COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Complex Numbers (z) : General form $\boxed{z = a + ib}$ (real numbers + imaginary number)

(real part) Re z

(imaginary part) Im z

a, b = real numbers

Note : Two complex numbers $z = a + ib$ and $z = c + id$ are equal if $a = c$ and $b = d$

Algebra of complex numbers :

1. Addition of two complex numbers :

(a) The closure law : $\boxed{z_1 + z_2}$ z_1, z_2 = two complex no.

(b) The commutative law : $\boxed{z_1 + z_2 = z_2 + z_1}$

(c) The associative law : $\boxed{(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)}$

(d) The existence of additive identity : $0 + i0$ denoted as 0 (zero complex no.) $\boxed{z + 0 = z}$ additive identity

(e) The existence of additive inverse : $-a + i(-b)$ denoted as $-z$ (negative of z) $\boxed{z + (-z) = 0}$ additive inverse

2. Difference of two complex numbers : $\boxed{z_1 - z_2 = z_1 + (-z_2)}$

3. Multiplication of two complex numbers : Let $z_1 = a + ib$ and $z_2 = c + id$, then, the product $z_1 z_2$ is $z_1 z_2 = (ac - bd) + i(ad + bc)$

(a) The closure law : $\boxed{z_1 z_2}$ z_1, z_2 = two complex no.

(b) The commutative law : $\boxed{z_1 z_2 = z_2 z_1}$

(c) The associative law : $\boxed{(z_1 z_2) z_3 = z_1 (z_2 z_3)}$

(d) The existence of multiplicative identity : $1 + i0$ denoted as 1 $\boxed{z \cdot 1 = z}$ multiplicative identity

(e) The existence of multiplicative inverse : $\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$ denoted as $\frac{1}{z}$ or z^{-1} $\boxed{z \cdot \frac{1}{z} = 1}$ multiplicative inverse

(f) The distribution law : (a) $\boxed{z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3}$

(b) $\boxed{(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3}$ z_1, z_2, z_3 = three complex no.

4. Division of two complex numbers : $\boxed{\frac{z_1}{z_2} = z_1 \frac{1}{z_2}}$ $z_2 \neq 0$

Power of i : $\boxed{i = \sqrt{-1}}$ $\boxed{i^2 = -1}$ $\boxed{i^3 = -i}$ $\boxed{i^4 = 1}$ $\boxed{i^5 = i}$ $\boxed{i^6 = -1}$

Note : Any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$

Identities

$\boxed{(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1 z_2}$

$\boxed{(z_1 - z_2)^2 = z_1^2 + z_2^2 - 2z_1 z_2}$

$\boxed{(z_1 + z_2)^3 = z_1^3 + 3z_1^2 z_2 + 3z_1 z_2^2 + z_2^3}$

$\boxed{(z_1 - z_2)^3 = z_1^3 - 3z_1^2 z_2 + 3z_1 z_2^2 - z_2^3}$

$\boxed{z_1^2 - z_2^2 = (z_1 + z_2)(z_1 - z_2)}$

Modulus : Let $z = a + ib$

Modulus of z $\boxed{|z| = \sqrt{a^2 + b^2}}$

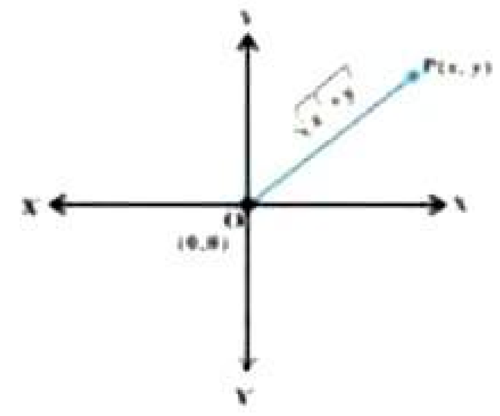
Conjugate : Let $z = a + ib$

conjugate of z $\boxed{\bar{z} = a - ib}$

Note : (a) $\boxed{|z_1 z_2| = |z_1||z_2|}$ (b) $\boxed{\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}}$ (c) $\boxed{\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}}$ $z_2 \neq 0$

(d) $\boxed{\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2}$ (e) $\boxed{\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2}$ (f) $\boxed{z\bar{z} = |z|^2}$

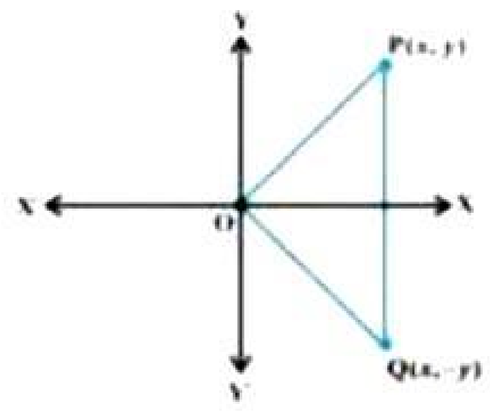
Argand Plane : The Plane having a complex number assigned to each of its point is called the complex plane on the argand plane.



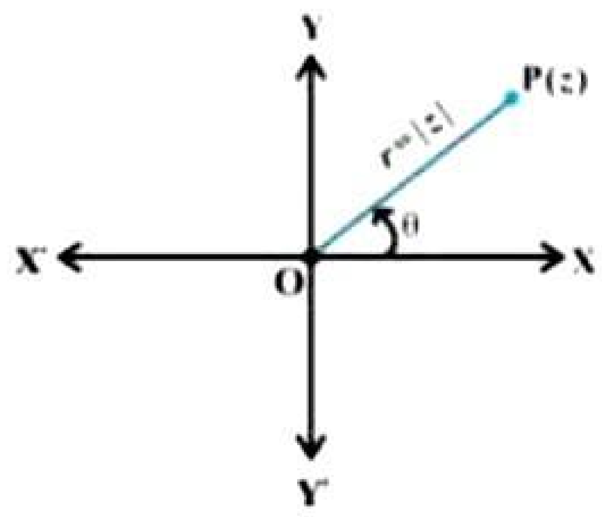
$x + iy = \sqrt{x^2+y^2}$ is the distance between the point $P(x, y)$ and the origin $O\ (0,0)$.

The x-axis and y-axis in the argand plane, respectivelly, the real axis and the imaginary axis.

The point $(x, -y)$ is the mirror image of the point (x, y) on the real axis.



Polar form of the complex no. : Let the point P represent the non-zero complex no. $z = x + iy$



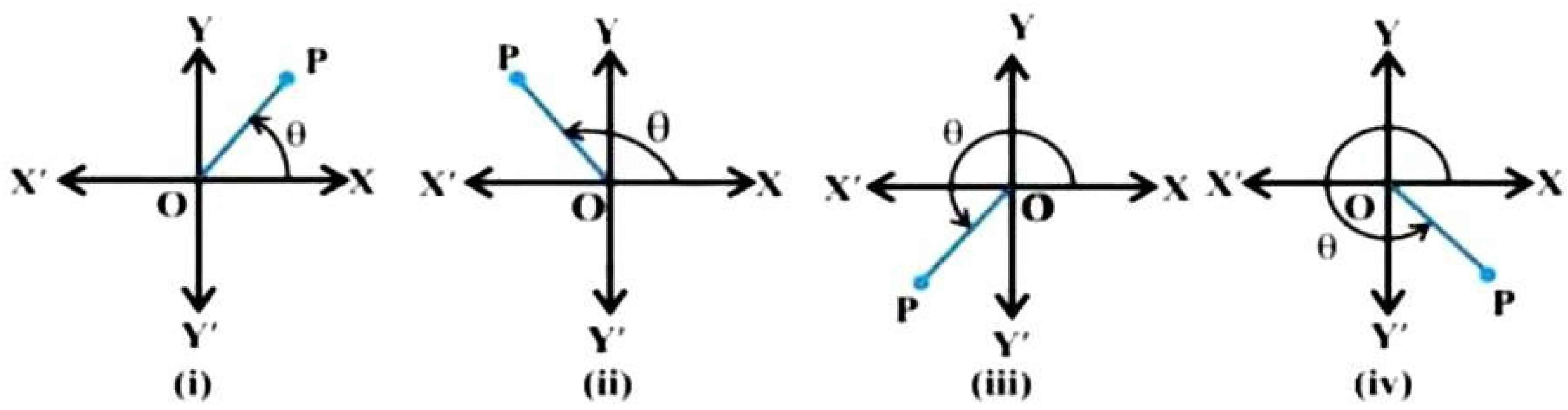
$z = r(\cos\theta + i\sin\theta)$ where $x = r\cos\theta$, $y = r\sin\theta$

$r = \sqrt{x^2+y^2} = |z|$ (modulus of z)

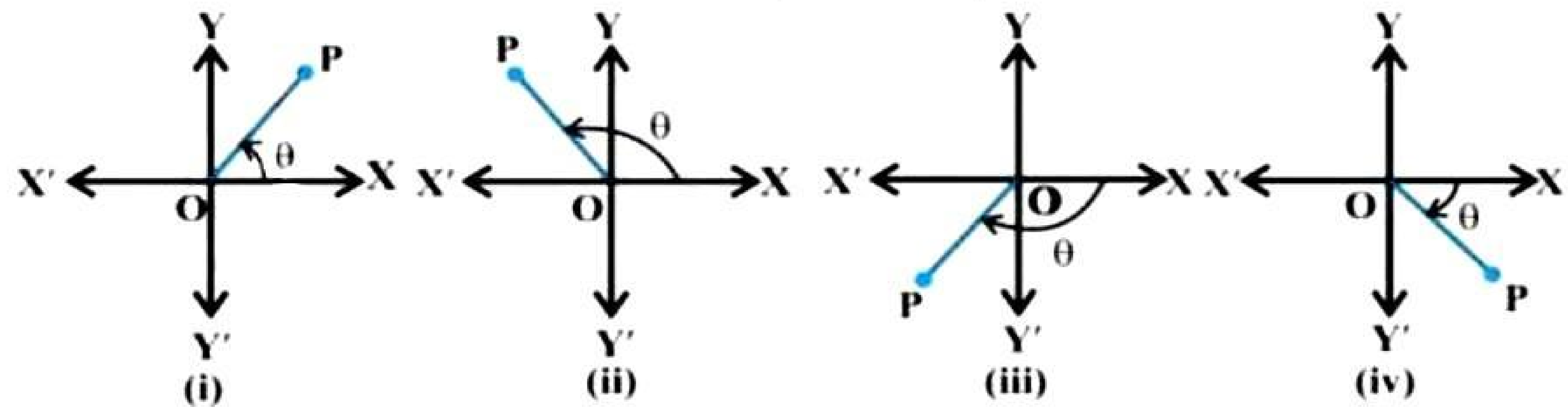
θ = arguement of z ($\arg z$)

For any complex no. $z \neq 0$, there corresponds only one value of θ in $0 \le \theta < 2\pi$

The value of θ, such that $-\pi < \theta \le \pi$ is called the principal arguement of z



$(0 \le \theta < 2\pi)$



$(-\pi < \theta \le \pi)$

Quadratic Equations $ax^2 + bx + c = 0$ where $a, b, c \in R$, $a \neq 0$, $b^2 - 4ac < 0$

then, the solution of the quadratic equation is,

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{4ac-b^2}\, i}{2a}}$$

Note : A polynomial equation has at least one root.

Note : A polynomial equation of degree n has n roots.