Mathematics - 2017

सामान्य निर्देश : General Instructions :

इस प्रश्न-पत्र में 29 प्रश्न हैं, जो तीन खण्डों-अ, ब और स में बंटे हुए हैं। खण्ड-अ में 10 प्रश्न हैं, जिनमें प्रत्येक 1 अंक का है, खण्ड-ब में 12 प्रश्न हैं जिनमें प्रत्येक 4 अंक का है तथा खण्ड-स में 7 प्रश्न हैं जिनमें प्रत्येक 6 अंक का है।

कैलकुलेटर के उपयोग की अनुमति नहीं है। आवश्यकता हो तो परीक्षार्थी के माँग पर लघुगणकीय अथवा सांख्यिकीय सारणी उपलब्ध करायी जा सकती है।

Section-A
(Objective Questions)

Q.1. **Let * be the binary operation on N defined by $a * b = L.C.M.$ of a and b. Find the value of 5 * 7.**

Ans. $a * b = L.C.M.\ of\ a \& b$

$a * 7 = L.C.M.\ of\ 5 \& 7 = 35$

Q.2. **Find the principal value of $\text{cosec}^{-1}(-\sqrt{2})$.**

Ans. $\text{cosec}^{-1}(-\sqrt{2}) = \propto$

$\text{cosec} \propto = -\sqrt{2} = \text{cosec}\left(\frac{3\pi}{4}\right)$

$\Rightarrow \quad \propto = \frac{3\pi}{4}.$

As we know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$\Rightarrow \quad \propto = -\frac{\pi}{4}$

$\therefore$ principal value of $\text{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

Q.3. **Construct a (2×2) matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$.**

Ans. $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$a_{11} = \frac{1}{1} = 1$

$a_{12} = \frac{1}{2}$

$a_{21} = \frac{2}{1} = 2$

$a_{22} = \frac{2}{2} = 1$

Required matrix

$\begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$

Q.4. **Find the values of x:**

$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}.$

Ans. $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix} \Rightarrow x = 6.$

Q.5. **Find the slope of tangent to the curve $y = 3x^3 - 4x$ at $x = 4$.**

Ans. $y = 3x^3 - 4x$

$\frac{dy}{dx} = 9x^2 - 4$

$\left(\frac{dy}{dx}\right)_{x=4} = 9(4)^2 - 4$

$= 144 - 4$

$= 140$

Q.6. **Find $\frac{dy}{dx}$:**

$y = \sin x^2$.

Ans. Let $y = \sin x^2$

putting $x^2 = t$

$\therefore \quad y = \sin t \ \& \ t = x^2$

$\frac{dy}{dt} = \cos t$

$\& \quad \frac{dt}{dx} = 2x$

$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \cos t . 2x$

$= 2x . \cos t$

$= 2x . \cos x^2.$

Q.7. **Find the value of $\int \tan x . \sec^2 x \, dx$.**

Ans. $\int \tan x \sec^2 x \, dx$

let $t = \tan x$

$\therefore \quad \frac{dt}{dx} = \sec^2 x$

$\therefore \quad dt = \sec^2 x\, dx$

$$\int t\,dt = \frac{t^2}{2} + C$$

$$= \frac{\tan^2 x}{2} + C$$

Q.8. Find a unit vector in the direction of vector $(\hat{i} + \hat{j} - \hat{k})$.

Ans. $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{(1)^2+(1)^2+(1)^2}} = \frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

Q.9. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

Ans. $\vec{a}.\vec{b} = ab\cos\theta$

$= |\vec{a}||\vec{b}|\cos\theta$

$10 = \sqrt{17}\sqrt{6}\cos\theta$

$10 = \sqrt{17}(\sqrt{6}\cos\theta)$

$\sqrt{6}\cos\theta$ is projection of $\vec{a}$ on $\vec{b}$

$|\vec{a}| = \sqrt{4+9+4} = \sqrt{17}$

$|\vec{b}| = \sqrt{1+4+1} = \sqrt{6}$

$\vec{a}.\vec{b} = 2+6+2 = 10$

$\Rightarrow \quad \sqrt{6}\cos\theta = \frac{10}{\sqrt{17}}$

Q.10. If a line makes equal angles with the x, y and z-axes, find its direction cosine.

Ans. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

Section-B

Q.11. If $f: R \rightarrow R$ be given by $f(x) = (7 - x^4)^{1/4}$ then find the value of $fof(x)$.

Ans. $f(x) = (7 - x^4)^{1/4}$

$fof(x) = fo(7 - x^4)^{1/4}$

$= [7 - \{(7-x^4)^{1/4}\}^4]^{1/4}$

$= [7 - (7 - x^4)]^{1/4}$

$= [7 - 7 + x^4]^{1/4}$

$= (x^4)^{1/4}$

$= x$

Q.12. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$.

Ans. Putting $x = \cos 2\theta$

R.H.S. $= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}(\cos 2\theta)$

$= \frac{\pi}{4} - \frac{1}{2}2\theta$

$= \frac{\pi}{4} - \theta$

L.H.S. $= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta} - \sqrt{1+\cos 2\theta}}{\sqrt{1+\cos\theta} - \sqrt{1-\cos 2\theta}}\right]$

$= \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}\right]$

$= \tan^{-1}\left[\frac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}\right]$

$= \tan^{-1}\left[\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right]$

$= \tan^{-1}\left[\frac{\cos\theta(1-\tan\theta)}{\cos\theta(1+\tan\theta)}\right]$

$= \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right]$

$= \tan^{-1}\left[\frac{\tan\pi/4 - \tan\theta}{1 + \tan\pi/4 . \tan\theta}\right]$ where $\tan\pi/4 = 1$

$= \tan^{-1}\left[\tan\left(\pi/4 - \theta\right)\right]$

R.H.S. $= \pi/4 - \theta$

$\therefore$ L.H.S. = R.H.S.

Q.13. Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$.

Ans. L.H.S. $D = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$ by $C_1 + (C_2 + C_3)$

$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$

$$= (1+x+x^2)\begin{vmatrix} 0 & x-1 & x^2-x \\ 0 & 1-x^2 & x-1 \\ 1 & x^2 & 1 \end{vmatrix} \text{ by } R_1 - R_2 \ \& \ R_2 - R_3$$

$$= (1+x+x^2)\left[(x-1)^2 - (1-x^2)(x^2-x)\right]$$

$$= (1+x+x^2)\left[x^2+1-2x-(x^2-x-x^4+x^3)\right]$$

$$= (1+x+x^2)[x^2+1-2x-x^2+x+x^4-x^3]$$

$$= (1+x+x)[1-x+x^4-x^3]$$

$$= 1-x+x^4+x^3+x-x^2+x^5-x^4+x^2-x^3+x^6-x^5$$

$$= 1-2x^3+x^6 = (1-x^3)^2 = RHS$$

$LHS = RHS$ *Proved.*

Q.14. Find the value of k so that the function $f(x)$ is continuous at $x=2$:

$$f(x) = \begin{cases} kx^2 & , x \le 2 \\ 3 & , x > 2 \end{cases}$$

Ans. For continuity

$$\underset{x\to 2-h}{Lt.} f(x) = \underset{x\to 2}{Lt.} f(x) = \underset{x\to 2+h}{Lt.} f(x)$$

or, $\underset{h\to 0}{Lt.} f(2-h) = \underset{h\to 0}{Lt.} K(2-h)^2 = K.4 = 4K.$

& $\underset{h\to 0}{Lt.} f(2) = \underset{h\to 0}{Lt.} K.(2)^2 = \underset{h\to 0}{Lt.}(K.4) = 4K.$

& $\underset{h\to 0}{Lt.} f(2+h) = \underset{h\to 0}{Lt.}(3) = 3$

for continuity, $4K = 3$

$\therefore \quad K = \frac{3}{4}$ *Ans.*

Q.15. If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

Ans. Here, $e^y(x+1) = 1$

Differentiating both sides w.r. to x we get

$$e^y\frac{dy}{dx}(x+1) + e^y(1+0) = 0$$

or, $e^y\frac{dy}{dx}(x+1) = -e^y$

or, $\frac{dy}{dx}(x+1) = -1$

$\therefore \quad \frac{dy}{dx} = -\frac{1}{(x+1)}$

$\therefore \quad \frac{d^2y}{dx^2} = \frac{d}{dx}\left[-(x+1)^{-1}\right] = (-)(-1)(x+1)^{-2} = \frac{1}{(x+1)^2}$

$$= \left(-\frac{1}{x+1}\right)^2 = \frac{1}{(x+1)^2} = \left(\frac{dy}{dx}\right)^2$$

$\therefore \quad \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ *Proved*

Or

Q. If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

Ans. If $(\cos x)^y = (\cos y)^x$, $\frac{dy}{dx} = ?$

taking log on both sides we get

$\log(\cos x)^y = \log(\cos y)^x$

or, $y\log(\cos x) = x\log(\cos y)$

differentiating both sides w.r. to x we get

$$\frac{dy}{dx}\log(\cos x) + y\frac{1}{\cos x}(-\sin x) = 1 - \log(\cos y) + x.\frac{1}{\cos y}(-\sin y)\left(\frac{dy}{dx}\right)$$

or, $\frac{dy}{dx}\log(\cos x) + x\tan y\frac{dy}{dx} = \log(\cos y) + \tan x$

or, $\frac{dy}{dx}[\log(\cos x) + x\tan y] = \log(\cos y) + y\tan x$

$$\frac{dy}{dx} = \frac{\log(\cos y) + y\tan x}{\log(\cos x) + x\tan y} \text{ Ans.}$$

Q.16. Prove that $y = \frac{4\sin\theta}{3+\cos\theta} - \theta$ is an increasing function of θ in $[0, \pi/2]$.

Ans. $y = \frac{4\sin\theta}{2+\cos\theta} - \theta = f(\theta)$,

θ	$f(\theta) = \frac{4\sin\theta}{2+\cos\theta} - \theta$
0	$f(0) = 0$
$\pi/4$	$f(\pi/4) = \frac{4}{2\sqrt{2}+1} - \frac{\pi}{4}$
$\pi/2$	$2 - \pi/2$

As we move from 0 to $\pi/2$ for the value of $f(0)$ increases,

Hence $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of a $[0, \pi/2]$

Or

Q. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius which is 10 cm.

Ans. Volume of the spherical balloon $v = \frac{4}{3}\pi r^3$

$r =$ radius of the sphere
$v =$ volume of the sphere

$\therefore \quad \frac{dv}{dt} = \frac{4}{3}\pi.3r^2.\frac{dr}{dt}$

$$=\frac{4}{3}\pi\,3(10)^2\frac{dr}{dt}$$

$$=400\pi\frac{dr}{dt}$$

Rate of change of volume is 400π times of the rate of change of radius.

Q.17. Find the value of $\int\frac{x+2}{\sqrt{x^2+2x+3}}dx.$

Ans. Let $x+2=A\frac{d}{dx}(x^2+2x+3)+B.$

$x+2=A(2x+2)+B.$

Equating co-efficient of 'x' & constant term on both side weget

$1=2A$

$\Rightarrow\quad A=\frac{1}{2}$

& $\quad 2=2A+B.$

$2=2\times\frac{1}{2}+B.$

$\Rightarrow\quad B=2-1=1$

$$\therefore\quad \int\frac{x+2}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int\frac{(2x+2)dx}{\sqrt{x^2+2x+3}}+\int\frac{dx}{\sqrt{x^2+2x+3}}.$$

$$=\frac{1}{2}I+I_2 \qquad ...(i)$$

In I_1 put $x^2+2x+3=t$

so that $(2x+2)=dt$

$$\therefore\quad I_1=\int\frac{(2x+x)dx}{\sqrt{x^2+2x+3}}=\int\frac{dx}{\sqrt{t}}=2\sqrt{t}+C_1$$

$$=2\sqrt{x^2+2x+3}+C_1 \qquad ...(ii)$$

Now, $I_2=\int\frac{dx}{\sqrt{x^2+2x+3}}=\int\frac{dx}{\sqrt{(\sqrt{2})^2+(x+1)^2}}$

$$=\log\left|(x+1)+\sqrt{x^2+2x+3}\right|$$

$$\therefore\quad I=\frac{1}{2}I_1+I_2$$

$$=\frac{1}{2}\times2\sqrt{x^2+2x+3}+\log(x+1)+\sqrt{x^2+2x+3}+C$$

Q.18. Find the value of $\int\frac{x}{(x-1)^2(x+2)}dx.$

Ans. $I=\int\frac{x}{(x-1)^2(x+2)}dx$

Here suppose,

$$\frac{x}{(x-1)^2(x+2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}$$

$$=\frac{A(x-1)(x+2)+B(x+2)+C(x-1)^2}{(x-1)^2(x+2)}$$

$$x=A(x-1)(x+2)+B(x+2)+C(x-1)^2 \qquad (i)$$

Put $x-1=0$

or, $\quad x=1$ in (i) we get

$1=B(1+2)=3B$

$\therefore\quad B=\frac{1}{3}$

Put $x+2=0$

or, $\quad x=-2$ in (i) we get

$-2=C(-2-1)^2=9C$

$\therefore\quad C=\frac{-2}{9}$

Put $x=0$ in (i) we get

$0=A(0-1)(0+2)+B(0+2)+C(0-1)^2$

or, $\quad 0=-2A+2B+C$

or, $\quad 0=-2A+2\left(\frac{1}{3}\right)+\left(-\frac{2}{9}\right)$

or, $\quad 2A=\frac{2}{3}-\frac{2}{9}=\frac{6-2}{9}=\frac{4}{9}$

or, $\quad A=\frac{4}{9}\times\frac{1}{2}=\frac{2}{9}$

Now, $I=\int\frac{x}{(x-1)^2(x+2)}dx=\int\frac{2dx}{9(x-1)}+\frac{1}{3}\int\frac{dx}{(x-1)^2}-\int\frac{2}{9}\frac{dx}{(x+2)}$

$$=\frac{2}{9}\int\frac{dx}{(x-1)}+\frac{1}{3}\int\frac{dx}{(x-1)^2}-\frac{2}{9}\int\frac{dx}{(x+2)}$$

$$=\frac{2}{9}\int\frac{dx}{x-1}+\frac{1}{3}\int(x-1)^{-2}dx-\frac{2}{9}\int\frac{dx}{(x+2)}$$

$$=\frac{2}{9}\log(x-1)+\frac{1}{3}\frac{(x-1)^{-1}}{(-1)(1)}-\frac{2}{9}\log(x+2)$$

$$=\frac{2}{9}\log(x-1)-\frac{1}{3}(x-1)-\frac{2}{9}\log(x+2)+C \text{ Ans.}$$

Q.19. Find the value of $\int_0^{\pi/2}\frac{\cos^5 x}{\sin^5 x+\cos^5 x}dx.$

Ans. $I=\int_0^{\pi/2}\frac{\cos^5 x}{\sin^5 x+\cos^5 x}dx$

$$=\int_0^{\pi/2}\frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\pi/2}\frac{\sin^5 x}{\cos^5 x+\sin^5(x)dx}=I$$

$$I+I=\int_0^{\pi/2}\left[\frac{\sin^5 x}{\cos^5 x+\sin^5 x}+\frac{\cos^5 x}{\sin^5 x+\cos^5 x}\right]dx$$

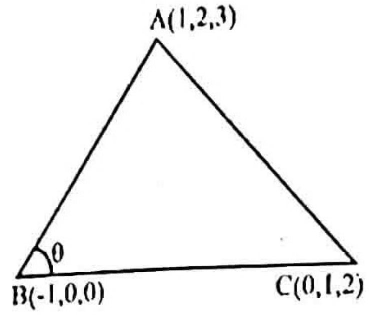
$$= \int_0^{\pi/2} \left[\frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x}\right] dx$$

$\therefore \quad 2I = \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \frac{\pi}{2}$

$\therefore \quad I = \frac{\pi}{4}$ Ans.

Q.20. If the vertices A, B and C of a triangle ABC are (1, 2, 3), (-1, 0, 0) and (0, 1, 2) respectively, then find $\angle ABC$.

Ans.



We have to find $\angle ABC$ which made by side AB and BC

Now,

$\overrightarrow{AB} = P.v\,of\ B - P.v\,of\ A$

$= -\hat{i} + 0\hat{j} + 0\hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= -2\hat{i} - 2\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = P.v\ of\ C - P.v\ of\ B$

$= 0\hat{i} + \hat{j} + 2\hat{k} - (-\hat{i} + 0.\hat{j} + 0\hat{k})$

$= \hat{i} + \hat{j} + 2\hat{k}$

$$\therefore \cos\theta = \left|\frac{\overrightarrow{AB}.\overrightarrow{BC}}{|\overrightarrow{AB}|.|\overrightarrow{BC}|}\right|$$

$$= \left|\frac{(-2\hat{i} - 2\hat{j} - 3\hat{k}).(\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{(-2)^2 + (-2)^2 + (-3)^2}.\sqrt{1^2 + 1^2 + 2^2}}\right|$$

$$= \left|\frac{-10}{\sqrt{17}.\sqrt{6}}\right|$$

$$\cos\theta = \frac{10}{\sqrt{102}}$$

$$\boxed{\therefore \theta = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)}$$

Q.21. Find the angle between the following pair of lines:

$\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j} - 2\hat{k})$ and

$\vec{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$.

Ans. Here $\vec{b}_1 = \vec{i} - \vec{j} - 2\vec{k}$

$\vec{b}_2 = 3\vec{i} - 5\vec{j} - 4\vec{k}$

the $\angle, \theta$ between the two line is given by

$$\cos\theta p = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \left|\frac{(\hat{i} - \hat{j} - 2\hat{k}):(3\hat{i} - 5\hat{j} - 4\hat{k})}{\sqrt{1+1+4}\sqrt{9+25+16}}\right|$$

$$= \left|\frac{3+5+8}{\sqrt{6}.\sqrt{50}}\right|$$

$$= \left|\frac{16}{\sqrt{300}}\right|$$

$$= \left|\frac{16}{10\sqrt{3}}\right|$$

$$= \frac{8}{5\sqrt{3}}$$

$$\cos\theta = \frac{8}{5\sqrt{3}}$$

$$\therefore \quad \theta = \cos^{-1}\frac{8}{5\sqrt{3}}.$$

Q.22. If a coin is tossed three times, find P(E/F), where

$E \to$ at least two heads

$F \to$ at most one head.

Ans. $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$E =$ at least two heads

$= \{HHH, HHT, HTH, THH\}$

$P(E) = 4$

$F =$ at most one head

$= \{HTT, THT, TTH\}$

$P(F) = 3$

$E \cap F = \{\ \} = \phi$

$P(E \cap F) = 0$

$$\therefore \quad P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{0}{3} = 0$$

Section-C

Q.23. Solve the system of linear equations using matrix method.

$x - y + 2z = 7,\ 3x + 4y - 5z = -5,\ 2x - y + 3z = 12.$

Ans. we know $AX = B \Rightarrow X = A^{-1}B$.

where,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix},\ X = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix},\ B = \begin{bmatrix} -7 \\ -5 \\ 12 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix} = 1\begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$$

$= 1(12 - 5) + 1(9 + 10) + 2(-3 - 8)$

$= 7 + 19 - 22 = 26 - 22 = 4$

co-efficient of $1 = \begin{vmatrix}4 & 5\\ 1 & 3\end{vmatrix} = 12 - 5 = 7$

" $1 = -\begin{vmatrix}3 & 5\\ 2 & 3\end{vmatrix} = -(9+10) = -19$

" $2 = \begin{vmatrix}3 & 4\\ 2 & 1\end{vmatrix} = 3 - 8 = -11$

" $3 = -\begin{vmatrix}1 & 2\\ 1 & 3\end{vmatrix} = -(3+2) = -1$

" $4 = \begin{vmatrix}1 & 2\\ 2 & 3\end{vmatrix} = (3-4) = -1$

" $-5 = -\begin{vmatrix}1 & -1\\ 2 & -1\end{vmatrix} = -(-1+2) = -1$

" $2 = \begin{vmatrix}-1 & 2\\ 4 & -5\end{vmatrix} = (5-8) = -3$

" $-1 = -\begin{vmatrix}1 & 2\\ 3 & -5\end{vmatrix} = -(-5-6) = 11$

" $3 = \begin{vmatrix}1 & -1\\ 3 & 4\end{vmatrix} = 4 + 3 = 7$

Matrix made by co-factor by

$$A = B = \begin{vmatrix}7 & -19 & -11\\ 1 & -1 & -1\\ -3 & 11 & 7\end{vmatrix}$$

$$Adj.\ A = B^{-1} = \begin{vmatrix}7 & 1 & -3\\ -1 & -1 & 11\\ -11 & -1 & 7\end{vmatrix}$$

$$A' = \frac{Adj.\ A}{|A|} = \frac{1}{4}\begin{vmatrix}7 & 1 & -3\\ -19 & -1 & 16\\ -11 & -1 & 7\end{vmatrix}$$

$$X = \begin{bmatrix}x\\ y\\ z\end{bmatrix} = A^{-1}B = \frac{1}{4}\begin{vmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{vmatrix}\begin{bmatrix}7\\ -5\\ 12\end{bmatrix}$$

$$= \frac{1}{4}\begin{vmatrix}7\times 7 & + & 1\times(-5) & + & (-3)(12)\\ (-19)7 & + & (-1)(-5) & + & 11(12)\\ (-11)7 & + & (-1)(-5) & + & 7(12)\end{vmatrix}$$

$$= \frac{1}{4}\begin{vmatrix}49 & -5 & -36\\ -133 & +5 & 132\\ -77 & +5 & 84\end{vmatrix}$$

$$= \frac{1}{4}\begin{bmatrix}8\\ 4\\ 12\end{bmatrix} = \begin{bmatrix}2\\ 1\\ 3\end{bmatrix}$$

$x = 2,\ y = 1\ \&\ z = 3$ Ans.

Or

Q. Obtain the inverse of the matrix using elementary operations :

$$A = \begin{vmatrix}1 & 3 & 2\\ 1 & 0 & 5\\ 2 & 5 & 0\end{vmatrix}$$

Q.24. Find the maximum and minimum value of given function:

$f(x) = 41 - 72x - 18x^2$

Ans. $f(x) = 41 - 72x - 18x^2$

$f'(x) = \frac{d}{dx}(f(x)) = 0 - 72 - 36x$

$f''(x) = \frac{d}{dx}f'(x) = -36$

for max^m. or min^m. value of $f'(x) = 0$

or, $-36x - 72 = 0$

$\Rightarrow x = \frac{-72}{36} = -2$

putting $x = -2$ in $f''(x)$ we get

$f''(x) = -36 < 0$

Hence function has maxumum value at $x = -2$.

& that value is $41 - 72(-2) - (18)(-2)^2$

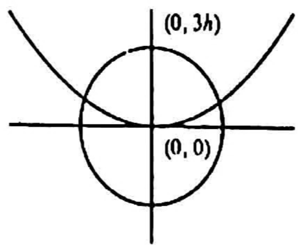
$= 41 + 144 - 72$

$= 113$ Ans.

Q.25. Find the area of the region bounded by the circle $4x^2 + 4y^2 = 9$ and the parabola $x^2 = 4y$.

Ans. circle is $4x^2 + 4y^2 = 9$ ..(i)

or $x^2 + y^2 = \frac{9}{4} = \left(\frac{3}{2}\right)^2$



Centre $= (0,0)$, Radius $= \frac{3}{2}$

Parabola is $x^2 = 4y$ ...(ii)

Intersection of circle & parabola can be obtained by putting $x^2 = 4y$ in (i)

$4(4y) + 4y^2 = 9$

or $16y + 4y^2 - 9 = 0$

or $4y^2 + 16y - 9 = 0$

or $4y^2-2y+18y-9=0$

or $2y(2y-1)+9(2y-1)=0$

$(2y-1)(2y+9)=0$

$\therefore \quad y=\frac{1}{2}$ & $y=\frac{-9}{2}$

when $y=\frac{-9}{2}$, from (ii) $x^2=4.\left(\frac{-9}{2}\right)=-18$

$\therefore \quad x=\sqrt{-18}$ imaginary (neglected)

when $y=\frac{1}{2}$, from (ii), $x^2=4\left(\frac{1}{2}\right)=2$

$\therefore \quad x=\pm\sqrt{2}$

pint of intersection $\left(\sqrt{2}, \frac{1}{2}\right)\left(-\sqrt{2}, \frac{1}{2}\right)$

Area bounded by (i) and (ii) will be

$$=\int_{-\sqrt{2}}^{\sqrt{2}} 4dx=\int_{-\sqrt{2}}^{\sqrt{2}} \frac{x}{4}dx=\left[\frac{x^3}{12}\right]_{-\sqrt{2}}^{\sqrt{2}}=\frac{2\sqrt{2}}{12}+\frac{2\sqrt{2}}{12}=\frac{4\sqrt{2}}{12}=\frac{\sqrt{2}}{3}$$

Area $=\frac{\sqrt{2}}{3}$ *sq. unit.*

Q.26. Find particular solution of the differential equation:

$$2xy-y^2-2x^2.\frac{dy}{dx}=0,\ y=2 \text{ when } x=1.$$

Ans. $2xy-y^2-2x^2\frac{dy}{dx}=0$

or $2xy-y^2=2x^2\frac{dy}{dx}$

or $\frac{dy}{dx}=\frac{2xy-y^2}{2x^2}$ ...(i) Homogeneous equation

put $y=Vx$

so let, $\frac{dy}{dx}=V.1+x\frac{dv}{dx}$ in (i) we get

$$v+x\frac{dv}{dx}=\frac{2xVx-V^2x^2}{2x^2}=\frac{x^2(2V-V^2)}{2x^2}$$

or $V+x\frac{dV}{dx}=\frac{2V-V^2}{2}$

or $x\frac{dV}{dx}=\frac{2V-V^2}{2}-V=\frac{2V-V^2-2V}{2}$

or $x\frac{dv}{dx}=-\frac{V^2}{2}$

or $\int-2\frac{dV}{V^2}=\int\frac{dx}{x}$

or $-2\int V^{-2}dV=\log x+C$

or $-2(V-1-1)=\log x+C$

or $\frac{2}{V}=\log x+C$

or $2=V\log x+CV$

or $2=\frac{y}{x}\log x+C\frac{y}{x}$

or $2x=y\log x+Cy$ ...(ii)

when $x=1,\ y=2$

$\therefore \quad 2.1=2\log 1+C\times 2$

or $2=0+2C$

$\therefore \quad C=1$

from (2) $2x=y\log x+y$ Ans

Or

Q. Find general solution of the differential equation :

$$\cos^2 x.\frac{dy}{dx}+y=\tan x.$$

Ans. $\cos^2 x\frac{dy}{dx}+y=\tan x$

or $\frac{dy}{dx}+\frac{y}{\cos^2 x}=\frac{\tan x}{\cos^2 x}$

or $\frac{dy}{dx}+y.\sec^2 x=\tan x.\sec^2 x$ ...(i)

here $P=\sec x,\ Q=\tan x.\sec^2 x$

$I.F.=_e\int Pdx=_e\int \sec^2 xdx=_e\tan x$

Multiplying by $_e\tan x$ on both sides of (i) and integration we get,

$y._e\tan x=\int {}_e\tan x.\tan x.\sec^2 xdx$ ...(ii)

$I=\int {}_e\tan x.\tan x.\sec^2 xdx$

Put in I, $\tan x=Z$ so that $\sec^2 xdx=dz$

$\therefore \quad I=\int e^z.z.dz$ integrating by parts we get

$I=z.e^z dz-\int(1.fe^z dz)dz$

$=ze^z-\int e^z dz$

$=ze^z-e^z$

$=e^z(z-1)=e^{\tan x}.(\tan x-1)$

from (ii) we get, $y.e^{\tan x}=e^{\tan x}.(\tan x-1)$

i.e. $y=\tan x-1+C$ Ans.

Q.27. Find the shortest distance between the lines

$$\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k}) \text{ and}$$

$$\vec{r}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k}).$$

Ans. $\vec{r}=6\vec{i}+2\vec{j}+2\vec{k}+\lambda(\vec{i}-2\vec{j}+2\vec{k})$ ...(i)

& $\vec{r}=-4\vec{i}-\vec{k}+\mu(3\vec{i}-2\vec{j}-2\vec{k})$ ...(ii) comparing (i) and (ii) with $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ respectively

we get, $\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{a}_2 = -4\hat{i} - 2\hat{k}$, $\vec{b}_2 = 2\hat{i} - 2\hat{j} - 2\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = -10\hat{i} - 2\hat{j} - 3\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (3\hat{i} - 2\hat{j} - 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$\therefore \quad |\vec{b}_1 \times \vec{b}_2| = \sqrt{64 + 64 + 16} = 12$

Hence the sortest distance between the given lines is given by

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{108}{12} = 9 \text{ Ans.}$$

Q.28. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamond. Find the probability of the lost card being a diamond.

Ans. Let E_1, E_2 & E_3 be the event that the balls are drawn from UrnA, UrnB and UrnC respectively.

E be the event of balls drawn are one white and red.

$$P(E) = E_1 = E_2 = E_3 = \frac{1}{3}$$

(i) $P(E_1) = \frac{1}{3}$ (ii) $P(E_2) = \frac{1}{3}$

(iii) $P\left(\frac{E}{E_1}\right) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$

$$= \frac{1 \times \frac{3!}{2! \times 1}}{\frac{6!}{2!(4!)}}$$

$$= \frac{\frac{3 \times 2 \times 1}{2 \times 1}}{\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 4 \times 3 \times 2 \times 1}}$$

$$= \frac{3 \times 2}{6 \times 5} = \frac{1}{5}$$

(iv) $P\left(\frac{E}{E_3}\right) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{\frac{4!}{3! \times 1!} \times \frac{3!}{2! \times 1!}}{\frac{12!}{10! \times 2!}}$

$$= \frac{\frac{4 \times 3!}{3! \times 1} \times \frac{3 \times 2!}{2! \times 1}}{\frac{12 \times 11 \times 10}{10! \times 2 \times 1}}$$

$$= \frac{12}{12 \times 11} \times 2 = \frac{2}{11}$$

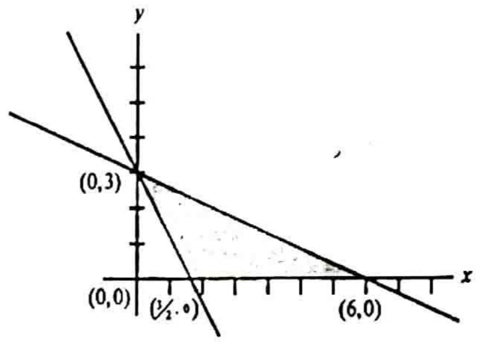
Q.29. Minimize $\quad Z = x + 2y$

subject to $\quad 2x + y \geq 3$

$\quad x + 2y \geq 6$

and $\quad x, y \geq 0.$

Ans.



Minimize $\quad z = x + 2y$

Subject to $\quad 2x + y \geq 3$

$\quad x + 2y \geq 6$

& $\quad x, y \geq 0$

Here $\quad 2x + y = 3$ 1st line

or $\quad \frac{2x}{3} + \frac{4}{3} = 1$

or $\quad \frac{x}{3/2} + \frac{4}{3} = 1$ 1st line

2nd line $\quad x + 2y = 6$

$\quad \frac{x}{6} + \frac{2}{6} = 1$

or $\quad \frac{x}{6} + \frac{4}{3} = 1$

Corner Point	$z = x + 2y$
(0, 3)	6
(6, 0)	6

Minimum $z = 6$ at all points on the line segment joining the points (6,0) and (0,3). *Ans.*

OOO