Differentiation

EXERCISE 3.1 [PAGES 90 - 91]

Exercise 3.1 | Q 1.1 | Page 91

Find $\frac{d^2y}{dx^2}$, if y = $\sqrt{x}$

Solution:

y = $\sqrt{x}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = \frac{1}{2} \cdot \frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(-\frac{1}{2}\right) \cdot x^{-\frac{3}{2}}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-1}{4}x^{-\frac{3}{2}}$$

Exercise 3.1 | Q 1.2 | Page 90

Find $\frac{d^2y}{dx^2}$, if y = x^5

Solution: y = x^5

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = 5x^4$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 5 \cdot \frac{d}{dx}\left(x^4\right)$$

$$= 5\left(4x^3\right)$$

$$\therefore \frac{d^2y}{dx^2} = 20x^3$$

Exercise 3.1 | Q 1.3 | Page 91

Find $\frac{d^2y}{dx^2}$, if y = x^{-7}

Solution:

y = x^{-7}

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = -7x^{-8}$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = -7 \cdot \frac{d}{dx}\left(x^{-8}\right)$$

$$= -7(-8)x^{-9}$$

$$\therefore \frac{d^2y}{dx^2} = 56x^{-9}$$

Exercise 3.1 | Q 2.1 | Page 91

Find $\frac{d^2y}{dx^2}$, if y = e^x

Solution:

y = e^x

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = e^x$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = e^x$$

Exercise 3.1 | Q 2.2 | Page 91

Find $\frac{d^2y}{dx^2}$, if y = $e^{(2x+1)}$

Solution:

y = $e^{(2x+1)}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = e^{(2x+1)} \cdot \frac{d}{dx}(2x+1)$$

$$\frac{dy}{dx} = e^{(2x+1)} \cdot (2+0)$$

$$\frac{dy}{dx} = 2e^{(2x+1)}$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 2 \cdot \frac{d}{dx} e^{(2x+1)}$$

$$= 2e^{(2x+1)} \cdot \frac{d}{dx}(2x + 1)$$

$$= 2e^{(2x+1)} \cdot (2 + 0)$$

$$\therefore \frac{d^2y}{dx^2} = 4e^{(2x+1)}$$

Exercise 3.1 | Q 2.3 | Page 91

Find $\frac{d^2y}{dx^2}$, if $y = e^{\log x}$

Solution:

$y = e^{\log x}$

$y = x$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = 1$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 0$$

Exercise 3.1 | Q 3.1 | Page 91

Find $\frac{dy}{dx}$ if, $y = e^{5x^2-2x+4}$

Solution:

$y = e^{5x^2-2x+4}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{5x^2-2x+4}\right)$$

$$= e^{5x^2-2x+4} \cdot \frac{d}{dx}(5x^2 - 2x + 4)$$

$$= e^{5x^2-2x+4} \cdot [5(2x) - 2 + 0]$$

$$\therefore \frac{dy}{dx} = (10x - 2) \cdot e^{5x^2-2x+4}$$

Exercise 3.1 | Q 3.2 | Page 91

Find $\frac{dy}{dx}$ if, y = $a^{(1+\log x)}$

Solution:

y = $a^{(1+\log x)}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}a^{(1+\log x)}$$

$$= a^{(1+\log x)} \cdot \log a \cdot \frac{d}{dx}(1 + \log x)$$

$$= a^{(1+\log x)} \cdot \log a \cdot \left(0 + \frac{1}{x}\right)$$

$$\therefore \frac{dy}{dx} = a^{(1+\log x)} \cdot \log a \cdot \frac{1}{x}$$

Exercise 3.1 | Q 3.3 | Page 91

Find $\frac{dy}{dx}$ if, y = $5^{(x+\log x)}$

Solution:

y = $5^{(x+\log x)}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[5^{(x+\log x)}\right]$$

$$= 5^{(x+\log x)} \cdot \log 5 \cdot \frac{d}{dx}(x + \log x)$$

$$\therefore \frac{dy}{dx} = 5^{(x+\log x)} \cdot \log 5 \cdot \left(1 + \frac{1}{x}\right)$$

EXERCISE 3.2 [PAGE 92]

Exercise 3.2 | Q 1.1 | Page 92

Find the rate of change of demand (x) of a commodity with respect to its price (y) if y = 12 + 10x + $25x^2$

Solution:

y = 12 + 10x + $25x^2$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(12 + 10x + 25x^2)$$

= 0 + 10 + 25(2x)

= 10 + 50x

Now by derivative of inverse function, the rate of change of demand (x) w.r.t. price (y) is

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}, \text{ where } \frac{dy}{dx} \neq 0$$

i.e. $\frac{dx}{dy} = \frac{1}{10 + 50x}$

Exercise 3.2 | Q 1.2 | Page 92

Find the rate of change of demand (x) of a commodity with respect to its price (y) if y = 18x + log(x - 4).

Solution:

y = 18x + log(x - 4)

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[18x + \log(x - 4)]$$

$$= \frac{d}{dx}(18x) + \frac{d}{dx}[\log(x - 4)]$$

$$= 18 + \frac{1}{x - 4} \cdot \frac{d}{dx}(x - 4)$$

$$= 18 + \frac{1}{x - 4} \cdot (1 - 0)$$

$$= 18 + \frac{1}{x - 4}$$

$$= \frac{18(x - 4) + 1}{x - 4}$$

$$= \frac{18x - 72 + 1}{x - 4}$$

$$\therefore \frac{dy}{dx} = \frac{18x - 71}{x - 4}$$

Now, by a derivative of inverse function, the rate of change of demand (x) w.r.t. price (y) is

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}, \text{ where } \frac{dy}{dx} \neq 0.$$

i.e. $\frac{dx}{dy} = \frac{1}{\frac{18x-71}{x - 4}} = \frac{x - 4}{18x - 71}$

Exercise 3.2 | Q 1.3 | Page 92

Find the rate of change of demand (x) of a commodity with respect to its price (y) if y = 25x + log(1 + x^2)

Solution: y = 25x + log(1 + x^2)

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[25x + \log\left(1 + x^2\right)\right]$$

$$= \frac{d}{dx}(25x) + \frac{d}{dx}\left[\log\left(1 + x^2\right)\right]$$

$$= 25 + \frac{1}{1 + x^2} \cdot \frac{d}{dx}\left(1 + x^2\right)$$

$$= 25 + \frac{1}{1 + x^2} \cdot (0 + 2x)$$

$$= 25 + \frac{2x}{1 + x^2}$$

$$= \frac{25\left(1 + x^2\right) + 2x}{1 + x^2}$$

$$\therefore \frac{dy}{dx} = \frac{25 + 25x^2 + 2x}{1 + x^2}$$

Now, by derivative of inverse function, the rate of change of demand (x) w.r.t. price (y) is

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}, \text{ where } \frac{dy}{dx} \neq 0.$$

$$\text{i.e. } \frac{dx}{dy} = \frac{1}{\frac{25+25x^2+2x}{1+x^2}} = \frac{1+x^2}{25x^2+2x+25}$$

Exercise 3.2 | Q 2.1 | Page 92

Find the marginal demand of a commodity where demand is x and price is y.

$y = x \cdot e^{-x} + 7$

Solution:

$y = x \cdot e^{-x} + 7$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x \cdot e^{-x} + 7\right)$$

$$= \frac{d}{dx}\left(x \cdot e^{-x}\right) + \frac{d}{dx}(7)$$

$$= x \cdot \frac{d}{dx}\left(e^{-x}\right) + e^{-x} \cdot \frac{d}{dx}(x) + 0$$

$$= x \cdot e^{-x} \cdot \frac{d}{dx}(-x) + e^{-x}(1)$$

$$= x \cdot e^{-x}(-1) + e^{-x}$$

$$= e^{-x}(-x+1)$$

$$\therefore \frac{dy}{dx} = \frac{-x+1}{e^x}$$

Now, by derivative of inverse function, the marginal demand of a commodity is

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}, \text{ where } \left(\frac{dy}{dx}\right) \neq 0$$

$$\text{i.e. } \frac{dx}{dy} = \frac{1}{\frac{-x+1}{e^x}} = \frac{e^x}{1-x}$$

Exercise 3.2 | Q 2.2 | Page 92

Find the marginal demand of a commodity where demand is x and price is y.

$$y = \frac{x+2}{x^2+1}$$

Solution:

$$y = \frac{x+2}{x^2+1}$$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x+2}{x^2+1}\right)$$

$$= \frac{\left(x^2+1\right) \cdot \frac{d}{dx}(x+2) - (x+2) \cdot \frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2}$$

$$= \frac{\left(x^2+1\right)(1+0) - (x+2)(2x+0)}{\left(x^2+1\right)^2}$$

$$= \frac{\left(x^2+1\right)(1) - (x+2)(2x)}{\left(x^2+1\right)^2}$$

$$= \frac{x^2+1-2x^2-4x}{\left(x^2+1\right)^2}$$

$$\therefore \frac{dy}{dx} = \frac{1-4x-x^2}{\left(x^2+1\right)^2}$$

Now, by derivative of inverse function, the marginal demand of a commodity is

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$, where $\frac{dy}{dx} \neq 0$

i.e., $$\frac{dx}{dy} = \frac{1}{\frac{1-4x-x^2}{(x^2+1)^2}} = \frac{(x^2+1)^2}{1-4x-x^2}$$

Exercise 3.2 | Q 2.3 | Page 92

Find the marginal demand of a commodity where demand is x and price is y.

y = $\frac{5x+9}{2x-10}$

Solution:

y = $\frac{5x+9}{2x-10}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{5x+9}{2x-10}\right)$$

$$= \frac{(2x-10)\cdot\frac{d}{dx}(5x+9) - (5x+9)\cdot\frac{d}{dx}(2x-10)}{(2x-10)^2}$$

$$= \frac{(2x-10)(5+0) - (5x+9)(2-0)}{(2x-10)^2}$$

$$= \frac{5(2x-10) - 2(5x+9)}{(2x-10)^2}$$

$$= \frac{10x - 50 - 10x - 18}{(2x - 10)^2}$$

$$\therefore \frac{dy}{dx} = \frac{-68}{(2x - 10)^2}$$

Now, by derivative of inverse function, the marginal demand of a commodity is

$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$, where $\frac{dy}{dx} \neq 0.$

i.e. $\frac{dx}{dy} = \frac{1}{\frac{-68}{(2x-10)^2}} = \frac{-(2x - 10)^2}{68}$

EXERCISE 3.3 [PAGE 94]

Exercise 3.3 | Q 1.1 | Page 94

Find $\frac{dy}{dx}$ if, y = $x^{x^{2x}}$

Solution:

y = $x^{x^{2x}}$

Taking logarithm of both sides, we get

log y = log$(x)^{x^{2x}}$

∴ log y = $x^{2x} \cdot \log x$

Differentiating both sides w.r.t.x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = x^{2x} \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x^{2x})$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = x^{2x} \cdot \frac{1}{x} + \log x \cdot \frac{d}{dx}(x^{2x}) \quad .....(i)$$

Let u = x^{2x}

Taking logarithm of both sides, we get

log u = $\log x^{2x} = 2x \cdot \log x$

Differentiating both sides w.r.t.x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = 2x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(2x)$$

$$\therefore \frac{1}{u} \cdot \frac{du}{dx} = 2x \cdot \frac{1}{x} + \log x \cdot (2)$$

$$\therefore \frac{1}{u} \cdot \frac{du}{dx} = 2 + 2 \log x$$

$$\therefore \frac{du}{dx} = u(2 + 2 \log x)$$

$$\therefore \frac{du}{dx} = 2u(1 + \log x)$$

$$\therefore \frac{du}{dx} = 2x^{2x}(1 + \log x) \quad ....(ii)$$

Substituting (ii) in (i), we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = x^{2x} \cdot \frac{1}{x} + (\log x)\left(2x^{2x}\right)(1 + \log x)$$

$$\therefore \frac{dy}{dx} = y\left[\frac{x^{2x}}{x} + 2x^{2x} \cdot \log x(1 + \log x)\right]$$

$$\therefore \frac{dy}{dx} = x^{x^{2x}} \cdot x^{2x} \log x\left[\frac{1}{x \log x} + 2(1 + \log x)\right]$$

Exercise 3.3 | Q 1.2 | Page 94

Find $\frac{dy}{dx}$ if, y = x^{e^x}

Solution:

y = x^{e^x}

Taking logarithm of both sides, we get

log y = log $x^{e^x} = e^x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^x)$$

$$= e^x \times \frac{1}{x} + (\log x)e^x$$

$$\therefore \frac{dy}{dx} = y \cdot e^x \left(\frac{1}{x} + \log x\right) = x^{e^x} e^x \left(\frac{1}{x} + \log x\right)$$

Exercise 3.3 | Q 1.3 | Page 94

Find $\frac{dy}{dx}$ if, y = e^{x^x}

Solution:

y = e^{x^x}

Taking the logarithm of both sides, we get

log y = log $e^{x^x} = x^x \log e$

∴ log y = x^x

Differentiating both sides w.r.t.x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(x^x)$$ .....(i)

Let u = x^x

Taking logarithm of both sides, we get

log u = $\log x^x = x \log x$

Differentiating both sides w. r. t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x)$$

$$\therefore \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{x} + \log x \cdot (1)$$

$\therefore \frac{1}{u} \cdot \frac{du}{dx}$ = 1 + log x

$$\therefore \frac{du}{dx} = u(1 + \log x)$$

$\therefore \frac{du}{dx} = x^x$(1 + log "x") .....(ii)

Substituting (ii) in (i), we get

$\frac{1}{y} \cdot \frac{dy}{dx} = x^x$(1 + log x)

$$\therefore \frac{dy}{dx} = y\, x^x(1 + \log x) = e^{x^x} \cdot x^x(1 + \log x)$$

Exercise 3.3 | Q 2.1 | Page 94

Find $\frac{dy}{dx}$ if, y = $\left(1 + \frac{1}{x}\right)^x$

Solution:

y = $\left(1+\frac{1}{x}\right)^{x}$

Taking logarithm of both sides, we get

log y = $\log\left(1+\frac{1}{x}\right)^{x}$

$\therefore$ log y = x $\log\left(1+\frac{1}{x}\right)$

Differentiating both sides w.r.t.x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx}=x\cdot\frac{d}{dx}\log\left(1+\frac{1}{x}\right)+\log\left(1+\frac{1}{x}\right)\cdot\frac{d}{dx}(x)$$

$$\therefore\ \frac{1}{y}\cdot\frac{dy}{dx}=x\cdot\frac{1}{1+\frac{1}{x}}\cdot\frac{d}{dx}\left(1+\frac{1}{x}\right)+\log\left(1+\frac{1}{x}\right)\cdot(1)$$

$$\therefore\ \frac{1}{y}\cdot\frac{dy}{dx}=\frac{x}{\frac{x+1}{x}}\cdot\left(0-\frac{1}{x^2}\right)+\log\left(1+\frac{1}{x}\right)$$

$$\therefore\ \frac{1}{y}\cdot\frac{dy}{dx}=\frac{x^2}{x+1}\cdot\left(\frac{-1}{x^2}\right)+\log\left(1+\frac{1}{x}\right)$$

$$\therefore\ \frac{1}{y}\cdot\frac{dy}{dx}=\frac{-1}{x+1}+\log\left(1+\frac{1}{x}\right)$$

$$\therefore\ \frac{dy}{dx}=y\left[\frac{-1}{x+1}+\log\left(1+\frac{1}{x}\right)\right]$$

$$\therefore\ \frac{dy}{dx}=\left(1+\frac{1}{x}\right)^{x}\cdot\left[\log\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]$$

Exercise 3.3 | Q 2.2 | Page 94

Find $\frac{dy}{dx}$ if, y = $(2x + 5)^x$

Solution: y = $(2x + 5)^x$

Taking logarithm of both sides, we get

log y = log $(2x + 5)^x$

∴ log y = x * log (2x + 5)

Differentiating both sides w.r.t.x, we get

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{d}{dx}[\log(2x+5)] + \log(2x+5) \cdot \frac{d}{dx}(x)$$

$$= x \cdot \frac{1}{2x+5} \cdot \frac{d}{dx}(2x+5) + \log(2x+5) \cdot (1)$$

$$= \frac{x}{2x+5} \cdot (2+0) + \log(2x+5)$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{2x}{2x+5} + \log(2x+5)$$

$$\therefore \frac{dy}{dx} = y\left[\frac{2x}{2x+5} + \log(2x+5)\right]$$

$$\therefore \frac{dy}{dx} = (2x+5)^x\left[\log(2x+5) + \frac{2x}{2x+5}\right]$$

Exercise 3.3 | Q 2.3 | Page 94

Find $\frac{dy}{dx}$ if, y = $\sqrt[3]{\frac{3x-1}{(2x+3)(5-x)^2}}$

Solution:

$$y = \sqrt[3]{\frac{3x-1}{(2x+3)(5-x)^2}}$$

$$= \frac{(3x-1)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{3}} \cdot (5-x)^{\frac{2}{3}}}$$

Taking logarithm of both sides, we get

$$\log y = \log\left[\frac{(3x-1)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{3}} \cdot (5-x)^{\frac{2}{3}}}\right]$$

$$= \log(3x-1)^{\frac{1}{3}} - \left[\log(2x+3)^{\frac{1}{3}} + \log(5-x)^{\frac{2}{3}}\right]$$

$$= \frac{1}{3}\log(3x-1) - \left[\frac{1}{3}\log(2x+3) + \frac{2}{3}\log(5-x)\right]$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3}\cdot\frac{d}{dx}[\log(3x-1)] - \frac{1}{3}\cdot\frac{d}{dx}[\log(2x+3)] - \frac{2}{3}\cdot\frac{d}{dx}[\log(5-x)]$$

$$= \frac{1}{3}\cdot\frac{1}{3x-1}\cdot\frac{d}{dx}(3x-1) - \frac{1}{3}\cdot\frac{1}{2x+3}\cdot\frac{d}{dx}(2x+3) - \frac{2}{3}\cdot\frac{1}{5\text{ - }x}\cdot\frac{d}{dx}(5-x)$$

$$= \frac{1}{3(3x-1)} \times 3 - \frac{1}{3(2x+3)} \times 2 - \frac{2}{3(5-x)} \times -1$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{3x-1} - \frac{2}{3(2x+3)} + \frac{2}{3(5-x)}$$

$$\therefore \frac{dy}{dx} = \frac{y}{3}\left[\frac{3}{3x-1} - \frac{2}{2x+3} + \frac{2}{5-x}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{3}\cdot\sqrt[3]{\frac{3x-1}{(2x+3)(5-x)^2}}\left[\frac{3}{3x-1} - \frac{2}{2x+3} + \frac{2}{5-x}\right]$$

Exercise 3.3 | Q 3.1 | Page 94

Find $\frac{dy}{dx}$ if, y = $\left(\log x^x\right) + x^{\log x}$

Solution:

y = $\left(\log x^x\right) + x^{\log x}$

Let u = $\left(\log x^x\right)$ and v = $x^{\log x}$

∴ y = u + v

Differentiating both sides w. r. t. x, we get

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{....(i)}$$

Now, u = $\left(\log x^x\right)$

Taking logarithm of both sides, we get

log u = log $\left(\log x^x\right)$ = x log (log x)

Differentiating both sides w. r. t. x, we get

$$\frac{d}{dx}(\log u) = x\frac{d}{dx}[\log(\log x)] + \log(\log x)\frac{d}{dx}(x)$$

$$\therefore \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{\log x} \cdot \frac{d}{dx}(\log x) + \log(\log x) \cdot 1$$

$$\therefore \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log(\log x)$$

$$\therefore \frac{du}{dx} = u\left[\frac{1}{\log x} + \log(\log x)\right]$$

$$\therefore \frac{du}{dx} = \left(\log x^x\right)\left[\frac{1}{\log x} + \log(\log x)\right] \quad \text{....(ii)}$$

$v = x^{\log x}$

Taking logarithm of both sides, we get

$\log v = \log\left(x^{\log x}\right) = \log x\ (\log x)$

$\therefore \log v = (\log x)^2$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{v}\cdot\frac{dv}{dx} = 2\log x\cdot\frac{d}{dx}(\log x)$$

$$\therefore \frac{1}{v}\cdot\frac{dv}{dx} = 2\log x\cdot\frac{1}{x}$$

$$\therefore \frac{dv}{dx} = v\left[\frac{2\log x}{x}\right]$$

$$\therefore \frac{dv}{dx} = x^{\log x}\left[\frac{2\log x}{x}\right] \quad ....(iii)$$

Substituting (ii) and (iii) in (i), we get

$$\frac{dy}{dx} = (\log x^x)\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{2\log x}{x}\right]$$

Exercise 3.3 | Q 3.2 | Page 94

Find $\frac{dy}{dx}$ if, $y = (x)^x + (a^x)$

Solution:

$y = (x)^x + (a^x)$

Let $u = (x)^x$ and $v = (a^x)$

$\therefore y = u + v$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad ....(i)$$

Now u = $(x)^x$

Taking logarithm of both sides, we get

log u = log $(x)^x$

∴ log u = $x \cdot \log x$

Differentiating both sides w.r.t.x, we get

$$\frac{1}{u}\frac{du}{dx} = x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x)$$

$$= x \cdot \frac{1}{x} + \log x \cdot (1)$$

$$\therefore \frac{1}{u}\frac{du}{dx} = 1 + \log x$$

$$\therefore \frac{du}{dx} = u(1 + \log x)$$

$$\therefore \frac{du}{dx} = (x)^x (1 + \log x) \quad ....(ii)$$

v = a^x

Differentiating both sides w.r.t.x, we get

$$\frac{dv}{dx} = a^x \cdot \log a \quad ....(iii)$$

Substituting (ii) and (iii) in (i), we get

$$\frac{dy}{dx} = x^x(1 + \log x) + a^x \cdot \log a$$

Exercise 3.3 | Q 3.3 | Page 94

Find $\frac{dy}{dx}$ if, y = $10^{x^x} + 10^{x^{10}} + 10^{10^x}$

Solution:

y = $10^{x^x} + 10^{x^{10}} + 10^{10^x}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(10^{x^x} + 10^{x^{10}} + 10^{10^x}\right)$$

$$= \frac{d}{dx}\left(10^{x^x}\right) + \frac{d}{dx}\left(10^{x^{10}}\right) + \frac{d}{dx}\left(10^{10^x}\right)$$

$$\therefore \frac{dy}{dx} = 10^{x^x} \cdot \log 10 \cdot \frac{d}{dx}(x^x) + 10^{x^{10}} \cdot \log 10 \cdot \frac{d}{dx}\left(x^{10}\right) + 10^{10^x} \cdot \log 10 \cdot \frac{d}{dx}(10^x)$$

$$= 10^{x^x} \cdot \log 10 \cdot x^x(1 + \log x) + 10^{x^{10}} \cdot \log 10 \cdot 10x^9 + 10^{10^x} \cdot \log 10 \cdot 10^x \log 10$$

$$\therefore \frac{dy}{dx} = 10^{x^x} \cdot x^x \cdot \log 10(1 + \log x) + 10^{x^{10}} \cdot 10x^9 \cdot \log 10 + 10^{10^x} \cdot 10^x(\log 10)^2$$

EXERCISE 3.4 [PAGE 95]

Exercise 3.4 | Q 1.1 | Page 95

Find $\frac{dy}{dx}$ if $, \sqrt{x} + \sqrt{y} = \sqrt{a}$

Solution:

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\therefore \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

Exercise 3.4 | Q 1.2 | Page 95

Find $\frac{dy}{dx}$ if, $x^3 + y^3 + 4x^3y = 0$

Solution:

$x^3 + y^3 + 4x^3y = 0$

Differentiating both sides w.r.t. x, we get

$$3x^2 + 3y^2\frac{dy}{dx} + 4\left[x^3\frac{dy}{dx} + y\frac{d}{dx}(x^3)\right] = 0$$

$$\therefore 3x^2 + 3y^2\frac{dy}{dx} + 4\left[x^3\frac{dy}{dx} + y(3x^2)\right] = 0$$

$$\therefore 3x^2 + 3y^2\frac{dy}{dx} + 4x^3\frac{dy}{dx} + 12x^2y = 0$$

$$\therefore (3y^2 + 4x^3)\frac{dy}{dx} = -(12x^2y + 3x^2)$$

$$\therefore \frac{dy}{dx} = \frac{-(12x^2y + 3x^2)}{(3y^2 + 4x^3)} = -\frac{3x^2(1 + 4y)}{3y^2 + 4x^2}$$

Exercise 3.4 | Q 1.3 | Page 95

Find $\frac{dy}{dx}$ if, $x^3 + x^2y + xy^2 + y^3 = 81$

Solution:

$x^3 + x^2y + xy^2 + y^3 = 81$

Differentiating both sides w.r.t. x, we get

$$3x^2 + x^2\frac{dy}{dx} + y\cdot\frac{d}{dx}(x^2) + x\cdot\frac{d}{dx}(y^2) + y^2\cdot\frac{d}{dx}(x) + 3y^2\cdot\frac{dy}{dx} = 0$$

$$\therefore 3x^2 + x^2\frac{dy}{dx} + y \cdot 2x + x \cdot 2y\frac{dy}{dx} + y^2 + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$\therefore \left(3x^2 + 2xy + y^2\right) + \left(x^2 + 2xy + 3y^2\right)\frac{dy}{dx} = 0$$

$$\therefore \left(x^2 + 2xy + 3y^2\right)\frac{dy}{dx} = -\left(3x^2 + 2xy + y^2\right)$$

$$\therefore \frac{dy}{dx} = -\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}$$

Exercise 3.4 | Q 2.1 | Page 95

Find $\frac{dy}{dx}$ if, $ye^x + xe^y = 1$

Solution:

$ye^x + xe^y = 1$

Differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}(ye^x) + \frac{d}{dx}(xe^y) = 0$$

$$\therefore y\frac{d}{dx}(e^x) + e^x\frac{dy}{dx} + x\frac{d}{dx}(e^y) + e^y\frac{d}{dx}(x) = 0$$

$$\therefore ye^x + (e^x)\frac{dy}{dx} + x(e^y)\frac{dy}{dx} + e^y$$

$$\therefore (e^x + xe^y)\frac{dy}{dx} = -(e^y + ye^x)$$

$$\therefore \frac{dy}{dx} = \frac{-(e^y + ye^x)}{e^x + xe^y}$$

Exercise 3.4 | Q 2.2 | Page 95

Find $\frac{dy}{dx}$ if, $x^y = e^{x - y}$

Solution:

$x^y = e^{x - y}$

Taking logarithm of both sides, we get

y log x = (x - y) log e = x - y

∴ y log x + y = x

∴ y(1 + log x) = x

∴ y = $\frac{x}{1 + \log x}$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{x}{1 + \log x}\right]$$

$$\therefore \frac{dy}{dx} = \frac{(1 + \log x)\frac{d}{dx}(x) - x\frac{d}{dx}(1 + \log x)}{(1 + \log x)^2}$$

$$= \frac{(1 + \log x) \times 1 - x \times \left(\frac{1}{x}\right)}{(1 + \log x)^2}$$

$$= \frac{1 + \log x - 1}{(1 + \log x)^2}$$

$$\therefore \frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$$

Exercise 3.4 | Q 2.3 | Page 95

Find $\frac{dy}{dx}$ if, xy = log (xy)

Solution: xy = log (xy)

Differentiating both sides w.r.t. x, we get

$$x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x) = \frac{1}{xy} \cdot \frac{d}{dx}(xy)$$

$$\therefore x \cdot \frac{dy}{dx} + y = \frac{1}{xy}\left(x\frac{dy}{dx} + y\right) = \frac{1}{y}\frac{dy}{dx} + \frac{1}{x}$$

$$\therefore \left(x - \frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{x} - y$$

$$\therefore -\left(\frac{1 - xy}{y}\right)\frac{dy}{dx} = \left(\frac{1 - xy}{x}\right)$$

$$\therefore \frac{dy}{dx} = \frac{-y}{x}$$

Exercise 3.4 | Q 3.1 | Page 95

Solve the following:

If $x^5 \cdot y^7 = (x + y)^{12}$ then show that, $\frac{dy}{dx} = \frac{y}{x}$

Solution:

$$x^5 \cdot y^7 = (x + y)^{12}$$

Taking logarithm of both sides, we get

$$\log\left(x^5 \cdot y^7\right) = \log(x + y)^{12}$$

$\therefore$ log x^5 + log y^7 = 12 log (x + y)

$\therefore$ 5 log x + 7 log y = 12 log (x + y)

Differentiating both sides w.r.t. x, we get

$$\frac{5}{x} + \frac{7}{y} \cdot \frac{dy}{dx} = 12 \cdot \frac{1}{x+y} \cdot \frac{d}{dx}(x+y)$$

$$\therefore \frac{5}{x} + \frac{7}{y} \cdot \frac{dy}{dx} = \frac{12}{x+y}\left[1 + \frac{dy}{dx}\right]$$

$$\therefore \frac{5}{x} + \frac{7}{y} \cdot \frac{dy}{dx} = \frac{12}{x+y} + \frac{12}{x+y} \cdot \frac{dy}{dx}$$

$$\therefore \left[\frac{7}{y} - \frac{12}{x+y}\right]\frac{dy}{dx} = \frac{12}{x+y} - \frac{5}{x}$$

$$\therefore \left[\frac{7x + 7y - 12y}{y(x+y)}\right]\frac{dy}{dx} = \frac{12x - 5x - 5y}{x(x+y)}$$

$$\therefore \left[\frac{7x - 5y}{y(x+y)}\right]\frac{dy}{dx} = \left[\frac{7x - 5y}{x(x+y)}\right]$$

$$\therefore \frac{dy}{dx} = \left[\frac{7x - 5y}{x(x+y)}\right] \times \frac{y(x+y)}{7x - 5y}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$

Exercise 3.4 | Q 3.2 | Page 95

Solve the following:

If log (x + y) = log (xy) + a then show that, $\frac{dy}{dx} = \frac{-y^2}{x^2}$.

Solution: log (x + y) = log (xy) + a

∴ log (x + y) = log x + log y + a

Differentiating both sides w.r.t. x, we get

$$\frac{1}{x+y}\cdot\frac{d}{dx}(x+y)=\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}$$

$$\therefore \frac{1}{x+y}\left(1+\frac{dy}{dx}\right)=\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx}\left(\frac{1}{y}-\frac{1}{x+y}\right)=\frac{1}{x+y}-\frac{1}{x}$$

$$\therefore \frac{dy}{dx}\left[\frac{x}{y(x+y)}\right]=\frac{-y}{x(x+y)}$$

$$\therefore \frac{dy}{dx}=-\frac{y^2}{x^2}$$

Exercise 3.4 | Q 3.3 | Page 95

Solve the following:

If $e^x+e^y=e^{x+y}$ then show that, $\frac{dy}{dx}=-e^{y-x}$.

Solution:

$e^x+e^y=e^{x+y}$ .....(i)

Differentiating both sides w.r.t. x, we get

$$e^x+e^y\frac{dy}{dx}=e^{x+y}\cdot\frac{d}{dx}(x+y)$$

$$\therefore e^x+e^y\frac{dy}{dx}=e^{x+y}\left[1+\frac{dy}{dx}\right]$$

$$\therefore \left(e^y-e^{x+y}\right)\frac{dy}{dx}=e^{x+y}-e^x$$

$$\therefore \left(e^y-e^x-e^y\right)\frac{dy}{dx}=\left(e^x+e^y-e^x\right)$$ ....[From (i)]

$$\therefore \left(-e^{x}\right)\frac{dy}{dx} = \left(e^{y}\right)$$

$$\therefore \frac{dy}{dx} = -e^{y-x}$$

EXERCISE 3.5 [PAGE 97]

Exercise 3.5 | Q 1.1 | Page 97

Find $\frac{dy}{dx}$, if $x = at^2$, $y = 2at$

Solution:

$x = at^2$

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt}\left(at^2\right) = a\frac{d}{dt}\left(t^2\right) = 2at$$

$y = 2at$

Differentiating both sides w.r.t. t, we get

$$\frac{dy}{dt} = \frac{d}{dt}(2at) = a\frac{d}{dt}(2t) = 2a$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2a}{2at} = \frac{1}{t}$$

$$\therefore \frac{dy}{dx} = \frac{1}{t}$$

Exercise 3.5 | Q 1.2 | Page 97

Find $\frac{dy}{dx}$, if $x = 2at^2$, $y = at^4$

Solution: $x = 2at^2$

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = 4at$$

$y = at^4$

Differentiating both sides w.r.t. t, we get

$$\frac{dy}{dt} = 4at^3$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dy}{dt}\right)} = \frac{4at^3}{4at} = t^2$$

Exercise 3.5 | Q 1.3 | Page 97

Find $\frac{dy}{dx}$, if $x = e^{3t}$, $y = e^{4t+5}$

Solution: $x = e^{3t}$

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = e^{3t} \cdot \frac{d}{dx}(3t) = e^{3t} \cdot (3) = 3e^{3t}$$

$y = e^{4t+5}$

Differentiating both sides w.r.t. t, we get

$$\frac{dy}{dt} = e^{4t+5} \cdot \frac{d}{dx}(4t+5) = e^{4t+5} \cdot (4+0)$$

$$= 4 \cdot e^{4t+5}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dy}{dt}\right)} = \frac{4 \cdot e^{4t+5}}{3e^{3t}} = \frac{4}{3}e^{t+5}$$

Exercise 3.5 | Q 2.1 | Page 97

Find $\frac{dy}{dx}$, if x = $\left(u + \frac{1}{u}\right)^2, y = (2)^{\left(u + \frac{1}{u}\right)}$

Solution:

x = $\left(u + \frac{1}{u}\right)^2$ .....(i)

Differentiating both sides w.r.t. u, we get

$$\frac{dx}{du} = 2\left(u + \frac{1}{u}\right) \cdot \frac{d}{dx}\left(u + \frac{1}{u}\right)$$

$$= 2\left(u + \frac{1}{u}\right)\left[1 + (-1)u^{-2}\right]$$

$$\therefore \frac{dx}{du} = 2\left(u + \frac{1}{u}\right)\left(1 - \frac{1}{u^2}\right)$$

$y = (2)^{\left(u + \frac{1}{u}\right)}$ .....(ii)

Differentiating both sides w.r.t. u, we get

$$\frac{dy}{du} = 2^{\left(u + \frac{1}{u}\right)} \log 2 \frac{d}{dx}\left(u + \frac{1}{u}\right)$$

$$\therefore \frac{dy}{du} = \log 2 \cdot 2^{\left(u + \frac{1}{u}\right)}\left(1 - \frac{1}{u^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{du}\right)}{\left(\frac{dx}{du}\right)} = \frac{2^{\left(u+\frac{1}{u}\right)} \log 2\left(1 - \frac{1}{u^2}\right)}{2\left(u + \frac{1}{u}\right)\left(1 - \frac{1}{u^2}\right)}$$

$$= \frac{2^{\left(u+\frac{1}{u}\right)} \log 2}{2\left(u + \frac{1}{u}\right)}$$

$$\therefore \frac{dy}{dx} = \frac{y \log 2}{2\sqrt{x}}$$ ....[From (i) and (ii)]

Exercise 3.5 | Q 2.2 | Page 97

Find $\frac{dy}{dx}$, if x = $\sqrt{1 + u^2}, y = \log\left(1 + u^2\right)$

Solution:

x = $\sqrt{1 + u^2}$

Differentiating both sides w.r.t. u, we get

$$\frac{dx}{du} = \frac{d}{du}\left(\sqrt{1 + u^2}\right)$$

$$= \frac{1}{2\sqrt{1 + u^2}} \cdot \frac{d}{dx}\left(1 + u^2\right)$$

$$= \frac{1}{2\sqrt{1 + u^2}} \times 2u$$

$$= \frac{u}{\sqrt{1 + u^2}}$$

$$y = \log\left(1 + u^2\right)$$

Differentiating both sides w.r.t. u, we get

$$\frac{dy}{du} = \frac{d}{dx}\left[\log\left(1 + u^2\right)\right]$$

$$= \frac{1}{1+u^2} \cdot \frac{d}{du}(1+u^2)$$

$$= \frac{1}{1+u^2} \times 2u$$

$$= \frac{2u}{1+u^2}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{du}\right)}{\left(\frac{dx}{du}\right)} = \frac{\left(\frac{2u}{1+u^2}\right)}{\left(\frac{u}{\sqrt{1+u^2}}\right)} = \frac{2}{1+u^2} \times \sqrt{1+u^2}$$

$$\therefore \frac{dy}{dx} = \frac{2}{\sqrt{1+u^2}}$$

Exercise 3.5 | Q 2.3 | Page 97

Find $\frac{dy}{dx}$, if Differentiate 5^x with respect to log x

Solution: Let u = 5^x and v = log x

u = 5^x

Differentiating both sides w.r.t.x, we get

$$\frac{du}{dx} = 5^x \cdot \log 5$$

v = log x

Differentiating both sides w.r.t.x, we get

$$\frac{dv}{dx} = \frac{1}{x}$$

$$\therefore \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = \frac{5^x \log 5}{\frac{1}{x}} = x \cdot 5^x (\log 5)$$

Exercise 3.5 | Q 3.1 | Page 97

Solve the following.

If x = $a\left(1-\frac{1}{t}\right), y=a\left(1+\frac{1}{t}\right)$, then show that $\frac{dy}{dx}=-1$

Solution:

x = $a\left(1-\frac{1}{t}\right)$

Differentiating both sides w.r.t. 't', we get

$$\frac{dx}{dt}=a\left[0-\left(\frac{-1}{t^2}\right)\right]=\frac{a}{t^2}$$

y = $a\left(1+\frac{1}{t}\right)$

Differentiating both sides w.r.t. 't', we get

$$\frac{dx}{dt}=a\left[0+\left(\frac{-1}{t^2}\right)\right]=\frac{\text{-a}}{t^2}$$

$$\therefore \frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{\frac{\text{-a}}{t^2}}{\frac{a}{t^2}}=-1$$

Exercise 3.5 | Q 3.2 | Page 97

Solve the following.

If x = $\frac{4t}{1+t^2}, y=3\left(\frac{1-t^2}{1+t^2}\right)$ then show that $\frac{dy}{dx}=\frac{-9x}{4y}$.

Solution:

$$x = \frac{4t}{1+t^2}$$

Differentiating both sides w.r.t. 't', we get

$$\frac{dx}{dt} = \frac{\left(1+t^2\right) \cdot \frac{d}{dx}(4t) - 4t \cdot \frac{d}{dx}\left(1+t^2\right)}{\left(1+t^2\right)^2}$$

$$= \frac{\left(1+t^2\right)(4) - 4t(0+2t)}{\left(1+t^2\right)^2}$$

$$= \frac{4+4t^2-8t^2}{\left(1+t^2\right)^2}$$

$$= \frac{4-4t^2}{\left(1+t^2\right)^2}$$

$$= \frac{4\left(1-t^2\right)}{\left(1+t^2\right)^2}$$

$$y = 3\left(\frac{1-t^2}{1+t^2}\right)$$

Differentiating both sides w.r.t. 't', we get

$$\frac{dx}{dt} = 3\frac{d}{dx}\left(\frac{1-t^2}{1+t^2}\right)$$

$$= 3\left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right) \cdot \frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= 3\left[\frac{\left(1+t^2\right)(0-2t) - \left(1-t^2\right)(0+2t)}{\left(1+t^2\right)^2}\right]$$

$$= 3\left[\frac{-2t\left(1+t^2\right)-2t\left(1-t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= 3(-2t)\left[\frac{1+t^2+1-t^2}{\left(1+t^2\right)^2}\right]$$

$$= -6t \times \frac{2}{\left(1+t^2\right)^2}$$

$$= \frac{-12t}{\left(1+t^2\right)^2}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\frac{-12t}{\left(1+t^2\right)^2}}{\frac{4\left(1-t^2\right)}{\left(1+t^2\right)^2}}$$

$$\therefore \frac{dy}{dx} = \frac{-3t}{1-t^2} \quad \text{....(i)}$$

Also $\frac{-9x}{4y} = \frac{-9\left(\frac{4t}{1+t^2}\right)}{4 \times 3\left(\frac{1-t^2}{1+t^2}\right)} = \frac{-3t}{1-t^2}$ ....(ii)

From (i) and (ii), we get

$$\frac{dy}{dx} = \frac{-9x}{4y}$$

Exercise 3.5 | Q 3.3 | Page 97

Solve the following.

If x = t . log t, y = t^t, then show that $\frac{dy}{dx} - y = 0$

Solution: x = t . log t ....(i)

$y = t^t$

Taking logarithm of both sides, we get

log y = t . log t

∴ log y = x ....[From (i)]

∴ $y = e^x$ ...(ii)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = e^x$$

$$\therefore \frac{dy}{dx} = y \qquad \text{....[From (ii)]}$$

$$\therefore \frac{dy}{dx} - y = 0$$

EXERCISE 3.6 [PAGE 98]

Exercise 3.6 | Q 1.1 | Page 98

Find $\frac{d^2y}{dx^2}$, if $y = \sqrt{x}$

Solution:

$y = \sqrt{x}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = \frac{1}{2} \cdot \frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(-\frac{1}{2}\right) \cdot x^{-\frac{3}{2}}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-1}{4}x^{-\frac{3}{2}}$$

Exercise 3.6 | Q 1.2 | Page 98

Find $\frac{d^2y}{dx^2}$, if y = x^5

Solution:

y = x^5

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = 5x^4$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 5 \cdot \frac{d}{dx}\left(x^4\right)$$

$$= 5\left(4x^3\right)$$

$$\therefore \frac{d^2y}{dx^2} = 20x^3$$

Exercise 3.6 | Q 1.3 | Page 98

Find $\frac{d^2y}{dx^2}$, if y = x^{-7}

Solution:

$y = x^{-7}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = -7x^{-8}$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = -7 \cdot \frac{d}{dx}\left(x^{-8}\right)$$

$$= -7(-8)x^{-9}$$

$$\therefore \frac{d^2y}{dx^2} = 56x^{-9}$$

Exercise 3.6 | Q 2.1 | Page 98

Find $\frac{d^2y}{dx^2}$, if $y = e^x$

Solution:

$y = e^x$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = e^x$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = e^x$$

Exercise 3.6 | Q 2.2 | Page 98

Find $\frac{d^2y}{dx^2}$, if y = $e^{(2x+1)}$

Solution:

y = $e^{(2x+1)}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = e^{(2x+1)} \cdot \frac{d}{dx}(2x + 1)$$

$$\frac{dy}{dx} = e^{(2x+1)} \cdot (2 + 0)$$

$$\frac{dy}{dx} = 2e^{(2x+1)}$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 2 \cdot \frac{d}{dx} e^{(2x+1)}$$

$$= 2e^{(2x+1)} \cdot \frac{d}{dx} (2x + 1)$$

$$= 2e^{(2x+1)} \cdot (2 + 0)$$

$$\therefore \frac{d^2y}{dx^2} = 4e^{(2x+1)}$$

Exercise 3.6 | Q 2.3 | Page 98

Find $\frac{d^2y}{dx^2}$, if y = $e^{\log x}$

Solution:

$y = e^{\log x}$

$y = x$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = 1$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 0$$

MISCELLANEOUS EXERCISE 3 [PAGES 99 - 101]

Miscellaneous Exercise 3 | Q 1.01 | Page 99

Choose the correct alternative.

If $y = (5x^3 - 4x^2 - 8x)^9$, then dy/dx =

1. **$9(5x^3 - 4x^2 - 8x)^8 (15x^2 - 8x - 8)$**
2. $9(5x^3 - 4x^2 - 8x)^9 (15x^2 - 8x - 8)$
3. $9(5x^3 - 4x^2 - 8x)^8 (5x^2 - 8x - 8)$
4. $9(5x^3 - 4x^2 - 8x)^9 (15x^2 - 8x - 8)$

Solution: $9(5x^3 - 4x^2 - 8x)^8 (15x^2 - 8x - 8)$

Explanation:

$y = (5x^3 - 4x^2 - 8x)^9$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[(5x^3 - 4x^2 - 8x)^9]$$

$$= 9\left(5x^3 - 4x^2 - 8x\right)^8 \cdot \frac{d}{dx}\left(5x^3 - 4x^2 - 8x\right)$$

$$= 9\left(5x^3 - 4x^2 - 8x\right)^8 \cdot \left[5\left(3x^2\right) - 4(2x) - 8\right]$$

$$\therefore \frac{dy}{dx} = 9\left(5x^3 - 4x^2 - 8x\right)^8 \cdot \left(15x^2 - 8x - 8\right)$$

Miscellaneous Exercise 3 | Q 1.02 | Page 99

Choose the correct alternative.

If y = $\sqrt{x + \frac{1}{x}}$, then $\frac{dy}{dx}$ =?

Options

$$\frac{x^2 - 1}{2x^2\sqrt{x^2 + 1}}$$

$$\frac{1 - x^2}{2x^2(x^2 + 1)}$$

$$\frac{x^2 - 1}{2x\sqrt{x}\sqrt{x^2 + 1}}$$

$$\frac{1 - x^2}{2x\sqrt{x}\sqrt{x^2 + 1}}$$

Solution:

$$\frac{x^2 - 1}{2x\sqrt{x}\sqrt{x^2 + 1}}$$

Explanation:

y = $\sqrt{x + \frac{1}{x}}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x + \frac{1}{x}}} \cdot \frac{d}{dx}\left(x + \frac{1}{x}\right)$$

$$= \frac{1}{2\sqrt{\frac{x^2+1}{x}}} \cdot \left(1 - \frac{1}{x^2}\right)$$

$$= \frac{\sqrt{x}}{2\sqrt{x^2+1}} \cdot \left(\frac{x^2-1}{x^2}\right)$$

$$= \frac{x^2-1}{2x\sqrt{x}\sqrt{x^2+1}}$$

Miscellaneous Exercise 3 | Q 1.03 | Page 99

Choose the correct alternative.

If y = $e^{\log x}$, then $\frac{dy}{dx} = ?$

Options

$$\frac{e^{\log x}}{x}$$

$$\frac{1}{x}$$

0

$$\frac{1}{2}$$

Solution:

$$\frac{e^{\log x}}{x}$$

Explanation:

$y = e^{\log x}$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = e^{\log x} \cdot \frac{d}{dx}(\log x)$$

$$= e^{\log x} \cdot \frac{1}{x}$$

$$= \frac{e^{\log x}}{x}$$

Miscellaneous Exercise 3 | Q 1.04 | Page 99

Choose the correct alternative.

If $y = 2x^2 + 2^2 + a^2$, then dy/dx=?

1. x
2. **4x**
3. 2x
4. -2x

Solution: 4x

Explanation:

$y = 2x^2 + 2^2 + a^2$

Differentiating both sides w.r.t.x, we get

Dy/dx = 2(2x) + 0 + 0 = 4x

Miscellaneous Exercise 3 | Q 1.05 | Page 99

Choose the correct alternative.

If y = 5^x . x^5, then dy/dx=?

1. 5^x. x^4 (5 + log 5)
2. 5^x. x^5 (5 + log 5)
3. **5^x . x^4 (5 + x log 5)**
4. 5^x. x^5 (5 + x log 5)

Solution: 5^x . x^4 (5 + x log 5)

Explanation:

y = 5^x . x^5

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = 5^x \cdot \frac{d}{dx}\left(x^5\right) + x^5 \cdot \frac{d}{dx}\left(5^x\right)$$

$$= 5^x \cdot \left(5x^4\right) + x^5\left(5^x \cdot \log 5\right)$$

$$= 5^x \cdot x^4\left(5 + x \log 5\right)$$

Miscellaneous Exercise 3 | Q 1.06 | Page 99

Choose the correct alternative.

If y = log $\left(\frac{e^x}{x^2}\right)$, then $\frac{dy}{dx}$ =?

Options

$\frac{2 - x}{x}$

$\frac{x - 2}{x}$

$\frac{e - x}{ex}$

$\frac{x - e}{ex}$

Solution:

$$\frac{x - 2}{x}$$

Explanation:

$y = \log\left(\frac{e^x}{x^2}\right)$

$= \log (e^x) - \log (x^2)$

$= x \log e - 2 \log x$

$= x(1) - 2 \log x$

$\therefore y = x - 2 \log x$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = 1 - 2\left(\frac{1}{x}\right) = \frac{x - 2}{x}$$

Miscellaneous Exercise 3 | Q 1.07 | Page 99

Choose the correct alternative.

If $ax^2 + 2hxy + by^2 = 0$ then $\frac{dy}{dx} = ?$

Options

$$\frac{(ax + hx)}{(hx + by)}$$

$$\frac{-(ax + hx)}{(hx + by)}$$

$$\frac{(ax - hx)}{(hx + by)}$$

$$\frac{(2ax + hy)}{(hx + 3by)}$$

Solution:

$$\frac{-(ax + hx)}{(hx + by)}$$

Explanation:

$ax^2 + 2hxy + by^2 = 0$

Differentiating both sides w.r.t.x, we get

$$a(2x) + 2h \cdot \frac{d}{dx}(xy) + b(2y)\frac{dy}{dx} = 0$$

$$\therefore 2ax + 2h\left[x \cdot \frac{dy}{dx} + y(1)\right] + 2by\frac{dy}{dx} = 0$$

$$\therefore 2ax + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} = 0$$

$$\therefore 2\frac{dy}{dx}(hx + by) = -2ax - 2hy$$

$$\therefore 2\frac{dy}{dx} = \frac{-2(ax + hy)}{(hx + by)}$$

$$\therefore \frac{dy}{dx} = \frac{-(ax + hx)}{(hx + by)}$$

Miscellaneous Exercise 3 | Q 1.08 | Page 99

Choose the correct alternative.

If $x^4.y^5=(x + y)m + 1$ then dy/dx=y/x then m = ?

1. 8
2. 4
3. 5
4. 20

Solution: 8

Miscellaneous Exercise 3 | Q 1.09 | Page 99

Choose the correct alternative.

If x = $\dfrac{e^t + e^{-t}}{2}, y = \dfrac{e^t - e^{-t}}{2}$ then $\dfrac{dy}{dx}$ = ?

1. -y/x
2. y/x
3. -x/y
4. **x/y**

Solution: x/y

Explanation:

x = $\dfrac{e^t + e^{-t}}{2}, y = \dfrac{e^t - e^{-t}}{2}$

$\therefore \dfrac{dx}{dt} = \dfrac{1}{2}(e^t - e^{-t})$ and $\dfrac{dy}{dx} = \dfrac{1}{2}(e^t + e^{-t})$

$\therefore \dfrac{dx}{dt} = y$ and "dy"/"dt" = "x"`

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{x}{y}$$

Miscellaneous Exercise 3 | Q 1.1 | Page 99

Choose the correct alternative.

If x = 2at^2 , y = 4at, then $\dfrac{dy}{dx}$ =?

Options

$-\dfrac{1}{2at^2}$

$\dfrac{1}{2at^3}$

$\dfrac{1}{t}$

$\dfrac{1}{4at^3}$

Solution:

$\frac{1}{t}$

Explanation:

$x = 2at^2$, $y = 4at$

$$\therefore \frac{dx}{dt} = 2a(2t) \text{ and } \frac{dy}{dx} = 4a$$

$$\therefore \frac{dx}{dt} = 4at \text{ and } \frac{dy}{dt} = 4a$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4a}{4at} = \frac{1}{t}$$

Miscellaneous Exercise 3 | Q 2.01 | Page 99

Fill in the Blank

If $3x^2y + 3xy^2 = 0$, then dy/dx = ________

Solution:

If $3x^2y + 3xy^2 = 0$, then $\frac{dy}{dx}$ = **-1**.

Explanation:

$3x^2y + 3xy^2 = 0$

Dividing both sides by 3xy, we get

$x + y = 0$

Differentiating both sides w.r.t.x, we get

$$1 + \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -1$$

Miscellaneous Exercise 3 | Q 2.02 | Page 99

Fill in the Blank

If $x^m \cdot y^n = (x + y)^{m+n}$, then $\frac{dy}{dx} = \frac{\square}{x}$

Solution:

If $x^m \cdot y^n = (x + y)^{m+n}$, then $\frac{dy}{dx} = \frac{y}{x}$

Miscellaneous Exercise 3 | Q 2.03 | Page 99

Fill in the Blank

If 0 = log(xy) + a, then $\frac{dy}{dx} = \frac{-y}{\square}$

Solution:

If 0 = log(xy) + a, then $\frac{dy}{dx} = \frac{-y}{x}$

Explanation:

0 = log(xy) + a

∴ log(xy) = - a

∴ log x + log y = - a

Differentiating both sides w.r.t.x, we get

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 0$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = -\frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{-y}{x}$$

Miscellaneous Exercise 3 | Q 2.04 | Page 99

Fill in the blank.

If x = t log t and y = t^t, then $\frac{dy}{dx}$ = ____

Solution:

If x = t log t and y = t^t, then $\frac{dy}{dx}$ = **y.**

Explanation:

x = t . log t ....(i)

y = t^t

Taking logarithm of both sides, we get

log y = t . log t

∴ log y = x ....[From (i)]

∴ y = e^x ...(ii)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = e^x$$

$$\therefore \frac{dy}{dx} = y$$ ....[From (ii)]

Miscellaneous Exercise 3 | Q 2.05 | Page 99

Fill in the blank.

If y = x . log x, then $\frac{d^2y}{dx^2}$ = _____

Solution:

If y = x . log x, then $\frac{d^2y}{dx^2} = \frac{1}{x}$

Miscellaneous Exercise 3 | Q 2.06 | Page 100

Fill in the blank.

If y = $[\log(x)]^2$ then $\frac{d^2y}{dx^2} =$ ____

Solution:

If y = $[\log(x)]^2$ then $\frac{d^2y}{dx^2} = \frac{-1}{x^2}$.

Explanation:

y = log x

$\therefore \frac{dy}{dx} = \frac{1}{x}$

$\therefore \frac{d^2y}{dx^2} = \frac{-1}{x^2}$

Miscellaneous Exercise 3 | Q 2.07 | Page 100

Fill in the blank.

If x = $y + \frac{1}{y}$, then $\frac{dy}{dx} =$ ___

Solution:

If x = $y + \frac{1}{y}$, then $\frac{dy}{dx} = \frac{y^2}{y^2 - 1}$

Explanation:

x = $y + \frac{1}{y}$

Differentiating both sides w.r.t. x, we get

$$1 = \frac{dy}{dx} + \left(\frac{-1}{y^2}\right) \cdot \frac{dy}{dx}$$

$$\therefore 1 = \frac{dy}{dx}\left(1 - \frac{1}{y^2}\right)$$

$$\therefore 1 = \frac{dy}{dx}\left(\frac{y^2 - 1}{y^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{y^2}{y^2 - 1}$$

Miscellaneous Exercise 3 | Q 2.08 | Page 100

Fill in the blank.

If y = e^{ax}, then $x \cdot \frac{dy}{dx} =$___

Solution:

If y = e^{ax}, then $x \cdot \frac{dy}{dx} =$ **axy**

Explanation:

y = e^{ax}

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = e^{ax} \cdot \frac{d}{dx}(ax)$$

$$= e^{ax} \cdot (a)$$

$$= a \cdot e^{ax}$$

$$\therefore \frac{dy}{dx} = ay$$

$$\therefore x\frac{dy}{dx} = axy$$

Miscellaneous Exercise 3 | Q 2.09 | Page 100

Fill in the blank.

If x = t log t and y = t^t, then $\frac{dy}{dx}$ = ___

Solution: If x = t log t and y = t^t, then dy/dx = **y.**

Explanation:

x = t . log t ....(i)

y = t^t

Taking logarithm of both sides, we get

log y = t . log t

∴ log y = x ....[From (i)]

∴ y = e^x ...(ii)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = e^x$$

$$\therefore \frac{dy}{dx} = y \quad \text{....[From (ii)]}$$

Miscellaneous Exercise 3 | Q 2.1 | Page 100

Fill in the blank.

If y = $\left(x + \sqrt{x^2 - 1}\right)^m$, then $(x^2 - 1)\dfrac{dy}{dx}$ = _____

Solution:

If y = $\left(x + \sqrt{x^2 - 1}\right)^m$, then $(x^2 - 1)\dfrac{dy}{dx}$ = **my**

Explanation:

y = $\left(x + \sqrt{x^2 - 1}\right)^m$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = m\left(x + \sqrt{x^2 - 1}\right)^{m - 1} \cdot \frac{d}{dx}\left(x + \sqrt{x^2 - 1}\right)$$

$$= m\frac{\left(x + \sqrt{x^2 - 1}\right)^m}{\left(x + \sqrt{x^2 - 1}\right)^1} \cdot \left[1 + \frac{1}{2\sqrt{x^2 - 1}} \cdot \frac{d}{dx}(x^2 - 1)\right]$$

$$= \frac{my}{x + \sqrt{x^2 - 1}} \times \left[\left(1 + \frac{1}{2\sqrt{x^2 - 1}}\right)(2x)\right]$$

$$= \frac{my}{x + \sqrt{x^2 - 1}} \times \left(1 + \frac{x}{\sqrt{x^2 - 1}}\right)$$

$$\therefore \frac{dy}{dx} = \frac{my}{x + \sqrt{x^2 - 1}} \times \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}$$

$$\therefore \frac{dy}{dx} = \frac{my}{\sqrt{x^2 - 1}}$$

$$\therefore \sqrt{x^2 - 1} \cdot \frac{dy}{dx} = my$$

Miscellaneous Exercise 3 | Q 3.1 | Page 100

State whether the following is True or False:

If f′ is the derivative of f, then the derivative of the inverse of f is the inverse of f′.

1. True
2. **False**

Solution: False

Miscellaneous Exercise 3 | Q 3.2 | Page 100

State whether the following is True or False:

The derivative of $\log_a x$, where a is constant is $\frac{1}{x \cdot \log a}$.

1. **True**
2. False

Solution: True

Miscellaneous Exercise 3 | Q 3.3 | Page 100

State whether the following is True or False:

The derivative of $f(x) = a^x$, where a is constant is $x.a^{x-1}$.

1. True
2. **False**

Solution: False

Miscellaneous Exercise 3 | Q 3.4 | Page 100

State whether the following is True or False:

The derivative of polynomial is polynomial.

1. **True**
2. False

Solution: True

Miscellaneous Exercise 3 | Q 3.5 | Page 100

State whether the following is True or False:

$$\frac{d}{dx}(10^x) = x \cdot 10^{x-1}$$

1. True
2. **False**

Solution: False

Miscellaneous Exercise 3 | Q 3.6 | Page 100

State whether the following is True or False:

If y = log x, then dy/dx=1/x

1. **True**
2. False

Solution: True

Miscellaneous Exercise 3 | Q 3.7 | Page 100

State whether the following is True or False:

If y = e^2, then dy/dx=2e

1. True
2. **False**

Solution: False

Miscellaneous Exercise 3 | Q 3.8 | Page 100

State whether the following is True or False:

The derivative of a^x is a^x . loga.

1. **True**
2. False

Solution: True

Miscellaneous Exercise 3 | Q 3.9 | Page 100

State whether the following is True or False:

The derivative of $x^m \cdot y^n = (x + y)^{m+n}$ is $\frac{x}{y}$

1. True
2. **False**

Solution: False

Miscellaneous Exercise 3 | Q 4.01 | Page 100

Solve the following:

If y = $(6x^3 - 3x^2 - 9x)^{10}$, find dy/dx

Solution: y = $(6x^3 - 3x^2 - 9x)^{10}$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(6x^3 - 3x^2 - 9x\right)^{10}\right]$$

$$= 10\left(6x^3 - 3x^2 - 9x\right)^9 \times \frac{d}{dx}\left(6x^3 - 3x^2 - 9x\right)$$

$$= 10\left(6x^3 - 3x^2 - 9x\right)^9 \times \left[6\left(3x^2\right) - 3(2x) - 9\right]$$

$$\therefore \frac{dy}{dx} = 10\left(6x^3 - 3x^2 - 9x\right)^9 \cdot \left(18x^2 - 6x - 9\right)$$

Miscellaneous Exercise 3 | Q 4.02 | Page 100

Solve the following:

If y = $\sqrt[5]{(3x^2+8x+5)^4}$, find $\frac{dy}{dx}$

Solution:

y = $\sqrt[5]{(3x^2+8x+5)^4}$

∴ y = $\left(3x^2+8x+5\right)^{\frac{4}{5}}$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(3x^2+8x+5\right)^{\frac{4}{5}}\right]$$

$$= \frac{4}{5}\left(3x^2+8x+5\right)^{-\frac{1}{5}} \cdot \frac{d}{dx}\left(3x^2+8x+5\right)$$

$$= \frac{4}{5}\left(3x^2+8x+5\right)^{-\frac{1}{5}} \cdot [3(2x)+8+0]$$

$$\therefore \frac{dy}{dx} = \frac{4}{5}\left(3x^2+8x+5\right)^{-\frac{1}{5}} \cdot (6x+8)$$

Miscellaneous Exercise 3 | Q 4.03 | Page 100

Solve the following:

If y = $[\log(\log(\log x))]^2$, find dy/dx

Solution: y = $[\log(\log(\log x))]^2$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[\log(\log(\log x))]^2$$

$$= 2[\log(\log(\log x))] \times \frac{d}{dx}[\log(\log(\log x))]$$

$$= 2[\log(\log(\log x))] \times \frac{1}{\log(\log x)} \times \frac{d}{dx}[\log(\log x)]$$

$$= 2[\log(\log(\log x))] \times \frac{1}{\log(\log x)} \times \frac{1}{\log x} \times \frac{d}{dx}(\log x)$$

$$= 2[\log(\log(\log x))] \times \frac{1}{\log(\log x)} \times \frac{1}{\log x} \times \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{2[\log(\log(\log x))]}{x(\log x)(\log(\log x))}$$

Miscellaneous Exercise 3 | Q 4.04 | Page 100

Find the rate of change of demand (x) of a commodity with respect to its price (y) if y = 25 + 30x – x^2.

Solution:

y = 25 + 30x – x^2.

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(25 + 30x - x^2\right) = 0 + 30 - 2x$$

$$\therefore \frac{dy}{dx} = 30 - 2x$$

Now, by the derivative of an inverse function, the rate of change of demand (x) w.r.t. price(y) is

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}, \text{ where } \frac{dy}{dx} \neq 0.$$

i.e. $\frac{dx}{dy} = \frac{1}{30 - 2x}$

Miscellaneous Exercise 3 | Q 4.05 | Page 100

Find the rate of change of demand (x) of a commodity with respect to its price (y) if y = $\frac{5x + 7}{2x - 13}$.

Solution:

y = $\frac{5x + 7}{2x - 13}$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{5x + 7}{2x - 13}\right)$$

$$= \frac{(2x - 13)\frac{d}{dx}(5x + 7) - (5x + 7)\frac{d}{dx}(2x - 13)}{(2x - 13)^2}$$

$$= \frac{(2x - 13)(5 \times 1 + 0) - (5x + 7)(2 \times 1 - 0)}{(2x - 13)^2}$$

$$= \frac{(2x - 13)(5) - (5x + 7)(2)}{(2x - 13)^2}$$

$$= \frac{10x - 65 - 10x - 14}{(2x - 13)^2}$$

$$\therefore \frac{dy}{dx} = \frac{-79}{(2x - 13)^2}$$

Now, by derivative of inverse function, the rate of change of demand (x) w.r.t. price(y) is

$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$, where $\frac{dy}{dx} \neq 0$

i.e. $\frac{dx}{dy} = \frac{1}{\frac{-79}{(2x-13)^2}}$

$$= \frac{-(2x - 13)^2}{79}$$

Miscellaneous Exercise 3 | Q 4.06 | Page 100

Find dy/dx, if y = x^x.

Solution: y = x^x.

Taking logarithm of both sides, we get

log y = log (x^x)

∴ log y = x log x

Differentiating both sides w.r.t.x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x)$$

$$= x \cdot \frac{1}{x} + \log x(1)$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = 1 + \log x$$

$$\therefore \frac{dy}{dx} = y(1 + \log x)$$

$$\therefore \frac{dy}{dx} = x^x(1 + \log x)$$

Miscellaneous Exercise 3 | Q 4.07 | Page 100

Find $\frac{dy}{dx}$, if y = 2^{x^x}.

Solution:

y = 2^{x^x}

Taking logarithm of both sides, we get

$$\frac{dy}{dx} = 2^{x^x} \cdot \log 2 \cdot \frac{d}{dx}(x^x) \quad \text{....(i)}$$

Let u = x^x

log u = log (x^x)

∴ log y = x log x

Differentiating both sides w.r.t.x, we get

$$\frac{1}{u}\cdot\frac{du}{dx} = x\cdot\frac{d}{dx}(\log x) + \log x\cdot\frac{d}{dx}(x)$$

$$= x\cdot\frac{1}{x} + \log x(1)$$

$$\therefore \frac{1}{u}\cdot\frac{du}{dx} = 1 + \log x$$

$$\therefore \frac{dy}{dx} = u(1 + \log x)$$

$$\therefore \frac{d}{dx}(x^x) = x^x(1 + \log x) \quad \text{...(ii)}$$

Substituting (ii) in (i), we get

$$\frac{dy}{dx} = 2^{x^x}\cdot\log 2\cdot x^x(1 + \log x)$$

$$\frac{dy}{dx} = 2^{x^x}\cdot x^x\cdot\log 2(1 + \log x)$$

Miscellaneous Exercise 3 | Q 4.08 | Page 100

Find $\frac{dy}{dx}$ if y = $\sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}$

Solution:

y = $\sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}$

$$= \frac{(3x - 4)^{\frac{3}{2}}}{(x+1)^{\frac{4}{2}} \cdot (x+2)^{\frac{1}{2}}}$$

Taking logarithm of both sides, we get

log y = log$\left[\frac{(3x - 4)^{\frac{3}{2}}}{(x+1)^{\frac{4}{2}} \cdot (x+2)^{\frac{1}{2}}}\right]$

$$= \log(3x-4)^{\frac{3}{2}} - \left[\log(x+1)^2 + \log(x+2)^{\frac{1}{2}}\right]$$

$$= \frac{3}{2}\log(3x-4) - 2\log(x+1) - \frac{1}{2}\log(x+2)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{2} \cdot \frac{d}{dx}[\log(3x-4)] - 2\frac{d}{dx}[\log(x+1)] - \frac{1}{2} \cdot \frac{d}{dx}[\log(x+2)]$$

$$= \frac{3}{2} \cdot \frac{1}{3x-4} \cdot \frac{d}{dx}(3x-4) - 2 \cdot \frac{1}{x+1} \cdot \frac{d}{dx}(x+1) - \frac{1}{2} \cdot \frac{1}{x+2} \cdot \frac{d}{dx}(x+2)$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{2(3x-4)} \times 3 - \frac{2}{x+1} \times 1 - \frac{1}{2(x+2)} \times 1$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = \frac{9}{2(3x-4)} - \frac{2}{x+1} - \frac{1}{2(x+2)}$$

$$\therefore \frac{dy}{dx} = \frac{y}{2}\left[\frac{9}{3x-4} - \frac{4}{x+1} - \frac{1}{x+2}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}\left[\frac{9}{3x-4} - \frac{4}{x+1} - \frac{1}{x+2}\right]$$

Miscellaneous Exercise 3 | Q 4.09 | Page 100

Find $\frac{dy}{dx}$ if y = $x^x + (7x-1)^x$

Solution:

y = $x^x + (7x - 1)^x$

Let u = x^x and v = $(7x - 1)^x$

∴ y = u + v

Differentiating both sides w.r.t.x, we get

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ....(i)

Now, u = x^x

Taking logarithm of both sides, we get

log u = log(x^x)

∴ log u = x. log x

Differentiating both sides w.r.t.x, we get

$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}$(x)

$= x \cdot \frac{1}{x} + \log x \cdot$ (1)

∴ $\frac{1}{u} \cdot \frac{du}{dx}$ = 1 + log x

∴ $\frac{du}{dx}$ = u(1 + log x)

∴ $\frac{d}{dx}(x^x) = x^x$(1 + log x) ....(ii)

Also, v = $(7x – 1)^x$

Taking logarithm of both sides, we get

log v = log(7x - 1)^x

∴ log v = x. log(7x – 1)

Differentiating both sides w.r.t.x, we get

$$\frac{1}{v} \cdot \frac{dv}{dx} = x \cdot \frac{d}{dx}\log(7x-1) + \log(7x-1) \cdot \frac{d}{dx}(x)$$

$$= x \cdot \frac{1}{7x-1} \cdot \frac{d}{dx}(7x-1) + \log(7x-1) \cdot (1)$$

$$\therefore \frac{1}{v} \cdot \frac{dv}{dx} = \frac{x}{7x-1}(7-0) + \log(7x-1)$$

$$\therefore \frac{dv}{dx} = v\left[\frac{7x}{7x-1} + \log(7x-1)\right]$$

$$\therefore \frac{dv}{dx} = (7x-1)^x\left[\frac{7x}{7x-1} + \log(7x-1)\right] \quad \text{....(iii)}$$

Substituting (ii) and (iii) in (i), we get

$$\frac{dy}{dx} = x^x(1+\log x) + (7x-1)^x\left[\log(7x-1) + \frac{7x}{7x-1}\right]$$

Miscellaneous Exercise 3 | Q 4.1 | Page 100

If y = $x^3 + 3xy^2 + 3x^2y$ Find $\frac{dy}{dx}$

Solution:

y = $x^3 + 3xy^2 + 3x^2y$

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + 3\frac{d}{dx}(xy^2) + 3\frac{d}{dx}(x^2y)$$

$$\therefore \frac{dy}{dx} = 3x^2 + 3\left[x \cdot \frac{d}{dx}(y^2) + y^2 \cdot \frac{d}{dx}(x)\right] + 3\left[x^2 \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(x^2)\right]$$

$$\therefore \frac{dy}{dx} = 3\left[x^2 + x \cdot 2y\frac{dy}{dx} + y^2(1) + x^2\frac{dy}{dx} + y(2x)\right]$$

$$\therefore \frac{dy}{dx} - 6xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 3\left(x^2 + y^2 + 2xy\right)$$

$$\therefore \frac{dy}{dx}\left(1 - 6xy - 3x^2\right) = 3\left(x^2 + y^2 + 2xy\right)$$

$$\therefore \frac{dy}{dx} = \frac{3\left(x^2 + y^2 + 2xy\right)}{1 - 6xy - 3x^2}$$

$$\therefore \frac{dy}{dx} = \frac{-3\left(x^2 + y^2 + 2xy\right)}{6xy + 3x^2 - 1}$$

Miscellaneous Exercise 3 | Q 4.11 | Page 100

If x^3+y^2+xy=7 Find dy/dx

Solution:

$$x^3y^3 = x^2 - y^2$$

Differentiating both sides w.r.t. x, we get

$$x^3\frac{d}{dx}y^3 + y^3\frac{d}{dx}x^3 = 2x - 2y\frac{dy}{dx}$$

$$\therefore x^3\left(3y^2\right)\frac{dy}{dx} + y^3\left(3x^2\right) = 2x - 2y\frac{dy}{dx}$$

$$\therefore 3x^3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 2x - 3x^2y^2$$

$$\therefore y\left(3x^3y + 2\right)\frac{dy}{dx} = x\left(2 - 3xy^3\right)$$

$$\therefore \frac{dy}{dx} = \frac{x\left(2 - 3xy^3\right)}{y(3x^3y + 2)}$$

$$\therefore \frac{dy}{dx} = \frac{x}{y}\left(\frac{2 - 3xy^3}{2 + 3x^3y}\right)$$

Miscellaneous Exercise 3 | Q 4.12 | Page 100

If $x^3y^3=x^2-y^2$, Find dy/dx

Solution:

$$x^3y^3 = x^2 - y^2$$

Differentiating both sides w.r.t. x, we get

$$x^3\frac{d}{dx}y^3 + y^3\frac{d}{dx}x^3 = 2x - 2y\frac{dy}{dx}$$

$$\therefore x^3\left(3y^2\right)\frac{dy}{dx} + y^3\left(3x^2\right) = 2x - 2y\frac{dy}{dx}$$

$$\therefore 3x^3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 2x - 3x^2y^2$$

$$\therefore y\left(3x^3y + 2\right)\frac{dy}{dx} = x\left(2 - 3xy^3\right)$$

$$\therefore \frac{dy}{dx} = \frac{x\left(2 - 3xy^3\right)}{y(3x^3y + 2)}$$

$$\therefore \frac{dy}{dx} = \frac{x}{y}\left(\frac{2 - 3xy^3}{2 + 3x^3y}\right)$$

Miscellaneous Exercise 3 | Q 4.13 | Page 100

If $x^7 \cdot y^9 = (x+y)^{16}$, then show that $\frac{dy}{dx} = \frac{y}{x}$

Solution:

$$x^7 \cdot y^9 = (x+y)^{16}$$

Taking logarithm of both sides, we get

$$\log x^7 \cdot y^9 = \log\,(x+y)^{16}$$

$\therefore \log x^7 + \log y^9 = 16 \log(x+y)$

$\therefore$ 7 log x + 9 log y = 16 log (x + y)

Differentiating both sides w.r.t. x, we get

$$7\left(\frac{1}{x}\right) + 9\left(\frac{1}{y}\right)\frac{dy}{dx} = 16\left(\frac{1}{x+y}\right)\frac{d}{dx}(x+y)$$

$$\therefore \frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\therefore \frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y} + \frac{16}{x+y}\frac{dy}{dx}$$

$$\therefore \frac{9}{y}\frac{dy}{dx} - \frac{16}{x+y}\frac{dy}{dx} = \frac{16}{x+y} - \frac{7}{x}$$

$$\therefore \left(\frac{9}{y} - \frac{16}{x+y}\right)\frac{dy}{dx} = \frac{16}{x+y} - \frac{7}{x}$$

$$\therefore \left[\frac{9x + 9y - 16y}{y(x+y)}\right]\frac{dy}{dx} = \frac{16x - 7x - 7y}{x(x+y)}$$

$$\therefore \left[\frac{9x - 7y}{y(x+y)}\right]\frac{dy}{dx} = \frac{9x - 7y}{x(x+y)}$$

$$\therefore \frac{dy}{dx} = \frac{9x - 7y}{x(x+y)} \times \frac{y(x+y)}{9x - 7y}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$

Miscellaneous Exercise 3 | Q 4.14 | Page 100

If $x^a \cdot y^b = (x+y)^{a+b}$, then show that $\dfrac{dy}{dx} = \dfrac{y}{x}$

Solution:

$$x^{a} \cdot y^{b} = (x + y)^{a+b}$$

Taking logarithm of both sides, we get

log $(x^{a} \cdot y^{b})$ = log $(x + y)^{a+b}$

$\therefore$ log $x^{a} + \log y^{b} = (a + b)\log(x + y)$

$\therefore$ a log x + b log y = (a + b) log (x + y)

Differentiating both sides w.r.t. x, we get

$$a\left(\frac{1}{x}\right) + b\left(\frac{1}{y}\right)\frac{dy}{dx} = (a + b)\left(\frac{1}{x + y}\right)\frac{d}{dx}(x + y)$$

$$\therefore \frac{a}{x} + \frac{b}{y}\frac{dy}{dx} = \frac{a + b}{x + y}\left(1 + \frac{dy}{dx}\right)$$

$$\therefore \frac{a}{x} + \frac{b}{y}\frac{dy}{dx} = \frac{a + b}{x + y} + \frac{a + b}{x + y}\frac{dy}{dx}$$

$$\therefore \frac{b}{y}\frac{dy}{dx} - \frac{a + b}{x + y}\frac{dy}{dx} = \frac{a + b}{x + y} - \frac{a}{x}$$

$$\therefore \left(\frac{b}{y} - \frac{a + b}{x + y}\right)\frac{dy}{dx} = \frac{a + b}{x + y} - \frac{a}{x}$$

$$\therefore \left[\frac{bx + by - ay - by}{y(x + y)}\right]\frac{dy}{dx} = \frac{ax + bx - ax - ay}{x(x + y)}$$

$$\therefore \left[\frac{bx - ay}{y(x + y)}\right]\frac{dy}{dx} = \frac{bx - ay}{x(x + y)}$$

$$\therefore \frac{dy}{dx} = \frac{bx - ay}{x(x + y)} \times \frac{y(x + y)}{bx - ay}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$

Miscellaneous Exercise 3 | Q 4.15 | Page 100

Find dy/dx if x = $5t^2$, y = 10t.

Solution: x = $5t^2$

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = 5(2t) = 10t$$

y = 10t

Differentiating both sides w.r.t. t, we get

$$\frac{dy}{dt} = 10(1) = 10$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\therefore \frac{dy}{dx} = \frac{10}{10t}$$

$$\therefore \frac{dy}{dx} = \frac{1}{t}$$

Miscellaneous Exercise 3 | Q 4.16 | Page 100

Find $\frac{dy}{dx}$ if x = e^{3t}, $y = e^{\sqrt{t}}$.

Solution:

x = e^{3t}

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = e^{3t} \cdot \frac{d}{dx}(3t)$$

$$= e^{3t} \cdot (3)$$

$$\therefore \frac{dx}{dt} = 3e^{3t}$$

$y = e^{\sqrt{t}}$

Differentiating both sides w.r.t. t, we get

$$\frac{dy}{dt} = e^{\sqrt{t}} \cdot \frac{d}{dx}\left(\sqrt{t}\right)$$

$$\frac{dy}{dt} = e^{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^{\sqrt{t}}}{\frac{2\sqrt{t}}{3e^{3t}}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{6\sqrt{t}} e^{\sqrt{t}-3t}$$

Miscellaneous Exercise 3 | Q 4.17 | Page 100

Differentiate log $(1 + x^2)$ with respect to a^x.

Solution: Let u = log $(1 + x^2)$ and v = a^x

u = log $(1 + x^2)$

Differentiating both sides w.r.t.x, we get

$$\frac{du}{dx} = \frac{1}{1+x^2} \cdot \frac{d}{dx}(1+x^2)$$

$$= \frac{1}{1+x^2} \cdot (0+2x)$$

$$\therefore \frac{du}{dx} = \frac{2x}{1+x^2}$$

v = a^x

Differentiating both sides w.r.t.x, we get

$$\frac{dv}{dx} = a^x \cdot \log a$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{2x}{1+x^2}}{a^x \cdot \log a}$$

$$\therefore \frac{du}{dv} = \frac{2x}{a^x \cdot \log a \cdot (1+x^2)}$$

Miscellaneous Exercise 3 | Q 4.18 | Page 101

Differentiate e^{4x+5} with respect to 10^{4x}.

Solution:

Let u = $e^{(4x+5)}$ and v = 10^{4x}.

u = $e^{(4x+5)}$

Differentiating both sides w.r.t.x, we get

$$\frac{du}{dx} = e^{(4x+5)} \cdot \frac{d}{dx}(4x+5)$$

$$= e^{(4x+5)} \cdot (4+0)$$

$$\therefore \frac{du}{dx} = 4 \cdot e^{(4x+5)}.$$

v = 10^{4x}

Differentiating both sides w.r.t.x, we get

$$\frac{dv}{dx} = 10^{4x} \cdot \log 10 \cdot \frac{d}{dx}(4x)$$

$$\therefore \frac{dv}{dx} = 10^{4x} \cdot (\log 10)(4)$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4 \cdot e^{(4x+5)}}{10^{4x} \cdot (\log 10)(4)}$$

$$\therefore \frac{du}{dv} = \frac{e^{(4x+5)}}{10^{4x} \cdot (\log 10)}$$

Miscellaneous Exercise 3 | Q 4.19 | Page 101

Find $\frac{d^2y}{dx^2}$, if y = log (x).

Solution: y = log x

Differentiating both sides w.r.t.x, we get

$$\frac{dy}{dx} = \frac{1}{x}$$

Again, differentiating both sides w.r.t. x , we get

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}$$

Miscellaneous Exercise 3 | Q 4.2 | Page 101

Find $\frac{d^2y}{dx^2}$, if y = 2at, x = at^2

Solution: x = at^2

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = a\frac{d}{dx}\left(t^2\right) = a(2t)$$

$$\therefore \frac{dx}{dt} = 2at \quad \text{....(i)}$$

y = 2at

Differentiating both sides w.r.t. t, we get

$$\frac{dy}{dt} = 2a\frac{d}{dt}(t)$$

∴ $\frac{dy}{dt}$ = 2a

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{-1}{t^2} \cdot \frac{dt}{dx} = \frac{-1}{t^2} \times \frac{1}{2at} \quad \text{....[From (i)]}$$

$$= \frac{-1}{2at^3}$$

Miscellaneous Exercise 3 | Q 4.21 | Page 101

Find $\frac{d^2y}{dx^2}$, if y = $x^2 \cdot e^x$

Solution:

y = $x^2 \cdot e^x$

Differentiating both sides w.r.t. t, we get

$$\frac{dy}{dx} = x^2 \cdot \frac{d}{dx}(e^x) \cdot \frac{d}{dx}(x^2)$$

$$= x^2 \cdot e^x + e^x(2x)$$

$$\frac{dy}{dx} = \left(x^2 + 2x\right) \cdot e^x$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \left(x^2 + 2x\right) \cdot \frac{d}{dx}\left(e^x\right) + e^x \cdot \frac{d}{dx}\left(x^2 + 2x\right)$$

$$= \left(x^2 + 2x\right) \cdot e^x + e^x(2x + 2)$$

$$= e^x\left(x^2 + 2x + 2x + 2\right)$$

$$\therefore \frac{d^2y}{dx^2} = e^x\left(x^2 + 4x + 2\right)$$

Miscellaneous Exercise 3 | Q 4.22 | Page 101

If $x^2 + 6xy + y^2 = 10$, then show that $\frac{d^2y}{dx^2} = \frac{80}{(3x + y)^3}$.

Solution: $x^2 + 6xy + y^2 = 10$ ....(i)

Differentiating both sides w.r.t. x, we get

$$2x + 6x \cdot \frac{dy}{dx} + 6y + 2y\frac{dy}{dx} = 0$$

$$\therefore (2x + 6y) + (6x + 2y)\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{x + 3y}{3x + y} \quad \text{....(ii)}$$

$$\therefore (3x + y)\frac{dy}{dx} = -(x + 3y)$$

Again, differentiating both sides w.r.t. x, we get

$$(3x + y)\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(3 + \frac{dy}{dx}\right) = -\left(1 + 3 \cdot \frac{dy}{dx}\right)$$

$$\therefore 3\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 + 1 + 3\frac{dy}{dx} = -\frac{d^2y}{dx^2}(y + 3x)$$

$$\therefore \left(\frac{dy}{dx}\right)^2 + 6\frac{dy}{dx} + 1 = -\frac{d^2y}{dx^2}(y + 3x)$$

$$\therefore \left[-\left(\frac{x+3y}{3x+y}\right)\right]^2 + 6\left[\frac{-(x+3y)}{3x+y}\right] + 1$$

$$= -\frac{d^2y}{dx^2}(y + 3x) \quad \text{....[From (ii)]}$$

By solving, we get

$$\frac{x^2 + 9y^2 + 6xy - 6xy - 18x^2 - 18y^2 - 54xy + y^2 + 9x^2 + 6xy}{(y+3x)^2} = -\frac{d^2y}{dx^2}(y + 3x)$$

$$\therefore -\frac{d^2y}{dx^2}(y+3x)^3 = -8x^2 - 8y^2 - 48xy$$

$$= -8\left(x^2 + y^2 + 6xy\right)$$

= - 8 × 10 .....[from (i)]

= - 80

$$\therefore -\frac{d^2y}{dx^2} = \frac{80}{(3x+y)^3}$$

Miscellaneous Exercise 3 | Q 4.23 | Page 101

If $ax^2 + 2hxy + by^2 = 0$, then show that $\frac{d^2y}{dx^2} = 0$

Solution: $ax^2 + 2hxy + by^2 = 0$ ....(i)

Differentiating both sides w.r.t. x, we get

$$a(2x) + 2h \cdot \frac{d}{dx}(xy) + b(2y)\frac{dy}{dx} = 0$$

$$\therefore 2ax + 2h\left[x \cdot \frac{dy}{dx} + y(1)\right] + 2by\frac{dy}{dx} = 0$$

$$\therefore 2ax + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} = 0$$

$$\therefore 2\frac{dy}{dx}(hx + by) = -2ax - 2hy$$

$$\therefore 2\frac{dy}{dx} = \frac{-2(ax + hy)}{hx + by}$$

$$\therefore \frac{dy}{dx} = \frac{-(ax + hy)}{hx + by} \quad \text{....(i)}$$

$ax^2 + 2hxy + by^2 = 0$

$\therefore ax^2 + hxy + hxy + by^2 = 0$

$\therefore x(ax + hy) + y(hx + by) = 0$

$\therefore y(hx + by) = - x(ax + hy)$

$$\therefore \frac{y}{x} = \frac{-(ax + hy)}{hx + by} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{dy}{dx} = \frac{y}{x} \quad \text{....(iii)}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{x \cdot \frac{dy}{dx} - y \cdot \frac{d}{dx}(x)}{x^2}$$

$$= \frac{x \cdot \left(\frac{y}{x}\right) - y(1)}{x^2} \quad \text{....[From (iii)]}$$

$$= \frac{y - y}{x^2}$$

$$= \frac{0}{x^2}$$

$$\therefore \frac{d^2y}{dx^2} = 0$$