MIND MAP : LEARNING MADE SIMPLE CHAPTER - 8

Binomial Theorem

Binomial Theorem for any Index

If n is negative integer, then n! is not defined. We state binomial theorem in another form

$$(a+b)^n = a^n + \frac{n}{1!}a^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + --- + \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}a^{n-r}b^r + \cdots\cdots + b^n$$

Here, $T_{r+1} = \frac{n\,(n-1)(n-2)\cdots\cdots(n-r+1)}{r!}a^{n-r}b^r$

General & Middle Terms in Expansion of $(a+b)^n$

The general term of an expansion $(a+b)^n$ is

$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r,\ 0 \le r \le n, r \in N$

Middle Terms:

1. In $(a+b)^n$, if n is even, then the no. of terms in the expansion is odd. Therefore, there is only one middle term and it is $\left(\frac{n+2}{2}\right)^{th}$ term.

2. In $(a+b)^n$, if n is odd then the no. of terms in the expansion is even. Therefore, there are two middle terms and those are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms.

Some Special Cases $(a+b)^n$

In the expansion of $(a+b)^n$,

(i) Taking a=x and b=−y, we obtain
$(x-y)^n = {}^nC_0 x^n - {}^nC_1 x^{n-1}y + {}^nC_2\, x^{n-2}y^2 + --- + (-1)^n\, {}^nC_n\, y^n$

(ii) Taking a=1, b=x, we obtain
$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + ------- + {}^nC_n x^n$

(iii) Taking a=1, b=−x, we obtain
$(1-x)^n = {}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 + ---- + (-1)^n\, {}^nC_n x^n$

Definition

If a, b$\in R$ and n$\in N$, then

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1}b^1 + {}^nC_2\, a^{n-2}b^2 + ------ + {}^nC_n\, a^0 b^n$$

- **Remarks:** If the index of the binomial is n then the expansion contains n+1 terms.
- In each term, the sum of indices of a and b is always n.
- Coefficients of the terms in binomial expansion equidistant from both the ends are equal.

$$(a-b)^n = {}^nC_0\, a^n b^0 - {}^nC_1 a^{n-1}b^1 + {}^nC_2\, a^{n-2}b^2 + ------ + (-1)^n\, {}^nC_n a^0 b^n$$

Binomial Coefficients

The coefficient ${}^nC_0, {}^nC_1, {}^nC_2$,------${}^nC_n$, in the expansion of $(a+b)^n$ are called binomial coefficients and denoted by C_0, C_1, C_2----C_x, respectively

Properties of binomial coefficients:

(i) $C_0 + C_1 + C_2 + ------ + C_n = 2^n$
(ii) $C_0 - C_1 + C_2 ------- + (-1)^n\, C_n = 0$
(iii) $C_0 + C_2 + C_4 + ----- = C_1 + C_3 + C_5 + ---- = 2^{n-1}$
(iv) ${}^nC_{r_1} = {}^nC_{r_2} \Rightarrow r_1 = r_2 \text{ or } r_1 + r_2 = n$
(v) ${}^nC_{r_1} + {}^nC_{r-1} = {}^{1+n}C_r$
(vi) ${}^nC_r = \frac{n}{r}\, {}^{n-1}C_{r-1}$