

# ★ Karnaugh Maps (Veitch Diagram).

\* 3-Variable K-Map:

$F(A, B, C)$

		BC			
		00	01	11	10
A	0	0	1	3	2
	1	4	5	7	6

Octet (group of 8 adj minterms).

↓ x  
Quad (group of 4 adj minterms).

↓ x  
Pair (group of 2 adj minterms).

Q: How many possible ways to get Quad of minterms?

Ans: **6.**

\* 4-Variable K-Map

		CD			
		00	01	11	10
AB	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10

Possible Quads  $\Rightarrow 24$

Possible Octets  $\Rightarrow 8$

Possible Pairs  $\Rightarrow 32$

(0, 2, 8, 10)  $\Rightarrow$  Quad

Columns Rows

✓ 1, 2

1, 2

✓ 2, 3

2, 3

✓ 3, 4

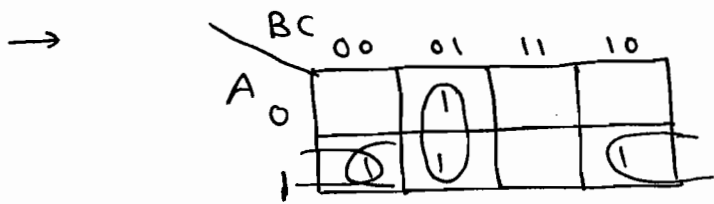
3, 4

✓ 4, 1

4, 1

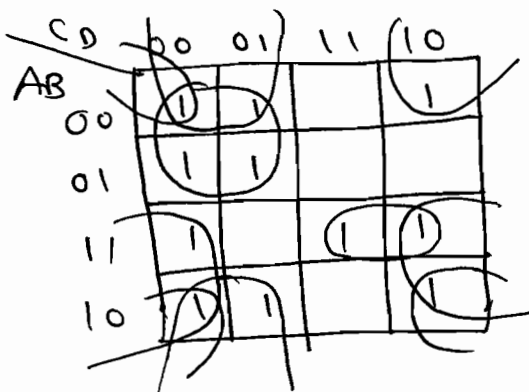
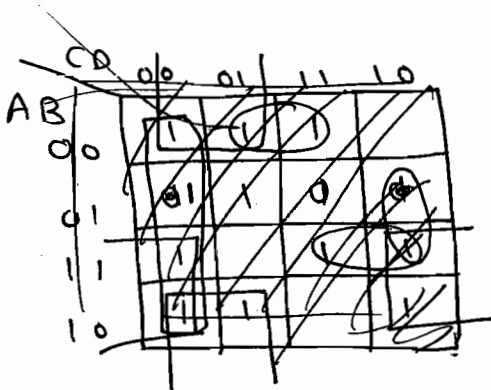
Ex-1 Simplify the following expression using mapping.

(a)  $F(A, B, C) = \sum m(1, 4, 5, 6)$

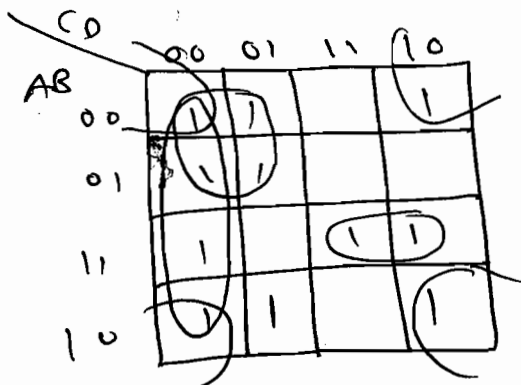


$F = A\bar{C} + \bar{B}C$

(b)  $F(A, B, C, D) = \sum m(0, 1, 2, 4, 5, 8, 9, 10, 12, 14, 15)$



(0, 9)



**NOTE**

The simplified expression obtained using SI  
K-map is minimal but not unique.

Ex-1 a)  $F(A, B, C) = \sum m(1, 2, 4, 7)$

b)  $F_1(A, B, C) = \sum m(0, 3, 5, 6)$

Ans: 5

⊙

	Bc	00	01	11	10
A	0		1		1
	1	1		1	0

	A	B	C
$m_1$	0	0	1
$m_2$	0	1	0
$m_4$	1	0	0
$m_7$	1	1	1

$F = A\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + ABC$

$F = \bar{B}(A\bar{C} + \bar{A}C) + A(\bar{B}C + BC)$

$F = A \oplus B \oplus C$

- All minterms have odd no. of 1's.
- All minterms have even no. of 0's.

b)

	Bc	00	01	11	10
A	0	1		1	
	1		1		1

$F = \sum m(1, 2, 4, 7)$   
 $\bar{F} = \sum m(0, 3, 5, 6)$   
 $F = \bar{F}$

$F = A \odot B \oplus C$

$F_1 = \bar{F}$

let,  $z = \overline{A \oplus B} = A \odot B$

$F_1 = \overline{A \oplus B \oplus C} = \bar{A} \odot \overline{B \oplus C}$

$F_1 = \bar{z} \oplus C$

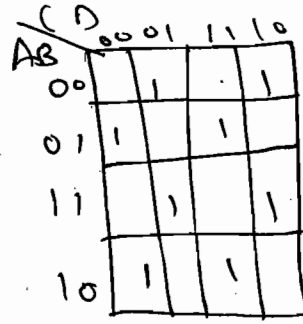
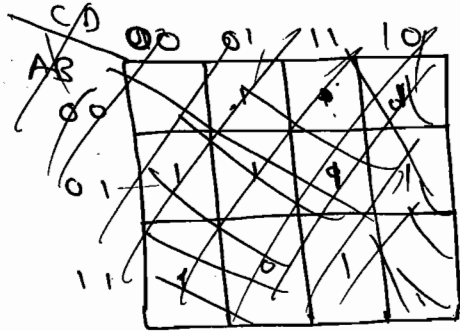
$F_1 = A \odot B \oplus C$  (or)

$F_1 = A \oplus B \odot C$

→ Minterms doesn't have odd no. zeros and even no. of 1's.

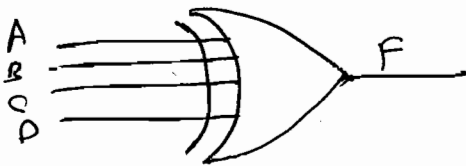
Ex-2 Represent  $F = A \oplus B \oplus C \oplus D$  in sum of minterms (Canonical form).

Ans:  $F = A \oplus B \oplus C \oplus D$ .



Ans: 8

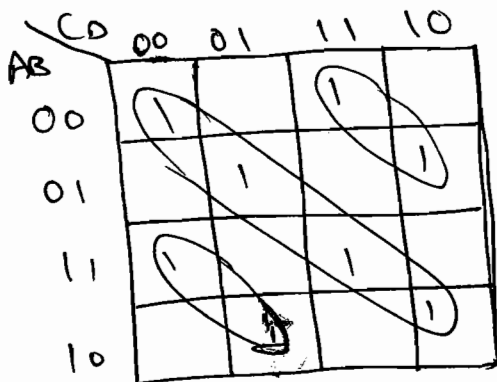
A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0



$$F = \sum m(1, 2, 4, 7, 8, 11, 13, 14)$$

Hint: All above minterms have odd no. of ones.

Ex-3 Simplify the following terms: mnp:



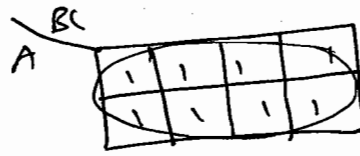
$$F = \sum m(0, 3, 5, 6, 9, 10, 12, 15)$$

\* All minterms contain even no. zeros

So, X-NOR gate.  
 $\therefore F = A \odot B \odot C \odot D$

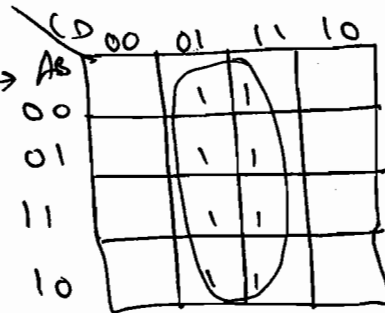
Ex-4 In n-variable k-map a group of  $2^m$  1's form how many literals the resulting minterms.

Ans:  $\rightarrow$  3-var k-map  $\rightarrow$



$F=1$   
So, 0 literals

$\rightarrow$  4-var k-map  $\rightarrow$



$F=0$   
So, 1 literal.

$\rightarrow$  n-var k-map  $\rightarrow$  there are  $n-3$  literals. require for 8 minterms.

for pair  $\rightarrow n-1$  literals.

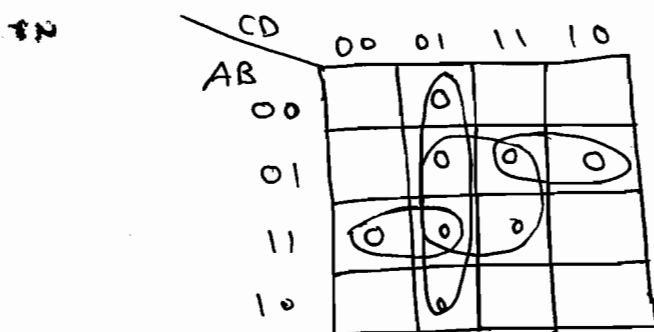
$\rightarrow$  for quad  $\rightarrow n-2$  literals.

for ~~min~~ octet  $\rightarrow n-3$  literals.

Ex-5 Determine the simplified pos expression of F where,  $F = \sum m(0, 2, 3, 4, 8, 10, 11, 14)$ .

Ans: for pos.

$\therefore F = \prod M(1, 5, 6, 7, 9, 12, 13, 15)$



for max term

0  $\rightarrow$  var  
var

$F = (\bar{B} + \bar{D}) \cdot (C + \bar{D}) \cdot (\bar{A} + \bar{B} + \bar{C}) (A + \bar{B} + \bar{C})$ .

\* Implicant:

- (i) Prime Implicant &
- (ii) Essential implicant

→ Implicant:

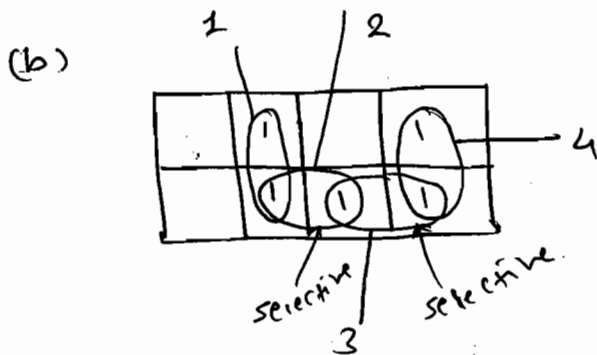
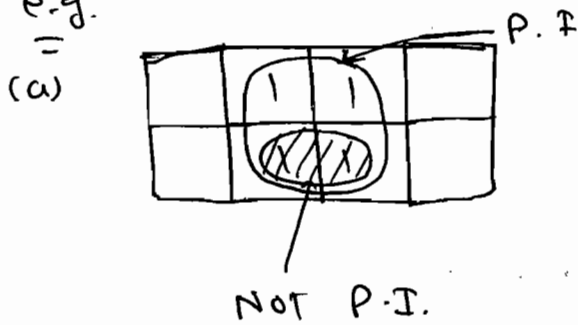
→ it is the set of all adjacent min terms.

E.g. octets, pairs.

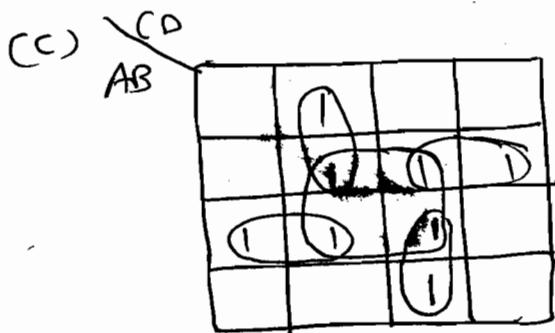
\* Prime implicant:

→ It is an implicant which is not a subset of another implicant.

e.g.  
=



All 4 are prime implicants.

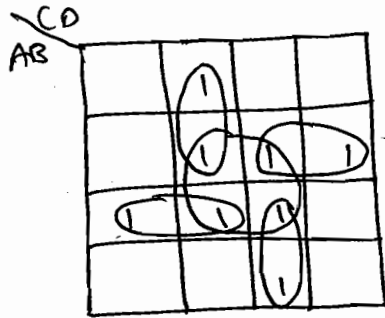



All are prime implicants.

## \* Essential Prime Implicants:

→ It is a prime implicant which contains at least 1 minterms which is not covered by another prime implicant.

e.g.



 ← is non-essential prime implicant and remaining are essential prime implicant.

## \* Non-Essential Prime Implicants:

① Redundant P.I (RPI).

→ It is a non-essential prime implicant whose minterms are covered by all essential P.I.

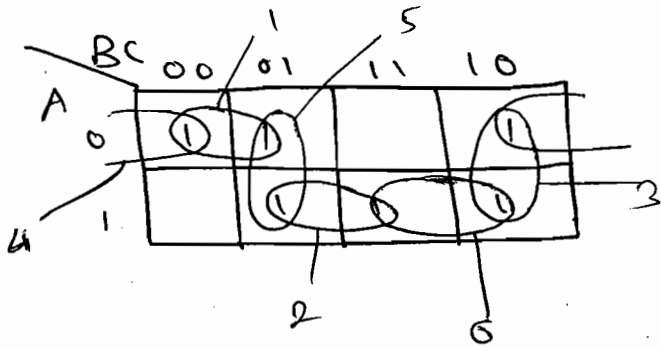
② Selective prime implicant:

→ It is a non-essential prime implicant whose minterms are covered by at least one non-essential P.I.

\* Minimal Expression = EPI's + (optional) SPI

Ex-1 Determine the essential P.I. and minimal expression for the following  $f^m$ .

Ans: ①  $F(A, B, C) = \sum m(0, 1, 2, 5, 6, 7)$ .



EP I's  $\rightarrow$  Null

RP I's  $\rightarrow$  Null

SP I's  $\rightarrow$  ①, ②, ③, ④

⑤, ⑥, ⑦

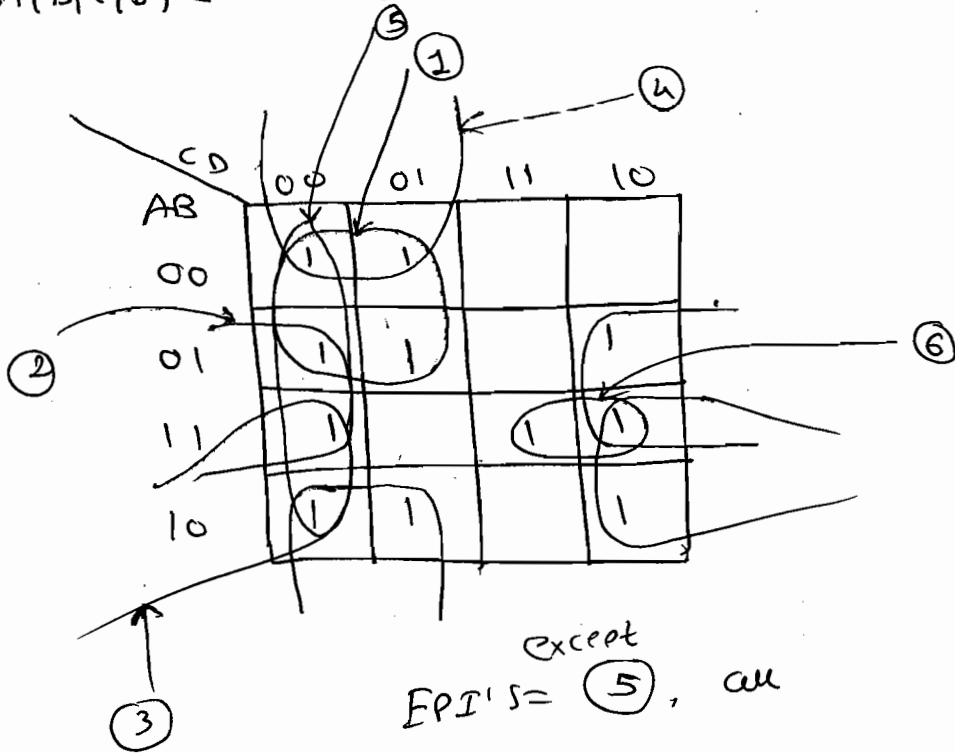
(Note)

Minimal expression = ① + ② + ③

(or)

= ④ + ⑤ + ⑥

②  $F(A, B, C, D) = \sum m(0, 1, 4, 5, 6, 8, 9, 10, 12, 14, 15)$



$\therefore F = ① + ② + ③ + ④ + ⑥ + ⑦$

$F = \bar{A}\bar{C} + B\bar{D} + A\bar{D} + \bar{C}\bar{B} + ABC$