e0}.$$

$$P(1) = P(0) = \frac{1}{2}.$$

$\therefore$



$\therefore$



$$P_{e1} = \int_{-\infty}^{1} f(z|1) \cdot dz$$

$$= \frac{1}{2} \times 1 \times 0.25$$

$$\therefore \boxed{P_{e1} = \frac{1}{8}}$$

$$P_{e0} = \int_{1}^{\infty} f(z|0)\, dz$$

$$\boxed{P_{e0} = 0}$$

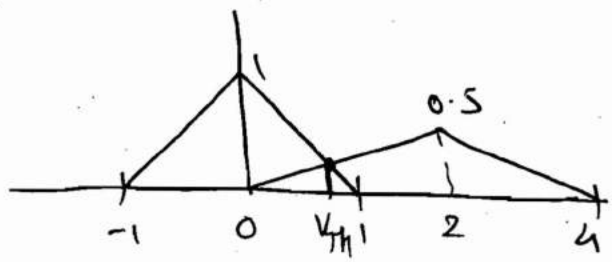
$$\therefore P_e = \left(\frac{1}{2} \times \frac{1}{8}\right) + \left(\frac{1}{2} \times 0\right)$$

$$\boxed{P_e = \frac{1}{16}.}$$

Q-2 The optimum threshold to achive the minimum bit error is ?

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$ (C) 1 (D) $3/2$.

Sol^n:



① $x+y=1$.
$y = \frac{x}{4}$.

$\Rightarrow$ $x + \frac{x}{4} = 1$
$\therefore x = \frac{4}{5}$.

$\therefore \boxed{V_{Th} = \frac{4}{5} V}$

E_b for ASK = E_b for message signal

*

ASK	PSK	FSK
$\Rightarrow S_1(t) = A_c \cos 2\pi f_c t$	$S_1(t) = A_c \cos 2\pi f_c t$	$S_1(t) = A_c \cos 2\pi f_1 t$
$S_2(t) = 0$	$S_2(t) = -A_c \cos 2\pi f_c t$	$S_2(t) = A_c \cos 2\pi f_2 t$
$E_d = \int_0^{T_b} (A_c \cos 2\pi f_c t)^2 dt$	$E_d = \int_0^{T_b} (2A_c \cos 2\pi f_c t)^2 dt$	$\boxed{E_d = A_c^2 T_b.}$
$\therefore \boxed{E_b = \frac{A_c^2 T_b}{2}}$	$\boxed{E_d = 2A_c^2 T_b.}$	
$\therefore \boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]}$	$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{N_0}}\right]}$	$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]}$
$\boxed{BW = 2R_b}$.	$\boxed{BW = 2R_b}$	$\boxed{BW = f_1 - f_2 + 2R_b}$

$x_2 > x_3 > x_4$

F.R.

So, $P(x_2) < P(x_3) < P(x_4)$.

⇒ The Probability of error for ASK is high when compared with FSK and PSK. The BW of the FSK signal is high when compared with ASK and PSK. So, the optimum technique in digital communication is PSK.

H.B

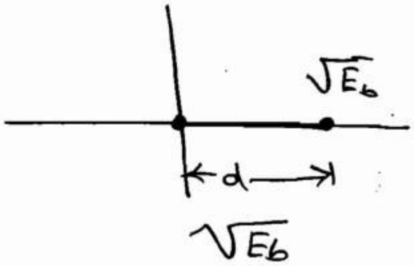
⇒ $E_b = \frac{A_c^2 T_b}{2}$.

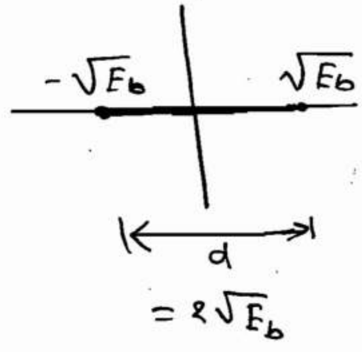
ASK	PSK	FSK
$\therefore P_e = Q\left[\sqrt{\frac{E_b}{2N_0}}\right]$	$P_e = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$	$P_e = Q\left[\sqrt{\frac{E_b}{N_0}}\right]$
$P_e = Q\left[\sqrt{\frac{E_b \cos^2\phi}{2N_0}}\right]$	$P_e = Q\left[\sqrt{\frac{2E_b \cos^2\phi}{N_0}}\right]$	$P_e = Q\left[\sqrt{\frac{E_b \cos^2\phi}{N_0}}\right]$

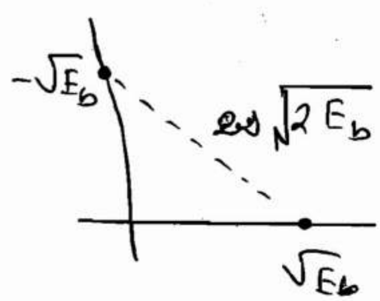
⇒ Due to phase shift in the local oscillator the prob. of error increases.

* ASK PSK FSK.

⇒



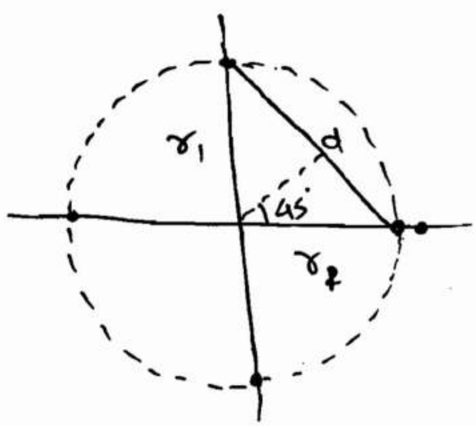




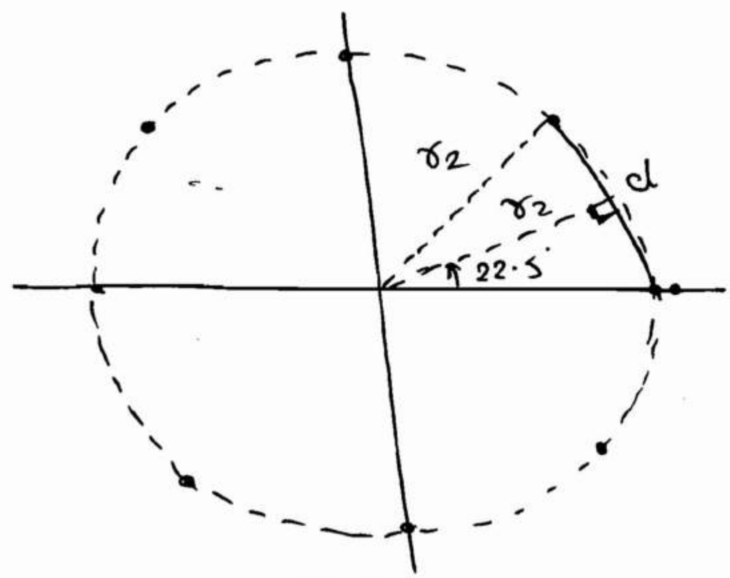
⇒ Based on the Constellation diagram the Prob. of error is.

$$\therefore \boxed{P_e = Q\left[\sqrt{\frac{d_{min}}{2N_0}}\right]} \leftarrow \text{H.B.}$$

$\boxed{Q}$ Consider the Constellation diagram of QPSK and 8-PSK as shown in figure.



QPSK



8-PSK

$\therefore$ ~~sin 22.5~~

$$\sin 45° = \frac{d/2}{r_1}$$

$$\therefore r_1 = \frac{d}{2\sin 45°}$$

$$\therefore E_1 = r_1^2 = \frac{d^2}{4\times\frac{1}{2}} = \frac{d^2}{2}$$

$$\therefore \sin 22.5° = \frac{d/2}{r_2}$$

$$\therefore r_2 = \frac{d}{2\sin 22.5°}$$

$$\therefore E_2 = r_2^2 = \frac{d^2}{4\times 0.3827}$$

$$\therefore \frac{E_2}{E_1} = \cancel{\frac{r_2}{\ldots}}\left(\frac{r_2}{r_1}\right)^2$$

$$\therefore \frac{E_2}{E_1} = \left(\frac{\cancel{0.707}\ 1.307}{\ldots}\right)^2$$

$$\therefore 10 \log_{10} \frac{E_2}{E_1} = 10 \log \left(\frac{1.307}{0.707} \right)^2 \; dB$$

$$= 10 \log (1.8486)^2 \; dB$$

$$(E_2)_{dB} - (E_1)_{dB} = 5.33.$$

$$\therefore \boxed{(E_2)_{dB} = 5.33 + (E_1)_{dB}.}$$