

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions

M.M., Min.

Single choice Objective (no negative marking) Q.1,2,3,4

(3 marks, 3 min.)

[12, 12]

Subjective Questions (no negative marking) Q.5,6,7

(4 marks, 5 min.)

[12, 15]

Match the Following (no negative marking) Q.8

(8 marks, 8 min.)

[8, 8]

1. Range of the function $f(x) = \frac{\ell nx}{\sqrt{x}}$ is

(A) $(-\infty, e)$

(B) $(-\infty, e^2)$

(C) $\left(-\infty, \frac{2}{e}\right]$

(D) $\left(-\infty, \frac{1}{e}\right]$

2. Let $\tan(2\pi |\sin \theta|) = \cot(2\pi |\cos \theta|)$, where $\theta \in \mathbb{R}$ and $f(x) = (|\sin \theta| + |\cos \theta|)^x$, $x \geq 1$. Then range of $f(x)$ does not include

(A) 1

(B) 2

(C) 3

(D) 4

3. Range of the function $f(x) = \sqrt{\sin^{-1} |\sin x| - \cos^{-1} |\cos x|}$ is

(A) $\{0\}$

(B) $\left[0, \sqrt{\pi/2}\right]$

(C) $\left[0, \sqrt{\pi}\right]$

(D) none of these

4. If $f(4) = g(4) = 2$, $f'(4) = 9$, $g'(4) = 6$, then $\lim_{x \rightarrow 4} \frac{\sqrt{f(x)} - \sqrt{g(x)}}{\sqrt{x} - 2}$ is equal to

(A) $3\sqrt{2}$

(B) $\frac{3}{\sqrt{2}}$

(C) 0

(D) does not exist

5. Evaluate :

$$(i) \lim_{x \rightarrow \infty} \frac{\cot^{-1}(\sqrt{x+1} - \sqrt{x})}{\sec^{-1}\left(\left(\frac{2x+1}{x-1}\right)^x\right)}$$

$$(ii) \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r}{1+r^2+r^4}$$

6. Let $f(x) = \begin{cases} \frac{\sin ax^2}{x^2}; & x \neq 0 \\ \frac{3}{4} + \frac{1}{4a}; & x = 0 \end{cases}$. For what values of a , $f(x)$ is continuous at $x = 0$?

7. Find all values of the parameter 'a' for which the inequality $4^x - a2^x - a + 3 \leq 0$ is satisfied by at least one real x .

8. **Column - I**

Column - II

(A) $\lim_{x \rightarrow \pi/2} [\sin^{-1} \sin x] =$

(p) -2

(B) $\lim_{x \rightarrow -\infty} [\tan^{-1} x] =$

(q) 0

(C) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x} =$

(r) 1

(D) $\lim_{x \rightarrow 0^+} \left[\frac{\sin |x|}{x} \right] =$

(s) does not exist.

(\because where $[.]$ denotes greatest integer function)

Answers Key

1. (C) 2. (A) 3. (A) 4. (A)

5. (i) 1 (ii) $\frac{1}{2}$ 6. $a = 1, a = \frac{-1}{4}$

7. $a \in [2, \infty)$ 8. $(A) \rightarrow r, (B) \rightarrow p, (C) \rightarrow s, (D) \rightarrow q$