Linear Equations in Two Variables

Ex. 3.1

Answer 1-i.

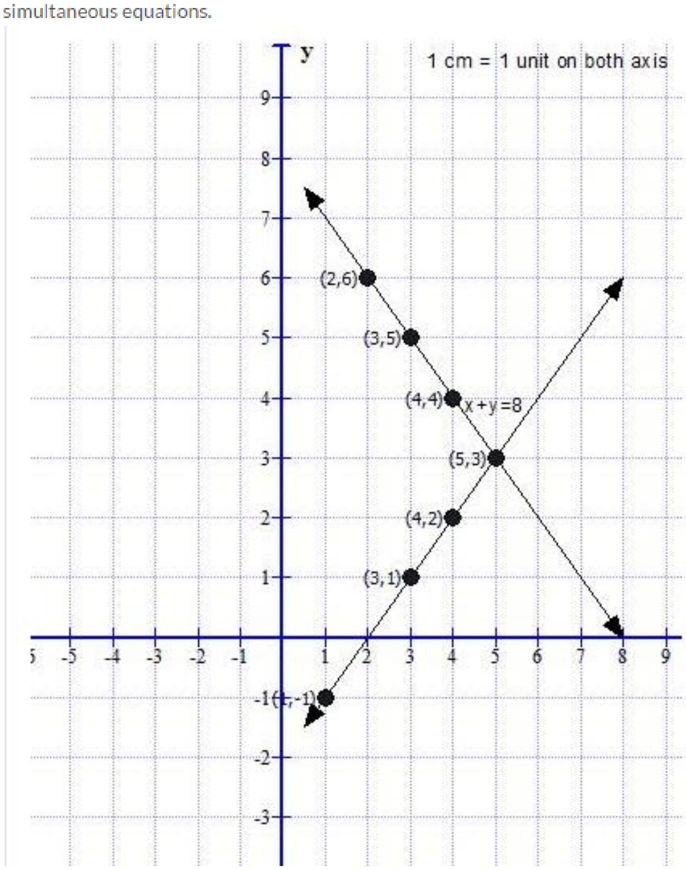
The first equation is, x + y = 8
∴ y = 8 - x

x	3	2	4
y	5	6	4
(x, y)	(3, 5)	(2,6)	(4,4)

The second equation is, x - y = 2
∴ y = x - 2

x	1	4	3
y	-1	2	1
(x, y)	(1, -1)	(4,2)	(3, 1)

Let us draw two lines corresponding to the two equations. The co-ordinates of the point of intersection of these lines is the solution of the given simultaneous equations.



The co-ordinates of the point of intersection are (5, 3).

Hence the solution of the given simultaneous equations is x = 5 and y = 3.

Answer 1-ii.

The first equation is

3x + 4y + 5 = 0

$\Rightarrow$ 4y = -3x - 5

$$\therefore y = \frac{-3x - 5}{4}$$

x	-3	1	5
Y	1	-2	-5
(x, y)	(- 3, 1)	(1, - 2)	(5, - 5)

The second equation is
y = x + 4

x	-4	0	3
Y	0	4	7
(x, y)	(-4, 0)	(0, 4)	(3, 7)

Let us draw two lines corresponding to the two equations. The co-ordinates of the point of intersection of these lines is the solution of the given simultaneous equations.

Answer 1-iii.

The first equation is 4x = y - 5

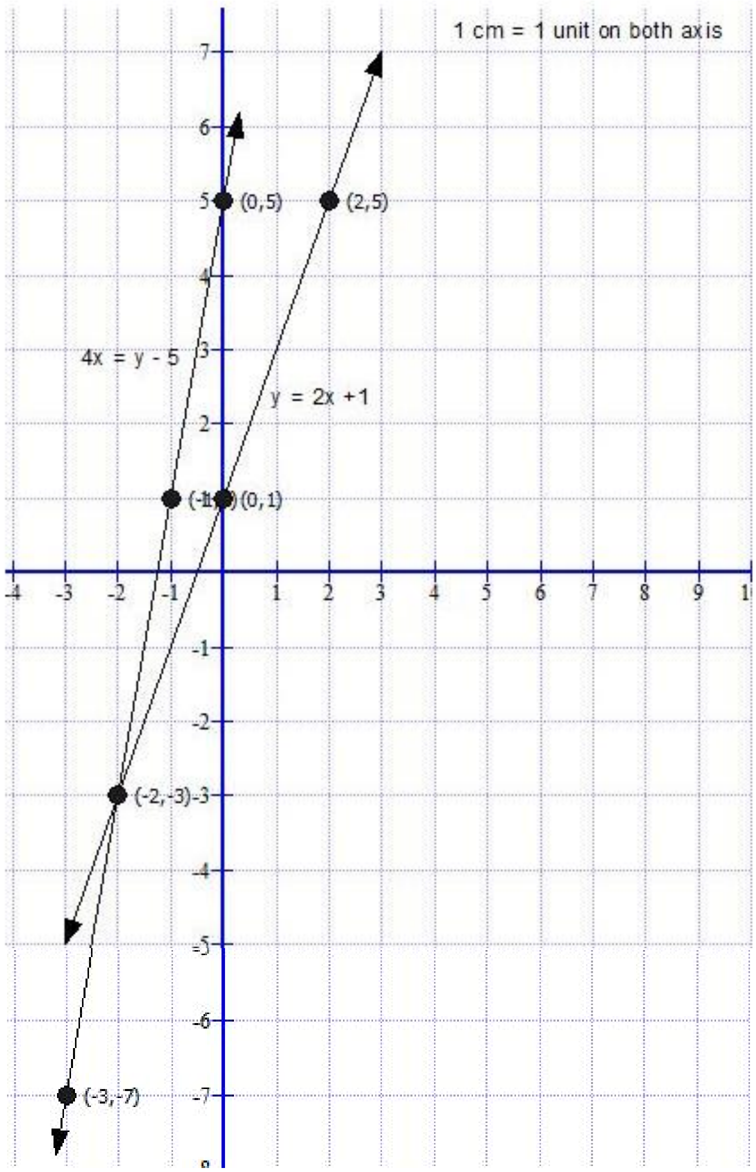
$\therefore$ y = 4x + 5

x	-3	-1	0
Y	-7	1	5
(x, y)	(-3, -7)	(-1, 1)	(0, 5)

The second equation is y = 2x + 1

x	-2	0	2
y	-3	1	5
(x, y)	(2, -3)	(0, 1)	(2, 5)

Let us draw two lines corresponding to the two equations. The co-ordinates of the point of intersection of these lines is the solution of the given simultaneous equations.



The co-ordinates of the point of intersection are
(-2, - 3).

Hence the solution of the given simultaneous equations is x = - 2 and y = - 3.

Answer 1-iv.

The first equation is x + 2y = 5

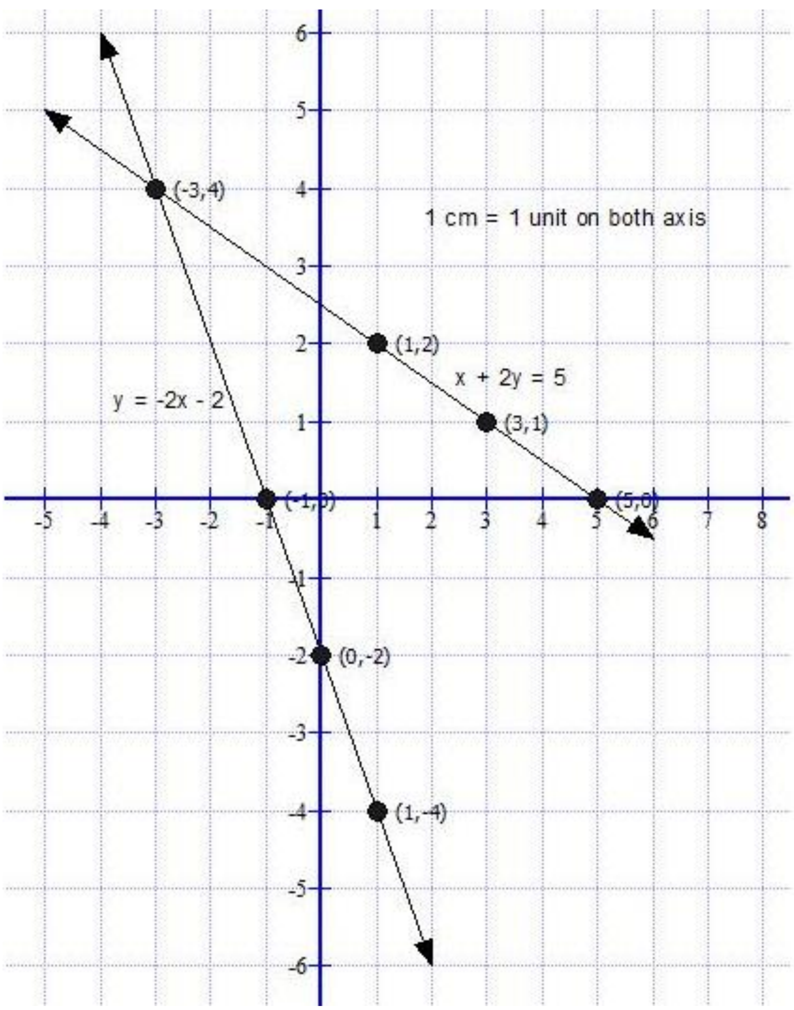
$$\therefore\ y = \frac{5 - x}{2}$$

X	5	1	3
Y	0	2	1
(x, y)	(5, 0)	(1, 2)	(3, 1)

The second equation is y = -2x - 2

X	0	-1	1
Y	-2	0	-4
(x, y)	(0, -2)	(-1, 0)	(1, - 4)

Let us draw two lines corresponding to the two equations. The co-ordinates of the point of intersection of these lines is the solution of the given simultaneous equations.



The co-ordinates of the point of intersection are (- 3, 4).

Hence the solution of the given simultaneous equations is x = -3 and y = 4.

Answer 1-v.

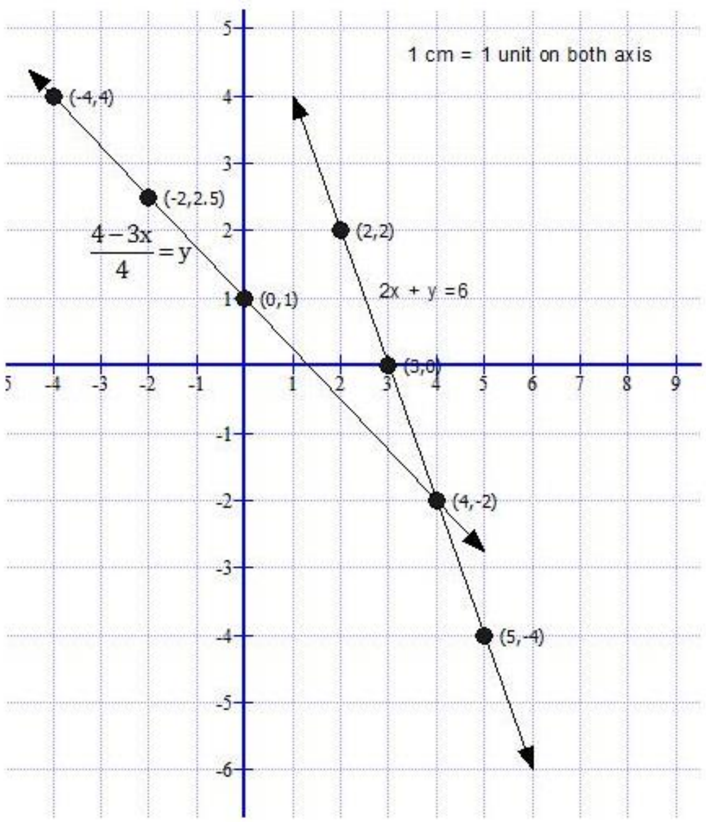
The first equation is
2x + y = 6
∴ y = 6 - 2x

X	2	3	5
Y	2	0	-4
(x, y)	(2,2)	(3, 0)	(5, - 4)

The second equation is

$$\therefore\ y = \frac{4 - 3x}{4}$$

x	0	-2	-4
y	1	2.5	4
(x, y)	(0, 1)	(-2, 2.5)	(-4,4)



The coordinates of the point of intersection are (4, - 2). Hence the solution of the given simultaneous equations is x = 4 and y = -2.

Ex. 3.2

Answer 1-ii.

The determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$\therefore$ In the determinant $\begin{vmatrix} -3 & 8 \\ 6 & 0 \end{vmatrix}$

$a = -3, \quad b = 8, \quad c = 6 \quad d = 0$

$\therefore \ ad - bc$

$= (-3) \times 0 - 8 \times 6$

$= 0 - 48 = -48$

Answer 1-iii.

The determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$\therefore$ In the determinant $\begin{vmatrix} 1.2 & 0.03 \\ 0.57 & -0.23 \end{vmatrix}$

$a = 1.2, \quad b = 0.03, \quad c = 0.57 \quad d = -0.23$

$\therefore \ ad - bc$

$= 1.2 \times (-0.23) - 0.03 \times 0.57$

$= -0.276 - 0.0171$

$= -0.2931$

Answer 1-iv.

The determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

In the determinant $\begin{vmatrix} 3\sqrt{6} & -4\sqrt{2} \\ 5\sqrt{3} & 2 \end{vmatrix}$

$a = 3\sqrt{6}, \quad b = -4\sqrt{2}, \quad c = 5\sqrt{3}, \quad d = 2$

$\therefore \ ad - bc$

$= 3\sqrt{6} \times 2 - \left[(-4\sqrt{2}) \times 5\sqrt{3}\right]$

$= 6\sqrt{6} + 20\sqrt{6}$

$= 26\sqrt{6}$

Answer 1-v.

$$\begin{vmatrix} \frac{-4}{7} & \frac{-6}{35} \\ 5 & \frac{-2}{5} \end{vmatrix}$$

$\therefore$ ad − bc

$$= \left[\left(\frac{-4}{7}\right) \times \left(\frac{-2}{5}\right)\right] - \left[\left(\frac{-6}{35}\right) \times 5\right]$$

$$= \frac{8}{35} + \frac{30}{35}$$

$$= \frac{8+30}{35} = \frac{38}{35}$$

Answer 2-i.

The Cramer's Rule:

If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ are the two simultaneous equations,

The given equations are,

$3x - y = 7, \ \therefore a_1 = 3, b_1 = -1, c_1 = 7$ and

$x + 4y = 11, \ \therefore a_2 = 1, b_2, = 4, c_2 = 11$

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - [(-1) \times 1]$$

$$= 12 + 1 = 13$$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = (7 \times 4) - [(-1) \times 11]$$

$$= 28 + 11 = 39$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = (3 \times 11) - (7 \times 1)$$

$$= 33 - 7 = 26$$

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{39}{13} = 3$$

$$y = \frac{D_y}{D} = \frac{26}{13} = 2$$

Hence, x = 3 and y = 2 is the solution of the given simultaneous equations.

Answer 2-ii.

The given simultaneous equations are,

$4x + 3y - 4 = 0$ and $6x = 8 - 5y$

$4x + 3y = 4$

$\therefore a_1 = 4, b_1 = 3, c_1 = 4$

$6x = 8 - 5y$

$\therefore 6x + 5y = 8$

$\therefore a_2 = 6, b_2 = 5, c_2 = 8$

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix} = (4 \times 5) - (3 \times 6)$$

$\therefore D = 20 - 18 = 2$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ 8 & 5 \end{vmatrix} = (4 \times 5) - (3 \times 8)$$

$\therefore D_x = 20 - 24 = -4$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = \begin{vmatrix} 4 & 4 \\ 6 & 8 \end{vmatrix} = (4 \times 8) - (4 \times 6)$$

$\therefore D_y = 32 - 24 = 8$

By Cramer's rule,

$$x = \frac{D_x}{D} = \frac{-4}{2} = -2$$

$$y = \frac{D_y}{D} = \frac{8}{2} = 4$$

$x = -2$ and $y = 4$ is the solution of the given simultaneous equations.

Answer 2-iii.

The given equations are,

$$y = \frac{5x - 10}{2}$$

$\therefore 2y - 5x = -10$

$\therefore 5x - 2y = 10$..............(1)

$\therefore a_1 = 5,\ b_1 = -2,\ c_1 = 10$

$4x + y = -5$.................(2)

$\therefore a_2 = 4,\ b_2 = 1,\ c_2 = -5$

From (1) and (2)

$$D_x = \begin{vmatrix} 10 & -2 \\ -5 & 1 \end{vmatrix} = (10 \times 1) - [-2 \times (-5)]$$

$= 10 - 10 = 0$

$$D_y = \begin{vmatrix} 5 & 10 \\ 4 & -5 \end{vmatrix} = (5 \times -5) - (10 \times 4)$$

$= -25 - 40 = -65$

$$D = \begin{vmatrix} 5 & -2 \\ 4 & 1 \end{vmatrix} = (5 \times 1) - (-2 \times 4)$$

$= 5 + 8 = 13$

By Cramer's rule

$$x = \frac{D_x}{D} = \frac{0}{-13} = 0 \text{ and}$$

$$y = \frac{D_y}{D} = \frac{-65}{13} = -5$$

x = 0 and y = -5 is the solution of the given simultaneous equations.

Answer 2-iv.

$3x+2y+11=0$

$\therefore 3x + 2y = -11$

$\therefore a_1 = 3,\ b_1 = 2,\ c_1 = -11$

$7x-4y=9$

$\therefore a_2 = 7,\ b_2 = -4,\ c_2 = 9$

$D = \begin{vmatrix} 3 & 2 \\ 7 & -4 \end{vmatrix} = (3 \times (-4)) - [7 \times 2]$

$= -12 - 14 = -26$

$D_x = \begin{vmatrix} -11 & 2 \\ 9 & -4 \end{vmatrix} = [(-11) \times (-4)] - [2 \times 9] = 44 - 18 = 26$

$D_y = \begin{vmatrix} 3 & -11 \\ 7 & 9 \end{vmatrix} = [3 \times 9] - [(-11) \times 7] = 27 + 77 = 104$

By Cramer's rule

$x = \frac{D_x}{D} = \frac{26}{-26} = -1$ and

$y = \frac{D_y}{D} = \frac{104}{-26} = -4$

x = -1 and y = -4 is the solution of the given simultaneous equations.

Answer 2-v.

$x + 18 = 2y \qquad \therefore x - 2y = -18$

$\therefore a_1 = 1,\ b_1 = -2,\ c_1 = -18$

$y = 2x - 9 \qquad \therefore -2x + y = -9$

$\therefore a_2 = -2,\ b_2 = 1,\ c_2 = -9$

$D = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = (1 \times 1) - [(-2) \times (-2)]$

$= 1 - 4 = -3$

$D_x = \begin{vmatrix} -18 & -2 \\ -9 & 1 \end{vmatrix} = [(-18) \times 1] - [(-2) \times (-9)]$

$= -18 - 18 = -36$

$D_y = \begin{vmatrix} 1 & -18 \\ -2 & -9 \end{vmatrix} = [1 \times (-9)] - [(-18) \times (-2)]$

$= -9 - 36 = -45$

By Cramer's rule,

$x = \frac{D_x}{D} = \frac{-36}{-3} = 12$ and

$y = \frac{D_y}{D} = \frac{-45}{-3} = 15$

x = 12 and y = 15 is the solution of the given simultaneous equations.

Answer 2-vi.

The given equations are,

$3x + y = 1$

$\therefore a_1 = 3,\ b_1 = 1,\ c_1 = 1$

and $2x = 11y + 3$

$\therefore 2x - 11y = 3$

$\therefore a_2 = 2,\ b_2 = -11,\ c_2 = 3$

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 2 & -11 \end{vmatrix} = [3 \times (-11)] - (1 \times 2)$$

$$= -33 - 2 = -35$$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 3 & -11 \end{vmatrix} = [1 \times (-11)] - (1 \times 3)$$

$$= -11 - 3 = -14$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = (3 \times 3) - (1 \times 2)$$

$$= 9 - 2 = 7$$

By Cramer's rule,

$$x = \frac{D_x}{D} = \frac{-14}{-35} = \frac{2}{5} \quad \text{and}$$

$$y = \frac{D_y}{D} = \frac{7}{-35} = -\frac{1}{5}$$

$x = \frac{2}{5}$ and $y = -\frac{1}{5}$ is the solution of the given simultaneous equations.

Ex. 3.3

Answer 1-i.

The given equations are,
$3x + 5y = 16;\ 4x - y = 6$
Comparing $3x + 5y = 16$ with $a_1x + b_1y = c_1$,
$a_1 = 3, b_1 = 5, c_1 = 16$
Comparing $4x - y = 6$ with $a_2x + b_2y = c_2$
$a_2 = 4, b_2 = -1, c_2 = 6$

$$\frac{a_1}{a_2} = \frac{3}{4},$$
$$\frac{b_1}{b_2} = \frac{5}{-1},$$
$$\frac{c_1}{c_2} = \frac{16}{6}$$

Here, $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

Hence, the given simultaneous equations have a unique solution.

Answer 1-ii.

The given equations are,
$3y = 2 - x;\ 3x = 6 - 9y$
That is $x + 3y = 2$ and $3x + 9y = 6$

Comparing $x + 3y = 2$ with $a_1x + b_1y = c_1$,
$a_1 = 1, b_1 = 3, c_1 = 2$
Comparing $3x + 9y = 6$ with $a_2x + b_2y = c_2$
$a_2 = 3, b_2 = 9, c_2 = 6$

$$\frac{a_1}{a_2} = \frac{1}{3},$$
$$\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3},$$
$$\frac{c_1}{c_2} = \frac{2}{6} = \frac{1}{3}$$

Here, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

Hence, the given simultaneous equations have infinitely many solutions.

Answer 1-iii.

The given equations are,
$3x - 7y = 15$; $6x = 14y + 10$, i.e. $6x - 14y = 10$

Comparing $3x - 7y = 15$ with $a_1x + b_1y = c_1$,
$a_1 = 3$, $b_1 = -7$, $c_1 = 15$
Comparing $6x - 14y = 10$ with $a_2x + b_2y = c_2$
$a_2 = 6$, $b_2 = -14$, $c_2 = 10$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2},$$
$$\frac{b_1}{b_2} = \frac{-7}{-14} = \frac{1}{2},$$
$$\frac{c_1}{c_2} = \frac{15}{10} = \frac{3}{2}$$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given simultaneous equations have no solution.

Answer 1-iv.

The given equations are,
$8y = x - 10$; $2x = 3y + 7$,
$\therefore x - 8y = 10$ and $2x - 3y = 7$

Comparing $x - 8y = 10$ with $a_1x + b_1y = c_1$,
$a_1 = 1$, $b_1 = -8$, $c_1 = 10$
Comparing $2x - 3y = 7$ with $a_2x + b_2y = c_2$
$a_2 = 2$, $b_2 = -3$, $c_2 = 7$

$$\frac{a_1}{a_2} = \frac{1}{2},$$
$$\frac{b_1}{b_2} = \frac{-8}{-3} = \frac{8}{3},$$
$$\frac{c_1}{c_2} = \frac{10}{7}$$

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given simultaneous equations have a unique solution.

Answer 1-v.

The given equations are,

$$\frac{x-2y}{3} = 1; \quad \text{and} \quad 2x - 4y = \frac{9}{2}$$

$$\frac{x-2y}{3} = 1$$

$$\therefore x - 2y = 3 \qquad \ldots(1)$$

$$2x - 4y = \frac{9}{2}$$

$$\therefore 4x - 8y = 9 \qquad \ldots(2)$$

Comparing x - 2y = 3 with $a_1x + b_1y = c_1$,
$a_1 = 1, b_1 = -2, c_1 = 3$
Comparing 4x - 8y = 9 with $a_2x + b_2y = c_2$
$a_2 = 4, b_2 = -8, c_2 = 9$

$$\frac{a_1}{a_2} = \frac{1}{4},$$

$$\frac{b_1}{b_2} = \frac{-2}{-8} = \frac{1}{4},$$

$$\frac{c_1}{c_2} = \frac{3}{9}$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given simultaneous equations have no solution.

Answer 1-vi.

The given equations are,

$$\frac{x}{2} + \frac{y}{3} = 4; \quad \frac{x}{4} + \frac{y}{6} = 2$$

$$\frac{x}{2} + \frac{y}{3} = 4$$

$$\therefore 3x + 2y = 24 \qquad (1)$$

$$\frac{x}{4} + \frac{y}{6} = 2$$

$$\therefore 6x + 4y = 48 \qquad (2)$$

Comparing 3x + 2y = 24 with $a_1x + b_1y = c_1$,
$a_1 = 3, b_1 = 2, c_1 = 24$
Comparing 6x + 4y = 48 with $a_2x + b_2y = c_2$
$a_2 = 6, b_2 = 4, c_2 = 48$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2},$$
$$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2},$$
$$\frac{c_1}{c_2} = \frac{24}{48} = \frac{1}{2}$$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given simultaneous equations have infinitely many solutions.

Answer 2-i.

The given equations are
$4x + y = 7$; $16x + ky = 28$
Comparing $4x + y = 7$ with $a_1x + b_1y = c_1$, $a_1 = 4$, $b_1 = 1$, $c_1 = 7$
Comparing $16x + ky = 28$ with $a_2x + b_2y = c_2$,
$a_2 = 16$, $b_2 = k$, $c_2 = 28$

$$\frac{a_1}{a_2} = \frac{4}{16} = \frac{1}{4}$$
$$\frac{b_1}{b_2} = \frac{1}{k}$$
$$\frac{c_1}{c_2} = \frac{7}{28} = \frac{1}{4}$$

The condition for simultaneous equations to have infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\therefore \frac{1}{4} = \frac{1}{k} = \frac{1}{4}$$
$$\therefore \frac{1}{k} = \frac{1}{4}$$
$$\therefore k = 4$$

Answer 2-ii.

The given equations are,
$4y = kx - 10 \therefore kx - 4y = 10$
$3x = 2y + 5 \therefore 3x - 2y = 5$

Comparing $kx - 4y = 10$ with $a_1x + b_1y = c_1$

$a_1 = k, b_1 = -4, c_1 = 10$
Comparing $3x - 2y = 5$ with $a_2x + b_2y = c_2$
$a_2 = 3; b_2 = -2; c_2 = 5$

$$\frac{a_1}{a_2} = \frac{k}{3}$$
$$\frac{b_1}{b_2} = \frac{-4}{-2} = \frac{2}{1}$$
$$\frac{c_1}{c_2} = \frac{10}{5} = \frac{2}{1}$$

The condition for simultaneous equations to have infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\therefore \frac{k}{3} = \frac{2}{1} = \frac{2}{1}$$
$$\therefore k = 6$$

The value of k is 6.

Answer 3-i.

The given equations are,
$kx + y = k - 2$
$9x + ky = k$

Comparing $kx + y = k - 2$ with $a_1x + b_1y = c_1$
$a_1 = k, b_1 = 1, c_1 = k - 2$
Comparing $9x + ky = k$, with $a_2x + b_2y = c_2$
$a_2 = 9, b_2 = k, c_2 = k$

$$\frac{a_1}{a_2} = \frac{k}{9}$$
$$\frac{b_1}{b_2} = \frac{1}{k}$$
$$\frac{c_1}{c_2} = \frac{k-2}{k}$$

The condition for simultaneous equations to have infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{9} = \frac{1}{k} = \frac{k-2}{k} \qquad \ldots(1)$$

$$\frac{k}{9} = \frac{1}{k}$$

$$k^2 = 9 \quad \therefore k = \pm 3$$

$$\text{Also,} = \frac{k-2}{k} = \frac{1}{k} \qquad \ldots\text{From}(1)$$

$$\therefore k - 2 = 1$$

$$\therefore k = 3$$

$\therefore$ k = 3 satisfies both the conditions.
The value of k is 3.

Answer 3-ii.

The given equations are,
$kx - y + 3 - k = 0 \quad \therefore kx - y = k - 3$ and
$4x - ky + k = 0 \quad \therefore 4x - ky = -k$

Comparing equations
$kx - y = k - 3$ with $a_1x + b_1y = c_1$,
$a_1 = k,\ b_1 = -1,\ c_1 = k - 3$ and
$4x - ky = -k$ with $a_2x + b_2y = c_2$,
$a_2 = 4,\ b_2 = -k\ \ c_2 = -k$

$$\frac{a_1}{a_2} = \frac{k}{4}$$

$$\frac{b_1}{b_2} = \frac{-1}{-k} = \frac{1}{k}$$

$$\frac{c_1}{c_2} = \frac{k-3}{-k}$$

The condition for simultaneous equations
to have infinitely many solutions is,

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \frac{k}{4} = \frac{1}{k} = \frac{k-3}{-k} \qquad \ldots(1)$$

$$\frac{k}{4} = \frac{1}{k} \therefore k^2 = 4 \quad \therefore k = \pm 2 \qquad \ldots(2)$$

$$\text{Also } \frac{1}{k} = \frac{k-3}{-k} \qquad \ldots\text{From}(1)$$

$$\therefore -1 = k - 3$$

$$\therefore k = 2$$

From (1) and (2), k = 2 satisfies both the conditions.
The value of k is 2.

Answer 4-i.

The given equations are

$3x + y = 10$ and $9x + py = 23$

Comparing the equation $3x + y = 10$ with $a_1x + b_1y = c_1$

$a_l = 3, b_l = 1, c_l = 10$

Comparing $9x + py = 23$, with $a_2x + b_2y = c_2$

$a_2 = 9, b_2 = p, c_2 = 23$

The condition for simultaneous equations to have a unique solution is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (1)

Here, $\frac{a_1}{a_2} = \frac{3}{9} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{1}{p}$

$\because \frac{1}{3} \neq \frac{1}{p}$ (1)

$\therefore p \neq 3$

Hence, the simulteneous equations will have a unique solution for all values of p except 3.

Answer 4-ii.

Comparing equations

$8x - py + 7 = 0$

$\therefore 8x - py = -7,$ with $a_1x + b_1y = c_1,$

$a_1 = 8, \; b_1 = -p, \; c_1, = -7$ and

$4x - 2y + 3 = 0$

$\therefore 4x - 2y = -3,$ with $a_2x + b_2y = c_2,$

$a_2 = 4, \; b_2 = -2, \; c_2 = -3$

The condition for simultaneous equations to have a unique solution is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (1)

Here, $\frac{a_1}{a_2} = \frac{8}{4} = \frac{2}{1}$

$\frac{b_1}{b_2} = \frac{-p}{-2} = \frac{p}{2}$

$\because \frac{2}{1} \neq \frac{p}{2}$ (1)

$\therefore p \neq 4$

Hence, the simulteneous equations will have a unique solution for all values of p except 4.

Ex. 3.4

Answer 1-i.

The given equations are, $\frac{1}{x}+\frac{1}{y}=8$; $\frac{4}{x}-\frac{2}{y}=2$

Substituting m for $\frac{1}{x}$ and n for $\frac{1}{y}$, in the given equations,

$\frac{1}{x}+\frac{1}{y} = 8$

$\therefore$ m + n = 8 ...(1)

$\frac{4}{x}-\frac{2}{y}=2$

$\therefore$ 4m - 2n = 2 ...(2)

Multiplying (1) by 2,

2(m + n) = 2(8)

$\therefore$ 2m + 2n = 16 ...(3)

Adding (2) and (3)

$$\begin{array}{rl} 4m - 2n &= 2 \\ \underline{2m + 2n} &\underline{= 16} \\ 6m &= 18 \end{array}$$

$\therefore$ m = 3

Substituting m = 3 in (1)

3 + n = 8

$\therefore$ n = 5

Resubstituting the values of m and n we get,

$\frac{1}{x} = 3$ and $\frac{1}{y} = 5$

Hence, $x = \frac{1}{3}$ and $y = \frac{1}{5}$

Answer 1-ii.

The given equations are, $\frac{2}{x}+\frac{6}{y}=13;\ \frac{3}{x}+\frac{4}{y}=12$

Substituting m for $\frac{1}{x}$ and n for $\frac{1}{y}$, in the given equations,

$\frac{2}{x}+\frac{6}{y}=13$

$\therefore\ 2m + 6n = 13 \qquad \ldots(1)$

$\frac{3}{x}+\frac{4}{y}=12$

$\therefore\ 3m + 4n = 12 \qquad \ldots(2)$

Multiplying (1) by 3,

$3(2m + 6n) = 3(13)$

$\therefore 6m + 18n = 39 \qquad \ldots(3)$

Multiplying (2) by 2,

$2(3m + 4n) = 2(12)$

$\therefore 6m + 8n = 24 \qquad \ldots(4)$

Subtracting (4) from (3)

$$\begin{array}{rcl} 6m + 18n &=& 39 \\ 6m + 8n &=& 24 \\ - \quad - & & - \\ \hline 10n &=& 15 \end{array}$$

$\therefore\ n = \frac{15}{10} = \frac{3}{2}$

Substituting $n = \frac{3}{2}$ in (1)

$2m + 6n = 13$

$\therefore 2m + 6\left(\frac{3}{2}\right) = 13$

$\therefore 2m + 9 = 13$

$\therefore\ 2m = 4,\ \therefore\ m = 2$

Resubstituting the values of m and n we get,

$\frac{1}{x} = 2$ and $\frac{1}{y} = \frac{3}{2}$

$\therefore x = \frac{1}{2}$ and $y = \frac{2}{3}$

Answer 1-iii.

The given equations are

$\frac{1}{3x}+\frac{1}{5y}=\frac{1}{15}$ and $\frac{1}{2x}+\frac{1}{3y}=\frac{1}{12}$

Substituting m for $\frac{1}{x}$ and n for $\frac{1}{y}$, in the given equations,

$\frac{1}{3x}+\frac{1}{5y}=\frac{1}{15}$

$\therefore \frac{m}{3}+\frac{n}{5}=\frac{1}{15}$ ...(1)

Multiplying (1) by 15,

$15\left(\frac{m}{3}+\frac{n}{5}\right)=15\left(\frac{1}{15}\right)$

$\therefore \underline{5m + 3n = 1}$ ...(2)

$\frac{1}{2x}+\frac{1}{3y}=\frac{1}{12}$

$\therefore \frac{m}{2}+\frac{n}{3}=\frac{1}{12}$ ...(3)

Multiplying (3) by 12

$12\left(\frac{m}{2}+\frac{n}{3}\right)=12\left(\frac{1}{12}\right)$

$\therefore \underline{6m + 4n = 1}$ ...(4)

Multiplying (2) by 4 and (4) by 3

$4(5m + 3n) = 4(1)$

$\therefore 20m + 12n = 4$ ...(5)

$3(6m + 4n) = 3(1)$

$\therefore 18m + 12n = 3$ ...(6)

Then subtracting (6) from (5)

$$\begin{array}{l} 20m + 12n = 4 \\ 18m + 12n = 3 \\ - \quad\; - \qquad - \\ \hline 2m \qquad\; = 1 \end{array}$$

$\therefore m = \frac{1}{2}$

Substituting m= $\frac{1}{2}$ in (4)

$6m + 4n = 1$

$\therefore 6\left(\frac{1}{2}\right) + 4n = 1$

$\therefore 4n = 1 - 3 = -2$

$\therefore n = -\frac{1}{2}$

Resubstituting the values of $m=\frac{1}{2}$ and $n=-\frac{1}{2}$,

$\frac{1}{x}=\frac{1}{2}$ and $\frac{1}{y}=-\frac{1}{2}$,

$\therefore x = 2$ and $y = -2$

Answer 1-iv.

The given equations are

$$\frac{27}{x-2} + \frac{31}{y+3} = 85; \quad \frac{31}{x-2} + \frac{27}{y+3} = 89$$

Substituting m for $\frac{1}{x-2}$ and n for $\frac{1}{y+3}$,

in the given equations,

$27m + 31n = 85 \quad \ldots(1)$

$31m + 27n = 89 \quad \ldots(2)$

Adding (1) and (2)

$$\begin{aligned} 27m + 31n &= 85 \\ 31m + 27n &= 89 \\ \hline 58m + 58n &= 174 \end{aligned}$$

$\therefore m + n = 3 \quad \ldots\text{(Dividing by 58)}\ldots(3)$

Subtracting (1) from (2)

$$\begin{aligned} 31m + 27n &= 89 \\ 27m + 31n &= 85 \\ - \quad - \quad &\quad - \\ \hline 4m \quad -4n &= 4 \end{aligned}$$

$\therefore m - n = 1 \quad \ldots\text{(Dividing by 4)}\ldots(4)$

Subtracting (1) from (2)

$$\begin{aligned} 31m + 27n &= 89 \\ 27m + 31n &= 85 \\ - \quad - \quad &\quad - \\ \hline 4m \quad -4n &= 4 \end{aligned}$$

$\therefore m - n = 1 \quad \ldots\text{(Dividing by 4)}\ldots(4)$

Adding equations (3) and (4)

$$\begin{aligned} m + n &= 3 \\ m - n &= 1 \\ \hline 2m \quad &= 4 \quad \therefore \quad m = 2 \end{aligned}$$

Substituting the value of m = 2 in (3)

$m + n = 3$

$\therefore 2 + n = 3$

$\therefore n = 1$

Resubstituting the value of $m = \frac{1}{x-2}$ and $n = \frac{1}{y+3}$,

$2 = \frac{1}{x-2}$ and $1 = \frac{1}{y+3}$,

$\therefore 2x - 4 = 1$ and $y + 3 = 1$

$\therefore 2x = 5$ and $y = -2$

Hence, the solution are $x = \frac{5}{2}$ and $y = -2$

Answer 1-v.

$\frac{16}{x+y} + \frac{2}{x-y} = 1;\quad \frac{8}{x+y} - \frac{12}{x-y} = 7$

Substituting m for $\frac{1}{x+y}$ and n for $\frac{1}{x-y}$,

in the given equations,

$16m + 2n = 1 \quad \ldots(1)$

$8m - 12n = 7 \quad \ldots(2)$

Multiplying (2) by 2

$16m - 24n = 14 \quad \ldots(3)$

Subtracting (3) from (1)

$16m + 2n = 1$

$16m - 24n = 14$

$-\quad +\quad -$

$+26n = -13$

$\therefore n = -\frac{1}{2}$

Substituting $n = \left(-\frac{1}{2}\right)$ in (1),

$16m + 2n = 1$

$\therefore 16m + 2\left(-\frac{1}{2}\right) = 1$

$\therefore 16m - 1 = 1$

$\therefore 16m = 2$

$\therefore m = \frac{1}{8}$

Resubstituting the value of $m = \frac{1}{8}$ and $n = -\frac{1}{2}$,

$\frac{1}{8} = \frac{1}{x+y}$

$\therefore x + y = 8 \quad \ldots(4)$

$-\frac{1}{2} = \frac{1}{x-y}$

$x - y = -2 \quad \ldots(5)$ Adding (4) and (5)

$x + y = 8$

$x - y = -2$

$2x = 6$

$\therefore x = 3$, substituting in (4)

$3 + y = 8$

$\therefore y = 5$

$\therefore x = 3$ and $y = 5$

Ex. 3.5

Answer 1-i.

Let the greater number be x
Let the smaller number be y
From condition 1:
$x + y = 60$ .... (1)
Thrice the smaller number is 3y
8 more than thrice the smaller number is 3y + 8
From condition 2:
$x = 3y + 8$
$\therefore x - 3y = 8$ .... (2)
Subtracting equation (2) from equation (1)
$x + y = 60$ .... (1)
$x - 3y = 8$ .... (2)
$-\ \ +\ \ -$
$4y = 52$
$\therefore y = 52 \div 4 = 13$
Substituting y = m =13 in (1)
$x + 13 = 60$
$\therefore x = 47$
∴The required numbers are 47 and 13.

Answer 1-ii.

Given: Let x be the length of the base.
Let y be the length of the congruent sides of the triangle.
∴Perimeter of the given triangle = x + 2y
But perimeter of the given triangle = 24 cm .... (Given)
$\therefore x + 2y = 24$ .... (1)
Twice the length of the base = 2x
And 13 cm less than twice the length of the base is
2x - 13
Hence, from the given condition,
$y = 2x - 13$ .... (2)
Substituting value of y from (2) in Equation (1)
$x + 2(2x - 13) = 24$
$\therefore x + 4x - 26 = 24$
$\therefore 5x = 50$
$\therefore x = 10$
Hence, length of the base = 10 cm
Substituting x = 10 in (1),
$10 + 2y = 24$
$\therefore 2y = 14$
$\therefore y = 7$
∴The lengths of the sides of the triangle are 7 cm, 7 cm, and 10 cm.

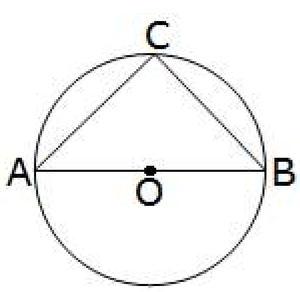
Answer 1-iii.

Let the greater acute angle of the triangle be x.
Let the smaller acute angle of the triangle be y.
From given condition,
$x - y = 20°$ .... (1)
Also, $x + y = 90°$ .... (2)
... (Sum of acute angles of a right angled triangle)
Adding (1) and (2)
$x - y = 20$ .... (1)
$\underline{x + y = 90}$ .... (2)
$2x = 110$
$\therefore x = 55$
Substituting x = 55 in (1),
$55 - y = 20$
$\therefore y = 35$
∴The measures of the acute angles of the right angled triangle are 55°and 35°.

Answer 1-iv.

Let the length of the yard be x.
Let the breadth of the yard be y.
From condition 1:
$x - y = 6$ .... (1)
Perimeter of the rectangle = 2(length + breadth)
∴Perimeter of the rectangular yard = $2(x + y)$
But, perimeter of the yard = 60 m .... (Given)
$\therefore 2(x + y) = 60$
$\therefore x + y = 30$ .... (2)
Adding (1) and (2),
$x - y = 6$ .... (1)
$\underline{x + y = 30}$ .... (2)
$2x = 36$
$\therefore x = 18$
Substituting x = 18 in (1),
$18 - y = 6$
$\therefore y = 12$
∴ The length of the yard measures 18 m and its breadth measures12 m.

Answer 1-v.



Seg AB is the diameter of a circle and C is the point on the circumference.
Hence, $\widehat{ACB}$ is a semicircle.
∴ ∠ ACB is a right angle .... (Angle in a semi-circle)
∴Δ ACB is a right angled triangle.
Sum of the acute angles of a right angled triangle =90° (Acute angles of right angled triangle are complimentary)
Let ∠ A, be x and ∠ B, be y
x + y = 90° .... (1)
From given condition:
∠ B is less by 10° than ∠A
Hence y is less by 10° than x.
∴ x - y = 10° .... (2)
On adding (1) and (2)
x + y = 90 .... (1)
x - y = 10 .... (2)
2x = 100
∴ x = 50
Substituting x = 50 in (1),
50 + y = 90
∴ y = 90 - 50
∴ y = 40
The measures of all the all the angles of ΔABC are
m∠A = 50°
m∠B = 40°
m∠C = 90°

Answer 1-vi.

Let the number of 10 rupee notes be x.
Let the number of 5 rupee notes be y.
Hence, from the first condition,
10x + 5y = 190
∴ 2x + y = 38 .... (1)
From the second condition,
5x + 10y = 185
∴ x + 2y = 37 .... (2)

Multiplying ...(2) by 2
2(x + 2y) = 2 (37)
2x + 2y = 74 .... (3)

Subtracting (1) from (3),
2x + 4y = 74 .... (3)
2x + y = 38 .... (1)
\- \- \-

3y = 36
∴y = 12
Substituting y = 12 in (1),
2x + 12 = 38
∴ 2x = 26
∴ x = 13

13 ten rupee notes and 12 five rupee notes were given to Durga.

Answer 1-vii.

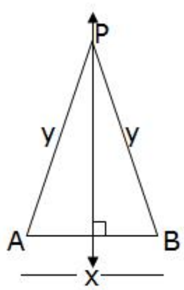
Let the starting salary of the man be Rs. x
Let the fixed annual increment be Rs. y
As per the given information, after 2 years the man's salary= Rs. 11000.
∴x + 2y = 11000, (Increment for two years = 2y) .... (1)
After 4 years, salary= Rs. 14000
∴x + 4y = 14000, (Increment for four years = 4y) .... (2)

Subtracting (1) from (2)

x + 4y = 14000 .... (2)
x + 2y = 11000 .... (1)
\- - -

0 +2y = 3000
∴ y = 1500
Substituting y = 1500 in (1),
∴ x + 2 (1500) = 11000
∴ x + 3000 = 11000
∴ x = 11000 - 3000
∴ x = 8000
∴The starting salary of the man was Rs. 8000 and his annual increment was Rs. 1500.

Answer 1-viii.



Let x be the length of seg AB
P is the point on the perpendicular bisector of AB.
Hence, P is equidistant from the end points of seg AB.
∴ AP = BP
Let AP = BP = y
From the first condition:
y = x + 7 .... (1)
Perimeter of ΔABP = x +y + y= x + 2y
But perimeter of ΔABP = 38 ....(Given)
∴ x + 2y = 38 ....(2)
Substituting y = x + 7 in (2),
∴x + 2(x + 7) = 38
∴x + 2x +14 = 38
∴3x = 24
∴x = 8
Substituting x = 8 in (2),
y = 8 + 7 = 15.
∴The sides of ΔABP are, AB = 15 cm, PA = PB = 8 cm.

Answer 1-ix.

Let the greater number be x
Let the smaller number be y
From the first condition:
$x + y = 97$ .... (1)
From the second condition:
$x \div y$ = quotient = 7 and remainder = 1.
Hence, $x = 7y + 1$ .... (2)
Substituting $x = 7y + 1$, in (1),
$\therefore 7y + 1 + y = 97$
$\therefore 8y = 96$
$\therefore y = 12$
Hence, the smaller number = 12
Substituting y = 12 in (1),
$\therefore x + 12 = 97$
$\therefore x = 85$
∴The two numbers are 85 and 12.

Answer 1-x.

Let the speed of the boat in still water be x km/h.
Let the speed of the stream be y km/h.
The speed of the boat downstream = speed of the boat + speed of the stream.
∴ The speed of the boat downstream = (x + y) km/h

The speed of boat upstream = speed of boat - speed of stream.
∴ The speed of the boat upstream = (x - y) km/h
From the first condition:
Time taken by boat:

$$\text{To travel 8 km upstream} = \frac{8}{x-y}$$

$$\text{To travel 32 km downstream} = \frac{32}{x+y}$$

$$\left[\text{Time} = \frac{\text{distance}}{\text{speed}}\right]$$

$$\text{Hence, total time taken by the boat} = \frac{8}{x-y} + \frac{32}{x+y}$$

But time taken by the boat to travel 8 km upstream and 32 km downstream is 6 hours.

$$\therefore \frac{8}{x-y} + \frac{32}{x+y} = 6 \qquad \ldots(1)$$

From the second condition:
Time taken by the boat:

$$\text{To travel 20 km upstream} = \frac{20}{x-y}$$

To travel 16 km downstream $= \dfrac{16}{x+y}$

$\left[\text{Time} = \dfrac{\text{distance}}{\text{speed}}\right]$

Hence, total time taken by boat

$= \dfrac{20}{x-y} + \dfrac{16}{x+y}$

But time taken by the boat to travel 20 km upstream and 16 km downstream is 7 hours.

$\dfrac{20}{x-y} + \dfrac{16}{x+y} = 7 \quad \ldots(2)$

Substituting, $\dfrac{1}{x-y}$ for m and $\dfrac{1}{x+y}$ for n, in ...(1) and ...(2)

$8m + 32n = 6$

$\therefore \ 4m + 16n = 3 \quad \ldots(3)$

$20m + 16n = 7 \quad \ldots(4)$

Subtracting (3) from (4)

$20m + 16n = 7 \quad \ldots(4)$

$4m + 16n = 3 \quad \ldots(3)$

$- \quad - \quad -$

$16m = 4$

$\therefore \ m = \dfrac{1}{4}$

Substituting $m = \dfrac{1}{4}$ in (3),

$4 \times \dfrac{1}{4} + 16n = 3 \quad \ldots(3)$

$\therefore 1 + 16n = 3$

$\therefore \ 16n = 2$

$\therefore \ n = \dfrac{1}{8}$

Resubstituting m for $\dfrac{1}{x-y}$ and n for $\dfrac{1}{x+y}$

$\dfrac{1}{x-y} = \dfrac{1}{4}$

$\therefore x - y = 4 \quad \ldots(5)$

$\dfrac{1}{x+y} = \dfrac{1}{8}$

$\therefore\ x + y = 8 \qquad \ldots(6)$ ∴

Adding $\ldots(5)$ and $\ldots(6)$

$x - y = 4 \qquad \ldots(5)$

$\underline{x + y = 8} \qquad \ldots(6)$

$\therefore\ 2x = 12$

$\therefore\ x = 6$ and $y = 2$

∴The speed of the boat in still water = 6 km/h and the speed of the stream is 2 km/h.