Arithmetic Progression II

Objective

To verify that the sum of first n natural numbers is $\frac{n(n+1)}{2}$ by graphical method.

The product of two polynomials say A and B represents a rectangle of sides A and B. Thus n(n+1) represents a rectangle of sides n and (n + 1).

Prerequisite Knowledge

1. Concept of natural numbers.
2. Area of squares and rectangles.

Materials Required

Graph papers, white chart paper, coloured pens, geometry box.

Procedure

Let us consider the sum of first n natural numbers
1 + 2 + 3 + 4 + + n (say n = 10).

1. Take a graph paper and paste it on a white chart paper.
2. Mark the rectangles 1, 2, 3 n, (n + 1) along the vertical line and 1,2, 3,…. n along the horLontal line.
3. Colour the rectangular strips of length 1 cm, 2 cm, 3 cm n cm each of width 1 cm.
4. Complete the rectangle with sides n and n+1. Name this rectangle as PQRS. Mark dot in each square as shown in fig. (i).
5. Count the coloured squares and total number of squares in rectangle PQRS.

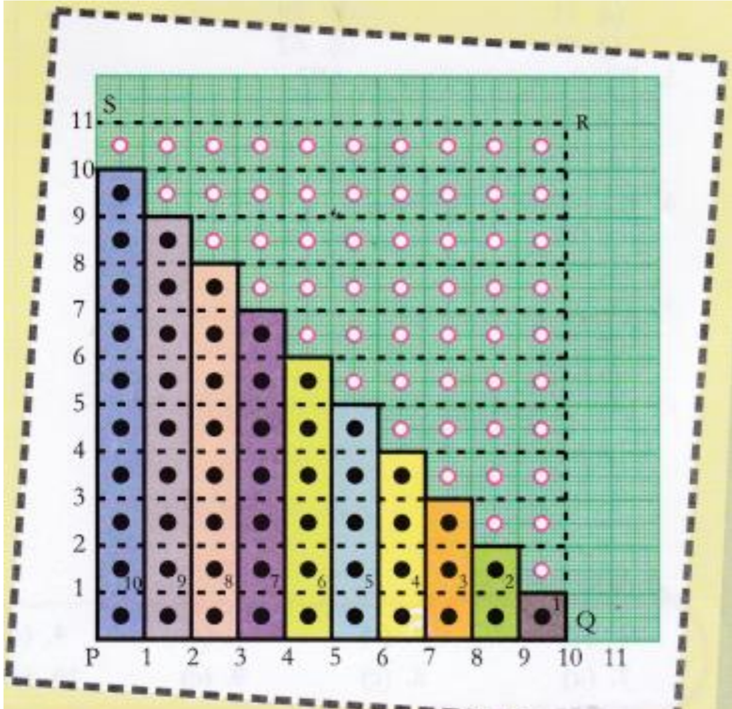

Fig. (i)

Observation

We observe, number of shaded squares = $\frac{1}{2}$ x total no. of squares
No. of shaded squares = 1+ 2 + 3 + … + n
Total squares = Area of rectangle = n (n + 1)
Therefore 1 + 2 + 3 + … + n = $\frac{1}{2}$ n(n + 1)

Mathematically

Area of rectangle PQRS = 10 x 11
Area of shaded region = $\frac{1}{2}$ x 10 x 11 = 55 **……………….(i)**
Also, area of shaded region = (1 x 1) + (2 x 1) + (3 x 1) +… + (10 x 1)
= 1+2 + 3 + … +10 = 55 **…………………….(ii)**
From (i) and (ii),
1+2 + 3 + … + 10= $\frac{1}{2}$ x 10 x 11 = 55
Verified that 1 + 2 + 3 + … + 10 = $\frac{1}{2}$ x 10 (10 + 1) by graphical method.

Result

It is verified graphically that 1 + 2 + 3 + … + n = $\frac{1}{2}$n(n+ 1) or sum of first n natural numbers = $\frac{1}{2}$ n(n + 1).

Learning Outcome

Students will develop a geometrical intuition of the formula for the sum of natural numbers starting from one.

Activity Time

1. Find the sum of first 100 natural numbers.
2. Find the sum of first 1000 natural numbers.
3. Evaluate 10 + 11 + 12 + … + 25.

Viva Voce

Question 1:
Are all natural numbers whole numbers ?
Answer:
Yes

Question 2:
Are all whole numbers natural numbers ?
Answer:
Except zero, all whole numbers are natural numbers.

Question 3:
Write down an AP having the sum of first 7 terms as zero.
Answer:
-3, -2, -1, 0, 1, 2, 3.

Question 4:
What does represent, where S„ represents the sum of n terms of anAP?
Answer:
The n th term of an AP.

Question 5:
What is the formula for the sum of n terms of an AP ?
Answer:
$S_n=\frac{n}{2}[2a+(n-1)d]$

Question 6:
What is the formula for the sum of n terms of an AP whose common difference is not given ? [First term (a) and last term (i) known]

Answer:
$S_n=\frac{n}{2}[a+l]$, where I represents the last term.

Question 7:
If $S_n = 3n^2+2n$, find the first term.
Answer:
5

Question 8:
What is the arithmetic mean of 4 and 8 ?
Answer:
6

Question 9:
What is the sum of first 10 natural numbers ?
Answer:
55

Question 10:
Find the common difference of an arithmetic progression of first 20 natural numbers.
Answer:
1

Multiple Choice Questions

Question 1:
Sum of first n terms of an AP is
(a) $\frac{n}{2}[2a+(n-1)d]$
(b) $\frac{n}{2}2n[a+(n-1)d]$
(c) $\frac{n}{2}[2a-(n-1)d]$
(d) $\frac{n}{2}[2a-(n+1)d]$

Question 2:
Sum of first n positive integers is
(a) $\frac{n(n-1)}{2}$
(b) $\frac{2n(n+1)}{2}$
(c) $\frac{n(n+1)}{2}$
(d) none of these

Question 3:
The sum of 0.70 + 0.71 + 0.72 + ….. + to 50 terms is
(a) 4.725
(b) 47.25
(c) 472.5
(d) none of these

Question 4:
If $a_n = 3 + 4n$ is n th termof an AP, then S_{15} is
(a) 525

(b) 325
(c) 425
(d) none of these

Question 5:
Sum of all odd numbers between 0 and 50 is
(a) 623
(b) 627
(c) 624
(d) 625

Question 6:
Sum of -37, -33, -29, … to 12 terms is
(a) -180
(b) 180
(c) 108
(d) -108

Question 7:
In an AP, given that $a_{12} = 37$ and $d = 3$. Find S_{12}.
(a) 246
(b) 642
(c) 264
(d) 624

Question 8:
In an AP, if $a = 8$, $a_n = 62$ and $S_n = 210$, then n is
(a) 4
(b) 6
(c) 5
(d) 7

Question 9:.
Sum of first 40 positive integers divisible by 6 is
(a) 4092
(b) 4029
(c) 4920
(d) 4290

Question 10:
Sum of first 15 multiples of 8 is
(a) 690
(b) 609

(r) 906
(d) 960

Answers

1. (a)
2. (c)
3. (b)
4. (a)
5 (d)
6. (a)
7. (a)
8. (b)
9. (c)
10. (d)