CBSE Test Paper 03

Chapter 5 Arithmetic Progression

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1. 200 logs are stacked in a such a way that 20 logs in the bottom row, 19 logs in the next row, 18 logs in the row next to it and so on. The total number of rows is **(1)**
   a. 16
   b. 12
   c. 15
   d. 10
2. Sum of n terms of the series, $\sqrt{2}+\ \sqrt{8}+\ \sqrt{18}+\sqrt{32}+\ldots\ldots\ldots\ldots$is **(1)**
   a. $\frac{n(n-1)}{2}$
   b. $\frac{n(n+1)}{\sqrt{2}}$
   c. $\frac{n(n+1)}{2}$
   d. $\frac{n(n-1)}{\sqrt{2}}$
3. A sum of Rs.700 is to be used to award 7 prizes. If each prize is Rs.20 less than its preceding prize, then the value of the first prize is **(1)**
   a. Rs.160
   b. Rs.100
   c. Rs.180
   d. Rs.200
4. The sum of odd numbers between 0 and 50 is **(1)**
   a. 625
   b. 600
   c. 500
   d. 2500
5. The common difference of an A.P. in which $a_{18}-a_{14}=32$ is **(1)**
   a. – 8
   b. 6
   c. 8
   d. – 6

6. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$. **(1)**

7. For an AP, if $a_{18} - a_{14} = 32$ then find the common difference d. **(1)**
8. Find the first four terms of an A.P. whose first term is - 2 and common difference is - 2. **(1)**
9. If the sum of n terms of an A.P. is $2n^2 + 5n$, then find the 4th term. **(1)**
10. What is the common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p} \ldots$? **(1)**
11. In the following AP's find the missing terms: 2, __, 26. **(2)**
12. Is 68 a term of the AP : 7, 10 , 13 ,....? **(2)**
13. Find 51 is a term of given A.P.or not where the A.P. is 5,8,11,14, ... **(2)**
14. In the following situation, does the list of numbers involved make an arithmetic progression, and why? The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum. **(3)**
15. If 7^{th} term of an A.P. is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term. **(3)**
16. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149. **(3)**
17. In an A.P. the first term is 8, n^{th} term is 33 and the sum to first n terms is 123. Find n and the common difference(d). **(3)**
18. Each year, a tree grows 5 cm less than it did the preceding year. If it grew by 1 m in the first year, then in how many years will it have ceased growing? **(4)**
19. If the sum of the first m terms of an AP be n and the sum of its first n terms be m then show that the sum of its first (m + n) terms is -(m + n). **(4)**
20. Find the sum of all integers between 100 and 550, which are divisible by 9. **(4)**

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Solution

1. a. 16

   Explanation: The number of logs in the row from bottom to the top are 20, 19, 18, ... which form an AP with first term 20 and common difference
   $19 - 20 = -1.$
   Let the 200 logs be arranged in n rows.
   Then $\mathrm{S}_n = 200$
   $\Rightarrow 200 = \frac{n}{2}[2a + (n-1)\,d]$
   $\Rightarrow 400 = n\,[2 \times 20 + (n-1)\,(-1)]$
   $\Rightarrow n^2 - 41n + 400 = 0$
   $\Rightarrow n^2 - 16n - 25n + 400 = 0$
   $\Rightarrow n\,(n-16) - 25\,(n-16) = 0$
   $\Rightarrow (n-25)\,(n-16) = 0$
   $\Rightarrow n - 25 = 0$ or $n - 16 = 0$
   $\Rightarrow n = 25$ or $n = 16$
   $n = 25$ is not poaaible as on calculating number of logs in 25th row, there is negative number of logs, which is not possible.
   Therefore, number of rows are 16.

2. b. $\frac{n(n+1)}{\sqrt{2}}$

   Explanation: Given: $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots\ldots\ldots\ldots$
   $\Rightarrow \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots\ldots\ldots\ldots\ldots\ldots$
   Here, $a = \sqrt{2}, d = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$
   $\therefore \mathrm{S}_n = \frac{n}{2}\,[2a + (n-1)\,d]$
   $\Rightarrow \mathrm{S}_n = \frac{n}{2}\left[2 \times \sqrt{2} + (n-1) \times \sqrt{2}\right]$
   $\Rightarrow \mathrm{S}_n = \frac{n}{2}\left[2\sqrt{2} + n\sqrt{2} - \sqrt{2}\right]$
   $\Rightarrow \mathrm{S}_n = \frac{n}{2}\left[\sqrt{2} + n\sqrt{2}\right]$
   $\Rightarrow \mathrm{S}_n = \frac{n\sqrt{2}}{2}(1+n)$ = $\frac{n(n+1)}{\sqrt{2}}$

3. a. Rs.160

Explanation: Let the first prize be a.

The seven prizes form an AP with first term a and common difference,

$d=-20$ n=7

Now the sum of all seven prizes = Rs. 700

$\therefore \mathrm{S}_n=700$

$\Rightarrow \frac{n}{2}[2a+(n-1)d]=700$

$\Rightarrow \frac{7}{2}[2a+(7-1)(-20)]=700$

$\Rightarrow 2a-120=200$

$\Rightarrow 2a=320$

$\Rightarrow a=160$

Therefore, the value of first prize is Rs. 160.

4. a. 625

Explanation: Odd numbers between 0 and 50 are 1, 3, 5, 7, ..., 49 Here

$a=1, d=3-1=2$ and

= $\frac{25}{2}\times 50$

S_{25} = 25×25

= 625

5. c. 8

Explanation: Given: $a_{18}-a_{14}=32$

$\Rightarrow a+(18-1)d-[a+(14-1)d]=32$

$\Rightarrow a+17d-a-13d=32$

$\Rightarrow 4d=32$

$\Rightarrow d=8$

6. Let a be the first term and d be the common difference of the given AP. Then,

S_n= $\frac{n}{2}\cdot$[2a+(n-l)d],

$\therefore 3(S_8-S_4)=3[\frac{8}{2}(2a+7d)-\frac{4}{2}(2a+3d)]$

= $3[4(2a+7d)-2(2a+3d)]=6(2a+11d)$

$=\frac{12}{2}\cdot(2a+11d)=S_{12}$.

Hence, S_{12}= 3(S_8 - S_4).

7. We know, n^{th} term of an AP is given by $a_n=a+(n-1)d$, where a is the first term and d is the common difference

Given, $a_{18}-a_{14}=32$

$\Rightarrow (a+(18-1)d)-(a+(14-1)d)=32$
$\Rightarrow (a+17d)-(a+13d)=32$
$\Rightarrow 4d=32$
$\Rightarrow d=8$

8. given a_1= -2, common difference d= -2

   a_1 = -2 ,

   $a_2 = a_1 + d$ = -2 + (-2) = -4

   $a_3 = a_2 + d$ = -4 + (-2) = -6

   $a_4 = a_3 + d$ = =-6 + (-2) = -8

   ∴ First four terms are - 2, - 4, - 6, - 8
9. Let the sum of n terms of A.P. = S_n.

   Given, $S_n = 2n^2 + 5n$

   Now, n^{th} term of A.P, = $S_n - S_{n-1}$

   or, $a_n = (2n^2 + 5n) - [2(n - 1)^2 + 5 (n - 1)]$

   $a_n= (2n^2 + 5n) - [2(n^2 -2n + 1) + 5 (n - 1)]$

   $a_n = 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5]$

   $a_n = 2n^2 + 5n - [2n^2+ n - 3]$

   $a_n = 2n^2 + 5n - 2n^2 - n + 3$

   $a_n = 4n + 3$

   4^{th} term a_4= $4 \times 4+3=16+3=19$
10. Common difference(d) = $a_2 - a_1$ = $\frac{(1-p-1)}{p}=\frac{-p}{p}=-1$

    therefore, d = -1
11. 2, __, 26
    We know that the difference between consecutive terms is equal in any A.P.
    Let the missing term be x .
    x - 2 = 26 - x ⇒ 2x = 28 ⇒ x = 14
    Therefore, missing term is 14.
12. Given AP 7,10,13...
    Here, first term $a=7$ and common difference $d=a_2-a_1=10-7=3$,

Assume that 68 is n^{th} term of given AP

We know that n^{th} term is given by $a_n = a + (n-1)d$

$\Rightarrow 68 = 7 + (n-1) \times 3$

$\Rightarrow 61 = (n-1) \times 3$

$\Rightarrow \frac{61}{3} + 1 = n$

$\Rightarrow n = \frac{64}{3} = 31\frac{1}{3}$

Since, n cannot be fraction, but a whole number. Therefore, 68 is not a term of given AP.

13. Given, A.P. is 5,8,11,14, ...

Here a = 5 and d = (8 - 5) = 3.

Let the nth term of the given AP be 51. Then,

$T_n = 51$

We know that $T_n = a + (n - 1)d$.

$\Rightarrow$ a + (n -1)d = 51

$\Rightarrow$ 5 + (n- 1) $\times$ 3 = 51 [Because, a = 5 and d = 3]

$\Rightarrow$5 + 3n - 3 = 51

$\Rightarrow$ 3n = 51 - 2

$\Rightarrow$ 3n = 49

$\Rightarrow$ n = $16\frac{1}{3}$.

But, the number of terms cannot be a fraction.

Therefore, 51 is not a term of the given Arithmetic progression.

14. Amount of money after 1 year $= Rs10000\left(1 + \frac{8}{100}\right) = a_1$

Amount of money after 2 year $= Rs10000\left(1 + \frac{8}{100}\right)^2 = a_2$

Amount of money after 3 year $= Rs10000\left(1 + \frac{8}{100}\right)^3 = a_3$

Amount of money after 4 year $= Rs10000\left(1 + \frac{8}{100}\right)^4 = a_4$

$a_2 - a_1 = Rs10000\left(1 + \frac{8}{100}\right)^2 - Rs10000\left(1 + \frac{8}{100}\right)$

$= Rs10000\left(1 + \frac{8}{100}\right)\left(1 + \frac{8}{100} - 1\right)$

$= 10000(1 + \frac{8}{100})(\frac{8}{100})$

$a_3 - a_2$

$= 10000\left(1+\frac{8}{100}\right)^2 - 10000\left(1+\frac{8}{100}\right)$
$= 10000\left(1+\frac{8}{100}\right)\left(1+\frac{8}{100}-1\right)$
$= 10000\left(1+\frac{8}{100}\right)\left(\frac{8}{100}\right)$

Since.$a_3 - a_2 \neq a_2 - a_1$.It does not form AP.

15. Let the first term be a and the common difference be d.

$t_n = a + (n-1)d$

Given, $t_7 = \frac{1}{9}$

$t_9 = \frac{1}{7}$

$a + 6d = \frac{1}{9}$ .....(i)

and $a + 8d = \frac{1}{7}$ ....(ii)

On subtracting eqn.(i) from (ii)

$a + 8d - a - 6d = \frac{1}{7} - \frac{1}{9}$

or, $2d = \frac{2}{63}$

or, $d = \frac{1}{63}$

Substituting the value of d in (ii) we get

$a + 8 \times \frac{1}{63} = \frac{1}{7}$

or, a = $\frac{1}{7} - \frac{8}{63}$

or, a = $\frac{9-8}{63} = \frac{1}{63}$

$t_{63} = a + 62d$

$\therefore \quad t_{63} = \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$

or, $t_{63} = \frac{63}{63} = 1$

Hence, t_{63}= 1.

16. Here, d = 7

a_{22} = 149

Let the first term of the AP be a.

We know that a_n = a + (n - 1)d

$\Rightarrow$ a_{22} = a + (22 - 1)d

$\Rightarrow$ a_{22} = a + 21d

$\Rightarrow$ 149 = a + (21) (7)

$\Rightarrow$ 149 = a + 147 $\Rightarrow$ a = 2

Again, we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_{22} = \frac{22}{2}[2(2) + (22-1)7]$
$\Rightarrow$ S_{22} = (11) [4 + 147]

$\Rightarrow$ S_{22} = (11) (151) $\Rightarrow$ S_{22} = 1661

Hence, the sum of the first 22 terms of the AP is 1661.

17. Given First term (a) = 8

    and, n^{th} term (a_n) = 33

    $\Rightarrow$ a + (n - 1)d = 33

    $\Rightarrow$ 8 + (n - 1)d = 33

    $\Rightarrow$ (n - 1)d = 33 - 8

    $\Rightarrow$ (n - 1)d = 25 .....(i)

    and, Sum of first n terms = 123
    $\Rightarrow \frac{n}{2}[a + a_n] = 123$
    $\Rightarrow \frac{n}{2}[8 + 33] = 123$
    $\Rightarrow \frac{n}{2} \times 41 = 123$
    $\Rightarrow n = \frac{123 \times 2}{41}$ $\Rightarrow$ n = 6
    Put value of n in equation (i)

    (6 - 1)d = 25

    $\Rightarrow$ 5d = 25
    $\Rightarrow d = \frac{25}{5} = 5$
18. Given that , tree grows 5 cm less than preceding year, and grew by 1m (100 cm) in the first year.
    means, 95cm in the 2nd year, 90 in the 3rd year , 85 in the fourth year and so on.
    growth in the year in which it will stop growing will be 0cm
    Therefore, The following sequence can be formed.
    i.e, $100, 95, 90, \ldots, 0$ which is an AP.
    Here, $a = 100, d = 95 - 100 = -5$ and $l = 0$
    Let $l = a_n = a + (n-1)d$
    Then, $0 = 100 + (n-1)(-5)$
    $0 = 100 - 5n + 5$
    $0 = 105 - 5n$
    $5n = 105 \Rightarrow n = 21$

Hence, Tree will ceased growing in 21 years.

19. Let a be the first term and d be the common difference of the given AP. Then,

S_m = n ⇒ $\frac{m}{2}$ [2a + (m-1)d] = n

⇒ 2am + m(m- 1)d - 2n ...... (i)

And, S_n = m ⇒ $\frac{n}{2}$ [2a + (n - 1)d] = m

⇒ 2an + n(n - 1)d = 2m ...... (ii)

On subtracting (ii) from (i), we get

2a(m-n) + [(m^2 - n^2) - (m - n)]d = 2(n - m)

⇒ (m - n)[2a + (m + n - 1)d] = 2(n - m)

⇒ 2a + (m + n- 1)d = -2 ..... (iii)

Sum of the first (m + n) terms of the given AP

= $\frac{(m+n)}{2}$ · {2a + (m + n - 1)d}

$= \frac{(m+n)}{2} \cdot (-2) = -(m+n)$ [using (iii)].

Hence, the sum of first (m + n) terms of the given AP is -(m + n).

20. According to the question,

All integers between 100 and 550, which are divisible by 9

= 108, 117, 126,.........., 549

First term (a) = 108

Common difference(d) = 117 - 108 = 9

Last term(a_n) = 549

⇒ a + (n - 1)d = 549

⇒ 108 + (n - 1)(9) = 549

⇒ 108 + 9n - 9 = 549

⇒ 9n = 549 + 9 - 108

⇒ 9n = 450

$\Rightarrow \quad n = \frac{450}{9} = 50$

Sum of 50 terms = $\frac{n}{2}[a + a_n]$

$= \frac{50}{2}[108 + 549]$

$= 25 \times 657$ = 16425