CENTRE OF MASS

1. Rigid body:
A body which does not under go deformation when it is subjeced to an external force is called rigid body

2. Non Rigid body:
A body which undergoes deformation when subjected to an external force is called non rigid body.

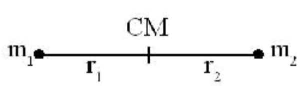
3. A body can execute 3 types of motions
(1) translatory motion (2) rotatory motion (3) vibratory motion

Translatory motion:
A body in which all the particles same displacement in same time is called translatory motion

Rotatory motion:
A body in which the particles move permanently around a fixed point then the motion of the body is called rotatory motion.

4. Centre of mass:
Centre of mass is a point with in the boundaries of a system or a body at which its entire mass appears to be concentrated.

5.



When two particles of masses m_1 and m_2 are located on a straight line, their centre of mass lies on the line joining the two particles.
Let r_1 and r_2 be the distances of the particles from their centre of mass respectively, then

$$m_1 r_1 = m_2 r_2$$

6. Coordinates of centre of mass
Let us consider a system of n particles of masses m_1, m_2, ..., m_n whose co-ordinates are (x_1, y_1, z_1) (x_2, y_2, z_2), ... , (x_n, y_n, z_n) respectively. Then co-ordinates of their centre of mass are

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + ...+ m_n x_n}{m_1 + m_2 + ...+ m_n}$$

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + ...+ m_n y_n}{m_1 + m_2 + ...+ m_n}$$

$$y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

and

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + ...+ m_n z_n}{m_1 + m_2 + ...+ m_n}$$

$$z_{cm} = \frac{1}{M}.\sum_{i=1}^{n} m_i z_i$$

7. Position vector of centre of mass
Consider two particles of mass m_1 and m_2 whose position are represented by position vectors $\vec{r_1}$ and $\vec{r_2}$ respectively at an instant, then

$$\vec{r}_{cm} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

8. Velocity and acceleration of centre of mass
Velocity of centre of mass

$$\vec{V}_{cm} = \frac{m_1 \vec{V_1} + m_2 \vec{V_2} + ...+ m_n \vec{V_n}}{m_1 + m_2 + ...+ m_n}$$

$$\vec{V}_{cm} = \frac{1}{M}.\sum_{i=1}^{n} m_i.\vec{V_i}$$

9. Moment of inertia of some bodies of regular shape

S.No. Body Axis Moment of inertia

1. Uniform rod of length ℓ Perpendicular to rod through its center $\frac{1}{12}M\ell^2$
2. Uniform rectangular lamina of length ℓ and breadth b Perpendicular to lamina and through its center $M\left(\frac{\ell^2 + b^2}{12}\right)$
3. Uniform circular ring of radius R Perpendicular to its plane and through the center MR^2

4. Uniform circular ring of radius R
Diameter $MR^2/2$

5. Uniform circular ring of disc of radius R
Perpendicular to its plane and through the center $\frac{1}{2}MR^2$

6. Uniform circular ring of disc of radius R
Diameter $\frac{1}{4}MR^2$

7. Hollow cylinder of radius R Axis of cylinder MR^2

8. Solid cylinder of radius R Axis of cylinder
$\frac{1}{2}MR^2$

9. Hollow sphere of radius R
Diameter $\frac{2}{3}MR^2$

10. Solid sphere of radius R Diameter
$\frac{2}{5}MR^2$

10. Acceleration of centre of mass

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ... + m_n\vec{a}_n}{m_1 + m_2 + ... + m_n}$$

$$\vec{a}_{cm} = \frac{1}{M}.\sum_{i=1}^{n} m_i.a_i$$

11. Characteristics of centre of mass

a. The position of centre of mass depends up on the shape of the body and the distribution of mass

b. Centre of mass always located towards massive part of the body

c. Matter may or may not present at centre of mass

d. If the origin is at the centre of mass sum of moment of masses of the system about centre of mass is zero.

e. Centre of mass canot be effected by the internal forces

f. When no external force acts on a system the velocity of centre of mass of the system remains constant.

g. The location of the centre of mass is independent of the reference frame used to locate it.

Motion of a cylinder rolling without slipping on an inclined plane

$$a = \frac{mg\sin\theta}{m + I/r^2}$$

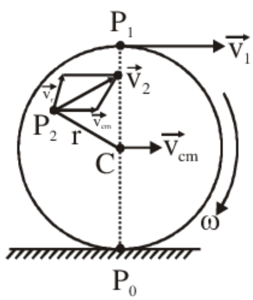
Linear Motion

Rotational Motion

1. Distance/ displacement (s)
2. Angle or angular displacement (θ)
3. Linear velocity, $v = \frac{ds}{dt}$
4. Angular velocity, $\omega = \frac{d\theta}{dt}$
5. Linear acceleration, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
6. Angular acceleration, $\alpha = \frac{d\omega}{et} = \frac{d^2\theta}{dt^2}$
7. Mass (m)
8. Moment of inertia (I)
9. Linear momentum, p = mv
10. Angular momentum, L = Iω
11. Force, F = ma
12. Torque = τ = Iα
13. Also force, F = $\frac{dp}{dt}$
14. Also, torque = $\tau = \frac{dL}{dt}$
15. Translational K.E. = $\frac{1}{2}mv^2 = \frac{p^2}{2m}$
16. Rotatinoal K.E. $= \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$
17. Work done, $W = Fs$
18. Work done, W = $\tau = \theta$
19. Power, P = Fv
20. Power, P = $\tau\omega$

12. Rolling Motion

$\vec{v}_{cm} = R\omega$



Kinetic energy of Rolling motion

$K = K_T + K_R$

K.E of translation $K_T = \frac{1}{2}mv_{cm}^2$

$K_R = \frac{1}{2}I\omega^2$

K.E of rolling body , $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$

$K = \frac{1}{2}mv_{cm}^2\left[1 + \frac{k^2}{R^2}\right]$

13. Centre of gravity

Centre of gravity is a point inside the body through which its whole weight acts

a. It refers weight of the body

b. At centre of gravity there must be matter present

c. Centre of mass and centre of gravity coincide for small bodies and they do not coincide for large bodies like planets