

Determinants

Question1

$$\text{Let } A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, B = [B_1, B_2, B_3], \text{ where } B_1, B_2, B_3 \text{ are column matrices, and } AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B , then $\alpha^3 + \beta^3$ is equal to

[27-Jan-2024 Shift 1]

Options:

Answer: 28

Solution:

$$x_1 = 1, y_1 = -1, z_1 = -1$$

$$AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$$x_2 = 2, y_2 = 1, z_2 = -2$$

$$AB_3 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$x_3 = 2, y_3 = 0, z_3 = -1$$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$\alpha = |B| = 3$$

$$\beta = 1$$

$$\alpha^3 + \beta^3 = 27 + 1 = 28$$

Question2

The values of α , for which
$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$
, lie in the interval

[27-Jan-2024 Shift 2]

Options:

A.

$(-2, 1)$

B.

$(-3, 0)$

C.

$\left(-\frac{3}{2}, \frac{3}{2}\right)$

D.

$(0, 3)$

Answer: B

Solution:

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2\alpha + 3) \left\{ \frac{7\alpha}{6} \right\} - (3\alpha + 1) \left\{ \frac{-7}{6} \right\} = 0$$

$$\Rightarrow (2\alpha + 3) \cdot \frac{7\alpha}{6} + (3\alpha + 1) \cdot \frac{7}{6} = 0$$

$$\Rightarrow 2\alpha^2 + 3\alpha + 3\alpha + 1 = 0$$

$$\Rightarrow 2\alpha^2 + 6\alpha + 1 = 0$$

$$\Rightarrow \alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$$

Hence option (2) is correct.

Question3

Let for any three distinct consecutive terms a, b, c of an A.P, the lines $ax + by + c = 0$ be concurrent at the point P and $Q(\alpha, \beta)$ be a point such that the system of equations

$$x + y + z = 6,$$

$$2x + 5y + az = \beta \text{ and}$$

$$x + 2y + 3z = 4,$$

has infinitely many solutions. Then $(PQ)^2$ is equal to ____

[29-Jan-2024 Shift 2]

Answer: 113

Solution:

$\because a, b, c$ are in A.P

$$\Rightarrow 2b = a + c \Rightarrow a - 2b + c = 0$$

$\therefore ax + by + c$ passes through fixed point $(1, -2)$

$$\therefore P = (1, -2)$$

For infinite solution,

$$D = D_1 = D_2 = D_3 = 0$$

$$D : \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow a = 8$$

$$D_1 : \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & a \\ 4 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \beta = 6$$

$$\therefore Q = (8, 6)$$

$$\therefore Q^2 = 113$$

Question4

If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to

[30-Jan-2024 Shift 1]

Options:

A.

0

B.

1

C.

2

D.

6

Answer: B

Solution:

$$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^4 x & \sin^2 2x \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$$

$$f(x) = 45$$

$$f'(x) = 0$$

Question5

Consider the system of linear equation $x + y + z = 4\mu$, $x + 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$ where $\lambda, \mu \in \mathbb{R}$. Which one of the following statements is NOT correct?

[30-Jan-2024 Shift 1]

Options:

A.

The system has unique solution if $\lambda \neq 1/2$ and $\mu \neq 1, 15$

B.

The system is inconsistent if $\lambda = 1/2$ and $\mu \neq 1$

C.

The system has infinite number of solutions if $\lambda = 1/2$ and $\mu = 15$

D.

The system is consistent if $\lambda \neq 1/2$ **Answer: B****Solution:**

$$x + y + z = 4\mu, x + 2y + 2\lambda z = 10\mu, x + 3y + 4\lambda^2 z = \mu^2 + 15$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda - 1)^2$$

For unique solution $\Delta \neq 0, 2\lambda - 1 \neq 0, \left(\lambda \neq \frac{1}{2}\right)$

$$\text{Let } \Delta = 0, \lambda = \frac{1}{2}$$

$$\begin{aligned} \Delta_y = 0, \Delta_x = \Delta_z &= \begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2 + 15 & 3 & 1 \end{vmatrix} \\ &= (\mu - 15)(\mu - 1) \end{aligned}$$

For infinite solution $\lambda = \frac{1}{2}, \mu = 1 \text{ or } 15$ **Question6****Consider the system of linear equations**

$$x + y + z = 5, x + 2y + \lambda^2 z = 9$$

 $x + 3y + \lambda z = \mu$, where $\lambda, \mu \in \mathbb{R}$. Then, which the following statement is NOT correct?**[30-Jan-2024 Shift 2]****Options:**

A.

System has infinite number of solution if $\lambda =$ and $\mu = 13$

B.

System is inconsistent if $\lambda = 1$ and $\mu \neq 13$

C.

System is consistent if $\lambda \neq 1$ and $\mu = 13$

D.

System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Answer: D

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$$

$$\lambda = 1, -\frac{1}{2}$$

$$\begin{vmatrix} 1 & 1 & 5 \\ 2 & \lambda^2 & 9 \\ 3 & \lambda & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$$

Infinite solution $\lambda = 1$ & $\mu = 13$

For unique solⁿ $\lambda \neq 1$

For no solⁿ $\lambda = 1$ & $\mu \neq 13$

If $\lambda \neq 1$ and $\mu \neq 13$

Considering the case when $\lambda = -\frac{1}{2}$ and $\mu \neq 13$ this will generate no solution case

Question7

If the system of linear equations

$$x - 2y + z = -4$$

$$2x + \alpha y + 3z = 5$$

$$3x - y + \beta z = 3$$

has infinitely many solutions, then $12\alpha + 13\beta$ is equal to

[31-Jan-2024 Shift 1]

Options:

A.

60

B.

64

C.

54

D.

58

Answer: D

Solution:

$$D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$$

$$= 1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha)$$

$$= \alpha\beta + 3 + 4\beta - 18 - 2 - 3\alpha$$

For infinite solutions $D = 0$, $D_1 = 0$, $D_2 = 0$ and

$$D_3 = 0$$

$$D = 0$$

$$\alpha\beta - 3\alpha + 4\beta = 17 \dots\dots\dots (1)$$

$$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 1(5\beta - 9) + 4(2\beta - 9) + 1(6 - 15) = 0$$

$$13\beta - 9 - 36 - 9 = 0$$

$$13\beta = 54, \beta = \frac{54}{13} \text{ put in (1)}$$

$$\frac{54}{13}\alpha - 3\alpha + 4\left(\frac{54}{13}\right) = 17$$

$$54\alpha - 39\alpha + 216 = 221$$

$$15\alpha = 5 \quad \alpha = \frac{1}{3}$$

$$\text{Now, } 12\alpha + 13\beta = 12 \cdot \frac{1}{3} + 13 \cdot \frac{54}{13}$$

$$= 4 + 54 = 58$$

Question8

$$\text{If } f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix} \text{ for all } x \in \mathbb{R}, \text{ then } 2f(0) + f'(0) \text{ is equal to}$$

[31-Jan-2024 Shift 1]

Options:

A.

48

B.

24

C.

42

D.

18

Answer: C

Solution:

$$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = 12$$

$$f'(x) = \begin{vmatrix} 3x^2 & 4x & 3 \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 6x & 2 & 3x^2 \\ x^3-x & 4 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ 3x^2-1 & 0 & 2x \end{vmatrix}$$

$$\therefore f'(0) = \begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= 24 - 6 = 18$$

$$\therefore 2f(0) + f'(0) = 42$$

Question9

Let A be a 3×3 real matrix such that

$$A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, A \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = 4 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\text{Then, the system } (A - 3I) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ has}$$

[31-Jan-2024 Shift 2]

Options:

A.

unique solution

B.

exactly two solutions

C.

no solution

D.

infinitely many solutions

Answer: A

Solution:

$$\text{Let } A = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$\text{Given } A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \dots\dots(1)$$

$$\therefore \begin{bmatrix} x_1 + z_1 \\ x_2 + z_2 \\ x_3 + z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\therefore x_1 + z_1 = 2 \dots\dots\dots(2)$$

$$x_2 + z_2 = 0 \dots\dots\dots(3)$$

$$x_3 + z_3 = 0 \dots\dots\dots(4)$$

$$\text{Given } A \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -x_1 + z_1 \\ -x_2 + z_2 \\ -x_3 + z_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow -x_1 + z_1 = -4 \dots\dots\dots(5)$$

$$-x_2 + z_2 = 0 \dots\dots\dots(6)$$

$$-x_3 + z_3 = 4 \dots\dots\dots(7)$$

$$\text{Given } A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\therefore y_1 = 0, y_2 = 2, y_3 = 0$$

$$\therefore \text{ from (2), (3), (4), (5), (6) and (7)}$$

$$x_1 = 3x, x_2 = 0, x_3 = -1$$

$$y_1 = 0, y_2 = 2, y_3 = 0$$

$$z_1 = -1, z_2 = 0, z_3 = 3$$

$$\therefore A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$\therefore \text{ Now } (A - 3I) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -z \\ -y \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$[z = -1], [y = -2], [x = -3]$$

Question10

If the system of equations

$$2x + 3y - z = 5$$

$$x + \alpha y + 3z = -4$$

$$3x - y + \beta z = 7$$

has infinitely many solutions, then $13\alpha\beta$ is equal to

[1-Feb-2024 Shift 1]

Options:

A.

1110

B.

1120

C.

1210

D.

1220

Answer: B

Solution:

Using family of planes

$$2x + 3y - z - 5 = k_1(x + \alpha y + 3z + 4) + k_2(3x - y + \beta z - 7)$$

$$2 = k_1 + 3k_2, 3 = k_1\alpha - k_2, -1 = 3k_1 + \beta k_2, -5 = 4k_1 - 7k_2$$

On solving we get

$$k_2 = \frac{13}{19}, k_1 = \frac{-1}{19}, \alpha = -70, \beta = \frac{-16}{13}$$

$$13\alpha\beta = 13(-70)\left(\frac{-16}{13}\right)$$

$$= 1120$$

Question11

Let the system of equations $x + 2y + 3z = 5, 2x + 3y + z = 9, 4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to :

[1-Feb-2024 Shift 2]

Options:

A.

28

B.

17

C.

22

D.

15

Answer: B

Solution:

$$x + 2y + 3z = 5$$

$$2x + 3y + z = 9$$

$$4x + 3y + \lambda z = \mu$$

for infinite following $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -13$$

$$\Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix} = 0 \Rightarrow \mu = 15$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 9 & 1 \\ 4 & 15 & -13 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = 0$$

for $\lambda = -13, \mu = 15$ system of equation has infinite solution hence $\lambda + 2\mu = 17$

Question12

If the system of equations

$$x + 2y + 3z = 3$$

$$4x + 3y - 4z = 4$$

$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to
[24-Jan-2023 Shift 2]

Options:

A. $\left(\frac{72}{5}, \frac{21}{5} \right)$

B. $\left(\frac{-72}{5}, \frac{-21}{5} \right)$

C. $\left(\frac{72}{5}, \frac{-21}{5} \right)$

D. $\left(\frac{-72}{5}, \frac{21}{5} \right)$

Answer: C

Solution:

Solution:

$$x + 2y + 3z = 3 \dots (i)$$

$$4x + 3y - 4z = 4 \dots (ii)$$

$$8x + 4y - \lambda z = 9 + \mu \dots (iii)$$

$$(i) \times 4 - (ii) \Rightarrow 5y + 16z = 8 \dots (iv)$$

$$(ii) \times 2 - (iii) \Rightarrow 2y + (\lambda - 8)z = -1 - \mu \dots (v)$$

$$(iv) \times 2 - (v) \times 5 \Rightarrow (32 - 5(\lambda - 8))z = 16 - 5(-1 - \mu)$$

$$\text{For infinite solutions } \Rightarrow 72 - 5\lambda = 0 \Rightarrow \lambda = \frac{72}{5}$$

$$21 + 5\mu = 0 \Rightarrow \mu = \frac{-21}{5}$$

$$\Rightarrow (\lambda, \mu) \equiv \left(\frac{72}{5}, \frac{-21}{5} \right)$$

Question 13

Let S_1 and S_2 be respectively the sets of all $a \in \mathbb{R} - \{0\}$ for which the system of linear equations

$$ax + 2ay - 3az = 1$$

$$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$$

$$(3a + 5)x + (a + 5)y + (a + 2)z = 3$$

has unique solution and infinitely many solutions. Then
[25-Jan-2023 Shift 1]

Options:

A. $n(S_1) = 2$ and S_2 is an infinite set

B. S_1 is an infinite set and $n(S_2) = 2$

C. $S_1 = \Phi$ and $S_2 = \mathbb{R} - \{0\}$

D. $S_1 = \mathbb{R} - \{0\}$ and $S_2 = \Phi$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$$= a(15a^2 + 31a + 36) = 0 \Rightarrow a = 0$$

$$\Delta \neq 0 \text{ for all } a \in \mathbb{R} - \{0\}$$

$$\text{Hence } S_1 = \mathbb{R} - \{0\} \quad S_2 = \Phi$$

Question14

Consider the following system of questions

$$\alpha x + 2y + z = 1$$

$$2\alpha x + 3y + z = 1$$

$$3x + \alpha y + 2z = \beta$$

For some $\alpha, \beta \in \mathbb{R}$. Then which of the following is NOT correct.
[29-Jan-2023 Shift 1]

Options:

- A. It has no solution if $\alpha = -1$ and $\beta \neq 2$
- B. It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$
- C. It has no solution for $\alpha = 3$ and for all $\beta \neq 2$
- D. It has a solution for all $\alpha \neq -1$ and $\beta = 2$

Answer: B

Solution:

Solution:

$$D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = 0 \Rightarrow \alpha = -1, 3$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ \alpha & 2 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 2$$

$$D_y = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & 2 & \beta \end{vmatrix} = 0$$

$$D_z = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix} = 0$$

$$\beta = 2, \alpha = -1$$

$$\alpha = -1, \beta = 2 \text{ Infinite solution}$$

Question15

Let the system of linear equations

$$x + y + kz = 2$$

$$2x + 3y - z = 1$$

$$3x + 4y + 2z = k$$

have infinitely many solutions. Then the system

$(k + 1)x + (2k - 1)y = 7$
 $(2k + 1)x + (k + 5)y = 10$ has:
[30-Jan-2023 Shift 1]

Options:

- A. infinitely many solutions
- B. unique solution satisfying $x - y = 1$
- C. no solution
- D. unique solution satisfying $x + y = 1$

Answer: D

Solution:

Solution:

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$\Rightarrow 1(10) - 1(7) + k(-1) = 0$
 $\Rightarrow k = 3$
For $k = 3$, 2nd system is
 $4x + 5y = 7 \dots (1)$
and $7x + 8y = 10 \dots (2)$
Clearly, they have a unique solution
 $(2) - (1) \Rightarrow 3x + 3y = 3$
 $\Rightarrow x + y = 1$

Question16

For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$$\begin{aligned} x - y + z &= 5 \\ 2x + 2y + \alpha z &= 8 \\ 3x - y + 4z &= \beta \end{aligned}$$

has infinitely many solutions. Then α and β are the roots of
[30-Jan-2023 Shift 2]

Options:

- A. $x^2 - 10x + 16 = 0$
- B. $x^2 + 18x + 56 = 0$
- C. $x^2 - 18x + 56 = 0$
- D. $x^2 + 14x + 24 = 0$

Answer: C

Solution:

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0; 8 + \alpha - 2(-4 + 1) + 3(-\alpha - 2) = 0$$

$$8 + \alpha + 6 - 3\alpha - 6 = 0$$

$$\alpha = 4$$

Question17

For the system of linear equations

$$x + y + z = 6$$

$$\alpha x + \beta y + 7z = 3$$

$$x + 2y + 3z = 14$$

which of the following is NOT true?

[31-Jan-2023 Shift 1]

Options:

A. If $\alpha = \beta = 7$, then the system has no solution

B. If $\alpha = \beta$ and $\alpha \neq 7$ then the system has a unique solution.

C. There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

D. For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions.

Answer: D

Solution:

By equation 1 and 3

$$y + 2z = 8$$

$$y = 8 - 2z$$

$$\text{And } x = -2 + z$$

Now putting in equation 2

$$\alpha(z - 2) + \beta(-2z + 8) + 7z = 3$$

$$\Rightarrow (\alpha - 2\beta + 7)z = 2\alpha - 8\beta + 3$$

So equations have unique solution if

$$\alpha - 2\beta + 7 \neq 0$$

And equations have no solution if

$$\alpha - 2\beta + 7 = 0 \text{ and } 2\alpha - 8\beta + 3 \neq 0$$

And equations have infinite solution if

$$\alpha - 2\beta + 7 = 0 \text{ and } 2\alpha - 8\beta + 3 = 0$$

Question18

Let S denote the set of all real values of λ such that the system of equations

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

is inconsistent, then $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$ is equal to
[1-Feb-2023 Shift 1]

Options:

- A. 2
- B. 12
- C. 4
- D. 6

Answer: D

Solution:

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda + 2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda + 2)[1(\lambda^2 - 1) - 1(\lambda - 1) + (1 - \lambda)] = 0$$

$$(\lambda + 2)(\lambda^2 - 2\lambda + 1) = 0.$$

$$(\lambda + 2)(\lambda - 1)^2 = 0 \Rightarrow \lambda = -2, \lambda = 1$$

at $\lambda = 1$ system has infinite solution, for inconsistent $\lambda = -2$
 so $\sum (|-2|^2 + |-2|) = 6$

Question19

For the system of linear equations $\alpha x + y + z = 1$, $x + \alpha y + z = 1$, $x + y + \alpha z = \beta$, which one of the following statements is NOT correct?

[1-Feb-2023 Shift 2]

Options:

- A. It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$
- B. It has no solution if $\alpha = -2$ and $\beta = 1$
- C. $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$
- D. It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

Answer: A

Solution:

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$$\alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha) = 0$$

$$\alpha^3 - 3\alpha + 2 = 0$$

$$\alpha^2(\alpha - 1) + \alpha(\alpha - 1) - 2(\alpha - 1) = 0$$

$$(\alpha - 1)(\alpha^2 + \alpha - 2) = 0$$

$$\alpha = 1, \alpha = -2, 1$$

$$\text{For } \alpha = 1, \beta = 1$$

$$x + y + z = 1$$

$$x + y + z = b \} \text{ infinite solution}$$

$$\text{For } \alpha = 2, \beta = 1$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 3 - 1 - 1 \Rightarrow x = \frac{1}{4}$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 2 - 1 = 1 \Rightarrow y = \frac{1}{4}$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2 - 1 = 1 \Rightarrow z = \frac{1}{4}$$

$$\text{For } \alpha = 2 \Rightarrow \text{unique solution}$$

Question20

If the system of equations

$$x + y + az = b$$

$$2x + 5y + 2z = 6$$

$$x + 2y + 3z = 3$$

has infinitely many solutions, then $2a + 3b$ is equal to :

[6-Apr-2023 shift 1]

Options:

A. 28

B. 20

C. 25

D. 23

Answer: D

Solution:

Solution:

$$x + y + az = b$$

$$2x + 5y + 2z = 6$$

$$x + 2y + 3z = 3$$

For ∞ solution

$$\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11 - 4 - a = 0 \Rightarrow a = 7$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 3 - 0 - b = 0 \Rightarrow b = 3$$

Hence $2a + 3b = 23$

Question 21

For the system of equations

$$\mathbf{x} + \mathbf{y} + \mathbf{z} = \mathbf{6}$$

$$\mathbf{x} + 2\mathbf{y} + \alpha\mathbf{z} = \mathbf{10}$$

$\mathbf{x} + 3\mathbf{y} + 5\mathbf{z} = \beta$, which one of the following is NOT true :
[6-Apr-2023 shift 2]

Options:

- A. System has a unique solution for $\alpha = 3, \beta \neq 14$.
- B. System has a unique solution for $\alpha = -3, \beta = 14$.
- C. System has no solution for $\alpha = 3, \beta = 24$.
- D. System has infinitely many solutions for $\alpha = 3, \beta = 14$.

Answer: A

Solution:

Solution:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix}$$

$$= (10 - 3\alpha) - (5 - \alpha) + (3 - 2)$$

$$= 6 - 2\alpha$$

$$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & \alpha \\ \beta & 3 & 5 \end{vmatrix}$$

$$= 6(10 - 3\alpha) - (50 - \alpha 13) + (30 - 2\beta)$$

$$= 40 - 18\alpha + \alpha\beta - 2\beta$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & \alpha \\ 1 & \beta & 5 \end{vmatrix}$$

$$= (50 - \alpha\beta) - 6(5 - \alpha) + (\beta - 10)$$

$$= 10 + 6\alpha + \beta - \alpha\beta$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 3 & \beta \end{vmatrix}$$

$$= (2\beta - 30) - (\beta - 10) + 6(1)$$

$$= \beta - 14$$

for Infinite solution

$$\Delta = 0, \Delta_x = \Delta_y = \Delta_z = 0$$

$x\alpha = 3, x\beta = 14$
For unique solution $\alpha \neq 3$
Ans. Option 1

Question22

Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$$x + y + \sqrt{3}z = 0$$

$$-x + (\tan \theta)y + \sqrt{7}z = 0$$

$$x + y + (\tan \theta)z = 0$$

has non-trivial solution. Then $\frac{120}{\pi} \sum_{\theta \in S} \theta$ is equal to

[8-Apr-2023 shift 2]

Options:

A. 20

B. 40

C. 30

D. 10

Answer: A

Solution:

Solution:

For non trivial solutions
 $D = 0$

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan \theta & \sqrt{7} \\ 1 & 1 & \tan \theta \end{vmatrix} = 0$$

$$\tan^2 \theta - (\sqrt{3} - 1) - \sqrt{3} = 0$$

$$\tan \theta = \sqrt{3}, -1$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{-2\pi}{3}, \frac{-\pi}{4}, \frac{3\pi}{4} \right\}$$

$$\frac{120}{\pi}(\sum \theta) = \frac{120}{\pi} \times \frac{\pi}{6} = 20 \text{ (Option 1)}$$

Question23

For the system of linear equations

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \alpha z = \beta$$

which of the following is NOT correct ?

[10-Apr-2023 shift 1]

Options:

- A. The system is inconsistent for $\alpha = -5$ and $\beta = 8$
- B. The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$
- C. The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$
- D. The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

Answer: B

Solution:

Solution:

Given

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \alpha z = \beta$$

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix} = 7\alpha + 35$$

$$\Delta = 7(\alpha + 5)$$

For unique solution $\Delta \neq 0$

$$\alpha \neq -5$$

For inconsistent & Infinite solution

$$\Delta = 0$$

$$\alpha + 5 = 0 \Rightarrow \alpha = -5$$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 7 & 2 & -1 \\ \beta & 5 & -5 \end{vmatrix} = -5(\beta - 9)$$

$$\Delta_2 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 7 & -1 \\ 4 & \beta & -5 \end{vmatrix} = 11(\beta - 9)$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 7 \\ 4 & 5 & \beta \end{vmatrix}$$

$$\Delta_3 = 7(\beta - 9)$$

For Inconsistent system :-

At least one Δ_1, Δ_2 & Δ_3 is not zero $\alpha = -5, \beta = 8$ option (A) True Infinite solution:

$$\Delta_1 = \Delta_2 = \Delta_3 = 0$$

From here $\beta - 9 = 0 \Rightarrow \beta = 9$ $\alpha = -5$ & option (D) True

$$\beta = 9$$

Unique solution

$\alpha \neq -5, \beta = 8 \rightarrow$ option (C) True

Option (B) False

For Infinitely many solution α must be -5 .

Question24

Let S be the set of values of λ , for which the system of equations $6\lambda x - 3y + 3z = 4\lambda^2$ $2x + 6\lambda y + 4z = 1$ $3x + 2y + 3\lambda z = \lambda$ has no solution.

Then $12 \sum_{\lambda \in S} |\lambda|$ is equal to _____.

[10-Apr-2023 shift 2]

Answer: 24

Solution:

Solution:

$$\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0$$

$$2\lambda(9\lambda^2 - 4) + (3\lambda - 6) + (2 - 9\lambda) = 0$$

$$18\lambda^3 - 14\lambda - 4 = 0$$

$$(\lambda - 1)(3\lambda + 1)(3\lambda + 2) = 0$$

$$\Rightarrow \lambda = 1, -1/3, -2/3$$

$$\text{For each values of } \lambda, \Delta_1 = \begin{vmatrix} 6\lambda & -3 & 4\lambda^2 \\ 2 & 6\lambda & 1 \\ 3 & 2 & \lambda \end{vmatrix} \neq 0$$

$$12 \left(1 + \frac{1}{3} + \frac{2}{3} \right) = 24$$

Question25

Let A be a 2×2 matrix with real entries such that $A' = \alpha A + I$, where $\alpha \in \mathbb{R} - \{-1, 1\}$. If $\det(A^2 - A) = 4$, then the sum of all possible values of α is equal to :

[11-Apr-2023 shift 1]

Options:

A. 0

B. $\frac{5}{2}$

C. 2

D. $\frac{3}{2}$

Answer: B

Solution:

Solution:

$$A^T = \alpha A + I$$

$$A = \alpha A^T + I$$

$$A = \alpha(\alpha A + I) + I$$

$$A = \alpha^2 A + (\alpha + 1)I$$

$$A(1 - \alpha^2) = (\alpha + 1)I$$

$$A = \frac{I}{1 - \alpha} \dots (1)$$

$$|A| = \frac{1}{(1 - \alpha)^2} \dots (2)$$

$$|A^2 - A| = |A| |A - I| \dots (3)$$

$$A - I = \frac{I}{1 - \alpha} - I = \frac{\alpha}{1 - \alpha} I$$

$$|A - I| = \left(\frac{\alpha}{1 - \alpha} \right)^2 \dots (4)$$

$$\text{Now } |A^2 - A| = 4$$

$$|A| |A - I| = 4$$

$$\Rightarrow \frac{1}{(1 - \alpha)^2} \frac{\alpha^2}{(1 - \alpha^2)} = 4$$

$$\Rightarrow \frac{\alpha}{(1 - \alpha)^2} = \pm 2$$

$$\Rightarrow 2(1 - \alpha)^2 = \pm \alpha$$

$$(C_1) 2(1 - \alpha)^2 = \alpha \quad (C_2) 2(1 - \alpha)^3 = -\alpha$$

$$\alpha_1 + \alpha_2 = \frac{5}{2}$$

Question26

If the system of linear equations

$$7x + 11y + \alpha z = 13$$

$$5x + 4y + 7z = \beta$$

$$175x + 194y + 57z = 361$$

has infinitely many solutions, then $\alpha + \beta + 2$ is equal to:

[11-Apr-2023 shift 2]

Options:

A. 3

B. 6

C. 5

D. 4

Answer: D

Solution:

Solution:

$$7x + 11y + \alpha z = 13$$

$$5x + 4y + 7z = \beta$$

$$175x + 194y + 57z = 361$$

4 sc condition of Infinite Many solution

$\Delta = 0$ & $\Delta x, \Delta y, \Delta z = 0$ check.

After solving we get $\alpha + 13 + 2 = 4$

Question27

If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x + 81)$, then $\lambda, \frac{\lambda}{3}$ are the roots of the equation

[11-Apr-2023 shift 2]

Options:

A. $4x^2 - 24x - 27 = 0$

B. $4x^2 + 24x + 27 = 0$

C. $4x^2 - 24x + 27 = 0$

D. $4x^2 + 24x - 27 = 0$

Answer: C

Solution:

Solution:

$$\begin{vmatrix} x+1 & x & x \\ x & x+d & x \\ x & x & x+d^2 \end{vmatrix} = \frac{9}{8}(103x+81)$$

Put $x = 0$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$$\lambda^3 = \frac{9^3}{8}$$

$$\lambda = \frac{9}{2}$$

$$\frac{\lambda}{3} = \frac{9}{2 \times 3} \Rightarrow \frac{3}{2}$$

$$\frac{\lambda}{3} = \frac{3}{2}$$

Option (C) $4x^2 - 24x + 27 = 0$

has Root $\frac{3}{2}, \frac{9}{2}$

Question28

Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^n D_k = 96$, then n is equal to

[12-Apr-2023 shift 1]

Answer: 6

Solution:

Solution:

$$D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$$\sum_{k=1}^n D_k = 96 \Rightarrow$$

$$\begin{vmatrix} \sum_{k=1}^n 1 & \sum 2k & \sum (2k-1) \\ n & n^2 + n + 2 & n^2 \\ n & n^2 + n & n^2 + n + 2 \end{vmatrix} = 96$$

$$\Rightarrow \begin{vmatrix} n & n^2 + n & n^2 \\ n & n^2 + n + 2 & n^2 \\ n & n^2 + n & n^2 + n + 2 \end{vmatrix} = 96$$

$$R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} n & n^2 + n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n + 2 \end{vmatrix} = 96$$

$$\Rightarrow n(2n + 4) = 96 \Rightarrow n(n + 2) = 48 \Rightarrow n = 6$$

Question29

For the system of linear equations

$$2x + 4y + 2az = b$$

$$x + 2y + 3z = 4$$

$$2x - 5y + 2z = 8$$

which of the following is NOT correct?

[13-Apr-2023 shift 1]

Options:

A. It has infinitely many solutions if $a = 3, b = 8$

B. It has unique solution if $a = b = 8$

C. It has unique solution if $a = b = 6$

D. It has infinitely many solutions if $a = 3, b = 6$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & 4 & 2a \\ 1 & 2 & 3 \\ 2 & -5 & 2 \end{vmatrix} = 18(3 - a)$$

$$\Delta_x = \begin{vmatrix} b & 4 & 2a \\ 4 & 2 & 3 \\ 8 & -5 & 2 \end{vmatrix} = (64 + 19b - 72a)$$

For unique solution $\Delta \neq 0$

$\Rightarrow a \neq 3$ and $b \in \mathbb{R}$

For infinitely many solution :

$$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$$

$$\Rightarrow a = 3 \quad \because \Delta = 0$$

and $b = 8 \quad \therefore \Delta_x = 0$

Question30

If the system of equations

$$2x + y - z = 5$$

$$2x - 5y + \lambda z = \mu$$

$$x + 2y - 5z = 7$$

has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to
[13-Apr-2023 shift 2]

Options:

A. 904

B. 916

C. 912

D. 920

Answer: B

Solution:

Solution:

$$\Delta = 0$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$$\Rightarrow 2(25 - 2\lambda) - 1(-10 - \lambda) - 1(4 + 5) = 0$$

$$\Rightarrow 51 - 3\lambda = 0$$

$$\Rightarrow \lambda = 17$$

$$\Delta_x = 0$$

$$\begin{vmatrix} 5 & 1 & -1 \\ \mu & -5 & 17 \\ 7 & 2 & -5 \end{vmatrix} = 0$$

$$\Rightarrow 5(25 - 34) - 1(-5\mu - 119) - 1(2\mu + 35) = 0$$

$$\Rightarrow -45 + 5\mu + 119 - 2\mu - 35 = 0$$

$$\Rightarrow 39 + 3\mu = 0 \Rightarrow \mu = -13$$

$$(\lambda + \mu)^2 + (\lambda - \mu)^2 = 4^2 + (30)^2$$

$$= 916$$

Question31

Let the system of linear equations

$$-x + 2y - 9z = 7$$

$$-x + 3y + 7z = 9$$

$$-2x + y + 5z = 8$$

$$-3x + y + 13z = \lambda$$

has a unique solution $x = \alpha$, $y = \beta$, $z = \gamma$. Then the distance of the point (α, β, γ) from the plane $2x - 2y + z = \lambda$ is

[15-Apr-2023 shift 1]

Options:

- A. 7
- B. 9
- C. 13
- D. 11

Answer: A

Solution:

Solution:

$$-x + 2y - 9z = 7 \quad (1)$$

$$-x + 3y - 7z = 9 \quad (2)$$

$$-2x + y + 5z = 8 \quad (3)$$

$$(2) - (1)$$

$$y + 16z = 2 \quad (4)$$

$$(3) - 2 \times (1)$$

$$-3y + 23z = -6 \quad (5)$$

$$3 \times (4) + (5)$$

$$71z = 0 \Rightarrow z = 0$$

$$y = 2$$

$$x = -3$$

$$(-3, 2, 0) \rightarrow (\alpha, \beta, \gamma)$$

$$\text{Put in } -3x + y + 13z = 1$$

$$\lambda = 9 + 2 = 11$$

$$d = \left| \frac{-6 - 4 - 11}{3} \right| = 7$$

Question32

Let the system of linear equations

$$x + y + \alpha z = 2$$

$$3x + y + z = 4$$

$$x + 2z = 1$$

have a unique solution (x^*, y^*, z^*) . If (α, x^*) , (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is

[24-Jun-2022-Shift-2]

Options:

- A. 4
- B. 3
- C. 2
- D. 1

Answer: C

Solution:

Given system of equations

$$x + y + az = 2 \dots (i)$$

$$3x - y - z = 4 \dots (ii)$$

$$x + 2z = 1 \dots (iii)$$

Solving (i), (ii) and (iii), we get

$$x = 1, y = 1, z = 0 \text{ (and for unique solution } a \neq -3 \text{)}$$

Now, $(a, 1)$, $(1, a)$ and $(1, -1)$ are collinear

$$\therefore \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow a(a+1) - 1(0) + 1(-1-a) = 0$$

$$\Rightarrow a^2 - 1 = 0$$

$$\therefore a = \pm 1$$

$$\therefore \text{Sum of absolute values of } a = 1 + 1 = 2$$

Question33

The system of equations

$$-kx + 3y - 14z = 25$$

$$-15x + 4y - kz = 3$$

$$-4x + y + 3z = 4$$

**is consistent for all k in the set
[25-Jun-2022-Shift-2]**

Options:

A. \mathbb{R}

B. $\mathbb{R} - \{-11, 13\}$

C. $\mathbb{R} - \{13\}$

D. $\mathbb{R} - \{-11, 11\}$

Answer: D

Solution:

Solution:

The system may be inconsistent if

$$\begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 0 \Rightarrow k = \pm 11$$

Hence if system is consistent then $k \in \mathbb{R} - \{11, -11\}$.

Question34

The ordered pair (a, b), for which the system of linear equations

$$3x - 2y + z = b$$

$$5x - 8y + 9z = 3$$

$$2x + y + az = -1$$

has no solution, is :

[26-Jun-2022-Shift-1]

Options:

A. $\left(3, \frac{1}{3}\right)$

B. $\left(-3, \frac{1}{3}\right)$

C. $\left(-3, -\frac{1}{3}\right)$

D. $\left(3, -\frac{1}{3}\right)$

Answer: C

Solution:

Solution:

$$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0 \Rightarrow -14a - 42 = 0 \Rightarrow a = -3$$

Now 3 (equation (1)) - (equation (2)) - 2 (equation (3)) is

$$3(3x - 2y + z - b) - (5x - 8y + 9z - 3) - 2(2x + y + az + 1) = 0$$

$$\Rightarrow -3b + 3 - 2 = 0 \Rightarrow b = \frac{1}{3}$$

So for no solution $a = -3$ and $b \neq \frac{1}{3}$

Question35

If the system of equations

$$\alpha x + y + z = 5, x + 2y + 3z = 4, x + 3y + 5z = \beta$$

has infinitely many solutions, then the ordered pair (α , β) is equal to :

[26-Jun-2022-Shift-2]

Options:

A. $(1, -3)$

B. $(-1, 3)$

C. $(1, 3)$

D. $(-1, -3)$

Answer: C

Solution:

Solution:

Given system of equations

$$\alpha x + y + z = 5$$

$x + 2y + 3z = 4$, has infinite solution

$$x + 3y + 5z = \beta$$

$$\therefore \Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0 \Rightarrow \alpha(1) - 1(2) + 1(1) = 0$$

$$\Rightarrow \alpha = 1$$

$$\text{and } \Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 5(1) - 1(20 - 3\beta) + 1(12 - 2\beta) = 0$$

$$\Rightarrow \beta = 3$$

$$\text{and } \Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 4 & 3 \\ 1 & \beta & 5 \end{vmatrix} = 0 \Rightarrow (20 - 3\beta) - 5(2) + 1(\beta - 4) = 0$$

$$\Rightarrow -2\beta + 6 = 0$$

$$\Rightarrow \beta = 3$$

Similarly,

$$\Rightarrow \Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 4 \\ 1 & 3 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 3$$

$$\therefore (\alpha, \beta) = (1, 3)$$

Question 36

Let the system of linear equations

$$x + 2y + z = 2$$

$$\alpha x + 3y - z = \alpha$$

$$-\alpha x + y + 2z = -\alpha$$

be inconsistent. Then α is equal to:

[27-Jun-2022-Shift-1]

Options:

A. $\frac{5}{2}$

B. $-\frac{5}{2}$

C. $\frac{7}{2}$

D. $-\frac{7}{2}$

Answer: D

Solution:

Solution:

$$x + 2y + z = 2$$

$$\alpha x + 3y - z = \alpha$$

$$-\alpha x + y + 2z = -\alpha$$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 1(6 + 1) - 2(2\alpha - \alpha) + 1(\alpha + 3\alpha) \\ = 7 + 2\alpha$$

$$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 14 + 2\alpha \neq 0 \text{ for } \alpha = -\frac{7}{2}$$

$$\therefore \text{For no solution } \alpha = -\frac{7}{2}$$

Question 37

Let for some real numbers α and β , $a = \alpha - i\beta$. If the system of equations $4ix + (1 + i)y = 0$ and $8\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)x + \bar{a}y = 0$ has more than one solution, then $\frac{\alpha}{\beta}$ is equal to
[27-Jun-2022-Shift-2]

Options:

A. $-2\sqrt{3}$

B. $2 - \sqrt{3}$

C. $2 + \sqrt{3}$

D. $-2 - \sqrt{3}$

Answer: B

Solution:

Solution:

Given $a = \alpha - i\beta$ and

$$4ix + (1 + i)y = 0 \dots\dots (i)$$

$$8\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)x + \bar{a}y = 0$$

By (i)

$$\frac{x}{y} = \frac{-(1 + i)}{4i} \dots\dots (iii)$$

By (ii)

$$\frac{x}{y} = \frac{-\bar{a}}{8\left(\frac{-1}{2} + \frac{\sqrt{3}i}{2}\right)} \dots\dots$$

Now by (iii) and (iv)

$$\frac{1+i}{4i} = \frac{\bar{a}}{4(-1+\sqrt{3}i)}$$

$$\Rightarrow \bar{a} = (\sqrt{3}-1) + (\sqrt{3}+1)i$$

$$\Rightarrow \alpha + i\beta = (\sqrt{3}-1) + (\sqrt{3}+1)i$$

$$\therefore \frac{\alpha}{\beta} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$$

Question38

If the system of linear equations

$$2x + 3y - z = -2$$

$$x + y + z = 4$$

$$x - y + |\lambda| z = 4\lambda - 4$$

where, $\lambda \in \mathbb{R}$, has no solution, then

[28-Jun-2022-Shift-1]

Options:

A. $\lambda = 7$

B. $\lambda = -7$

C. $\lambda = 8$

D. $\lambda^2 = 1$

Answer: B

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \lambda \end{vmatrix} = 7$$

But at $\lambda = 7$, $D_x = D_y = D_z = 0$

$$P_1 : 2x + 3y - z = -2$$

$$P_2 : x + y + z = 4$$

$$P_3 : x - y + |\lambda| z = 4\lambda - 4$$

So clearly $5P_2 - 2P_1 = P_3$, so at $\lambda = 7$, system of equation is having infinite solutions.

So $\lambda = -7$ is correct answer.

Question39

If the system of linear equations

$$2x - 3y = \gamma + 5,$$

$$\alpha x + 5y = \beta + 1,$$

where $\alpha, \beta, \gamma \in \mathbb{R}$ has infinitely many solutions then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to_____

[28-Jun-2022-Shift-2]

Answer: 58

Solution:

Solution:

If $2x - 3y = \gamma + 5$ and $\alpha x + 5y = \beta + 1$ have infinitely many solutions then

$$\frac{2}{\alpha} = \frac{-3}{5} = \frac{\gamma + 5}{\beta + 1}$$

$$\Rightarrow \alpha = -\frac{10}{3} \text{ and } 3\beta + 5\gamma = -28$$

$$\text{So } |9\alpha + 3\beta + 5\gamma| = |-30 - 28| = 58$$

Question40

If the system of linear equations

$$2x + y - z = 7$$

$$x - 3y + 2z = 1$$

$x + 4y + \delta z = k$, where $\delta, k \in \mathbb{R}$ has infinitely many solutions, then $\delta + k$ is equal to:

[29-Jun-2022-Shift-1]

Options:

A. -3

B. 3

C. 6

D. 9

Answer: B

Solution:

Solution:

$$2x + y - z = 7$$

$$x - 3y + 2z = 1$$

$$x + 4y + \delta z = k$$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = -7\delta - 21 = 0$$

$$\delta = -3$$

$$\Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ k & 4 & -3 \end{vmatrix}$$

$$\Rightarrow 6 - k = 0 \Rightarrow k = 6$$

$$\delta + k = -3 + 6 = 3$$

Question41

The number of values of α for which the system of equations:

$$x + y + z = \alpha$$

$$\alpha x + 2\alpha y + 3z = -1$$

$$x + 3\alpha y + 5z = 4$$

is inconsistent, is

[24-Jun-2022-Shift-1]

Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix}$$

$$= 1(10\alpha - 9\alpha) - 1(5\alpha - 3) + 1(3\alpha^2 - 2\alpha)$$

$$= \alpha - 5\alpha + 3 + 3\alpha^2 - 2\alpha$$

$$= 3\alpha^2 - 6\alpha + 3$$

For inconsistency $\Delta = 0$ i.e. $\alpha = 1$

Now check for $\alpha = 1$

$$x + y + z = 1 \quad \dots (i)$$

$$x + 2y + 3z = -1 \quad \dots (ii)$$

$$\text{By (ii)} \times 2 - \text{(i)} \times 1$$

$$x + 3y + 5z = -3$$

so equations are inconsistent for $\alpha = 1$

Question42

The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

$$3(\sin 3\theta)x - y + z = 2$$

$$3(\cos 2\theta)x + 4y + 3z = 3$$

$$6x + 7y + 7z = 9$$

has no solution, is :

[25-Jul-2022-Shift-1]

Options:

A. 6

B. 7

C. 8

D. 9

Answer: B

Solution:

Solution:

Given,

$$3(\sin 3\theta)x - y + z = 2$$

$$3(\cos 2\theta)x + 4y + 3z = 3$$

$$6x + 7y + 7z = 9$$

For no solutions determinant of coefficient will be $= 0$

$$\therefore D = \begin{vmatrix} 3 \sin 3\theta & -1 & 1 \\ 3 \cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$$\Rightarrow 3 \sin 3\theta(28 - 21) + 1(21 \cos 2\theta - 18) + 1(21 \cos 2\theta - 24) = 0$$

$$\Rightarrow 21 \sin 3\theta + 42 \cos 2\theta - 42 = 0$$

$$\Rightarrow \sin 3\theta + 2 \cos 2\theta - 2 = 0$$

$$\Rightarrow 3 \sin \theta - 4 \sin^3 \theta + 2(1 - 2 \sin^2 \theta) - 2 = 0$$

$$\Rightarrow 3 \sin \theta - 4 \sin^3 \theta - 4 \sin^2 \theta = 0$$

$$\Rightarrow 4 \sin^3 \theta + 4 \sin^2 \theta - 3 \sin \theta = 0$$

$$\Rightarrow \sin \theta(4 \sin^2 \theta + 4 \sin \theta - 3) = 0$$

$$\therefore \sin \theta = 0$$

$$\Rightarrow \theta = \pi, 2\pi, 3\pi \text{ when } \theta \in (0, 4\pi)$$

or,

$$4 \sin^2 \theta + 4 \sin \theta - 3 = 0$$

$$\Rightarrow 4 \sin 2\theta + 6 \sin \theta - 2 \sin \theta - 3 = 0$$

$$\Rightarrow 2 \sin \theta(2 \sin \theta + 3) - 1(2 \sin \theta + 3) = 0$$

$$\Rightarrow (2 \sin \theta - 1)(2 \sin \theta + 3) = 0$$

$$\therefore \sin \theta = \frac{1}{2}$$

or,

$$\sin \theta = -\frac{3}{2} \text{ [not possible as } \sin \in [-1, 1] \text{]}$$

$$\therefore \sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$\therefore \text{Possible values of } \theta = \pi, 2\pi, 3\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

\therefore Total 7 values of θ possible.

Question43

The number of real values of λ , such that the system of linear equations

$$2x - 3y + 5z = 9$$

$$x + 3y - z = -18$$

$$3x - y + (\lambda^2 - |\lambda|)z = 16$$

has no solutions, is

[25-Jul-2022-Shift-2]

Options:

A. 0

B. 1

C. 2

D. 4

Answer: C

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - |\lambda| \end{vmatrix} = 2(3\lambda^2 - 3|\lambda| - 1) + 3(\lambda^2 - |\lambda| - 3) + 5(-1 - 9)$$

$$= 9\lambda^2 - 9|\lambda| - 43$$

$$= 9|\lambda|^2 - 9|\lambda| - 43$$

$\Delta = 0$ for 2 values of $|\lambda|$ out of which one is -ve and other is +ve

So, 2 values of λ satisfy the system of equations to obtain no solution.

Question44

If the system of linear equations.

$$8x + y + 4z = -2$$

$$x + y + z = 0$$

$$\lambda x - 3y = \mu$$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane

$$8x + y + 4z + 2 = 0 \text{ is :}$$

[26-Jul-2022-Shift-1]

Options:

A. $3\sqrt{5}$

B. 4

C. $\frac{26}{9}$

D. $\frac{10}{3}$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix}$$

$$= 8(3) - 1(-\lambda) + 4(-3 - \lambda)$$

$$= 24 + \lambda - 12 - 4\lambda$$

$$= 12 - 3\lambda$$

So for $\lambda = 4$, it is having infinitely many solutions.

$$\Delta_x = \begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 1 \\ \mu & -3 & 0 \end{vmatrix}$$

$$= -2(3) - 1(-\mu) + 4(-\mu)$$

$$= -6 - 3\mu = 0$$

For $\mu = -2$

$$\text{Distance of } \left(4, -2, \frac{-1}{2}\right) \text{ from } 8x - y - 4z + 2 = 0 = \frac{32 - 2 - 2 - 2}{\sqrt{64 + 1 + 16}} = \frac{10}{3} \text{ units}$$

Question45

Let p and $p + 2$ be prime numbers and let $\Delta =$

$$\begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

Then the sum of the maximum values of α and β , such that p^α and $(p+2)^\beta$ divide Δ , is _____.
[29-Jul-2022-Shift-1]

Answer: 4

Solution:

Solution:

$$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

$$= p! \cdot (p+1)! \cdot (p+2)! \begin{vmatrix} 1 & (p+1) & (p+1)(p+2) \\ 1 & (p+2) & (p+2)(p+3) \\ 1 & (p+3) & (p+3)(p+4) \end{vmatrix}$$

$$= p! \cdot (p+1)! \cdot (p+2)! \begin{vmatrix} 1 & p+1 & p^2+3p+2 \\ 0 & 1 & 2p+4 \\ 0 & 1 & 2p+6 \end{vmatrix}$$

$$= 2(p!) \cdot ((p+1)!) \cdot ((p+2)!) \\ = 2(p+1) \cdot (p!)^2 \cdot ((p+2)!) \\ = 2(p+1)^2 \cdot (p!)^3 \cdot ((p+2)!) \\ \therefore \text{Maximum value of } \alpha \text{ is 3 and } \beta \text{ is 1.} \\ \therefore \alpha + \beta = 4$$

Question46

If the system of equations
 $x + y + z = 6$
 $2x + 5y + \alpha z = \beta$
 $x + 2y + 3z = 14$
has infinitely many solutions, then $\alpha + \beta$ is equal to
[29-Jul-2022-Shift-2]

Options:

- A. 8
- B. 36
- C. 44
- D. 48

Answer: C

Solution:

Solution:
Given,
 $x + y + z = 6 \dots (1)$
 $2x + 5y + \alpha z = \beta \dots (2)$
 $x + 2y + 3z = 14 \dots (3)$
System of equation have infinite many solutions.
 $\therefore \Delta_x = \Delta_y = \Delta_z = 0$ and $\Delta = 0$

$$\text{Now, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 - C_3$$

$$C_2 \rightarrow C_2 - C_3$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 2 - \alpha & 5 - \alpha & \alpha \\ -2 & -1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow -2 + \alpha + 10 - 2\alpha = 0$$

$$\Rightarrow 8 - \alpha = 0$$

$$\Rightarrow \alpha = 8$$

$$\text{Now, } x + y + z = 6$$

$$2x + 5y + 8z = \beta$$

$$x + 2y + 3z = 14$$

$$\therefore \Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & 8 \\ 14 & 2 & 3 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 - 6C_3$$

$$C_2 \rightarrow C_2 - C_3$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ \beta - 48 & -3 & 8 \\ -4 & -1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow -\beta + 48 - 12 = 0$$

$$\Rightarrow \beta = 36$$

$$\therefore \alpha + \beta = 8 + 36 = 44$$

Question47

The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

[26 Feb 2021 Shift 1]

Options:

A. $(a+2)(a+3)(a+4)$

B. -2

C. $(a+1)(a+2)(a+3)$

D. 0

Answer: B

Solution:

Solution:

$$\text{Given, } A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$\text{Apply } R_2 \rightarrow R_2 - R_1$$

$$\text{Apply } R_3 \rightarrow R_3 - R_1$$

$$A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

Now, expanding along third column,

$$A = 1[4(a+2) - (4a+10)] = 4a+8-4a-10 = -2$$

Question48

Let A be a 3×3 matrix with $\det(A) = 4$. Let R_i denote the i th row of A. If

a matrix **B** is obtained by performing the operation $R_2 \rightarrow 2R_2 + 5R_3$ on **2A**, then $\det(B)$ is equal to
[25 Feb 2021 Shift 2]

Options:

A. 16

B. 80

C. 64

D. 128

Answer: C

Solution:

Solution:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\text{Then, } 2A = \begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{bmatrix}$$

Now, perform the operation
 $R_2 \rightarrow 2R_2 + 5R_3$ on $2A$, we get

$$B = \begin{bmatrix} 2a & 2b & 2c \\ 4d + 10g & 4e + 10h & 4f + 10i \\ 2g & 2h & 2i \end{bmatrix}$$

Using property of invariance to calculate $|B|$, apply
 $R_2 \rightarrow R_2 - 5R_3$

$$\begin{aligned} |B| &= \begin{vmatrix} 2a & 2b & 2c \\ 4d & 4e & 4f \\ 2g & 2h & 2i \end{vmatrix} = 2 \times 4 \times 2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \quad [\because \det(A) = 4] \\ &= 16 \times \det(A) \\ &= 16 \times 4 = 64 \end{aligned}$$

Question49

Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers, such that

$x + y + z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is

.....

[25 Feb 2021 Shift 1]

Answer: 7

Solution:

Solution:

$$\text{Here, } A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = x^3 + y^3 + z^3 - 3xyz$$

$$|A^2| = |I_3| = 1$$

$$\therefore |(x^3 + y^3 + z^3 - 3xyz)^2| = 1$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1$$

$$\Rightarrow x^3 + y^3 + z^3 = 1 + 3xyz \quad [\because x + y + z > 0]$$

$$\Rightarrow = 1 + 3(2)$$

$$= 7 \quad [\because xyz = 2]$$

Question 50

Consider the following system of equations

$$x + 2y - 3z = a$$

$$2x + 6y - 11z = b$$

$$x - 2y + 7z = c$$

where, a, b and c are real constants. Then, the system of equations [26 Feb 2021 Shift 2]

Options:

A. has a unique solution, when $5a = 2b + c$

B. has infinite number of solutions when $5a = 2b + c$

C. has no solution for all a, b and c

D. has a unique solution for all a, b and c

Answer: B

Solution:

Solution:

Given, system of equation can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \text{Then,}$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$$

$$= 1(42 - 22) - 2(14 + 11) - 3(-4 - 6)$$

$$= 20 - 50 + 30 = 0$$

$$\begin{aligned}
 |A_1| &= \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix} \\
 &= a(42 - 22) - 2(7b + 11c) \\
 &\quad - 3(-2b - 6c) \\
 &= 20a - 14b - 22c + 6b + 18c \\
 &= 20a - 8b - 4c = 4(5a - 2b - c)
 \end{aligned}$$

$$\begin{aligned}
 |A_2| &= \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix} \\
 &= 1(7b + 11c) - a(14 + 11) - 3(2c - b) \\
 &= -25a + 10b + 5c = -5(5a - 2b - c)
 \end{aligned}$$

$$\begin{aligned}
 |A_3| &= \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix} \\
 &= 1(42 - 22) - 2(14 + 11) - 3(-4 - 6) \\
 &= 20 - 50 + 30 = 0
 \end{aligned}$$

$$\begin{aligned}
 |A_1| &= \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix} \\
 &= a(42 - 22) - 2(7b + 11c) \\
 &\quad - 3(-2b - 6c) \\
 &= 20a - 14b - 22c + 6b + 18c \\
 &= 20a - 8b - 4c = 4(5a - 2b - c)
 \end{aligned}$$

$$\begin{aligned}
 |A_2| &= \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix} \\
 &= 1(7b + 11c) - a(14 + 11) - 3(2c - b) \\
 &= -25a + 10b + 5c = -5(5a - 2b - c)
 \end{aligned}$$

$$\begin{aligned}
 |A_3| &= \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix} \\
 &= -10a + 4b + 2c \\
 &= -2(5a - 2b - c)
 \end{aligned}$$

For infinite solution,
 $|A| = |A_1| = |A_2| = |A_3| = 0$
 $\Rightarrow 5a - 2b - c = 0 \Rightarrow 5a = 2b + c$

Question 51

The following system of linear equations

$$2x + 3y + 2z = 9$$

$$3x + 2y + 2z = 9$$

$$x - y + 4z = 8$$

[25 Feb 2021 Shift 2]

Options:

A. does not have any solution

B. has a unique solution

C. has infinitely many solutions

D. has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

Answer: B

Solution:

Solution:

The given system of equations is non-homogeneous and it can be written as,

$$\begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 8 \end{bmatrix}$$

i.e., $AX = B$

$$\begin{aligned} \text{Now, } |A| &= 2(8 + 2) - 3(12 - 2) + 2(-3 - 2) \\ &= 20 - 30 - 10 = -20 \neq 0 \end{aligned}$$

$\therefore |A| \neq 0$, then this system have unique solution.

Question52

If the system of equations

$$kx + y + 2z = 1$$

$$-2x - 2y - 4z = 3$$

**$3x - y - 2z = 2$ has infinitely many solutions, then k is equal to
[25 Feb 2021 Shift 1]**

Answer: 21

Solution:

Given equations, $kx + y + 2z = 1$

$$3x - y - 2z = 2$$

$$-2x - 2y - 4z = 3$$

For infinitely many solutions,

$$\Delta = \Delta x = \Delta y = \Delta z = 0$$

$$\text{Here, } \Delta y = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$$

$$\Rightarrow k(-8 + 6) - 1(-12 - 4) + 2(9 + 4) = 0$$

$$\Rightarrow -2k + 16 + 26 = 0$$

$$\Rightarrow 2k = 42$$

$$\therefore k = 21$$

Question53

**Let A and B be 3×3 real matrices, such that A is symmetric matrix and B is skew-symmetric matrix. Then, the system of linear equations $(A^2B^2 - B^2A^2)X = 0$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has
[24 Feb 2021 Shift 2]**

Options:

- A. no solution
- B. exactly two solutions
- C. infinitely many solutions
- D. a unique solution

Answer: C**Solution:****Solution:**

Given, A be a 3×3 matrix. A is symmetric and B is skew-symmetric.

$$\therefore A^T = A, B^T = -B$$

$$\text{Let } A^2B^2 - B^2A^2 = P$$

$$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$$

$$= (B^2)^T(A^2)^T - (A^2)^T(B^2)^T$$

$$= B^2A^2 - A^2B^2 = -(A^2B^2 - B^2A^2) = -P$$

$$P^T = -P$$

P is skew-symmetric. $\therefore |P| = 0$

Hence, $PX = 0$ have infinite solutions.

Question54

For the system of linear equations

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbb{R}$, consider the following statements

- (A) The system has unique solution, if $k \neq 2, k \neq -2$**
- (B) The system has unique solution, if $k = -2$**
- (C) The system has unique solution, if $k = 2$**
- (D) The system has no solution, if $k = 2$**
- (E) The system has infinite number of solutions, if $k \neq -2$**

Which of the following statements are correct?

[24 Feb 2021 Shift 2]

Options:

- A. (C) and (D)
- B. (B) and (E)
- C. (A) and (E)
- D. (A) and (D)

Answer: D**Solution:****Solution:**

Given, $x - 2y + 0z = 1$

$$x - y + kx = -2$$

$$0x + ky + 4z = 6$$

$$\text{Here, } \Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 1(-4 - k^2) + 2(4)$$

$$= -4 - k^2 + 8 = 4 - k^2$$

$$\Delta_x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = 1(-4 - k^2) + 2(-8 - 6k)$$

$$= -4 - k^2 - 16 - 12k = -k^2 - 12k - 20$$

If $\Delta \neq 0$, then it has unique solution i.e. $4 - k^2 \neq 0$
 $\Rightarrow k \neq \pm 2$ for unique solution.

Also at $k = 2$

$$\Delta_x = -2^2 - 12 \times 2 - 20 = -48 \neq 0$$

Then, in this case it has no solution.

Hence, statement (A) and statement (D) both are correct.

$\Rightarrow k \neq \pm 2$ for unique solution. Also at $k = 2$

Then, in this case it has no solution.

Hence, statement (A) and statement (D) both are correct.

Question55

The system of linear equations :

$3x - 2y - kz = 10$; $2x - 4y - 2z = 6$; $x + 2y - z = 5$ is inconsistent if :

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Options:

A. $k = 3, m = \frac{4}{5}$

B. $k \neq 3, m \in \mathbb{R}$

C. $k \neq 3, m \neq \frac{4}{5}$

D. $k = 3, m \neq \frac{4}{5}$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 24 + 2(0) - k(8) = 0 \Rightarrow k = 3$$

$$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix}$$

$$= 10(8) - 2(-10m + 6) - 3(12 + 20m)$$

$$= 8(4 - 5m)$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix}$$

$$= 3(-6 + 10m) - 10(0) - 3(10m - 6)$$

$$= 0$$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix}$$

$$= 3(-20m - 12) - 2(6 - 10m) + 10(8)$$

$$= -40m + 32 = 8(4 - 5m)$$

For inconsistent,

$$k = 3m \neq \frac{4}{5}$$

Question 56

Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in \mathbb{R}$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to __

24 Feb 2021 Shift 1

Answer: 17

Solution:

Solution:

Given that

$$PQ = kI$$

$$|P| \cdot |Q| = k^3$$

$$\Rightarrow |P| = 2k \neq 0 \Rightarrow P \text{ is an invertible matrix}$$

$$\therefore PQ = kI$$

$$\therefore Q = kP^{-1}I$$

$$\therefore Q = \frac{\text{adj.}P}{2}$$

$$\therefore q_{23} = -\frac{k}{8}$$

$$\therefore \frac{(3\alpha + 4)}{2} = -\frac{k}{8} \Rightarrow k = 4$$

$$\therefore |P| = 2k \Rightarrow k = 10 + 6\alpha$$

Put value of k in (i)... we get $\alpha = -1$

$$\therefore \alpha^2 + k^2 = 1 + 16 = 17.$$

Question 57

The solutions of the equation

$$\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix}$$

$= 0$, ($0 < x < \pi$), are

[18 Mar 2021 Shift 1]

Options:

A. $\frac{\pi}{12}, \frac{\pi}{6}$

B. $\frac{\pi}{6}, \frac{5\pi}{6}$

C. $\frac{5\pi}{12}, \frac{7\pi}{12}$

D. $\frac{7\pi}{12}, \frac{11\pi}{12}$

Answer: D

Solution:

Solution:

Given,
$$\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

($0 < x < \pi$)

Applying $R_1 \rightarrow R_1 + R_2$,

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 - C_2$,

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 + \cos^2 x & \cos^2 x \\ 0 & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

$$\Rightarrow 2 + 8 \sin 2x - 4 \sin 2x = 0 \text{ (expanding along } C_1 \text{)}$$

$$\Rightarrow 4 \sin 2x = -2 \Rightarrow \sin 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} \Rightarrow 2x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\Rightarrow x = \frac{7\pi}{12}, \frac{11\pi}{12}$$

[Note You can also solve by applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - C_3$]

Question58

If $A = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$ and $\det \left(A^2 - \frac{1}{2}I \right) = 0$, then a possible value of α is

[17 Mar 2021 Shift 1]

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: C

Solution:

Solution:

$$A = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$$

$$\text{and } \det \left(A^2 - \frac{1}{2}I \right) = 0$$

$$\therefore A^2 = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix} \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} \sin^2 \alpha & 0 \\ 0 & \sin^2 \alpha \end{pmatrix}$$

$$\Rightarrow \frac{I}{2} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\left(A^2 - \frac{1}{2}I \right) = \begin{pmatrix} \sin^2 \alpha - 1/2 & 0 \\ 0 & \sin^2 \alpha - 1/2 \end{pmatrix}$$

$$\therefore \det \left(A^2 - \frac{1}{2}I \right) = \begin{vmatrix} \sin^2 \alpha - 1/2 & 0 \\ 0 & \sin^2 \alpha - 1/2 \end{vmatrix}$$

$$\left(\sin^2 \alpha - \frac{1}{2} \right)^2 = 0$$

$$\sin \alpha = \pm \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4} \text{ is one possibility.}$$

Question59

If x, y, z are in arithmetic progression with common difference

d , $x \neq 3d$, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then

the value of k^2 is
[17 Mar 2021 Shift 2]

Options:

A. 72

B. 12

C. 36

D. 6

Answer: A

Solution:

Solution:

Method (I)

Given, x, y and z are in AP with common difference = d

 $\therefore x =$ First term

y = Second term of AP = First term + Common difference

 $\Rightarrow y = x + d \dots(i)$

and z = Third term of AP = Second term + Common difference

 $\Rightarrow z = (x + d) + d = x + 2d \dots(ii)$ Also, given $x \neq 3d \dots(iii)$

$$\text{and } \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_1 + R_3 - 2R_2$, we have

$$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & k - 6\sqrt{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$$

 $\Rightarrow (k - 6\sqrt{2})(3z - 5x) = 0$ (Expanding along R_2)Either $k - 6\sqrt{2} = 0$ or $3z - 5x = 0$ $\Rightarrow k = 6\sqrt{2}$ or $3(x + 2d) - 5x = 0$ [from Eq. (ii)] $\Rightarrow x = 3d$ which is not possible as in Eq. (iii). $\therefore k = 6\sqrt{2}$ is only one solution.Hence, $k^2 = (6\sqrt{2})^2$ $\Rightarrow k^2 = 72$

Question60

If $1, \log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant

$$\begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x - 1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix} \text{ is}$$

[17 Mar 2021 Shift 2]**Answer: 2****Solution:**Given $1, \log_{10}(4^x - 2), \log_{10}\left(4^x + \frac{18}{5}\right)$ are in A.P.

$$\therefore 2\log_{10}(4^x - 2) = 1 + \log_{10}\left(4^x + \frac{18}{5}\right) = \log_{10}10 + \log_{10}\left(4^x + \frac{18}{5}\right)$$

$$\Rightarrow \log_{10}(4^x - 2)^2 = \log_{10}\left(10 \times \left(4^x + \frac{18}{5}\right)\right)$$

$$\Rightarrow (4^x - 2)^2 = 10 \times 4^x + 36$$

$$\Rightarrow (4^x)^2 - 4(4^x) + 4 = 10 \times 4^x + 36$$

$$\Rightarrow (4^x)^2 - 14(4^x) - 32 = 0 \Rightarrow (4^x - 16)(4^x + 2) = 0$$

$\Rightarrow 4^x = 16$ or $4^x = -2$ (Rejected because $4^x > 0, \forall x \in \mathbb{R}$)
 $\Rightarrow 4^x = 4^2 \Rightarrow x = 2$

$$\therefore \begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 2$$

Question61

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, such that $AB = B$ and $a + d = 2021$, then the value of $ad - bc$ is equal to
[17 Mar 2021 Shift 2]

Answer: 2020

Solution:

Solution:

Given,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

i.e. $B \neq 0$

and $AB = B$

$$\Rightarrow AB - B = 0 \Rightarrow B(A - I) = 0$$

$$\Rightarrow |(A - I)B| = 0$$

$\therefore B \neq 0$

$$\therefore |A - I| = 0 \Rightarrow \begin{vmatrix} (a-1) & b \\ c & (d-1) \end{vmatrix} = 0$$

$$\Rightarrow (a-1)(d-1) - bc = 0 \Rightarrow ad - bc = 2020$$

Question62

The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$, $x \in \mathbb{R}$ is

[16 Mar 2021 Shift 2]

Options:

A. $\sqrt{7}$

B. $\frac{3}{4}$

C. $\sqrt{5}$

D. 5

Answer: C

Solution:

Solution:

$$\text{Given, } f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$$

On applying $C_1 \rightarrow C_1 + C_2$, we get

$$f(x) = \begin{vmatrix} \sin^2 x + 1 + \cos^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x + \cos^2 x & \cos^2 x & \cos 2x \\ \sin^2 x + \cos^2 x & \cos^2 x & \sin 2x \end{vmatrix}$$

$$f(x) = \begin{vmatrix} 2 & 1 + \cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

On applying $R_1 \rightarrow R_1 - R_2$

$$f(x) = \begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$$f(x) = -1(2 \sin 2x - \cos 2x)$$

As, we know that, if $f(\theta) = A \sin \theta + B \cos \theta$

$$\text{Then, } -\sqrt{A^2 + B^2} \leq f(\theta) \leq \sqrt{A^2 + B^2}$$

$$\text{Here, we have, } f(x) = \cos 2x - 2 \sin 2x$$

$$-\sqrt{2^2 + 1^2} \leq f(x) \leq \sqrt{2^2 + 1^2}$$

$$-\sqrt{5} \leq f(x) \leq \sqrt{5}$$

So, maximum value of $f(x)$ is $\sqrt{5}$.

Question63

Let α, β, γ be the real roots of the equation,
 $x^3 + ax^2 + bx + c = 0$, ($a, b, c \in \mathbb{R}$ and $a, b \neq 0$). If the system of
equations (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + w + \alpha w = 0$;
 $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is
[18 Mar 2021 Shift 1]

Options:

A. 5

B. 3

C. 1

D. 0

Answer: B

Solution:

Solution:

Given, α, β, γ are the real roots of $x^3 + ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$ and $a, b \neq 0$

$\therefore \alpha + \beta + \gamma =$ Sum of roots taken one at a time $= -a$

$\alpha\beta + \beta\gamma + \gamma\alpha =$ Sum of roots taken two at a time $= b$

$\alpha\beta\gamma =$ Product of roots $= -c$

Also, given system of equations in u, v, w

$$\left. \begin{aligned} \alpha u + \beta v + \gamma w &= 0 \\ \beta u + \gamma v + \alpha w &= 0 \\ \gamma u + \alpha v + \beta w &= 0 \end{aligned} \right\}$$

has non-trivial solution.

$$\therefore \Delta = 0 \Rightarrow \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \alpha(\beta\gamma - \alpha^2) - \beta(\beta^2 - \gamma\alpha) + \gamma(\alpha\beta - \gamma^2) = 0 \text{ (expanding along } R_1 \text{)}$$

$$\Rightarrow \alpha\beta\gamma - \alpha^3 - \beta^3 + \alpha\beta\gamma + \alpha\beta\gamma - \gamma^3 = 0$$

$$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$

Then (using standard result),

Either $\alpha + \beta + \gamma = 0$ or $\alpha = \beta = \gamma$

If $\alpha + \beta + \gamma = 0$, then $-a = 0$

$\Rightarrow a = 0$ which is not possible according to given condition.

$\therefore \alpha + \beta + \gamma = 0$ (not possible)

Now,

$$\alpha + \beta + \gamma = -a$$

$$\Rightarrow \alpha + \alpha + \alpha = -a \quad (\because \alpha = \beta = \gamma)$$

$$\Rightarrow a = -3\alpha \dots (i)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = b$$

$$\Rightarrow b = 3\alpha^2 \dots (ii)$$

Using Eqs. (i) and (ii),

$$\frac{a^2}{b} = 3$$

Question64

Let the system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbb{R}$$

has a non-trivial solution. Then which of the following is true?

[18 Mar 2021 Shift 2]

Options:

A. $\mu = 6, \lambda \in \mathbb{R}$

B. $\lambda = 2, \mu \in \mathbb{R}$

C. $\lambda = 3, \mu \in \mathbb{R}$

D. $\mu = -6, \lambda \in \mathbb{R}$

Answer: A

Solution:

Solution:

Given, system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0$$

For non-trivial solution, $\Delta = 0$

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 4(-3-2) - \lambda(6-\mu) + 2(4+\mu) = 0$$

$$\Rightarrow -\lambda(6-\mu) - 2(6-\mu) = 0$$

$$\Rightarrow (6-\mu)(\lambda+2) = 0$$

$$\Rightarrow \lambda = -2 \text{ and } \mu \in \mathbb{R} \text{ or } \mu = 6 \text{ and } \lambda \in \mathbb{R}.$$

Question65

The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + zk = k^2$ has no solution, if k is equal to
[17 Mar 2021 Shift 1]

Options:

A. 0

B. 1

C. -1

D. -2

Answer: D

Solution:**Solution:**

$$kx + y + z = 1$$

$$x + ky + z = k$$

$$x + y + kz = k^2$$

For this set of equations to have no solution, $\Delta = 0$

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = k(k^2 - 1) - 1(k - 1) + (1 - k)$$

$$= k^3 - k - k + 1 + 1 - k = k^3 - 3k + 2$$

Now, $\Delta = 0$

$$\Rightarrow k^3 - 3k + 2 = 0 \Rightarrow (k-1)(k^2 + k - 2) = 0$$

$$\Rightarrow (k-1)(k-1)(k+2) = 0$$

$$\therefore k = 1, -2$$

$$\left. \begin{array}{l} x + y + z = 1 \\ \text{If } k = 1 \quad x + y + z = 1 \\ x + y + z = 1 \end{array} \right\}$$

There are same equations and they will have infinite solutions.

$$\text{So, } K = -2$$

Question66

Let

$$f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix},$$

$x \in [0, \pi]$ Then the maximum value of $f(x)$ is equal to
[27 Jul 2021 Shift 1]

Answer: 6

Solution:

Solution:

$$\begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}$$

$$(R_1 \rightarrow R_1 - R_2 \text{ \& } R_2 \rightarrow R_2 - R_3)$$

$$-2(\cos^2 x) + 2(2 + 2 \cos 2x + \sin^2 x)$$

$$4 + 4 \cos 2x - 2(\cos^2 x - \sin^2 x)$$

$$f(x) = 4 + 2 \cos 2x$$

$$f(x)_{\max} = 4 + 2 = 6$$

Question67

Let

$$M = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\} \right\}.$$

Define $f : M \rightarrow \mathbb{Z}$, as $f(A) = \det(A)$, for all $A \in M$ where \mathbb{Z} is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to

[25 Jul 2021 Shift 1]

Answer: 16

Solution:

$$|A| = ad - bc = 15$$

where $a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\}$

Case I $ad = 9$ & $bc = -6$

For ad possible pairs are $(3, 3), (-3, -3)$

For bc possible pairs are $(3, -2), (-3, 2), (-2, 3), (2, -3)$

So total matrix = $2 \times 4 = 8$

Case II $ad = 6$ & $bc = -9$
 Similarly total matrix $= 2 \times 4 = 8$
 \Rightarrow Total such matrices are $= 16$

Question68

The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval

$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is:

[25 Jul 2021 Shift 2]

Options:

A. 4

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

Apply : $R_1 \rightarrow R_1 - R_2$ & $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x - \cos x)^2 (\sin x + 2 \cos x) = 0$$

$$\therefore x = \frac{\pi}{4}$$

Question69

Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in \mathbb{R}$ be written as $P + Q$ where P is a symmetric

matrix and Q is skew symmetric matrix. If $\det(Q) = 9$, then the modulus of the sum of all possible values of determinant of P is equal to :

[20 Jul 2021 Shift 1]

Options:

- A. 36
- B. 24
- C. 45
- D. 18

Answer: A

Solution:

Solution:

$$A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}, a \in \mathbb{R}$$

$$\text{and } P = \frac{A + A^T}{2} = \begin{bmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{bmatrix}$$

$$\text{and } P = \frac{A - A^T}{2} = \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$$

$$\text{As, } \det(Q) = 9$$

$$\Rightarrow (a-3)^2 = 36$$

$$\Rightarrow a = 3 \pm 6$$

$$\therefore a = 9, -3$$

$$\therefore \det.(P) = \begin{vmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{vmatrix}$$

$$= 0 - \frac{(a-3)^2}{4} = 0, \text{ for } a = -3$$

$$= 0 - \frac{(a-3)^2}{4} = -\frac{1}{4}(12)(12), \text{ for } a = 9$$

$$\therefore \text{Modulus of the sum of all possible values of } \det.(P) = |-36| + |0| = 36 \text{ Ans.}$$

$$\Rightarrow \text{Option (1) is correct}$$

Question70

Let $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = \begin{cases} 1 & , \text{ if } i = j \\ -x & , \text{ if } |i - j| = 1 \\ 2x + 1 & , \text{ otherwise} \end{cases}$

Let a function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \det(A)$ Then the sum of maximum and minimum values of f on \mathbb{R} is equal to:
[20 Jul 2021 Shift 1]

Options:

A. $-\frac{20}{27}$

B. $\frac{88}{27}$

C. $\frac{20}{27}$

D. $-\frac{88}{27}$

Answer: D

Solution:

Solution:

$$A = \begin{bmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{bmatrix}$$

$$|A| = 4x^3 - 4x^2 - 4x = f(x)$$

$$f'(x) = 4(3x^2 - 2x - 1) = 0$$

$$\Rightarrow x = 1; x = -\frac{1}{3}$$

$$\therefore f(1) = -4; f\left(-\frac{1}{3}\right) = \frac{20}{27}$$

$$\text{Sum} = -4 + \frac{20}{27} = -\frac{88}{27}$$

Question71

Let a, b, c, d be in arithmetic progression with common difference λ . If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2 \text{ then value of } \lambda^2 \text{ is equal to } \underline{\hspace{2cm}}.$$

[20 Jul 2021 Shift 1]

Answer: 1

Solution:

Solution:

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$$

$$C_2 \rightarrow C_2 - C_3$$

$$\Rightarrow \begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix} = 2$$

$$R_2 \rightarrow R_2 - R_1, \quad R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \lambda \begin{vmatrix} x - 2\lambda & 1 & x + a \\ 2\lambda - 1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix} = 2$$

$$\Rightarrow 1(4\lambda^2 - 4\lambda^2 + 2\lambda) = 2$$

$$\Rightarrow \lambda^2 = 1$$

Question72

For real numbers α and β , consider the following system of linear equations :

$x + y - z = 2$, $x + 2y + \alpha z = 1$, $2x - y + z = \beta$ If the system has infinite solutions, then $\alpha + \beta$ is equal to _____ .

[27 Jul 2021 Shift 1]

Answer: 5

Solution:

Solution:

For infinite solutions

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = 3(2 + \alpha) = 0$$

$$\Rightarrow \alpha = -2$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix} = 0$$

$$1(1 + 2\beta) - 2(1 + 4) - (\beta - 2) = 0$$

$$\beta - 7 = 0$$

$$\beta = 7$$

$$\therefore \alpha + \beta = 5 \text{ Ans.}$$

Question73

The values of a and b , for which the system of equations

$$2x + 3y + 6z = 8$$

$$x + 2y + az = 5$$

$$3x + 5y + 9z = b$$

has no solution, are :

[25 Jul 2021 Shift 1]

Options:

A. $a = 3, b \neq 13$

B. $a \neq 3, b \neq 13$

C. $a \neq 3, b = 3$

D. $a = 3, b = 13$

Answer: A**Solution:****Solution:**

$$D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$$

$$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$$

If $a = 3, b \neq 13$, no solution.

Question74

The values of λ and μ such that the system of equations $x + y + z = 6, 3x + 5y + 5z = 26, x + 2y + \lambda z = \mu$ has no solution, are :
[22 Jul 2021 Shift 2]

Options:

A. $\lambda = 3, \mu = 5$

B. $\lambda = 3, \mu \neq 10$

C. $\lambda \neq 2, \mu = 10$

D. $\lambda = 2, \mu \neq 10$

Answer: D**Solution:****Solution:**

$x + y + z = 6 \dots\dots(i)$

$3x + 5y + 5z = 26 \dots\dots(ii)$

$x + 2y + \lambda z = \mu \dots\dots(iii)$

$5 \times (i) - (ii) \Rightarrow 2x = 4 \Rightarrow x = 2$

 \therefore from (i) and (iii)

$y + z = 4 \dots\dots(iv)$

$2y + \lambda z = \mu - 2$

$(v) - 2 \times (iv)$

$\Rightarrow (\lambda - 2)z = \mu - 10$

$\Rightarrow z = \frac{\mu - 10}{\lambda - 2} \text{ \& } y = 4 - \frac{\mu - 10}{\lambda - 2}$

 \therefore For no solution $\lambda = 2$ and $\mu \neq 10$.

Question75

The value of $k \in \mathbb{R}$, for which the following system of linear equations

$$3x - y + 4z = 3$$

$$x + 2y - 3z = -2$$

$$6x + 5y + kz = -3$$

has infinitely many solutions, is :

[20 Jul 2021 Shift 2]

Options:

A. 3

B. -5

C. 5

D. -3

Answer: B

Solution:

Solution:

$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & K \end{vmatrix} = 0$$

$$\Rightarrow 3(2K + 15) + K + 18 - 28 = 0$$

$$\Rightarrow 7K + 35 = 0 \Rightarrow K = -5$$

Question76

Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where $[t]$ denotes the greatest integer

less than or equal to t . If $\det(A) = 192$, then the set of values of x is the interval

[27 Aug 2021 Shift 2]

Options:

A. [68, 69]

B. [62, 63]

C. [65, 66]

D. [60, 61]

Answer: B

Solution:

Solution:

$$\text{Given, } A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$$

$$A = \begin{pmatrix} [x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4 \end{pmatrix} \quad (\because [x+n] = n + [x], n \in \mathbb{I})$$

Applying $R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix}$$

$$\det(A) = 1([x] + 4 + [x] + 2) - 1(-[x])$$

$$= 3[x] + 6$$

$$\because \det(A) = 192$$

$$\Rightarrow 3[x] + 6 = 192$$

$$\Rightarrow [x] = 62$$

$$\Rightarrow 62 \leq x < 63$$

$$\Rightarrow x \in [62, 63)$$

Question77

Let $A(a, 0)$, $B(b, 2b + 1)$ and $C(0, b)$, $b \neq 0$, $|b| \neq 1$, be points such that the area of ΔABC is 1 sq. unit, then the sum of all possible values of a is
[27 Aug 2021 Shift 2]

Options:

A. $\frac{-2b}{b+1}$

B. $\frac{2b}{b+1}$

C. $\frac{2b^2}{b+1}$

D. $\frac{-2b^2}{b+1}$

Answer: D

Solution:

Solution:

$A(a, 0)$, $B(b, 2b + 1)$, $C(0, b)$

$$\text{Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 1$$

$$\Rightarrow \frac{1}{2}[a(b+1) + b^2] = \pm 1$$

$$\Rightarrow a = \frac{2-b^2}{b+1} \text{ or } \frac{-2-b^2}{b+1}$$

Sum of all possible values of $a = \frac{-2b^2}{b+1}$

Question78

If the following system of linear equations $2x + y + z = 5$, $x - y + z = 3$ and $x + y + az = b$ has no solution, then
[31 Aug 2021 Shift 1]

Options:

A. $a = -\frac{1}{3}$ and $b \neq \frac{7}{3}$

B. $a \neq \frac{1}{3}$ and $b = \frac{7}{3}$

C. $a \neq -\frac{1}{3}$ and $b = \frac{7}{3}$

D. $a = \frac{1}{3}$ and $b \neq \frac{7}{3}$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 2(-a-1) - 1(a-1) + (1+1) \\ = 1 - 3a$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 2(-b-3) - 1$$

$$(b-3) + 5(1+1) = 7 - 3b$$

$$\text{Now, } z = \frac{\Delta_3}{\Delta}$$

If $\Delta = 0$ and $\Delta_3 \neq 0$, then no solution

$$1 - 3a = 0$$

$$\Rightarrow 7 - 3b \neq 0$$

$$a = \frac{1}{3}, b \neq \frac{7}{3}$$

Question79

If $\alpha + \beta + \gamma = 2\pi$, then the system of equations
 $x + (\cos \gamma)y + (\cos \beta)z = 0$
 $(\cos \gamma)x + y + (\cos \alpha)z = 0$
 $(\cos \beta)x + (\cos \alpha)y + z = 0$
has :
[31 Aug 2021 Shift 2]

Options:

- A. no solution
- B. infinitely many solution
- C. exactly two solutions
- D. a unique solution

Answer: B

Solution:

Solution:

Given $\alpha + \beta + \gamma = 2\pi$

$$\Delta = \begin{vmatrix} 1 & \cos \gamma & \cos \beta \\ \cos \gamma & 1 & \cos \alpha \\ \cos \beta & \cos \alpha & 1 \end{vmatrix}$$

$$= 1 - \cos^2 \alpha - \cos \gamma (\cos \gamma - \cos \alpha \cos \beta) + \cos \beta (\cos \alpha \cos \gamma - \cos \beta)$$

$$= 1 - \cos^2 \alpha - \cos^2 \beta - \cos^2 \gamma + 2 \cos \alpha \cos \beta \cos \gamma$$

$$= -\cos(\alpha + \beta) \cos(\alpha - \beta) - \cos \gamma (\cos(2\pi - (\alpha + \beta)) - 2 \cos \alpha \cos \beta)$$

$$= -\cos(2\pi - \gamma) \cos(\alpha - \beta) - \cos \gamma (\cos(\alpha + \beta) - 2 \cos \alpha \cos \beta)$$

$$= -\cos \gamma \cos(\alpha - \beta) + \cos \gamma (\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$= -\cos \gamma \cos(\alpha - \beta) + \cos \gamma \cos(\alpha - \beta)$$

$$= 0$$

Question 80

If the system of linear equations

$$2x + y - z = 3$$

$$x - y - z = \alpha$$

$$3x + 3y + \beta z = 3$$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to

[27 Aug 2021 Shift 1]

Answer: 5

Solution:

Solution:

Given, system of equation

$$2x + y - z = 3$$

$$x - y - z = \alpha$$

$$3x + 3y + \beta z = 3$$

has infinitely many solutions,

if $\Delta = 0, \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\text{Now, } \Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & -1 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 2(-\beta + 3) - 1(\beta + 3) - 1(3 + 3) = 0$$

$$\Rightarrow \beta = -1$$

$$\Delta_1 = \begin{vmatrix} 3 & 1 & -1 \\ \alpha & -1 & -1 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 3(1 + 3) - 1(-\alpha + 3) - 1(3\alpha + 3) = 0$$

$$\Rightarrow 12 + \alpha - 3 - 3\alpha - 3 = 0$$

$$\Rightarrow \alpha = 3$$

$$\text{Also, } \Delta_2 = \begin{vmatrix} 2 & 3 & -1 \\ 1 & \alpha & -1 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 2(-\alpha + 3) - 3(-1 + 3) - 1(3 - 3\alpha) = 0$$

$$\Rightarrow \alpha = 3$$

$$\text{and } \Delta_3 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & \alpha \\ 3 & 3 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 2(-3 - 3\alpha) - 1(3 - 3\alpha) + 3(3 + 3) = 0$$

$$\Rightarrow -3\alpha + 9 = 0$$

$$\Rightarrow \alpha = 3$$

$$\therefore \alpha = 3, \beta = -1$$

$$\text{So, } \alpha + \beta - \alpha\beta = 3 - 1 - 3(-1) = 5$$

Question81

Let $|\lambda|$ be the greatest integer less than or equal to λ . The set of all values of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + |\lambda|)z = |\lambda|$ has a solution is [27 Aug 2021 Shift 2]

Options:

A. \mathbb{R}

B. $(-\infty, -9) \cup (-9, \infty)$

C. $[-9, -8)$

D. $(-\infty, -9) \cup [-8, \infty)$

Answer: A

Solution:

Solution:

Given, system of equations

$$x + y + z = 4$$

$$3x + 2y + 5z = 3$$

$$9x + 4y + (28 + |\lambda|)z = |\lambda|$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28 + |\lambda| \end{vmatrix}$$

$$= 1(56 + 2|\lambda| - 20) - 1(84 + 3|\lambda| - 45) + 1(-6)$$

$$= -(|\lambda| + 9)$$

If $\Delta \neq 0$ i.e. $|\lambda| + 9 \neq 0$, then system of equation has unique solution.

If $|\lambda| + 9 = 0$, then $\Delta_1 = \Delta_2 = \Delta_3 = 0$, the system of equation has infinite solution.

$$\Rightarrow \lambda \in \mathbb{R}$$

Question82

Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$$(1 + \cos^2\theta)x + \sin^2\theta y + 4 \sin 3\theta z = 0$$

$$\cos^2\theta x + (1 + \sin^2\theta)y + 4 \sin 3\theta z = 0$$

$$\cos^2\theta x + \sin^2\theta y + (1 + 4 \sin 3\theta)z = 0$$

has a non-trivial solution, then the value of θ is

[26 Aug 2021 Shift 1]

Options:

A. $\frac{4\pi}{9}$

B. $\frac{7\pi}{18}$

C. $\frac{\pi}{18}$

D. $\frac{5\pi}{18}$

Answer: B

Solution:

Solution:

$$\Rightarrow \begin{vmatrix} 1 + \cos^2\theta & \sin^2\theta & 4 \sin 3\theta \\ \cos^2\theta & 1 + \sin^2\theta & 4 \sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4 \sin 3\theta \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 - R_3$, we get

$$\begin{vmatrix} 1 & 0 & -1 \\ \cos^2\theta & 1 + \sin^2\theta & 4 \sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4 \sin 3\theta \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 - C_3$, we get

$$\begin{vmatrix} 1 & 0 & 0 \\ \cos^2\theta & 1 + \sin^2\theta & 4 \sin 3\theta + \cos^2\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4 \sin 3\theta + \cos^2\theta \end{vmatrix} = 0$$

$$\Rightarrow (1 + \sin^2\theta)(1 + 4 \sin 3\theta + \cos^2\theta) - \sin^2\theta(4 \sin 3\theta + \cos^2\theta) = 0$$

$$\Rightarrow 1 + 4 \sin 3\theta + \cos^2\theta + \sin^2\theta + 4 \sin^2\theta \sin 3\theta + \sin^2\theta \cos^2\theta - 4 \sin^2\theta \sin 3\theta - \sin^2\theta \cos^2\theta = 0$$

$$\Rightarrow 1 + 4 \sin 3\theta + \cos^2\theta + \sin^2\theta = 0$$

$$\Rightarrow 1 + 4 \sin 3\theta + 1 = 0$$

$$\Rightarrow 4 \sin 3\theta + 2 = 0$$

$$\Rightarrow \sin 3\theta = \frac{-1}{2}$$

$$\Rightarrow 3\theta = \left(\pi + \frac{\pi}{6}\right)$$

$$\Rightarrow \theta = \frac{7\pi}{18}$$

Question83

Two fair dice are thrown. The numbers on them are taken as λ and μ , and a system of linear equations $x + y + z = 5$, $x + 2y + 3z = \mu$ and $x + 3y + \lambda z = 1$ is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then [26 Aug 2021 Shift 2]

Options:

A. $p = \frac{1}{6}$ and $q = \frac{1}{36}$

B. $p = \frac{5}{6}$ and $q = \frac{5}{36}$

C. $p = \frac{5}{6}$ and $q = \frac{1}{36}$

D. $p = \frac{1}{6}$ and $q = \frac{5}{36}$

Answer: B

Solution:

Solution:

$$\begin{aligned} x + y + z &= 5 \\ x + 2y + 3z &= \mu \\ x + 3y + \lambda z &= 1 \end{aligned}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = (2\lambda - 9) + (3 - \lambda) + (3 - 2) = \lambda - 5$$

For unique solution $\Delta \neq 0$

$$\Rightarrow \lambda \neq 5$$

And Δ_1 or Δ_2 or $\Delta_3 \neq 0$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} = (2 - 3\mu) + (\mu - 1) + 5 = 6 - 2\mu$$

If $\Delta_3 \neq 0$ and $\Delta = 0$, then no solution

$$\mu \neq 3 \text{ and } \lambda = 5$$

$$p = \text{Probability of unique solution} = \frac{5}{6}$$

$$q = \text{Probability of no solution} = \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$$

Question 84

Consider the system of linear equations

$$-x + y + 2z = 0$$

$$3x - ay + 5z = 1$$

$$2x - 2y - az = 7$$

Let S_1 be the set of all $a \in \mathbb{R}$ for which the system is inconsistent and S_2 be the set of all $a \in \mathbb{R}$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2

respectively, then
[1 Sep 2021 Shift 2]

Options:

- A. $n(S_1) = 2$ and $n(S_2) = 2$
- B. $n(S_1) = 1$ and $n(S_2) = 0$
- C. $n(S_1) = 2$ and $n(S_2) = 0$
- D. $n(S_1) = 0$ and $n(S_2) = 2$

Answer: C

Solution:

Solution:

For in consistent system of equations

[$\Delta = 0$ and atleast one is non-zero in Δ_1 , Δ_2 and Δ_3]

$$\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix} = 0$$

$$\Rightarrow a^2 - 7a + 12 = 0$$

$$\Rightarrow a = 3, 4$$

$$\Delta_x = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix} = 15a + 31$$

$$\Delta_x \neq 0 \text{ for } a = 3, 4$$

$$\Rightarrow n(S_1) = 2$$

Now, for infinitely many solutions.

If $\Delta = 0$ also $\Delta_x = \Delta_y = \Delta_z = 0$

Which is not possible for any real value of a

$$\Rightarrow n(S_2) = 0$$

Question85

If the system of linear equations,

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$3x + 2y + \lambda z = \mu$$

has more than two solutions, then $\mu - \lambda^2$ is equal to _____.

[NA Jan. 7, 2020 (II)]

Answer: 13

Solution:

$x + y + z = 6$ (i)
 $x + 2y + 3z = 10$ (ii)
 $3x + 2y + \lambda z = \mu$ (iii)
 From (i) and (ii),
 If $z = 0 \Rightarrow x + y = 6$ and $x + 2y = 10$
 $\Rightarrow y = 4, x = 2$
 $(2, 4, 0)$
 If $y = 0 \Rightarrow x + z = 6$ and $x + 3z = 10$
 $\Rightarrow z = 2$ and $x = 4$
 $(4, 0, 2)$
 So, $3x + 2y + \lambda z = \mu$, must pass through $(2, 4, 0)$ and $(4, 0, 2)$
 So, $6 + 8 = \mu \Rightarrow \mu = 14$
 and $12 + 2\lambda = \mu$
 $12 + 2\lambda = 14 \Rightarrow \lambda = 1$
 So, $\mu - \lambda^2 = 14 - 1 = 13$

Question86

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that
 $b_{ij} = (3)^{(i+j-2)} a_{ij}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then
 the determinant of A is:
[Jan. 7, 2020 (II)]

Options:

- A. $1/3$
- B. 3
- C. $1/81$
- D. $1/9$

Answer: D

Solution:

Solution:

It is given that $|B| = 81$

$$\therefore |B| = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0 a_{11} & 3^1 a_{12} & 3^2 a_{13} \\ 3^1 a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}$$

$$\Rightarrow 81 = 3^3 \cdot 3^2 \cdot 3^1 |A|$$

$$\Rightarrow 3^4 = 3^6 |A| \Rightarrow |A| = \frac{1}{9}$$

Question87

Let two points be $A(1, -1)$ and $B(0, 2)$. If a point $P(x', y')$ be such that
 the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x + y - 4\lambda = 0$,
 then a value of λ is:
[Jan.8,2020 (I)]

Options:

A. 4

B. 3

C. 1

D. -3

Answer: B

Solution:

Solution:

$$D = \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix} = 5$$

$$\Rightarrow -2(1 - x') + (y' + x') = \pm 10$$

$$\Rightarrow -2 + 2x' + y' + x' = \pm 10$$

$$\Rightarrow 3x' + y' = 12 \text{ or } 3x' + y' = -8$$

$$\therefore \lambda = 3, -2$$

Question 88

If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to:
[Jan. 9, 2020 (I)]

Options:

A. 8

B. 16

C. 72

D. 2

Answer: A

Solution:

Solution:

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = ((9 + 4) - 1(3 - 4) + 2(-1 - 3))$$

$$= 13 + 1 - 8 = 6$$

$$|\text{adj } B| = |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2} = |A|^4 = (36)^2$$

$$|C| = |3A| = 3^3 \times 6$$

$$\text{Hence, } \frac{|\text{adj } B|}{|C|} = \frac{36 \times 36}{3^3 \times 6} = 8$$

Question89

The following system of linear equations

$$7x + 6y - 2z = 0$$

$$3x + 4y + 2z = 0$$

$$x - 2y - 6z = 0, \text{ has}$$

[Jan. 9, 2020 (II)]

Options:

A. infinitely many solutions, (x, y, z) satisfying $y = 2z$.

B. no solution.

C. infinitely many solutions, (x, y, z) satisfying $x = 2z$.

D. only the trivial solution.

Answer: C

Solution:

Solution:

The given system of linear equations

$$7x + 6y - 2z = 0 \dots\dots(i)$$

$$3x + 4y + 2z = 0 \dots\dots(ii)$$

$$x - 2y - 6z = 0 \dots\dots(iii)$$

Now, determinant of coefficient matrix

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$$

$$= 7(-20) - 6(-20) - 2(-10)$$

$$= -140 + 120 + 20 = 0$$

So, there are infinite non-trivial solutions.

From eqn. (i) $+3 \times$ (iii); we get

$$10x - 20z = 0 \Rightarrow x = 2z$$

Hence, there are infinitely many solutions (x, y, z) satisfying $x = 2z$

Question90

For which of the following ordered pairs (μ, δ) , the system of linear equations

$$x + 2y + 3z = 1$$

$$3x + 4y + 5z = \mu$$

$$4x + 4y + 4z = \delta$$

is inconsistent?

[Jan. 8, 2020 (I)]

Options:

A. (4,3)

B. (4,6)

C. (1,0)

D. (3,4)

Answer: A

Solution:

Solution:

From the given linear equation, we get

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix} \quad (R_3 \rightarrow R_3 - 2R_2 + 3R_3)$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Now, let $P_3 = 4x + 4y + 4z - \delta = 0$. If the system has solutions it will have infinite solution.

So, $P_3 \equiv \alpha P_1 + \beta P_2$

Hence, $3\alpha + \beta = 4$ and $4\alpha + 2\beta = 4$

$\Rightarrow \alpha = 2$ and $\beta = -2$

So, for infinite solution $2\mu - 2 = \delta$

\Rightarrow For $2\mu \neq \delta + 2$ system is inconsistent

Question91

The system of linear equations

$$\lambda x + 2y + 2z = 5$$

$$2\lambda x + 3y + 5z = 8$$

$$4x + \lambda y + 6z = 10 \text{ has :}$$

[Jan. 8, 2020 (II)]

Options:

A. no solution when $\lambda = 8$

B. a unique solution when $\lambda = -8$

C. no solution when $\lambda = 2$

D. infinitely many solutions when $\lambda = 2$

Answer: C

Solution:

Solution:

$$D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$$

$$D = \lambda^2 + 6\lambda - 16$$

$$D = (\lambda + 8)(2 - \lambda)$$

For no solutions, $D = 0$

$$\Rightarrow \lambda = -8, 2$$

when $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$$

$$= 5[18 - 10] - 2[48 - 50] + 2(16 - 30)$$

$$= 40 + 4 - 28 \neq 0$$

There exist no solutions for $\lambda = 2$

Question92

If the system of linear equations

$$2x + 2ay + az = 0$$

$$2x + 3by + bz = 0$$

$$2x + 4cy + cz = 0$$

where $a, b, c \in \mathbb{R}$ are non-zero and distinct; has a non-zero solution, then:

[Jan. 7, 2020 (I)]

Options:

A. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

B. a, b, c are in G.P.

C. $a + b + c = 0$

D. a, b, c are in A.P.

Answer: A

Solution:

Solution:

For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow (3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$$

$$\Rightarrow -bc + 2ac - ab = 0$$

$$\Rightarrow ab + bc + 2ac$$

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ in A.P.}$$

Question93

Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$. If $B = A + A^4$, then $\det(B)$:

[Sep. 06, 2020 (II)]

Options:

A. is one

B. lies in (2,3)

C. is zero

D. lies in (1,2)

Answer: D

Solution:

Solution:

$$\therefore A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in \mathbb{N}$$

$$\therefore B = A + A^4$$

$$= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} \cos \frac{\pi}{5} + \cos \frac{4\pi}{5} & \sin \frac{\pi}{5} + \sin \frac{4\pi}{5} \\ -\sin \frac{\pi}{5} - \sin \frac{4\pi}{5} & \cos \frac{\pi}{5} + \cos \frac{4\pi}{5} \end{bmatrix}$$

$$\text{Then, } \det(B) = 2 \sin\left(\frac{\pi}{5}\right) \cdot \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$$

$$= \frac{\sqrt{10 - 2\sqrt{5}}}{2} \approx 2.352 \approx 1.175$$

$$\therefore \det B \in (1, 2)$$

Question94

$$\text{If } \Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D \text{ then } B + C \text{ is equal to :}$$

[Sep. 03, 2020 (I)]

Options:

A. -1

B. 1

C. -3

D. 9

Answer: C

Solution:

Solution:

$$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$$
$$\Rightarrow \Delta = \begin{vmatrix} x-2 & x-1 & x-1 \\ 2x-3 & x-1 & x-1 \\ 3x-5 & 2x-3 & 5x-9 \end{vmatrix} \begin{bmatrix} C_3 \rightarrow C_3 - C_2 \\ C_2 \rightarrow C_2 - C_1 \end{bmatrix}$$
$$\Rightarrow \Delta = \begin{vmatrix} x-2 & x-1 & x-1 \\ x-1 & 0 & 0 \\ 3x-5 & 2x-3 & 5x-9 \end{vmatrix} [R_2 \rightarrow R_2 - R_1]$$
$$\Rightarrow \Delta = -(x-1)[(x-1)(5x-9) - (x-1)(2x-3)]$$
$$\Rightarrow \Delta = -(x-1)[(5x^2 - 14x + 9) - (2x^2 - 5x + 3)]$$
$$= -3x^3 + 12x^2 - 15x + 6$$

So, $B + C = -3$

Question95

If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2} \right] \rightarrow \mathbb{R}$,

defined by $f(\theta) = \begin{vmatrix} -\sin^2\theta & -1 - \sin^2\theta & 1 \\ -\cos^2\theta & -1 - \cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$ are m and M respectively, then

the ordered pair (m, M) is equal to :
[Sep. 05, 2020 (I)]

Options:

A. $(0, 2\sqrt{2})$

B. $(-4, 0)$

C. $(-4, 4)$

D. $(0, 4)$

Answer: B

Solution:

Solution:

Applying $C_2 \rightarrow C_2 - C_1$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1 & 1 \\ -\cos^2\theta & -1 & 1 \\ 12 & -2 & -2 \end{vmatrix}$$
$$= 4(\cos^2\theta - \sin^2\theta)$$

$$= 4 \cos 2 \theta, \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2} \right)$$

$$\text{Max. } f(\theta) = M = 0$$

$$\text{Min. } f(\theta) = m = -4$$

$$\text{So, } (m, M) = (-4, 0)$$

Question96

$$\text{Let } a - 2b + c = 1.$$

$$\text{If } f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}, \text{ then:}$$

[Jan. 9,2020 (II)]

Options:

$$\text{A. } f(-50) = 501$$

$$\text{B. } f(-50) = -1$$

$$\text{C. } f(50) = -501$$

$$\text{D. } f(50) = 1$$

Answer: D

Solution:

Solution:

$$f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$$R_1 = R_1 + R_3 - 2R_2$$

$$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$$\Rightarrow f(x) = 1 \Rightarrow f(50) = 1$$

Question97

$$\text{If } a + x = b + y = c + z + 1, \text{ where } a, b, c, x, y, z \text{ are non-zero distinct real}$$

$$\text{numbers, then } \begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} \text{ is equal to}$$

[Sep. 05, 2020 (II)]

Options:

$$\text{A. } y(b - a)$$

B. $y(a - b)$

C. 0

D. $y(a - c)$

Answer: B

Solution:

Solution:

Use properties of determinant

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} + y \begin{vmatrix} x & 1 & x+a \\ y & 1 & y+b \\ z & 1 & z+c \end{vmatrix}$$

$$= 0 + y \begin{vmatrix} x & 1 & x+a \\ y-x & 0 & 0 \\ z-x & 0 & -1 \end{vmatrix} \left[\begin{array}{l} R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - R_1 \end{array} \right]$$

$$= -y(x-y) = -y(b-a) = y(a-b)$$

Question98

Let A be a 3×3 matrix such that $\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and

$B = \text{adj}(\text{adj } A)$. If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to:

[Sep. 03, 2020 (II)]

Options:

A. $\left(3, \frac{1}{81}\right)$

B. $\left(9, \frac{1}{9}\right)$

C. (3,81)

D. $\left(9, \frac{1}{81}\right)$

Answer: A

Solution:

Solution:

$$|\text{adj } A| = |A|^2 = 9$$

$$[\because |\text{adj } A| = |A|^{n-1}]$$

$$\Rightarrow |A| = \pm 3 = \lambda \Rightarrow |\lambda| = 3$$

$$\Rightarrow |B| = |\text{adj } A|^2 = 81$$

$$\mu = |(B^{-1})^T| = |B^{-1}| = |B|^{-1} = \frac{1}{81}$$

Question99

The values of λ and μ for which the system of linear equations

$$x + y + z = 2$$

$$x + 2y + 3z = 5$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions are, respectively:

[Sep. 06, 2020 (I)]

Options:

A. 6 and 8

B. 5 and 7

C. 5 and 8

D. 4 and 9

Answer: C

Solution:

Solution:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 5$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0 \Rightarrow \lambda = 8$$

Question100

The sum of distinct values of λ for wheih the system of equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

has non-zero solutions, is _____.

[NA Sep. 06, 2020 (II)]

Answer: 3

Solution:

For non-zero solution, $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$$\Rightarrow 6\lambda^3 - 36\lambda^2 + 54\lambda = 0$$

$$\Rightarrow 6\lambda[\lambda^2 - 6\lambda + 9] = 0$$

$$\Rightarrow \lambda = 0, \lambda = 3 \text{ [Distinct values]}$$

Then, the sum of distinct values of $\lambda = 0 + 3 = 3$.

Question101

Let $\lambda \in \mathbb{R}$. The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

[Sep. 05, 2020 (I)]

Options:

A. exactly one negative value of λ

B. exactly one positive value of λ

C. every value of λ

D. exactly two value of λ

Answer: A

Solution:

Solution:

$$\therefore \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0 \Rightarrow 3\lambda^2 - 7\lambda - 12 = 0$$

$$\Rightarrow \lambda = 3 \text{ or } -\frac{2}{3}$$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3 - \lambda)$$

$$\therefore \text{When } \lambda = -\frac{2}{3}, D_1 \neq 0.$$

Hence, equations will be inconsistent when $\lambda = -\frac{2}{3}$.

Question102

If the system of linear equations

$$x + y + 3z = 0$$

$$x + 3y + k^2z = 0$$

$$3x + y + 3z = 0$$

has a non-zero solution (x, y, z) for some $k \in \mathbb{R}$, then $x + \left(\frac{y}{z}\right)$ is equal to :

[Sep. 05, 2020 (II)]

Options:

A. -3

B. 9

C. 3

D. -9

Answer: A

Solution:

Solution:

Since, system of linear equations has non-zero solution
 $\therefore \Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 1(9 - k^2) - 1(3 - 3k^2) + 3(1 - 9) = 0$$

$$\Rightarrow 9 - k^2 - 3 + 3k^2 - 24 = 0$$

$$\Rightarrow 2k^2 = 18 \Rightarrow k^2 = 9, k = \pm 3$$

So, equations are

$$x + y + 3z = 0 \dots\dots(i)$$

$$x + 3y + 9z = 0 \dots\dots(ii)$$

$$3x + y + 3z = 0 \dots\dots(iii)$$

Now, from equation (i) - (ii),

$$-2y - 6z = 0 \Rightarrow y = -3z \Rightarrow \frac{y}{z} = -3 \dots\dots(iv)$$

Now, from equation (i) - (iii),

$$-2x = 0 \Rightarrow x = 0$$

$$\text{So, } x + \frac{y}{z} = 0 - 3 = -3$$

Question103

If the system of equations $x - 2y + 3z = 9$, $2x + y + z = b$, $x - 7y + az = 24$, has infinitely many solutions, then $a - b$ is equal to _____.

[NA Sep. 04, 2020 (I)]

Answer: 5

Solution:

For infinitely many solutions,

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$$\Rightarrow (a + 7) - 2(1 - 2a) + 3(-15) = 0$$

$$\Rightarrow a = 8$$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & 9 \\ 2 & 1 & b \\ 1 & -7 & 24 \end{vmatrix} = 0$$

$$\Rightarrow (24 + 7b) - 2(b - 48) + 9(-15) = 0$$

$$\Rightarrow b = 3$$

$$\therefore a - b = 5$$

Question104

Suppose the vectors x_1 , x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1 , b_2 and b_3 respectively. If

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\text{and } b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{ then the determinant of A is equal to:}$$

[Sep. 04, 2020 (II)]

Options:

A. 4

B. 2

C. $\frac{1}{2}$

D. $\frac{3}{2}$

Answer: B

Solution:

Given that $Ax = b$ has solutions x_1 , x_2 , x_3 and b is equal to b_1 , b_2 and b_3

$$\therefore x_1 + y_1 + z_1 = 1$$

$$\Rightarrow 2y_1 + z_1 = 2 \Rightarrow z_1 = 2$$

Determinant of coefficient matrix

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2$$

Question 105

If the system of equations

$$x + y + z = 2$$

$$2x + 4y - z = 6$$

$$3x + 2y + \lambda z = \mu$$

has infinitely many solutions, then :

[Sep. 04, 2020 (II)]

Options:

A. $\lambda + 2\mu = 14$

B. $2\lambda - \mu = 5$

C. $\lambda - 2\mu = -5$

D. $2\lambda + \mu = 14$

Answer: D

Solution:

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \quad [\because \text{Equation has many solutions}]$$

$$\Rightarrow -15 + 6 + 2\lambda = 0 \Rightarrow \lambda = \frac{9}{2}$$

$$\therefore D_z = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 2\mu \end{vmatrix} = 0 \Rightarrow \mu = 5$$

$$\therefore 2\lambda + \mu = 14$$

Question 106

Let S be the set of all integer solutions, (x, y, z), of the system of equations

$$x - 2y + 5z = 0$$

$$-2x + 4y + z = 0$$

$$-7x + 14y + 9z = 0$$

such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then, the number of elements in the set S is equal to _____.

[NA Sep. 03, 2020 (II)]

Answer: 8

Solution:

The given system of equations

$$x - 2y + 5z = 0 \dots\dots(i)$$

$$-2x + 4y + z = 0 \dots\dots(ii)$$

$$-7x + 14y + 9z = 0 \dots\dots(iii)$$

From equation, $2 \times (i) + (ii) \Rightarrow z = 0$

Put $z = 0$ in equation (i), we get $x = 2y$

$$\therefore 15 \leq x^2 + y^2 + z^2 \leq 150$$

$$\Rightarrow 15 \leq 4y^2 + y^2 \leq 150$$

$$[\because x = 2y, z = 0]$$

$$\Rightarrow 3 \leq y^2 \leq 30$$

$$\Rightarrow y = \pm 2, \pm 3, \pm 4, \pm 5$$

$$\Rightarrow 8 \text{ solutions.}$$

Question107

Let S be the set of all $\lambda \in \mathbb{R}$ for which the system of linear equations

$$2x - y + 2z = 2$$

$$x - 2y + \lambda z = -4$$

$$x + \lambda y + z = 4$$

has no solution. Then the set S

[Sep. 02, 2020 (I)]

Options:

A. contains more than two elements.

B. is an empty set.

C. is a singleton.

D. contains exactly two elements.

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = -(\lambda - 1)(2\lambda + 1)$$

$$\Delta_1 = \begin{vmatrix} 2 & -1 & 2 \\ -4 & -2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = -2(\lambda^2 + 6\lambda - 4)$$

For no solution $\Delta = 0$ and at least one of Δ_1 , Δ_2 and Δ_3 is non-zero.

$$\therefore \Delta = 0 \Rightarrow \lambda = 1, -\frac{1}{2} \text{ and } \Delta_1 \neq 0$$

$$\text{Hence, } S = \left\{ 1, -\frac{1}{2} \right\}$$

Question108

Let $A = \{ X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1 \}$ where

P = $\begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, then the set A :

[Sep. 02, 2020 (II)]

Options:

- A. is a singleton
- B. is an empty set
- C. contains more than two elements
- D. contains exactly two elements

Answer: D

Solution:

Solution:

$$\because |P| = 1(-3 + 36) - 2(2 + 4) + 1(-18 - 3) = 0$$

Given that $PX = 0$

\therefore System of equations

$$x + 2y + z = 0; 2x - 3y + 4z = 0$$

and $x + 9y - z = 0$ has infinitely many solution.

Let $z = k \in \mathbb{R}$ and solve above equations, we get

$$x = -\frac{11k}{7}, y = \frac{2k}{7}, z = k$$

$$\text{But given that } x^2 + y^2 + z^2 = 1$$

$$\therefore k = \pm \frac{7}{\sqrt{174}}$$

\therefore Two solutions only.

Question 109

If **A** = $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then the matrix **A**⁻⁵⁰ when $\theta = \frac{\pi}{12}$, is equal to:

[Jan 09, 2019 (I)]

Options:

A. $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

B. $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

C. $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

D. $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \Rightarrow |A| = 1$$

$$\text{adj}(A) = \begin{bmatrix} +\cos \theta & -\sin \theta \\ +\sin \theta & +\cos \theta \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} = B$$

$$B^2 = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow B^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

$$\Rightarrow A^{-50} = B^{50} = \begin{bmatrix} \cos(50\theta) & \sin(50\theta) \\ -\sin(50\theta) & \cos(50\theta) \end{bmatrix}$$

$$(A^{-50})_{\theta = \frac{\pi}{12}} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\left[\therefore \cos\left(\frac{50\pi}{12}\right) = \cos\left(4\pi\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \right]$$

Question 110

If

$$A = \begin{bmatrix} e^t & e^{-t} \cos t & e^{-t} \sin t \\ e^t & -e^{-t} \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\ e^t & 2e^{-t} \sin t & -2e^{-t} \cos t \end{bmatrix}$$

then A is:

[Jan. 09, 2019 (II)]

Options:

A. invertible for all $t \in \mathbb{R}$.

B. invertible only if $t = \pi$.

C. not invertible for any $t \in \mathbb{R}$.

D. invertible only if $t = \frac{\pi}{2}$.

Answer: A

Solution:

Solution:

$$\det(A) = |A|$$

$$A = \begin{vmatrix} e^t & e^{-t} \cos t & e^{-t} \sin t \\ e^t & -e^{-t} \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\ e^t & 2e^{-t} \sin t & -2e^{-t} \cos t \end{vmatrix}$$

$$= e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2 \sin t & -2 \cos t \end{vmatrix}$$

$$= e^{-t} \begin{vmatrix} 0 & 2 \cos t + \sin t & 2 \sin t - \cos t \\ 0 & -\cos t - 3 \sin t & -\sin t + 3 \cos t \\ 1 & 2 \sin t & -2 \cos t \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 + R_3$$

$$= e^{-t} \begin{vmatrix} 0 & -5 \sin t & 5 \cos t \\ 0 & -\cos t - 3 \sin t & -\sin t + 3 \cos t \\ 1 & 2 \sin t & -2 \cos t \end{vmatrix}$$

$$R_1 \rightarrow R_1 + 2R_2$$

$$= e^{-t} [(-5 \sin t)(-\sin t + 3 \cos t) - 5 \cos t(-\cos t - 3 \sin t)]$$

$$= 5e^{-t} \neq 0, \forall t \in \mathbb{R}$$

$\therefore A$ is invertible.

Question 111

Let $d \in \mathbb{R}$, and

$$A = \begin{bmatrix} -2 & 4 + d & (\sin \theta) - 2 \\ 1 & (\sin \theta) + 2 & d \\ 5 & (2 \sin \theta) - d & (-\sin \theta) + 2 + 2d \end{bmatrix}$$

$\theta \in [0, 2\pi]$. If the minimum value of $\det(A)$ is 8, then a value of d is:
[Jan 10, 2019 (I)]

Options:

A. -5

B. -7

C. $2(\sqrt{2} + 1)$

D. $2(\sqrt{2} + 2)$

Answer: A

Solution:

Solution:

$$\det A = \begin{vmatrix} -2 & 4+d & \sin \theta^{-2} \\ 1 & \sin \theta + 2 & d \\ 5 & (2 \sin \theta) - d & -\sin \theta + 2 + 2d \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - 2R_2 + R_1$

$$\begin{aligned} \text{we get } \det(A) &= \begin{vmatrix} -2 & 4+d & \sin \theta - 2 \\ 1 & \sin \theta + 2 & d \\ 1 & 0 & 0 \end{vmatrix} \\ &= d(4+d) - (\sin^2 \theta - 4) \\ \Rightarrow \det(A) &= d^2 + 4d + 4 - \sin^2 \theta = (d+2)^2 - \sin^2 \theta \\ \text{Minimum value of } \det(A) &\text{ is attained when } \sin^2 \theta = 1 \\ \therefore (d+2)^2 - 1 &= 8 \Rightarrow (d+2)^2 = 9 \Rightarrow d+2 = \pm 3 \\ \Rightarrow d &= -5 \text{ or } 1 \end{aligned}$$

Question 112

Let $a_1, a_2, a_3, \dots, a_{10}$ be in G.P. with $a_i > 0$ for $i = 1, 2, \dots, 10$ and S be the set of pairs (r, k) , $r, k \in \mathbb{N}$ (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S , is :
[Jan. 10, 2019 (II)]

Options:

- A. 4
- B. infinitely many
- C. 2
- D. 10

Answer: B

Solution:

Solution:

Let common ratio of G.P. be R

$$\Rightarrow a_2 = a_1 R, a_3 = a_1 R^2, \dots, a_{10} = a_1 R^9$$

$$C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3$$

$$\Delta = \begin{vmatrix} \ln \left(\frac{a_1^r a_2^k}{a_2^r a_3^k} \right) & \ln \left(\frac{a_2^r a_3^k}{a_3^r a_4^k} \right) & \ln a_3^r a_4^k \\ \ln \left(\frac{a_4^r a_5^k}{a_5^r a_6^k} \right) & \ln \left(\frac{a_5^r a_6^k}{a_6^r a_7^k} \right) & \ln a_6^r a_7^k \\ \ln \left(\frac{a_7^r a_8^k}{a_8^r a_9^k} \right) & \ln \left(\frac{a_8^r a_9^k}{a_9^r a_{10}^k} \right) & \ln a_9^r a_{10}^k \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \ln 1 R^{r+k} & \ln \frac{1}{R^{r+k}} & \ln a_3^r a_4^k \\ \ln \frac{1}{R^{r+k}} & \ln \frac{1}{R^{r+k}} & \ln a_6^r a_7^k \\ \ln \frac{1}{R^{r+k}} & \ln \frac{1}{R^{r+k}} & \ln a_9^r a_{10}^k \end{vmatrix} = 0$$

$\forall r, k \in \mathbb{N}$

Hence, number of elements in S is infinitely many.

Question 113

Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is:

[Jan. 10, 2019 (II)]

Options:

A. $2\sqrt{3}$

B. $-2\sqrt{3}$

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: A

Solution:

Solution:

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{vmatrix} \\ &= 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1) \\ &= 2b^2 + 4 - b^2 - 1 = b^2 + 3 \\ \frac{|A|}{b} &= b + \frac{3}{b} \\ \therefore \frac{b + \frac{3}{b}}{2} &\geq \left(b \frac{3}{b}\right)^{\frac{1}{2}} \Rightarrow b + \frac{3}{b} \geq 2\sqrt{3} \end{aligned}$$

$$\therefore \frac{|A|}{b} \geq 2\sqrt{3}$$

Minimum value of $\frac{|A|}{b}$ is $2\sqrt{3}$.

Question 114

If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)(x+a+b+c)^2$, $x \neq 0$ and $a+b+c \neq 0$, then x is equal to:
[Jan. 11, 2019 (II)]

Options:

- A. abc
- B. $-(a+b+c)$
- C. $2(a+b+c)$
- D. $-2(a+b+c)$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$C_1 \rightarrow C_1 - C_3, C_2 \rightarrow C_2 - C_3$$

$$\Delta = (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$$

$$= (a+b+c)(a+b+c)^2$$

$$\text{Hence, } x = -2(a+b+c)$$

Question 115

If $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$ then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4} \right)$, $\det(A)$ lies in the interval :
[Jan. 12, 2019 (II)]

Options:

A. $\left(1, \frac{5}{2} \right]$

B. $\left[\frac{5}{2}, 4 \right)$

C. $\left(0, \frac{3}{2} \right]$

D. $\left(\frac{3}{2}, 3 \right]$

Answer: D

Solution:

Solution:

$$\begin{aligned} A &= \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 2 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix} \quad R_1 \rightarrow R_1 + R_3 \\ &= 2(\sin^2 \theta + 1) \\ \text{Since } \theta &\in \left(\frac{3\pi}{4}, \frac{5\pi}{4} \right) \Rightarrow \sin^2 \theta \in \left(0, \frac{1}{2} \right) \\ \therefore \det(A) &\in [2, 3) \\ [2, 3) &\subset \left(\frac{3}{2}, 3 \right] \end{aligned}$$

Question 116

An ordered pair (α, β) for which the system of linear equations
 $(1 + \alpha)x + \beta y + z = 2$
 $ax + (1 + \beta)y + z = 3$
 $\alpha x + \beta y + 2z = 2$
has a unique solution, is :
[Jan. 12, 2019 (I)]

Options:

A. (2,4)

B. (-3,1)

C. (-4,2)

D. (1,-3)

Answer: A

Solution:

Solution:

∴ The system of linear equations has a unique solution.

∴ $\Delta \neq 0$

$$\Delta = \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\Rightarrow \begin{vmatrix} 1+\alpha+\beta+1 & \beta & 1 \\ \alpha+1+\beta+1 & 1+\beta & 1 \\ \alpha+\beta+2 & \beta & 2 \end{vmatrix} \neq 0 \quad [C_1 \rightarrow C_1 + C_2 + C_3]$$

$$\Rightarrow (\alpha + \beta + 2) \begin{vmatrix} 1 & \beta & 1 \\ 1 & 1+\beta & 1 \\ 1 & \beta & 2 \end{vmatrix} \neq 0$$

$$\Rightarrow (\alpha + \beta + 2) \begin{vmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \neq 0$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow (\alpha + \beta + 2)1(1) \neq 0$$

$$\Rightarrow \alpha + \beta + 2 \neq 0$$

∴ Ordered pair (2,4) satisfies this condition

∴ $\alpha = 2$ and $\beta = 4$.

Question 117

The set of all values of λ for which the system of linear equations

$$x - 2y - 2z = \lambda x$$

$$x + 2y + z = \lambda y$$

$$-x - y = \lambda z$$

has a non-trivial solution :

[Jan. 12, 2019 (II)]

Options:

A. is a singleton

B. contains exactly two elements

C. is an empty set

D. contains more than two elements

Answer: A

Solution:

Solution:

Consider the given system of linear equations

$$x(1 - \lambda) - 2y - 2z = 0$$

$$x + (2 - \lambda)y + z = 0$$

$$-x - y - \lambda z = 0$$

Now, for a non-trivial solution, the determinant of coefficient matrix is zero.

$$\begin{vmatrix} 1 - \lambda & -2 & -2 \\ 1 & 2 - \lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1 - \lambda)^3 = 0$$

$$\lambda = 1$$

Question 118

If the system of linear equations

$$2x + 2y + 3z = a$$

$$3x - y + 5z = b$$

$$x - 3y + 2z = c$$

where, a, b, c are non-zero real numbers, has more than one solution, then :

[Jan. 11, 2019 (I)]

Options:

A. $b - c + a = 0$

B. $b - c - a = 0$

C. $a + b + c = 0$

D. $b + c - a = 0$

Answer: B

Solution:

Solution:

\because System of equations has more than one solution $\therefore \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ for infinite solution

$$\Delta_1 = \begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix} = a(13) + 2(5c - 2b) + 3(-3b + c)$$

$$= 13a - 13b + 13c = 0$$

$$\text{i.e., } a - b + c = 0$$

$$\text{or } b - c - a = 0$$

Question 119

The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$$x + 3y + 7z = 0$$

$$-x + 4y + 7z = 0$$

$$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$$

**has a non-trivial solution, is:
[Jan. 10, 2019 (II)]**

Options:

- A. three
- B. two
- C. four
- D. one

Answer: B

Solution:

Solution:

Since, the system of linear equations has, non-trivial solution then determinant of coefficient matrix = 0

$$\text{i.e., } \begin{vmatrix} \sin 3\theta & \cos 2\theta & 2 \\ 1 & 3 & 7 \\ -1 & 4 & 7 \end{vmatrix} = 0$$

$$\sin 3\theta(21 - 28) - \cos 2\theta(7 + 7) + 2(4 + 3) = 0$$

$$\sin 3\theta + 2 \cos 2\theta - 2 = 0$$

$$3 \sin \theta - 4 \sin^3 \theta + 2 - 4 \sin^2 \theta - 2 = 0$$

$$4 \sin^3 \theta + 4 \sin^2 \theta - 3 \sin \theta = 0$$

$$\sin \theta(4 \sin^2 \theta + 4 \sin \theta - 3) = 0$$

$$\sin \theta(4 \sin^2 \theta + 6 \sin \theta - 2 \sin \theta - 3) = 0$$

$$\sin \theta[2 \sin \theta(2 \sin \theta - 1) + 3(2 \sin \theta - 1)] = 0$$

$$\sin \theta(2 \sin \theta - 1)(2 \sin \theta + 3) = 0$$

$$\sin \theta = 0, \sin \theta = \frac{1}{2} \left(\because \sin \theta \neq -\frac{3}{2} \right)$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Hence, for two values of θ , system of equations has non-trivial solution

Question 120

If the system of equations

$$\mathbf{x} + \mathbf{y} + \mathbf{z} = \mathbf{5}$$

$$\mathbf{x} + 2\mathbf{y} + 3\mathbf{z} = \mathbf{9}$$

$$\mathbf{x} + 3\mathbf{y} + \alpha\mathbf{z} = \beta$$

has infinitely many solutions, then $\beta - \alpha$ equals:

[Jan 10, 2019 (I)]

Options:

- A. 21
- B. 8
- C. 18
- D. 5

Answer: B

Solution:

Solution:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 2\alpha - 9 - \alpha + 3 + 1 = \alpha - 5$$

$$\begin{aligned}\Delta_1 &= \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & \alpha \end{vmatrix} = 5(2\alpha - 9) - 1(9\alpha - 3\beta) + (27 - 2\beta) \\ &= 10\alpha - 45 - 9\alpha + 3\beta + 27 - 2\beta \\ &= \alpha + \alpha - 18\end{aligned}$$

$$\begin{aligned}\Delta_2 &= \begin{vmatrix} 1 & 5 & 1 \\ 1 & 9 & 3 \\ 1 & \beta & \alpha \end{vmatrix} = 9\alpha - 3\beta - 5(\alpha - 3) + 1(\beta - 9) \\ &= 9\alpha - 3\beta - 5\alpha + 15 + \beta - 9 = 4\alpha - 2\beta + 6\end{aligned}$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 2\beta - 27 - \beta + 9 + 5 = \beta - 13$$

Since, the system of equations has infinite many solutions.

Hence,

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$$

$$\Rightarrow \alpha = 5, \beta = 13 \Rightarrow \beta - \alpha = 8$$

Question 121

If the system of linear equations

$$\mathbf{x} - 4\mathbf{y} + 7\mathbf{z} = \mathbf{g}$$

$$3\mathbf{y} - 5\mathbf{z} = \mathbf{h}$$

$$-2\mathbf{x} + 5\mathbf{y} - 9\mathbf{z} = \mathbf{k}$$

is consistent, then :

[Jan. 09, 2019 (II)]

Options:

A. $g + 2h + k = 0$

B. $g + h + 2k = 0$

C. $2g + h + k = 0$

D. $g + h + k = 0$

Answer: C

Solution:

Solution:

Consider the system of linear equations

$$x - 4y + 7z = g \text{(i)}$$

$$3y - 5z = h \text{(ii)}$$

$$-2x + 5y - 9z = k \text{(iii)}$$

Multiply equation (i) by 2 and add equation (i), equation (ii) and equation (iii)

$$\Rightarrow 0 = 2g + h + k. \therefore 2g + h + k = 0$$

then system of equation is consistent.

Question122

Let A and B be two invertible matrices of order 3×3 . If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to :
[Jan. 11, 2019 (II)]

Options:

A. $\frac{1}{4}$

B. 1

C. $\frac{1}{16}$

D. 16

Answer: C

Solution:

Solution:

Let $|A| = a$, $|B| = b$

$$\Rightarrow |A^T| = a|A^{-1}| = \frac{1}{a}, |B^T| = b, |B^{-1}| = \frac{1}{b}$$

$$\therefore |ABA^T| = 8 \Rightarrow |A| |B| |A^T| = 8 \dots (1)$$

$$\Rightarrow a \cdot b \cdot a = 8 \Rightarrow a^2 b = 8$$

$$\therefore |AB^{-1}| = 8 \Rightarrow |A| |B^{-1}| = 8 \Rightarrow a \cdot \frac{1}{b} = 8 \dots (2)$$

From (1) & (2)

$$a = 4, b = \frac{1}{2}$$

$$\text{Then, } |BA^{-1}B^T| = |B| |A^{-1}| |B^T| = b \cdot \frac{1}{a} \cdot b = \frac{b^2}{a} = \frac{1}{16}$$

Question123

$$\text{If } \Delta_1 = \begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$$

and

$$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix},$$

$x \neq 0$ then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

[April 10, 2019 (I)]

Options:

- A. $\Delta_1 - \Delta_2 = -2x^3$
- B. $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$
- C. $\Delta_1 \times \Delta_2 = -2(x^3 + x - 1)$
- D. $\Delta_1 + \Delta_2 = -2x^3$

Answer: D

Solution:

Solution:

$$\Delta_1 = \begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$$

$$= (x - x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta) + \cos \theta(-\sin \theta + x \cos \theta)$$

$$= -x^3 - x + x \sin^2 \theta + \sin \theta \cos \theta - \cos \theta \sin \theta + x \cos^2 \theta$$

$$= -x^3 - x + x = -x^3$$

Similarly, $\Delta_2 = -x^3$

Then, $\Delta_1 + \Delta_2 = -2x^3$

Question 124

The sum of the real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ is equal to

[April 2019]

Options:

- A. 0
- B. 6
- C. -4
- D. 1

Answer: A

Solution:

Solution:

Given equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

On expansion of determinant along R , we get

$$x[(-3x)(x+2) - 2x(x-3)] + 6[2(x+2) + 3(x-3)]$$

$$- 1[9(2x) - (-3x)(-3)] = 0$$

$$\Rightarrow x[-3x^2 - 6x - 2x^2 + 6x] + 6[2x + 4 + 3x - 9] - 1[4x - 9x] = 0$$

$\Rightarrow x(-5x^2) + 6(5x - 5) - 1(-5x) = 0$
 $\Rightarrow -5x^3 + 30x - 30 + 5x = 0$
 $\Rightarrow 5x^3 - 35x + 30 = 0 \Rightarrow x^3 - 7x + 6 = 0.$
 Since all roots are real

$$\therefore \text{Sum of roots} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3} = 0$$

Question125

A value of $\theta \in (0, \pi / 3)$, for which

$$\begin{vmatrix} 1 + \cos^2 \theta & \sin^2 \theta & 4 \cos 6 \theta \\ \cos^2 \theta & 1 + \sin^2 \theta & 4 \cos 6 \theta \\ \cos^2 \theta & \sin^2 \theta & 1 + 4 \cos 6 \theta \end{vmatrix} = 0,$$

is

[April 12, 2019 (II)]

Options:

A. $\frac{\pi}{9}$

B. $\frac{\pi}{18}$

C. $\frac{7\pi}{24}$

D. $\frac{7\pi}{36}$

Answer: A

Solution:

Solution:

$$C_1 \rightarrow C_1 + C_2$$

$$\begin{vmatrix} 2 & \sin^2 \theta & 4 \cos 6 \theta \\ 2 & 1 + \sin^2 \theta & 4 \cos 6 \theta \\ 1 & \sin^2 \theta & 1 + 4 \cos 6 \theta \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2 \theta & (1 + 4 \cos 6 \theta) \end{vmatrix} = 0$$

On expanding, we get $2 + 4 \cos 6 \theta = 0$

$$\cos 6 \theta = -\frac{1}{2} \because \theta \in \left(0, \frac{\pi}{3}\right) \Rightarrow 6\theta \in (0, 2\pi)$$

$$\text{Therefore, } 6\theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3} \Rightarrow \theta = \frac{\pi}{9} \text{ or } \frac{2\pi}{9}$$

Question126

Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$

in \mathbb{R} , $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to:

[April 09, 2019 (I)]

Options:

A. $y(y^2 - 1)$

B. $y(y^2 - 3)$

C. y^3

D. $y^3 - 1$

Answer: C

Solution:

Solution:

Let $\alpha = \omega$ and $\beta = \omega^2$ are roots of $x^2 + x + 1 = 0$

$$\text{\& Let } \Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} y+1+\omega+\omega^2 & \omega & \omega^2 \\ y+1+\omega+\omega^2 & y+\omega^2 & 1 \\ 1+\omega+\omega^2+y & 1 & y+\omega \end{vmatrix}$$

$$\Delta = \begin{vmatrix} y & \omega & \omega^2 \\ y & y+\omega^2 & 1 \\ y & 1 & y+\omega \end{vmatrix} \quad (\because 1+\omega+\omega^2=0)$$

$$\Delta = y \begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & y+\omega^2 & 1 \\ 1 & 1 & y+\omega \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ & $R_3 \rightarrow R_3 - R_1$

$$\Delta = y \begin{vmatrix} y+\omega^2-\omega & 1-\omega^2 \\ 1-\omega & y+\omega-\omega^2 \end{vmatrix}$$

$$\Rightarrow \Delta = y[y - (\omega - \omega^2)(y + (\omega - \omega^2)) - (1 - \omega)(1 - \omega^2)]$$

$$\Rightarrow \Delta = y[y^2 - (\omega - \omega^2)^2 - 1 + \omega^2 + \omega - \omega^3]$$

$$\Rightarrow \Delta = y[y^2 - \omega^2 - \omega^4 + 2\omega^3 - 1 + \omega^2 + \omega^4 - \omega^3] (\because \omega^4 = \omega)$$

$$\Rightarrow \Delta = y(y^2) = y^3$$

Question 127

Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If

$\det(A) \in [2, 16]$, then c lies in the interval :
[April 08, 2019 (II)]

Options:

- A. [2,3)
- B. $(2 + 2^{3/4}, 4)$
- C. [4,6]
- D. $[3, 2 + 2^{3/4}]$

Answer: C

Solution:

Solution:

Consider, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix}$$

$$= (b-2)(c-2)(c-b)$$

$\therefore 2, b, c$ are in A.P.

$$\therefore (b-2) = (c-b) = d \text{ and } c-2 = 2d$$

$$\Rightarrow |A| = d \cdot 2d \cdot d = 2d^3$$

$$\therefore |A| \in [2, 16] \Rightarrow 1 \leq d^3 \leq 8 \Rightarrow 1 \leq d \leq 2$$

$$4 \leq 2d + 2 \leq 6 \Rightarrow 4 \leq c \leq 6$$

Question 128

If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of

all values of α for which $\det(A) + 1 = 0$, is:
[April 12, 2019 (I)]

Options:

- A. 0
- B. -1

C. 1

D. 2

Answer: C

Solution:

Solution:

$$\because B = A^{-1} \Rightarrow |B| = \frac{1}{|A|}$$

$$\text{Now, } |B| = \begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = 2\alpha^2 - 2\alpha - 25$$

$$\text{Given, } \det(A) + 1 = 0$$

$$\Rightarrow \frac{1}{2\alpha^2 - 2\alpha - 25} + 1 = 0$$

$$\Rightarrow \frac{2\alpha^2 - 2\alpha - 24}{2\alpha^2 - 2\alpha - 25} = 0$$

$$\Rightarrow \alpha = 4, -3 \Rightarrow \text{Sum of values} = 1$$

Question 129

If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$ then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is:

[April 09, 2019 (II)]

Options:

A. $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$

Answer: B

Solution:

Solution:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \dots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\dots+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix} \Rightarrow (n-1)\frac{n}{2} = 78 \Rightarrow n^2 - n - 15 = 0$$

$$\Rightarrow n = 13$$

$$\text{Now, the matrix } \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$$

$$\text{Then, the required inverse of } \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$$

Question130

If the system of linear equations

$$x + y + z = 5$$

$$x + 2y + 2z = 6$$

$x + 3y + \lambda z = \mu$, ($\lambda, \mu \in \mathbb{R}$), has infinitely many solutions, then the value of $\lambda + \mu$ is:

[April 10, 2019 (I)]

Options:

A. 12

B. 9

C. 7

D. 10

Answer: D

Solution:

Solution:

Given system of linear equations: $x + y + z = 5$; $x + 2y + 2z = 6$ and $x + 3y + \lambda z = \mu$ have infinite solution.

$$\therefore \Delta = 0, \Delta x = \Delta y = \Delta z = 0$$

$$\text{Now, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 2) + 1(3 - 2) = 0$$

$$\Rightarrow 2\lambda - 6 - \lambda + 2 + 1 = 0 \Rightarrow \lambda = 3$$

$$\Delta y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & 3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu - 5 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 1(2 - \mu + 5) = 0 \Rightarrow \mu = 7$$

$$\therefore \lambda + \mu = 10$$

Question131

Let λ be a real number for which the system of linear equations:

$$x + y + z = 6$$

$$4x + \lambda y - \lambda z = \lambda - 2$$

$$3x + 2y - 4z = -5$$

has infinitely many solutions. Then λ is a root of the quadratic equation :

[April 10, 2019 (II)]

Options:

A. $\lambda^2 + 3\lambda - 4 = 0$

B. $\lambda^2 - 3\lambda - 4 = 0$

C. $\lambda^2 + \lambda - 6 = 0$

D. $\lambda^2 - \lambda - 6 = 0$

Answer: D

Solution:

Solution:

\therefore system of equations has infinitely many solutions.

$$\therefore \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\text{Here, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 - C_2 \text{ \& } C_2 \rightarrow C_2 - C_3$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4 - \lambda & 2\lambda & -\lambda \\ 1 & 6 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

$$\text{Now, for } \lambda = 3, \Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ \lambda - 2 & \lambda & -\lambda \\ -5 & 2 & -4 \end{vmatrix} = 0$$

$$\text{For } \lambda = 3, \Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 4 & \lambda - 2 & -\lambda \\ 3 & -5 & -4 \end{vmatrix} = 0$$

$$\text{For } \lambda = 3, \Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 4 & \lambda & \lambda - 2 \\ 3 & 2 & -5 \end{vmatrix} = 0$$

\therefore for $\lambda = 3$, system of equations has infinitely many solutions.

Question 132

If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and

$2x - y + z = 0$ has a non-trivial solution (x, y, z) , then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to:

[April 09, 2019 (II)]

Options:

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $-\frac{1}{4}$

D. -4

Answer: B

Solution:

Solution:

Given system of equations has a non-trivial solution.

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow k = \frac{9}{2}$$

\therefore equations are $2x + 3y - z = 0$ (i)

$2x - y + z = 0$ (ii)

$2x + 9y - 4z = 0$ (iii)

By (i) - (ii), $2y = z$

$\therefore z = -4x$ and $2x + y = 0$

$$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{-1}{2} + \frac{1}{2} - 4 + \frac{9}{2} = \frac{1}{2}$$

Question133

The greatest value of $c \in \mathbb{R}$ for which the system of linear equations $\mathbf{x} - cy - cz = 0$; $\mathbf{cx} - y + cz = 0$; $\mathbf{cx} + cy - z = 0$ has a non-trivial solution, is:

[April 08, 2019 (I)]

Options:

A. -1

B. $\frac{1}{2}$

C. 2

D. 0

Answer: B

Solution:

Solution:

If the system of equations has non-trivial solutions, then the determinant of coefficient matrix is zero

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

$$\Rightarrow (1 - c^2) + c(-c - c^2) - c(c^2 + c) = 0$$

$$\Rightarrow (1 + c)(1 - c) - 2c^2(1 + c) = 0$$

$$\Rightarrow (1+c)(1-c-2c^2) = 0$$

$$\Rightarrow (1+c)^2(1-2c) = 0$$

$$\Rightarrow c = -1 \text{ or } \frac{1}{2}$$

Hence, the greatest value of c is $\frac{1}{2}$ for which the system of linear equations has non-trivial solution.

Question 134

If the system of linear equations

$$x - 2y + kz = 1$$

$$2x + y + z = 2$$

$$3x - y - kz = 3$$

has a solution (x, y, z) , $z \neq 0$, then (x, y) lies on the straight line whose equation is :

[April 08, 2019 (II)]

Options:

A. $3x - 4y - 1 = 0$

B. $4x - 3y - 4 = 0$

C. $4x - 3y - 1 = 0$

D. $3x - 4y - 4 = 0$

Answer: B

Solution:

Solution:

Given system of linear equations,

$$x - 2y + kz = 1, 2x + y + z = 2, 3x - y - kz = 3$$

$$\Delta = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix}$$

$$= 1(-k+1) + 2(-2k-3) + k(-2-3)$$

$$= -k+1-4k-6-5k = -10k-5 = -5(2k+1)$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = -5(2k+1)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & k \\ 2 & 2 & 1 \\ 3 & 3 & -k \end{vmatrix} = 0, \Delta_3 = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$$\because z \neq 0 \Rightarrow \Delta = 0$$

$$\Rightarrow -5(2k+1) = 0 \Rightarrow k = -\frac{1}{2}$$

\therefore System of equation has infinite many solutions.

$$\text{Let } z = \lambda \neq 0 \text{ then } x = \frac{10-3\lambda}{10} \text{ and } y = -\frac{2\lambda}{5}$$

$$\therefore (x, y) \text{ must lie on line } 4x - 3y - 4 = 0$$

Question135

If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to :
[2018]

Options:

A. (-4,3)

B. (-4,5)

C. (4,5)

D. (-4,-5)

Answer: B

Solution:

Solution:

$$\text{Here, } \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$$

$$\text{Put } x=0 \Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A^3 = (-4)^3$$

$$\Rightarrow A = -4$$

$$\Rightarrow \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx-4)(x+4)^2$$

Now take x common from both the sides

$$\therefore \begin{vmatrix} 1-\frac{4}{x} & 2x & 2x \\ 2x & 1-\frac{4}{x} & 2x \\ 2x & 2x & 1-\frac{4}{x} \end{vmatrix} = \left(B-\frac{4}{x}\right)\left(1+\frac{4}{x}\right)^2$$

Now take $x \rightarrow \infty$, then $\frac{1}{x} \rightarrow 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$$

\therefore ordered pair (A, B) is (-4,5)

Question136

Let A be a matrix such that $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$.

Then A^2 equals

[Online April 15, 2018]

Options:

A. $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$

C. $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

D. $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

Answer: D

Solution:

Solution:

Since $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$

suppose the scalar matrix is $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$$\therefore A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^{-1}$$

$$[\because AB = C \Rightarrow ABB^{-1} = CB^{-1} \Rightarrow A = CB^{-1}]$$

$$\Rightarrow A = \frac{1}{3} \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} k & -\frac{2}{3}k \\ 0 & \frac{k}{3} \end{bmatrix} \dots\dots(1)$$

$$\because |3A| = 108$$

$$\Rightarrow 108 = \begin{vmatrix} 3k & -2k \\ 0 & k \end{vmatrix}$$

$$\Rightarrow 3k^2 = 108 \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$$

For $k = 6$

$$A = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix} \dots\dots \text{From (1)}$$

$$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$$

For $k = -6$

$$\Rightarrow A = \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix} \dots\dots \text{From (1)}$$

$$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$$

Question137

Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$ then $\alpha + \beta$ is equal to [Online April 15, 2018]

Options:

- A. 8
- B. 12
- C. 13
- D. 7

Answer: A

Solution:

Solution:

We have

$$(A - 3I)(A - 5I) = O$$

$$\Rightarrow A^2 - 8A + 15I = O$$

Multiplying both sides by A^{-1} , we get;

$$A^{-1}A \cdot A - 8A^{-1}A + 15A^{-1}I = A^{-1}O$$

$$\Rightarrow A - 8I + 15A^{-1} = O$$

$$A + 15A^{-1} = 8I$$

$$\frac{A}{2} + \frac{15A^{-1}}{2} = 4I$$

$$\therefore \alpha + \beta = \frac{1}{2} + \frac{15}{2} = \frac{16}{2} = 8$$

Question138

If the system of linear equations

$$x + ky + 3z = 0$$

$$3x + ky - 2z = 0$$

$$2x + 4y - 3z = 0$$

has a non-zero solution (x, y, z) , then $\frac{xz}{y^2}$ is equal to :

[2018]

Options:

- A. 10
- B. - 30
- C. 30
- D. - 10

Answer: A

Solution:

Solution:

For non zero solution of the system of linear equations;

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$$\Rightarrow k = 11$$

Now equations become

$$x + 11y + 3z = 0 \dots\dots(1)$$

$$3x + 11y - 2z = 0 \dots\dots(2)$$

$$2x + 4y - 3z = 0 \dots\dots(3)$$

Adding equations (1) & (3) we get

$$3x + 15y = 0$$

$$\Rightarrow x = -5y$$

Now put $x = -5y$ in equation (1), we get

$$-5y + 11y + 3z = 0$$

$$\Rightarrow z = -2y$$

$$\therefore \frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$$

Question139

The number of values of k for which the system of linear equations, $(k + 2)x + 10y = k$, $kx + (k + 3)y = k - 1$ has no solution, is [Online April 16, 2018]

Options:

- A. Infinitely many
- B. 3
- C. 1
- D. 2

Answer: C

Solution:

Solution:

Here, the equations are;

$$(k + 2)x + 10y = k$$

$$\& kx + (k + 3)y = k - 1.$$

These equations can be written in the form of $Ax = B$ as

$$\begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ k-1 \end{bmatrix}$$

For the system to have no solution

$$|A| = 0$$

$$\Rightarrow \begin{vmatrix} k+2 & 10 \\ k & k+3 \end{vmatrix} = 0 \Rightarrow (k+2)(k+3) - k \times 10 = 0$$

$$\Rightarrow k^2 - 5k + 6 = (k-2)(k-3) = 0$$

$$\therefore k = 2, 3$$

For $k = 2$, equations become:

$$4x + 10y = 2$$

$$\& 2x + 5y = 1$$

& hence infinite number of solutions.

For $k = 3$, equations becomes;

$$5x + 10y = 3$$

$$3x + 6y = 2$$

& hence no solution.

\therefore required number of values of k is 1

Question140

Let S be the set of all real values of k for which the system of linear equations

$$x + y + z = 2$$

$$2x + y - z = 3$$

$$3x + 2y + kz = 4$$

has a unique solution. Then S is

[Online April 15, 2018]

Options:

A. an empty set

B. equal to $\mathbb{R} - \{0\}$

C. equal to $\{0\}$

D. equal to \mathbb{R}

Answer: B

Solution:

Solution:

The system of linear equations is:

$$x + y + z = 2$$

$$2x + y - z = 3$$

$$3x + 2y + kz = 4$$

As, system has unique solution.

$$\text{So, } \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

$$\Rightarrow k + 2 - (2k + 3) + 1 \neq 0$$

$$\Rightarrow k \neq 0$$

$$\text{Hence, } k \in \mathbb{R} - \{0\} \equiv S$$

Question141

If the system of linear equations

$$\mathbf{x} + \mathbf{ay} + \mathbf{z} = \mathbf{3}$$

$$\mathbf{x} + \mathbf{2y} + \mathbf{2z} = \mathbf{6}$$

$$\mathbf{x} + \mathbf{5y} + \mathbf{3z} = \mathbf{b}$$

has no solution, then

[Online April 15, 2018]

Options:

A. $a = 1, b \neq 9$

B. $a \neq -1, b = 9$

C. $a = -1, b = 9$

D. $a = -1, b \neq 9$

Answer: D

Solution:

Solution:

As the system of equations has no solution then Δ should be zero and at least one of Δ_1 , Δ_2 and Δ_3 should not be zero.

$$\therefore \Delta = \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$$

$$\Rightarrow -a - 1 = 0 \Rightarrow a = -1$$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 2 \\ 1 & b & 3 \end{vmatrix} \neq 0$$

$$\Rightarrow b \neq 9$$

Question142

If

$$\mathbf{S} = \left\{ \mathbf{x} \in [0, 2\pi]: \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\},$$

then $\sum_{x \in S} \tan \left(\frac{\pi}{3} + x \right)$ is equal to

[Online April 8, 2017]

Options:

A. $4 + 2\sqrt{3}$

B. $-2 + \sqrt{3}$

C. $-2 - \sqrt{3}$

D. $-4 - 2\sqrt{3}$

Answer: C

Solution:

Solution:

Since the given determinant is equal to zero.

$$\Rightarrow 0(0 - \cos x \sin x) - \cos x(0 - \cos^2 x) - \sin x(\sin^2 x - 0) = 0$$

$$\Rightarrow \cos^3 x - \sin^3 x = 0$$

$$\Rightarrow \tan^3 x = 1 \Rightarrow \tan x = 1$$

$$\therefore \sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right) = \sum_{x \in S} \frac{\tan \pi/3 + \tan x}{1 - \tan \pi/3 \cdot \tan x}$$

$$\sum_{x \in S} \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$$

$$\sum_{x \in S} \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \Rightarrow \sum_{x \in S} \frac{1 + 3 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

Question 143

Let k be an integer such that triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point:
[2017]

Options:

A. $\left(2, \frac{1}{2}\right)$

B. $\left(2, -\frac{1}{2}\right)$

C. $\left(1, \frac{3}{4}\right)$

D. $\left(1, -\frac{3}{4}\right)$

Answer: A

Solution:

Solution:

Let $A(k, -3k)$, $B(5, k)$ and $C(-k + 2)$,
we have

$$\frac{1}{2} \left| \begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} \right| = 28$$

$$\Rightarrow 5k^2 + 13k - 46 = 0$$

$$\text{or } 5k^2 + 13k + 66 = 0$$

$$\text{Now, } 5k^2 + 13k - 46 = 0$$

$$\Rightarrow k = \frac{-13 \pm \sqrt{1089}}{10} \therefore k = \frac{-23}{5}; k = 2$$

since k is an integer, $\therefore k = 2$

$$\text{Also } 5k^2 + 13k + 66 = 0$$

$$\Rightarrow k = \frac{-13 \pm \sqrt{-1151}}{10}$$

So no real solution exist
 $A(2, -6)$, $B(5, 2)$ and $C(-2, 2)$
 For orthocentre $H(\alpha, \beta)$
 $BH \perp AC$

$$\therefore \left(\frac{\beta - 2}{\alpha - 5} \right) \left(\frac{8}{-4} \right) = -1$$

$$\Rightarrow \alpha - 2\beta = 1 \dots\dots(1)$$

Also $CH \perp AB$

$$\therefore \left(\frac{\beta - 2}{\alpha + 2} \right) \left(\frac{8}{3} \right) = -1$$

$$\Rightarrow 3\alpha + 8\beta = 1 \dots\dots(2)$$

Solving (1) and (2), we get

$$\alpha = 2, \beta = \frac{1}{2}$$

orthocentre is $\left(2, \frac{1}{2} \right)$

Question144

Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k, \text{ then } k \text{ is equal to}$$

[2017]

Options:

A. 1

B. $-z$

C. z

D. -1

Answer: B

Solution:

Solution:

Given $2\omega + 1 = z$;

$$\text{and } z = \sqrt{3}i \Rightarrow \omega = \frac{\sqrt{3}i - 1}{2}$$

$\Rightarrow \omega$ is complex cube root of unity Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$$\begin{aligned} &= \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} \\ &= 3(-1 - \omega - \omega) = -3(1 + 2\omega) = -3z \\ &\Rightarrow k = -z \end{aligned}$$

Question145

If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $\text{adj}(3A^2 + 12A)$ is equal to:

[2017]

Options:

A. $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

B. $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

C. $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

D. $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

Answer: C

Solution:

Solution:

We have $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} \Rightarrow 3A^2 = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix}$$

$$\text{Also } 12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$\therefore 3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

Question 146

Let A be any 3×3 invertible matrix. Then which one of the following is not always true?

[Online April 8, 2017]

Options:

A. $\text{adj}(A) = |A| \cdot A^{-1}$

B. $\text{adj}(\text{adj}(A)) = |A| \cdot A$

C. $\text{adj}(\text{adj}(A)) = |A|^2 \cdot (\text{adj}(A))^{-1}$

D. $\text{adj}(\text{adj}(A)) = |A| \cdot (\text{adj}(A))^{-1}$

Answer: B

Question 147

If S is the set of distinct values of ' b ' for which the following system of linear equations

$$x + y + z = 1$$

$$x + ay + z = 1$$

$$ax + by + z = 0$$

has no solution, then S is :

[2017]

Options:

A. a singleton

B. an empty set

C. an infinite set

D. a finite set containing two or more elements

Answer: A

Solution:

Solution:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1[a - b] - 1[1 - a] + 1[b - a^2] = 0$$

$$\Rightarrow (a - 1)^2 = 0$$

$$\Rightarrow a = 1$$

For $a = 1$, First two equations are identical

i.e., $x + y + z = 1$

To have no solution with $x + by + z = 0$

$$b = 1$$

So $b = \{1\} \Rightarrow$ It is singleton set.

Question 148

The number of real values of λ for which the system of linear equations

$$2x + 4y - \lambda z = 0$$

$$4x + \lambda y + 2z = 0$$

$$\lambda x + 2y + 2z = 0$$

has infinitely many solutions, is :
[Online April 8, 2017]

Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

Since the given system of linear equations has infinitely many solutions.

$$\therefore \begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 + 4\lambda - 40 = 0$$

λ has only 1 real root.

Question149

If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix

$(A^{2016} - 2A^{2015} - A^{2014})$ is:

[Online April 10, 2016]

Options:

A. -175

B. 2014

C. 2016

D. -25

Answer: D

Solution:

Solution:

$$A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix} \text{ and } |A| = 1$$

$$\text{Now, } A^{2016} - 2A^{2015} - A^{2014} = A^{2014}(A^2 - 2A - I) \\ \Rightarrow A^{2016} - 2A^{2015} - A^{2014} = A^{2014} | A^2 - 2A - I |$$

$$= |A|^{2014} \begin{vmatrix} 20 & 5 \\ -15 & -5 \end{vmatrix} = -25$$

Question150

The number of distinct real roots of the equation,

$$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is

[Online April 9, 2016]

Options:

A. 1

B. 4

C. 2

D. 3

Answer: C

Solution:

Solution:

$$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 - R_2$$

$$R_2 \rightarrow -R_2 - R_3$$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & \cos x - \sin x & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$$C_2 \rightarrow C_2 + C_3$$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Expanding using second row

$$2 \sin x (\sin x - \cos x)^2 = 0$$

$$\sin x = 0 \text{ or } \sin x = \cos x$$

$$x = 0 \text{ or } x = \frac{\pi}{4}$$

Question151

If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{adj} A = AA^T$, then $5a + b$ is equal to:

[2016]

Options:

- A. 4
- B. 13
- C. -1
- D. 5

Answer: D

Solution:

Solution:

Given that $A(\text{adj} A) = AA^T$

Pre-multiply by A^{-1} both side, we get

$$\Rightarrow A^{-1}A(\text{adj} A) = A^{-1}AA^T$$

$$\text{adj} A = A^T$$

$$\Rightarrow \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$$

$$\Rightarrow a = \frac{2}{5} \text{ and } b = 3$$

$$\Rightarrow 5a + b = 5$$

Question152

Let A be a 3×3 matrix such that $A^2 - 5A + 7I = 0$.

Statement-I: $A^{-1} = \frac{1}{7}(5I - A)$

Statement-II: the polynomial $A^3 - 2A^2 - 3A + \alpha$ can be reduced to $5(A - 4I)$

Then :

[Online April 10, 2016]

Options:

- A. Both the statements are true.
- B. Both the statements are false.
- C. Statement-I is true, but Statement-II is false.
- D. Statement I is false, but Statement-II is true.

Answer: A

Solution:

Solution:

$$A^2 - 5A = -7I$$

$$AAA^{-1} - 5AA^{-1} = -7IA^{-1}$$

$$AI - 5I = -7A^{-1}$$

$$A - 5I = -7A^{-1}$$

$$A^{-1} = \frac{1}{7}(5I - A)$$

$$\begin{aligned} A^3 - 2A^2 - 3A + I &= A(5A - 7I) - 2A^2 - 3A + I \\ &= 5A^2 - 7A - 2A^2 - 3A + I = 3A^2 - 10A + I \\ &= 3(5A - 7I) - 10A + I = 5A - 20I = 5(A - 4I) \end{aligned}$$

Question153

The system of linear equations

$$\mathbf{x} + \lambda \mathbf{y} - \mathbf{z} = \mathbf{0}$$

$$\lambda \mathbf{x} - \mathbf{y} - \mathbf{z} = \mathbf{0}$$

$$\mathbf{x} + \mathbf{y} - \lambda \mathbf{z} = \mathbf{0}$$

has a non-trivial solution for:
[2016]

Options:

A. exactly two values of λ .

B. exactly three values of λ .

C. infinitely many values of λ .

D. exactly one value of λ .

Answer: B

Solution:

Solution:

For non-trivial solution,

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$-\lambda(\lambda + 1)(\lambda - 1) = 0 \Rightarrow \lambda = 0, +1, -1$$

Question154

if $\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = ax - 12$, then 'a' is equal to:

[Online April 11, 2015]

Options:

A. 24

B. -12

C. -24

D. 12

Answer: A

Solution:

Solution:

$$\text{Let } \begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = ax - 12$$

Put $x = -1$, we get

$$\begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & 0 \\ 2 & -3 & -3 \end{vmatrix} = -a - 12$$

$$\Rightarrow -3(6 + 6) = -a - 12 \Rightarrow -36 + 12 = a$$

$$\Rightarrow a = 24$$

Question 155

The least value of the product xyz for which the determinant

is non-negative, is:
[Online April 10, 2015]

$$\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$$

Options:

A. $-2\sqrt{2}$

B. -1

C. $-16\sqrt{2}$

D. -8

Answer: D

Solution:

Solution:

$$\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} \geq 0$$

$$xyz - x - y - z + 2 \geq 0$$

$$xyz + 2 \geq x + y + z \geq 3(xyz)^{1/3}$$

$$xyz + 2 - 3(xyz)^{1/3} \geq 0$$

$$\text{Let } (xyz) = t^3$$

$$t^3 - 3t + 2 \geq 0$$

$$(t + 2)(t - 1)^2 \geq 0$$

$$[t = -2] t^3 = -8$$

Question156

If A is a 3×3 matrix such that $|5 \cdot \text{adj } A| = 5$, then $|A|$ is equal to :
[Online April 11, 2015]

Options:

A. $\pm \frac{1}{5}$

B. $\pm \frac{1}{25}$

C. ± 1

D. ± 5

Answer: A

Solution:

Solution:

$$\begin{aligned} |5 \cdot \text{adj } A| &= 5 \Rightarrow 5^3 \cdot |A|^{3-1} = 5 \\ \Rightarrow 125 |A|^2 &= 5 \Rightarrow |A| = \pm \frac{1}{5} \end{aligned}$$

Question157

The set of all values of λ for which the system of linear equations:

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$

$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$

$$-x_1 + 2x_2 = \lambda x_3$$

has a non-trivial solution,
[2015]

Options:

A. contains two elements.

B. contains more than two elements

C. is an empty set.

D. is a singleton

Answer: A

Solution:

$$\left. \begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned} \right\}$$

$$\Rightarrow (2 - \lambda)x_1 - 2x_2 + x_3 = 0$$

$$2x_1 - (3 + \lambda)x_2 + 2x_3 = 0$$

$$-x_1 + 2x_2 - \lambda x_3 = 0$$

For non-trivial solution,

$$\Delta = 0$$

$$\text{i.e. } \begin{vmatrix} 2 - \lambda & -2 & 1 \\ 2 & -(3 + \lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2 - \lambda)[\lambda(3 + \lambda) - 4] + 2[-2\lambda + 2] + 1[4 - (3 + \lambda)] = 0$$

$$\Rightarrow \lambda^3 + \lambda^2 - 5\lambda + 3 = 0$$

$$\Rightarrow \lambda = 1, 1, 3$$

Hence λ has 2 values.

Question 158

If $f(\theta) = \begin{vmatrix} 1 & \cos \theta & 1 \\ -\sin \theta & 1 & -\cos \theta \\ -1 & \sin \theta & 1 \end{vmatrix}$ and A and B are respectively the maximum

and the minimum values of $f(\theta)$, then (A, B) is equal to:

[Online April 12, 2014]

Options:

A. (3, -1)

B. $(4, 2 - \sqrt{2})$

C. $(2 + \sqrt{2}, 2 - \sqrt{2})$

D. $(2 + \sqrt{2}, -1)$

Answer: C

Solution:

Solution:

$$\text{Let } f(\theta) = \begin{vmatrix} 1 & \cos \theta & 1 \\ -\sin \theta & 1 & -\cos \theta \\ -1 & \sin \theta & 1 \end{vmatrix}$$

$$= (1 + \sin \theta \cos \theta) - \cos \theta(-\sin \theta - \cos \theta) + 1(-\sin^2 \theta + 1)$$

$$= 1 + \sin \theta \cos \theta + \sin \theta \cos \theta + \cos^2 \theta - \sin^2 \theta + 1$$

$$= 2 + 2 \sin \theta \cos \theta + \cos 2\theta$$

$$= 2 + \sin 2\theta + \cos 2\theta$$

Now, maximum value of (1)

$$\text{is } 2 + \sqrt{1^2 + 1^2} = 2 + \sqrt{2}$$

and minimum value of (1) is

$$2 - \sqrt{1^2 + 1^2} = 2 - \sqrt{2}$$

Question159

If B is a 3×3 matrix such that $B^2 = 0$, then $\det. [(I + B)^{50} - 50B]$ is equal to:

[Online April 9, 2014]

Options:

- A. 1
- B. 2
- C. 3
- D. 50

Answer: A

Solution:

Solution:

$$\begin{aligned} & \det[(I + B)^{50} - 50B] \\ &= \det[{}^{50}C_0 I + {}^{50}C_1 B + {}^{50}C_2 B^2 + {}^{50}C_3 B^3 + \dots + {}^{50}C_{50} B^{50} - 50B] \\ & \{ \text{All terms having } B^n, 2 \leq n \leq 50 \\ & \text{will be zero because given that } B^2 = 0 \} \\ &= \det[I + 50B - 50B] = \det[I] = 1 \end{aligned}$$

Question160

If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1 - \alpha)^2(1 - \beta)^2(\alpha - \beta)^2$, then K is equal to:
[2014]

Options:

- A. 1
- B. -1
- C. $\alpha\beta$
- D. $\frac{1}{\alpha\beta}$

Answer: A

Solution:

$$\begin{aligned}
 \text{Consider } & \begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} \\
 = & \begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix} \\
 = & \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \quad [\because |A| = |A^1|] \\
 = & \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2 = [(1-\alpha)(1-\beta)(\alpha-\beta)]^2 \\
 \text{So, } K &= 1
 \end{aligned}$$

Question 161

If

$$\Delta_r = \begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n-4) \end{vmatrix}$$

then the value of $\sum_{r=1}^{n-1} \Delta_r$
[Online April 19, 2014]

Options:

- A. depends only on a
- B. depends only on n
- C. depends both on a and n
- D. is independent of both a and n

Answer: D

Solution:

Solution:

$$\begin{aligned}
 \sum_{r=1}^{n-1} r &= 1 + 2 + 3 + \dots + (n-1) = \frac{n(n-1)}{2} \\
 \sum_{r=1}^{n-1} (2r-1) &= 1 + 3 + 5 + \dots + [2(n-1)-1] = (n-1)^2 \\
 \sum_{r=1}^{n-1} (3r-2) &= 1 + 4 + 7 + \dots + (3n-3-2) = \frac{(n-1)(3n-4)}{2}
 \end{aligned}$$

$$\therefore \sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \sum r & \sum (2r-1) & \sum (3r-2) \\ \frac{n}{2} & n-1 & a \\ n(n-1) & 2 & \frac{(n-1)^2}{(n-1)(3n-4)} \end{vmatrix}$$

$\sum_{r=1}^{n-1} \Delta_r$ consists of $(n-1)$ determinants in L.H.S. and in R.H.S every constituent of first row consists of $(n-1)$ elements and hence it can be splitted into sum of $(n-1)$ determinants.

$$\therefore \sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \end{vmatrix} = 0$$

($\because R_1$ and R_3 are identical)

Hence, value of $\sum_{r=1}^{n-1} \Delta_r$ is independent of both a and n .

Question 162

If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}, \lambda \neq 0$ then k is equal to

[Online April 12, 2014]

Options:

- A. $4\lambda abc$
- B. $-4\lambda abc$
- C. $4\lambda^2$
- D. $-4\lambda^2$

Answer: C

Solution:

Solution:

$$\text{Let } \Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

Apply $R_2 \rightarrow R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 - (a-\lambda)^2 & (b+\lambda)^2 - (b-\lambda)^2 & (c+\lambda)^2 - (c-\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} \quad (\because (x+y)^2 - (x-y)^2 = 4xy)$$

Taking out 4 common from R_2

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ a^2 + \lambda^2 - 2a\lambda & b^2 + \lambda^2 - 2b\lambda & c^2 + \lambda^2 - 2c\lambda \end{vmatrix}$$

Apply $R_3 \rightarrow [R_3 - (R_1 - 2R_2)]$

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ \lambda^2 & \lambda^2 & \lambda^2 \end{vmatrix}$$

Taking out λ common from R_2 and λ^2 from R_3

$$= 4\lambda(\lambda^2) \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow k = 4\lambda^2$$

Question 163

If A is an 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals:
[2014]

Options:

- A. B^{-1}
- B. $(B^{-1})'$
- C. $I + B$
- D. I

Answer: D

Solution:

Solution:

$$\begin{aligned} BB' &= B(A^{-1}A')' = B(A')'(A^{-1})' \\ &= BA(A^{-1})' = (A^{-1}A')(A(A^{-1})') \\ &= A^{-1}A \cdot A' \cdot (A^{-1})' \quad \{ \text{as } AA' = A'A \} \\ &= I(A^{-1}A)' = I \cdot I = I^2 = I \end{aligned}$$

Question 164

Let A be a 3×3 matrix such that

$$A \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{Then } A^{-1} \text{ is:}$$

[Online April 11, 2014]

Options:

A. $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix}$

Answer: A

Solution:

Solution:

$$\text{Given } A \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Applying $C_1 \leftrightarrow C_3$

$$A \begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Again Applying $\rightarrow C_2 \leftrightarrow C_3$

$$A \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

pre-multiplying both sides by A^{-1}

$$A^{-1}A \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix} = A^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix} = A^{-1}I = A^{-1}$$

($\because A^{-1}A = I$ and $I =$ Identity matrix)

$$\text{Hence, } A^{-1} = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

Question 165

If a, b, c are non-zero real numbers and if the system of equations

$$(a - 1)x = y + z$$

$$(b - 1)y = z + x$$

$$(c - 1)z = x + y$$

has a non-trivial solution, then $ab + bc + ca$ equals:

[Online April 9, 2014]

Options:

A. $a + b + c$

B. abc

C. 1

D. -1

Answer: B

Solution:

Solution:

Given system of equations can be written as

$$(a - 1)x - y - z = 0$$

$$-x + (b - 1)y - z = 0$$

$$-x - y + (c - 1)z = 0$$

For non-trivial solution, we have

$$\begin{vmatrix} a-1 & -1 & -1 \\ -1 & b-1 & -1 \\ -1 & -1 & c-1 \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_3$$

$$\begin{vmatrix} a-1 & -1 & -1 \\ 0 & b & -c \\ -1 & -1 & c-1 \end{vmatrix} = 0$$

$$C_2 \rightarrow C_2 - C_3$$

$$\begin{vmatrix} a-1 & 0 & -1 \\ 0 & b+c & -c \\ -1 & -c & c-1 \end{vmatrix} = 0$$

$$\text{Apply } R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} a-1 & 0 & -1 \\ 0 & b+c & -c \\ -a & -c & c \end{vmatrix} = 0$$

$$\begin{aligned} \Rightarrow (a-1)[bc+c^2-c^2]-1[a(b+c)] &= 0 \\ \Rightarrow (a-1)[bc]-ab-ac &= 0 \\ \Rightarrow abc-bc-ab-ac &= 0 \\ \Rightarrow ab+bc+ca &= abc \end{aligned}$$

Question 166

Let

$$S = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} : a_{ij} \in \{0, 1, 2\}, a_{11} = a_{22} \right\}$$

Then the number of non-singular matrices in the set S is :
[Online April 25, 2013]

Options:

- A. 27
- B. 24
- C. 10
- D. 20

Answer: D

Solution:

Solution:

The matrices in the form $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} : a_{ij} \in \{0, 1, 2\}, a_{11} = a_{22}$ are $\begin{bmatrix} 0 & 0/1/2 \\ 0/1/2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0/1/2 \\ 0/1/2 & 1 \end{bmatrix},$
 $\begin{bmatrix} 2 & 0/1/2 \\ 0/1/2 & 2 \end{bmatrix}$

At any place, 0 / 1 / 2 means 0, 1 or 2 will be the element at that place.

Hence there are total $27 = 3 \times 3 + 3 \times 3 + 3 \times 3$ matrices of the above form. Out of which the matrices which are

singular are $\begin{bmatrix} 0 & 0/1/2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1/2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Hence there are total $7 (= 3 + 2 + 1 + 1)$ singular matrices. Therefore number of all non-singular matrices in the given form $= 27 - 7 = 20$

Question 167

Let A, other than I or -I, be a 2×2 real matrix such that $A^2 = I$, I being the unit matrix. Let $\text{Tr}(A)$ be the sum of diagonal elements of A.

Statement-1: $\text{Tr}(A) = 0$

Statement-2: $\det(A) = -1$

[Online April 23, 2013]

Options:

- A. Statement-1 is true; Statement-2 is false.

B. Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

D. Statement-1 is false; Statement-2 is true.

Answer: B

Solution:

Solution:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$b(a + d) = 0, b = 0 \text{ or } a = -d \dots\dots(1)$$

$$c(a + d) = 0, c = 0 \text{ or } a = -d \dots\dots(2)$$

$$a^2 + bc = 1, bc + d^2 = 1 \dots\dots(3)$$

‘a’ and ‘d’ are diagonal elements $a + d = 0$

statement-1 is correct.

Now, $\det(A) = ad - bc$

$$(3) \ a^2 + bc = 1 \text{ and } d^2 + bc = 1$$

Now, from So, $a^2 - d^2 = 0$

$$\text{Adding } a^2 + d^2 + 2bc = 2$$

$$\Rightarrow (a + d)^2 - 2ad + 2bc = 2$$

$$\text{or } 0 - 2(ad - bc) = 2$$

$$\text{So, } ad - bc = 1 \Rightarrow \det(A) = -1$$

So, statement -2 is also true.

But statement -2 is not the correct explanation of statement- 1

Question168

If a, b, c are sides of a scalene triangle, then the value of

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

is:

[Online April 9, 2013]

Options:

A. non - negative

B. negative

C. positive

D. non-positive

Answer: B

Solution:

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$$

$$= (a+b+c)[ab+bc+ca-a^2-b^2-c^2]$$

$$= -(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

Since a, b, c are sides of a scalene triangle, therefore at least two of the a, b, c will be unequal.

$$\therefore (a-b)^2 + (b-c)^2 + (c-a)^2 > 0$$

$$\text{Also } a+b+c > 0$$

$$\therefore -(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] < 0$$

Question169

If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α

is equal to :
[2013]

Options:

A. 4

B. 11

C. 5

D. 0

Answer: B

Solution:

Solution:

$$|P| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 2\alpha - 6$$

$$\text{Now, adj } A = P \Rightarrow |\text{adj } A| = |P|$$

$$\Rightarrow |A|^2 = |P|$$

$$\Rightarrow |P| = 16$$

$$\Rightarrow 2\alpha - 6 = 16$$

$$\Rightarrow \alpha = 11$$

Question170

The number of values of k, for which the system of equations:
 $(k+1)x + 8y = 4k$
 $kx + (k+3)y = 3k - 1$

**has no solution, is
[2013]**

Options:

A. infinite

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

Since, system of equations have no solution

$$\therefore \frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1} \quad (\because \text{System has no solution})$$

$$\Rightarrow k^2 + 4k + 3 = 8k \Rightarrow k^2 - 4k + 3 = 0$$

$$\Rightarrow k = 1, 3$$

$$\text{If } k = 1 \text{ then } \frac{8}{1+3} \neq \frac{4.1}{2} \text{ which is false}$$

$$\text{and if } k = 3 \text{ then } \frac{8}{6} \neq \frac{4.3}{9-1} \text{ which is true, therefore } k = 3$$

Hence for only one value of k. System has no solution.

Question171

Consider the system of equations:

$x + ay = 0$, $y + az = 0$ and $z + ax = 0$. Then the set of all real values of 'a' for which the system has a unique solution is:

[Online April 25, 2013]

Options:

A. $\mathbb{R} - \{1\}$

B. $\mathbb{R} - \{-1\}$

C. $\{1, -1\}$

D. $\{1, 0, -1\}$

Answer: B

Solution:

Solution:

Given system of equations is homogeneous which is

$$x + ay = 0$$

$$y + az = 0$$

$$z + ax = 0$$

It can be written in matrix form as

$$A = \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix}$$

Now, $|A| = [1 - a(-a^2)] = 1 + a^3 \neq 0$

So, system has only trivial solution.

Now, $|A| = 0$ only when $a = -1$

So, system of equations has infinitely many solutions which is not possible because it is given that system has a unique solution.

Hence set of all real values of 'a' is $\mathbb{R} - \{-1\}$

Question 172

Statement-1: The system of linear equations

$$x + (\sin \alpha)y + (\cos \alpha)z = 0$$

$$x + (\cos \alpha)y + (\sin \alpha)z = 0$$

$$x - (\sin \alpha)y - (\cos \alpha)z = 0$$

has a non-trivial solution for only one value of α lying in the interval $\left(0, \frac{\pi}{2}\right)$.

Statement- 2 : The equation in α

$$\begin{vmatrix} \cos \alpha & \sin \alpha & \cos \alpha \\ \sin \alpha & \cos \alpha & \sin \alpha \\ \cos \alpha & -\sin \alpha & -\cos \alpha \end{vmatrix} = 0 \text{ has only one solution lying in the interval } \left(0, \frac{\pi}{2}\right).$$

[Online April 23, 2013]

Options:

A. Statement-1 is true, Statement-2 is true, Statement-2 is not correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

C. Statement-1 is true, Statement-2 is false.

D. Statement-1 is false, Statement-2 is true

Answer: C

Solution:

Solution:

$$\Delta_1 = \begin{vmatrix} 1 & \sin \alpha & \cos \alpha \\ 1 & \cos \alpha & \sin \alpha \\ 1 & -\sin \alpha & \cos \alpha \end{vmatrix} = 0$$

$$\begin{aligned} &= \begin{vmatrix} 0 & \sin \alpha - \cos \alpha & \cos \alpha - \sin \alpha \\ 0 & \cos \alpha + \sin \alpha & \sin \alpha - \cos \alpha \\ 1 & -\sin \alpha & \cos \alpha \end{vmatrix} = (\sin \alpha - \cos \alpha)^2 - (\cos^2 \alpha - \sin^2 \alpha) \\ &= \sin^2 \alpha + \cos^2 \alpha - 2 \sin \alpha \cdot \cos \alpha - \cos^2 \alpha + \sin^2 \alpha \end{aligned}$$

$$= 2\sin^2\alpha - 2\sin\alpha \cdot \cos\alpha$$

$$= 2\sin\alpha(\sin\alpha - \cos\alpha)$$

Now, $\sin\alpha - \cos\alpha = 0$ for only

$$\alpha = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

$$\therefore \Delta_1 = 2(\sin\alpha) \times 0 = 0,$$

since value of $\sin\alpha$ is finite for $\alpha \in \left(0, \frac{\pi}{2}\right)$

Hence non-trivial solution for only one value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & \sin\alpha & \cos\alpha \\ 0 & \cos\alpha & \sin\alpha \\ 2\cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow 2\cos\alpha(\sin^2\alpha - \cos^2\alpha) = 0$$

$$\therefore \cos\alpha = 0 \text{ or } \sin^2\alpha - \cos^2\alpha = 0$$

But $\cos\alpha = 0$ not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

$\therefore \sin^2\alpha - \cos^2\alpha = 0 \Rightarrow \sin\alpha = -\cos\alpha$, which is also not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

Hence, there is no solution.

Question173

If the system of linear equations :

$$\mathbf{x_1 + 2x_2 + 3x_3 = 6}$$

$$\mathbf{x_1 + 3x_2 + 5x_3 = 9}$$

$$\mathbf{2x_1 + 5x_2 + ax_3 = b}$$

is consistent and has infinite number of solutions, then :

[Online April 22, 2013]

Options:

A. $a = 8$, b can be any real number

B. $b = 15$, a can be any real number

C. $a \in \mathbb{R} - \{8\}$ and $b \in \mathbb{R} - \{15\}$

D. $a = 8$, $b = 15$

Answer: D

Solution:

Solution:

Given system of equations can be written in matrix form as $AX = B$ where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{pmatrix} \text{ and } B = \begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix}$$

Since, system is consistent and has infinitely many solutions

$$\therefore (\text{adj. } A) B = 0$$

$$\Rightarrow \begin{pmatrix} 3a - 25 & 15 - 2a & 1 \\ 10 - a & a - 6 & -2 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow -6 - 9 + b = 0 \Rightarrow b = 15$$

$$\text{and } 6(10 - a) + 9(a - 6) - 2(b) = 0$$

$$\Rightarrow 60 - 6a + 9a - 54 - 30 = 0$$

$$\Rightarrow 3a = 24 \Rightarrow a = 8$$

Hence, $a = 8$, $b = 15$.

Question 174

If a, b, c , are non zero complex numbers satisfying $a^2 + b^2 + c^2 = 0$ and

$$\begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix} = k a^2 b^2 c^2, \text{ then } k \text{ is equal to}$$

[Online May 19, 2012]

Options:

A. 1

B. 3

C. 4

D. 2

Answer: C

Solution:

Solution:

$$\text{Let } \Delta = \begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix}$$

Multiply C_1 by a , C_2 by b and C_3 by c and hence divide by abc .

$$= \frac{1}{abc} \begin{vmatrix} a(b^2 + c^2) & ab^2 & ac^2 \\ a^2b & b(c^2 + a^2) & bc^2 \\ a^2c & b^2c & c(a^2 + b^2) \end{vmatrix}$$

Take out a, b, c common from R_1, R_2 and R_3 respectively.

$$\therefore \Delta = \frac{abc}{abc} \begin{vmatrix} b^2 + c^2 & b^2 & c^2 \\ a^2 & c^2 + a^2 & c^2 \\ a^2 & b^2 & a^2 + b^2 \end{vmatrix}$$

Apply $C_1 \rightarrow C_1 - C_2 - C_3$

$$\Delta = \begin{vmatrix} 0 & b^2 & c^2 \\ -2c^2 & c^2 + a^2 & c^2 \\ -2b^2 & b^2 & a^2 + b^2 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & c^2 + a^2 & c^2 \\ b^2 & b^2 & a^2 + b^2 \end{vmatrix}$$

Apply $C_2 - C_1$ and $C_3 - C_1$

$$= -2 \begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & a^2 & 0 \\ b^2 & 0 & a^2 \end{vmatrix} = -2[-b^2(c^2 a^2) + c^2(-a^2 b^2)]$$

$$= 2a^2 b^2 c^2 + 2a^2 b^2 c^2 = 4a^2 b^2 c^2$$

But $\Delta = k a^2 b^2 c^2 \therefore k = 4$

Question 175

If $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix} = \alpha(a+b)(b+c)(c+a) \neq 0$ then α is equal to

[Online May 12, 2012]

Options:

A. $a + b + c$

B. abc

C. 4

D. 1

Answer: C

Solution:

Solution:

$$\text{Let } \Delta = \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}$$

Applying $C_1 + C_3$ and $C_2 + C_3$

$$\Delta = \begin{vmatrix} -a+c & 2a+b+c & a+c \\ 2b+a+c & -b+c & b+c \\ a-c & b-c & -2c \end{vmatrix}$$

Now, applying $R_1 + R_3$ and $R_2 + R_3$

$$\Delta = \begin{vmatrix} 0 & 2(a+b) & a-c \\ 2(a+b) & 0 & b-c \\ a-c & b-c & -2c \end{vmatrix}$$

On expanding, we get

$$\Delta = -2(a+b)\{-2c[2(a+b)] - (a-c)(b-c)\} + (a-c)[2(a+b)(b-c)]$$

$$\Delta = 8c(a+b)(a+b) + 4(a+b)(a-c)(b-c)$$

$$= 4(a+b)[2ac + 2bc + ab - bc - ac + c^2]$$

$$= 4(a+b)[ac + bc + ab + c^2]$$

$$= 4(a+b)[c(a+c) + b(a+c)]$$

$$= 4(a+b)(b+c)(c+a)$$

$$= \alpha(a+b)(b+c)(c+a)$$

Hence, $\alpha = 4$

Question176

The area of the triangle whose vertices are complex numbers z , iz , $z + iz$ in the Argand diagram is
[Online May 12, 2012]

Options:

- A. $2 |z|^2$
- B. $\frac{1}{2} |z|^2$
- C. $4 |z|^2$
- D. $|z|^2$

Answer: B

Solution:

Solution:

Vertices of triangle in complex form is

z , iz , $z + iz$

In cartesian form vertices are

(x, y) , $(-y, x)$ and $(x - y, x + y)$

$$\begin{aligned}\therefore \text{Area of triangle} &= \frac{1}{2} \begin{vmatrix} x & y & 1 \\ -y & x & 1 \\ x - y & x + y & 1 \end{vmatrix} \\ &= \frac{1}{2} [x(x - x - y) - y(-y - x + y) + 1(-yx - y^2 - x^2 + xy)] \\ &= \frac{1}{2} [-xy + xy - y^2 - x^2] = \frac{1}{2} (x^2 + y^2) \\ (\because \text{Area can not be negative}) \\ &= \frac{1}{2} |z|^2 \quad (\because z = x + iy, |z|^2 = x^2 + y^2)\end{aligned}$$

Question177

The area of triangle formed by the lines joining the vertex of the parabola, $x^2 = 8y$, to the extremities of its latus rectum is
[Online May 12, 2012]

Options:

- A. 2
- B. 8
- C. 1
- D. 4

Answer: B

Solution:

Solution:

Given parabola is $x^2 = 8y$

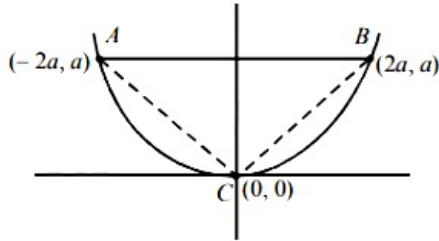
$$\Rightarrow 4a = 8 \Rightarrow a = 2$$

To find: Area of $\triangle ABC$

$$A = (-2a, a) = (-4, 2)$$

$$B = (2a, a) = (4, 2)$$

$$C = (0, 0)$$



$$\begin{aligned}\therefore \text{Area} &= \frac{1}{2} \begin{vmatrix} -4 & 2 & 1 \\ 4 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}[-4(2) - 2(4) + 1(0)] \\ &= \frac{-16}{2} = -8 \approx 8 \text{ sq. unit } (\because \text{area cannot be negative})\end{aligned}$$

Question178

Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $(P^2 + Q^2)$ is equal to :
[2012]

Options:

- A. -2
- B. 1
- C. 0
- D. -1

Answer: C

Solution:

Solution:

Given that $P^3 = Q^3$ (1)

and $P^2Q = Q^2P$ (2)

Subtracting (1) and (2), we get

$$P^3 - P^2Q = Q^3 - Q^2P$$

$$\Rightarrow P^2(P - Q) + Q^2(P - Q) = 0$$

$$\Rightarrow (P^2 + Q^2)(P - Q) = 0$$

$$\because P \neq Q, \therefore P^2 + Q^2 = 0$$

$$\text{Hence } |P^2 + Q^2| = 0$$

Question179

Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that

$Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :

[2012]

Options:

A. $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

B. $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

C. $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

D. $\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

Answer: D

Solution:

Solution:

$$\text{Let } Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\text{Then, } Au_1 + Au_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \dots\dots(1)$$

$$\text{Given that } A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

$$\Rightarrow |A| = 1(1) - 0(2) + 0(4 - 3) = 1$$

$$C_{11} = 1 \quad C_{21} = 0 \quad C_{31} = 0$$

$$C_{12} = -2 \quad C_{22} = 1 \quad C_{32} = 0$$

$$C_{13} = 1 \quad C_{23} = -2 \quad C_{33} = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

We know,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\Rightarrow A^{-1} = \text{adj}(A) (\because |A| = 1)$$

Now, from equation (1), we have

$$u_1 + u_2 = A^{-1} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

Question 180

If A^T denotes the transpose of the matrix $A = \begin{bmatrix} 0 & 0 & a \\ 0 & b & c \\ d & e & f \end{bmatrix}$, where

a, b, c, d, e and f are integers such that $abd \neq 0$, then the number of such matrices for which $A^{-1} = A^T$ is

[Online May 19, 2012]

Options:

A. $2(3!)$

B. $3(2!)$

C. 2^3

D. 3^2

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} 0 & 0 & a \\ 0 & b & c \\ d & e & f \end{bmatrix}, |A| = -abd \neq 0$$

$$c_{11} = +(bf - ce), c_{12} = -(-\infty) = cd, c_{13} = +(-bd) = -bd$$

$$c_{21} = -(-ea) = ae, c_{22} = +(-ad) = -ad, c_{23} = -(0) = 0$$

$$c_{31} = +(-ab) = -ab, c_{32} = -(0) = 0, c_{33} = 0$$

$$\text{Adj}A = \begin{bmatrix} (bf - ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A) = \frac{1}{abd} \begin{bmatrix} (bf - ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix}$$

$$\text{Now } A^{-1} = A^T$$

$$\Rightarrow \frac{1}{-abd} \begin{bmatrix} (bf - ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (bf - ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -abd^2 \\ 0 & -ab^2d & -abde \\ -a^2bd & -abcd & -abdf \end{bmatrix}$$

$$\therefore bf - ce = ae = cd = 0 \dots\dots(i)$$

$$abd^2 = ab, ab^2d = ad, a^2bd = bd \dots\dots(ii)$$

$$abde = abcd = abdf = 0 \dots\dots(iii)$$

From (ii),

$$(abd^2) \cdot (ab^2d) \cdot (a^2bd) = ab \cdot ad \cdot bd$$

$$\Rightarrow (abd)^4 - (abd)^2 = 0$$

$$\Rightarrow (abd)^2[(abd)^2 - 1] = 0$$

$$\text{because } abd \neq 0, \therefore abd = \pm 1 \dots\dots(iv)$$

From (iii) and (iv),

$$e = c = f = 0 \dots\dots(v)$$

From (i) and (v),

$$bf = ae = cd = 0 \dots\dots(iv)$$

From (iv), (v) and (vi), it is clear that a, b, d can be any non-zero integer such that $abd = \pm 1$

But it is only possible, if $a = b = d = \pm 1$

Hence, there are 2 choices for each a, b and d. therefore, there are $2 \times 2 \times 2$ choices for a, b and d. Hence number of required matrices = $2 \times 2 \times 2 = (2)^3$

Question181

Let A and B be real matrices of the form $\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$ and $\begin{bmatrix} 0 & \gamma \\ \delta & 0 \end{bmatrix}$, respectively.

Statement 1: AB - BA is always an invertible matrix.

Statement 2: AB - BA is never an identity matrix.

[Online May 12, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is false, Statement 2 is true.

C. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

D. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

Answer: A

Solution:

Solution:

Let A and B be real matrices such that $\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$ and $\begin{bmatrix} 0 & \gamma \\ \delta & 0 \end{bmatrix}$

$$\text{Now, } AB = \begin{bmatrix} 0 & \alpha\gamma \\ \beta\delta & 0 \end{bmatrix}$$

$$\text{and } BA = \begin{bmatrix} 0 & \gamma\beta \\ \delta\alpha & 0 \end{bmatrix}$$

Statement - 1 :

$$AB - BA = \begin{bmatrix} 0 & \gamma(\alpha - \beta) \\ \delta(\beta - \alpha) & 0 \end{bmatrix}$$

$$|AB - BA| = (\alpha - \beta)^2 \gamma \delta \neq 0$$

$\therefore AB - BA$ is always an invertible matrix.

Hence, statement -1 is true.

But $AB - BA$ can be identity matrix if $\gamma = -\delta$ or $\delta = -\gamma$

So, statement -2 is false.

Question182

Statement 1: If the system of equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$ has a non-trivial solution, then the value of k is $\frac{31}{2}$.

Statement 2: A system of three homogeneous equations in three variables has a non trivial solution if the determinant of the coefficient matrix is zero.

[Online May 26, 2012]

Options:

A. Statement 1 is false, Statement 2 is true.

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1.

D. Statement 1 is true, Statement 2 is false.

Answer: A

Solution:

Solution:

(a) Given system of equations is $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$ Since, system has non-trivial

$$\text{solution:} \therefore \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$$\Rightarrow 1(-4k + 6) - k(-12 + 4) + 3(9 - 2k) = 0$$

$$\Rightarrow 4k + 33 - 6k = 0 \Rightarrow k = \frac{33}{2}$$

Hence, statement - 1 is false.

Statement- 2 is the property.
It is a true statement.

Question183

If the system of equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = 0$$

has a unique solution, then λ is not equal to

[Online May 7, 2012]

Options:

A. 1

B. 0

C. 2

D. 3

Answer: D

Solution:

Solution:

Given system of equations is

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = 0$$

It has unique solution.

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} \neq 0$$

$$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 3) + 1(2 - 2) \neq 0$$

$$\Rightarrow 2\lambda - 6 - \lambda + 3 \neq 0 \Rightarrow \lambda - 3 \neq 0 \Rightarrow \lambda \neq 3$$

Question184

Statement -1:

Determinant of a skew-symmetric matrix of order 3 is zero.

Statement -2:

For any matrix A, $\det(A)^T = \det(A)$ and $\det(-A) = -\det(A)$.

Where $\det(B)$ denotes the determinant of matrix B. Then :

[2011RS]

Options:

A. Both statements are true

B. Both statements are false

C. Statement-1 is false and statement-2 is true

D. Statement-1 is true and statement-2 is false

Answer: D

Solution:

Solution:

We know that determinant of skew symmetric matrix of odd order is zero.

So, statement-1 is true.

We know that $\det(A^T) = \det(A)$

$\det(-A) = -(-1)^n \det(A)$

where A is a $n \times n$ order matrix.

So, statement- 2 is false.

Question 185

Consider the following relation R on the set of real square matrices of order 3.

$R = \{ (A, B) \mid A = P^{-1}BP \text{ for some invertible matrix } P \}$

Statement- 1: R is equivalence relation.

Statement- 2 : For any two invertible 3×3 matrices M and

N , $(MN)^{-1} = N^{-1}M^{-1}$.

[2011 RS]

Options:

A. Statement-1 is true, statement-2 is true and statement 2 is a correct explanation for statement-1.

B. Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

C. Statement-1 is true, statement-2 is false.

D. Statement-1 is false, statement-2 is true.

Answer: B

Solution:

Solution:

For reflexive

$A = P^{-1}AP$ is true

For $P = I$, which is an invertible matrix.

$(A, A) \in R$

$\therefore R$ is reflexive.

For symmetry

As $(A, B) \in R$ for matrix P

$A = P^{-1}BP$

$\Rightarrow PAP^{-1} = B$

$\Rightarrow B = PAP^{-1}$

$\Rightarrow B = (P^{-1})^{-1}A(P^{-1})$

$\therefore (B, A) \in R$ for matrix P^{-1}

$\therefore R$ is symmetric.

For transitivity

$A = P^{-1}BP$
 and $B = P^{-1}CP$
 $\Rightarrow A = P^{-1}(P^{-1}CP)P$
 $\Rightarrow A = (P^{-1})^2CP^2$
 $\Rightarrow A = (P^2)^{-1}C(P^2)$
 $\therefore (A, C) \in R$ for matrix P^2
 $\therefore R$ is transitive.
 So R is equivalence.
 So, statement- 1 is true.
 We know that if A and B are two invertible matrices of order n , then
 $(AB)^{-1} = B^{-1}A^{-1}$
 So, statement- 2 is true.

Question 186

If the trivial solution is the only solution of the system of equations
 $x - ky + z = 0$
 $kx + 3y - kz = 0$
 $3x + y - z = 0$
 then the set of all values of k is
 [2011 RS]

Options:

- A. $R - \{2, -3\}$
- B. $R - \{2\}$
- C. $R - \{-3\}$
- D. $\{2, -3\}$

Answer: A

Solution:

Solution:

$$x - ky + z = 0$$

$$kx + 3y - kz = 0$$

$$3x + y - z = 0$$

The given that system of equations have trivial solution,

$$\therefore \begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$$\Rightarrow 1(-3 + k) + k(-k + 3k) + 1(k - 9) \neq 0$$

$$\Rightarrow k - 3 + 2k^2 + k - 9 \neq 0$$

$$\Rightarrow k^2 + k - 6 \neq 0 \Rightarrow k = -3, k \neq 2$$

So, the equation will have only trivial solution,
 when $k \in R - \{2, -3\}$

Question 187

The number of values of k for which the linear equations
 $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero
 solution is

[2011]

Options:

A. 2

B. 1

C. zero

D. 3

Answer: A

Solution:

Solution:

Given that system of equations have non-zero solution

$$\Delta = 0$$

$$\Rightarrow \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 4(4 - 2) - k(k - 2) + 2(2k - 8) = 0$$

$$\Rightarrow 8 - k^2 + 2k + 4k - 16 = 0$$

$$k^2 - 6k + 8 = 0$$

$$\Rightarrow (k - 4)(k - 2) = 0 \Rightarrow k = 4, 2$$

Question 188

Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define

$\text{Tr}(A)$ = sum of diagonal elements of A and

$|A|$ = determinant of matrix A

Statement -1: $\text{Tr}(A) = 0$

Statement -2 : $|A| = 1$

[2010]

Options:

A. Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement -1.

B. Statement -1 is true, Statement -2 is false.

C. Statement -1 is false, Statement -2 is true .

D. Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

Answer: B

Solution:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \neq 0$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = I$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$$

$$ab + bd = ac + cd = 0$$

$$c \neq 0 \text{ and } b \neq 0 \Rightarrow a + d = 0$$

$$|A| = ad - bc = -a^2 - bc = -1$$

$$\text{Also if } A \neq I, \text{ then } \text{tr}(A) = a + d = 0.$$

\therefore Statement- 1 true and statement- 2 false.

Question189

Consider the system of linear equations;

$$x_1 + 2x_2 + x_3 = 3$$

$$2x_1 + 3x_2 + x_3 = 3$$

$$3x_1 + 5x_2 + 2x_3 = 1$$

**The system has
[2010]**

Options:

A. exactly 3 solutions

B. a unique solution

C. no solution

D. infinite number of solutions

Answer: C

Solution:

Solution:

$$D_x = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$$

\Rightarrow Given system, does not have any solution.

\Rightarrow No solution

Question190

Let a, b, c be such that $b(a + c) \neq 0$

if
$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^nc \end{vmatrix} = 0$$
 then the value of n

is :
[2009]

Options:

- A. any even integer
- B. any odd integer
- C. any integer
- D. ero

Answer: B

Solution:

Solution:

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^nc \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^nc \end{vmatrix} = 0$$

(Taking transpose of second determinant)

$C_1 \leftrightarrow C_3$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2}a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2}c & c+1 & c-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow [1 + (-1)^{n+2}] \Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$C_2 - C_1, C_3 - C_1$

$$\Rightarrow [1 + (-1)^{n+2}] \begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0R_1 + R_3$$

$$\Rightarrow [1 + (-1)^{n+2}] \begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow [1 + (-1)^{n+2}](a+c)(2b+1+2b-1) = 0$$

$$\Rightarrow 4b(a+c)[1 + (-1)^{n+2}] = 0$$

$$\Rightarrow 1 + (-1)^{n+2} = 0 \text{ as } b(a+c) \neq 0$$

$\Rightarrow n$ should be an odd integer.

Question 191

Let A be a 2×2 matrix

Statement -1: $\text{adj}(\text{adj } A) = A$

Statement -2: $|\text{adj } A| = |A|$

[2009]

Options:

A. Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement -1 is false, Statement-2 is true.

D. Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

Answer: A

Solution:

Solution:

We know that if A is square matrix of order n then

$$\text{adj}(\text{adj } A) = |A|^{n-2} A$$

$$= |A|^0 A = A$$

$$\text{Also } |\text{adj } A| = |A|^{n-1} = |A|^{2-1} = |A|$$

\therefore Both the statements are true but statement- 2 is not a correct explanation for statement-1

Question 192

Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by $\text{tr}(A)$, the sum of diagonal entries of A . Assume that $A^2 = I$

Statement-1 : If $A \neq I$ and $A \neq -I$, then $\det(A) = -1$

Statement-2 : If $A \neq I$ and $A \neq -I$, then $\text{tr}(A) \neq 0$

[2008]

Options:

A. Statement -1 is false, Statement-2 is true

B. Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

C. Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

D. Statement -1 is true, Statement-2 is false

Answer: D

Solution:

Solution:

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Given that $A^2 = I$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow a^2 + bc = 1 \text{ and } ab + bd = 0$$

$$ac + cd = 0 \text{ and } bc + d^2 = 1$$

From these four equations,

$$a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$$

$$\text{and } b(a + d) = 0 = c(a + d) \Rightarrow a = -d$$

$$|A| = ad - bc = -a^2 - bc = -1$$

$$\text{Also if } A \neq I \text{ then } \text{tr}(A) = a + d = 0$$

\therefore Statement 2 is false..

Question193

Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

[2008]

Options:

A. If $\det A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers

B. If $\det A \neq \pm 1$, then A^{-1} exists and all its entries are non integers

C. If $\det A = \pm 1$, then A^{-1} exists but all its entries are integers

D. If $\det A = \pm 1$, then A^{-1} need not exists

Answer: C

Solution:

Solution:

Given that all entries of square matrix A are integers, therefore all cofactors should also be integers.

If $\det A = \pm 1$ then A^{-1} exists. Also all entries of A^{-1} are integers.

Question194

Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$.

Then $a^2 + b^2 + c^2 + 2abc$ is equal to [2008]

Options:

A. 2

B. -1

C. 0

D. 1

Answer: D

Solution:

Solution:

The given equations are

$$-x + cy + bz = 0$$

$$cx - y + az = 0$$

$$bx + ay - z = 0$$

Given that x, y, z are not all zero

∴ The above system have non-zero solution

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\Rightarrow -1(1 - a^2) - c(-c - ab) + b(ac + b) = 0$$

$$\Rightarrow -1 + a^2 + b^2 + c^2 + 2abc = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

Question195

$$\text{Let } A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix} . \text{ If } |A^2| = 25, \text{ then } |\alpha| \text{ equals}$$

[2007]

Options:

A. 1 / 5

B. 5

C. 5^2

D. 1

Answer: A

Solution:

Solution:

Given that $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. and $|A^2| = 25$

$$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$$

$$\therefore |A^2| = 25(25\alpha^2)$$

$$\therefore 25 = 25(25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$$

Question196

If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0$, $y \neq 0$, then D is

[2007]

Options:

- A. divisible by x but not y
- B. divisible by y but not x
- C. divisible by neither x nor y
- D. divisible by both x and y

Answer: D

Solution:

Solution:

$$\text{Given that, } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$$

Hence, D is divisible by both x and y

Question197

If $a^2 + b^2 + c^2 = -2$ and

$$f(x) = \begin{vmatrix} 1 + a^2x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix}$$

then $f(x)$ is a polynomial of degree
[2005]

Options:

- A. 1
- B. 0
- C. 3
- D. 2

Answer: D

Solution:

Solution:

Applying, $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$f(x) = \begin{vmatrix} 1 + (a^2 + b^2 + c^2 + 2)x & (1 + b^2)x & (1 + c^2)x \\ 1 + (a^2 + b^2 + c^2 + 2)x & 1 + b^2x & (1 + c^2)x \\ 1 + (a^2 + b^2 + c^2 + 2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix}$$

$[\because a^2 + b^2 + c^2 = -2]$

$$= \begin{vmatrix} 1 & (1 + b^2)x & (1 + c^2)x \\ 1 & 1 + b^2x & (1 + c^2)x \\ 1 & (1 + b^2)x & 1 + c^2x \end{vmatrix}$$

Applying, $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

$$\therefore f(x) = \begin{vmatrix} 1 & (1 + b^2)x & (1 + c^2)x \\ 0 & 1 - x & 0 \\ 0 & 0 & 1 - x \end{vmatrix}$$

$$f(x) = (x - 1)^2$$

Hence degree = 2

Question 198

If $a_1, a_2, a_3, \dots, a_n, \dots$ are in G. P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

is equal to

[2005]

Options:

- A. 1
- B. 0
- C. 4
- D. 2

Answer: B

Solution:

Solution:
Let r be the common ratio of an G.P., then

$$\begin{aligned} & \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} \\ &= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix} \\ &= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_1 + (n+7)\log r \end{vmatrix} \end{aligned}$$

Applying $C_3 \rightarrow C_3 + C_1$, we get

$$\begin{aligned} &= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix} \\ &= 0 \end{aligned}$$

Question199

If $A^2 - A + I = 0$, then the inverse of A is
[2005]

Options:

- A. $A + I$
- B. A
- C. $A - I$
- D. $I - A$

Answer: D

Solution:

Solution:

Given that $A^2 - A + I = 0$

Pre-multiply by A^{-1} both side, we get

$$A^{-1}A^2 - A^{-1}A + A^{-1} \cdot I = A^{-1} \cdot 0$$

$$\Rightarrow A - I + A^{-1} = 0 \text{ or } A^{-1} = I - A.$$

Question 200

The system of equations

$$\alpha x + y + z = \alpha - 1$$

$$x + \alpha y + z = \alpha - 1$$

$$x + y + \alpha z = \alpha - 1$$

has infinite solutions, if α is [2005]

Options:

A. - 2

B. either - 2 or 1

C. not - 2

D. 1

Answer: A

Solution:

Solution:

$$\alpha x + y + z = \alpha - 1$$

$$x + \alpha y + z = \alpha - 1$$

$$x + y + \alpha z = \alpha - 1$$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$$

$$= \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha)$$

$$= \alpha(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1)$$

$$= (\alpha - 1)[\alpha^2 + \alpha - 1 - 1]$$

$$= (\alpha - 1)[\alpha^2 + \alpha - 2]$$

$$= (\alpha - 1)[\alpha^2 + 2\alpha - \alpha - 2]$$

$$= (\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)]$$

$$= (\alpha - 1)^2(\alpha + 2)$$

\therefore Equations has infinite solutions

$$\therefore \Delta = 0$$

$$\Rightarrow (\alpha - 1) = 0, \alpha + 2 = 0$$

$$\Rightarrow \alpha = -2, 1;$$

But $\alpha \neq 1$.

$$\therefore \alpha = -2$$

Question201

If $a_1, a_2, a_3, \dots, a_n, \dots$ are in G.P., then the value of the determinant

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}, \text{ is}$$

[2004]

Options:

A. -2

B. 1

C. 2

D. 0

Answer: D

Solution:

Solution:

Let r be the common ratio of an G.P., then

$$\begin{aligned} & \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} \\ &= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix} \\ &= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_1 + (n+7)\log r \end{vmatrix} \end{aligned}$$

Applying $C_3 \rightarrow C_3 + C_1$, we get

$$\begin{aligned} &= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix} \\ &= 0 \end{aligned}$$

Question202

Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$ If B is the inverse of

matrix A, then α is
[2004]

Options:

- A. 5
- B. -1
- C. 2
- D. -2

Answer: A

Solution:

Solution:

Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow B = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

Given that $B = A^{-1} \Rightarrow AB = I$

$\Rightarrow \frac{1}{10} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \frac{1}{10} \begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$

Question203

Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is
[2004]

Options:

- A. $A^2 = I$
- B. $A = (-1)I$, where I is a unit matrix
- C. A^{-1} does not exist
- D. A is a zeromatrix

Answer: A

Solution:

Solution:

Given that

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$$\therefore A^{-1} \text{ exists, further } (-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$$

$$\begin{aligned} \text{Also } A^2 &= \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \end{aligned}$$

Question204

If $1, \omega, \omega^2$ are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal

to
[2003]

Options:

A. ω^2

B. 0

C. 1

D. ω

Answer: B

Solution:

Solution:

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$

Expand through R_1

$$\begin{aligned} &= 1(\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n}) \\ &= \omega^{3n} - 1 - 0 + \omega^{3n} - \omega^{6n} \end{aligned}$$

$$= 1 - 1 + 1 - 1 = 0 [\because \omega^{3n} = 1]$$

Question205

If the system of linear equations $x + 2ay + az = 0$; $x + 3by + bz = 0$; $x + 4cy + cz = 0$ has a non - zero solution, then a, b, c.
[2003]

Options:

- A. satisfy $a + 2b + 3c = 0$
- B. are in A.P
- C. are in G..P
- D. are in H.P.

Answer: D

Solution:

Solution:
For homogeneous system of equations to have nonzero solution, $\Delta = 0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - 2C_3$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0 \quad R_3 \rightarrow R_3 - R_1, R_2 \rightarrow R_2 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 0 & b & b - a \\ 0 & 2c & c - a \end{vmatrix} = 0$$

$$\Rightarrow bc - ab = 2bc - 2ac$$

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

\therefore a, b, c are in Harmonic Progression.

Question206

If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is $-ve$, then

$$\begin{vmatrix} a & b & ax + b \\ b & c & bx + c \\ ax + b & bx + c & 0 \end{vmatrix} \text{ is equal to}$$

[2002]

Options:

- A. +ve
- B. $(ac - b^2)(ax^2 + 2bx + c)$
- C. -ve
- D. 0

Answer: C

Solution:

Solution:

Given that
$$\begin{vmatrix} a & b & ax + b \\ b & c & bx + c \\ ax + b & bx + c & 0 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - (xR_1 + R_2)$

$$= \begin{vmatrix} a & b & ax + b \\ b & c & bx + c \\ 0 & 0 & -(ax^2 + 2bx + c) \end{vmatrix}$$

$$= (ax^2 + 2bx + c)(b^2 - ac) = (+)(-) = -ve.$$

[Given that discriminant of $ax^2 + 2bx + c$ is $-ve$
 $\therefore 4b^2 - 4ac < 0 \Rightarrow b^2 - ac < 0$]

Question207

l, m, n are the p^{th} , q^{th} and r^{th} term of a G. P. all positive, then

$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} \text{ equals}$$

[2002]

Options:

- A. -1
- B. 2
- C. 1
- D. 0

Answer: D

Solution:

$$\begin{aligned} l &= AR^{p-1} \Rightarrow \log l = \log A + (p-1) \log R \\ m &= AR^{q-1} \Rightarrow \log m = \log A + (q-1) \log R \\ n &= AR^{r-1} \Rightarrow \log n = \log A + (r-1) \log R \end{aligned}$$

Now,
$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$$

Operating

$$C_1 - (\log R)C_2 + (\log R - \log A)C_3 = \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0$$
