

Topic : Definite Integration

Type of Questions

M.M., Min.

Single choice Objective ('-1' negative marking) Q.2,3,4

(3 marks, 3 min.)

[9, 9]

Multiple choice objective ('-1' negative marking) Q.5

(5 marks, 4 min.)

[5, 4]

Subjective Questions ('-1' negative marking) Q.1,6,7

(4 marks, 5 min.)

[12, 15]

Match the Following (no negative marking) Q.8

(8 marks, 8 min.)

[8, 8]

1. Evaluate : $\int_{-1}^3 \left(\tan^{-1} \frac{x}{x^2+1} + \tan^{-1} \frac{x^2+1}{x} \right) dx$

2. Given $\int_0^{\pi/2} \frac{dx}{1+\sin x + \cos x} = \ln 2$, then the value of $\int_0^{\pi/2} \frac{\sin x}{1+\sin x + \cos x} dx$ is equal to:

(A) $\frac{1}{2} \ln 2$

(B) $\frac{\pi}{2} - \ln 2$

(C) $\frac{\pi}{4} - \frac{1}{2} \ln 2$

(D) $\frac{\pi}{2} + \ln 2$

3. If $I = \int_0^{\pi/2} \ln(\sin x) dx$ then $\int_{-\pi/4}^{\pi/4} \ln(\sin x + \cos x) dx =$

(A) $\frac{I}{2}$

(B) $\frac{I}{4}$

(C) $\frac{I}{\sqrt{2}}$

(D) I

4. Let $u = \int_0^1 \frac{\ln(x+1)}{x^2+1} dx$ and $v = \int_0^{\pi/2} \ln(\sin 2x) dx$, then

(A) $u = 4v$

(B) $4u + v = 0$

(C) $u + 4v = 0$

(D) $2u + v = 0$

5. If $A_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$; $B_n = \int_0^{\pi/2} \left(\frac{\sin nx}{\sin x} \right)^2 dx$; for $n \in \mathbb{N}$, then :

(A) $A_{n+1} = A_n$

(B) $B_{n+1} = B_n$

(C) $A_{n+1} - A_n = B_{n+1}$

(D) $B_{n+1} - B_n = A_{n+1}$

6. Prove that $\int_0^1 \tan^{-1} \left(\frac{1}{1-x+x^2} \right) dx = 2 \int_0^1 \tan^{-1} x dx$. Hence or otherwise,

evaluate the integral $\int_0^1 \tan^{-1} (1-x+x^2) dx$

7. Given that, $F(x) = \frac{1}{x^2} \int_4^x (4t^2 - 2F'(t)) dt$, find $F'(4)$.

8. Column – 1

Column – 2

(A) The value of $\int_{-1}^1 \max(|x|, x^2, x^4) dx$ is equal to

(p) -1

(B) If for a continuous $f(x)$, $\int_{-a}^a f(x) dx = k \int_0^a (f(x) + f(-x)) dx$, then k is/are

(q) $\frac{17}{3}$

(C) If $\int_0^\infty e^{-2x} (\sin 2x + \cos 2x) dx = -\frac{k}{2}$, then value of k is/are

(r) $\frac{5}{2}$

(D) $\lim_{x \rightarrow 0} \frac{\int_0^x (\sin^2 4t + t^2) dt}{x^3}$ is equal to

(s) 1

Answers Key

1. π 2. (C) 3. (A) 4. (B)

5. (A)(D) 6. $\ln 2$ 7. $32/9$

8. (A) $\rightarrow$ (s), (B) $\rightarrow$ (p, s), (C) $\rightarrow$ (p), (D) $\rightarrow$ (q)