CHANGE OF AXES

- Change of axes or transformation of axes is of three types :
  - Translation
  - Rotation
  - General Transformation
  - TRANSLATION : Shifting the origin to some other point without changing the direction of axes.
  - ROTATION : Rotating the system of coordinate axes through an angle without changing the position of the origin.
  - GENERAL TRANSFORMATION : Applying both translation and rotation.
- The equations showing the relations between the old (x,y) and the new (X,Y) coordinates of any point are called equations of transformation.
- When the origin is translated to (x_1, y_1), the equations of transformation are $x = X+x_1$, $y = Y+y_1$
- When the axes are rotated through an angle 'θ', the equations of transformation are given by

	X	Y
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

i.e., $x = X\cos\theta - Y\sin\theta$, $X = x\cos\theta + y\sin\theta$

$y = X\sin\theta + Y\cos\theta$, $Y = -x\sin\theta + y\cos\theta$

- The equations of general transformation are given by

	X	Y
$x - x_1$	$\cos\theta$	$-\sin\theta$
$y - y_1$	$\sin\theta$	$\cos\theta$

i.e., $x - x_1 = X\cos\theta - Y\sin\theta$,

$y - y_1 = X\sin\theta + Y\cos\theta$,

$X = (x-x_1)\cos\theta + (y-y_1)\sin\theta$

$Y = -(x-x_1)\sin\theta + (y-y_1)\cos\theta$

Where (x_1, y_1) is the new origin and θ is the angle of rotation.

- Transformation is used in reducing the general equation of any curve to the desired form. For example
  - To eliminate first degree terms, we employ translation.
  - To eliminate the term containing 'xy', we employ rotation.
- The point to which the origin has to be shifted to eliminate first degree terms (x, y terms) in $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is obtained by solving $\frac{\partial S}{\partial x} = 0, \frac{\partial S}{\partial y} = 0$
  - The first degree terms are removed from the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ by translation of axes to the point $\left[\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right]$. In this case, the transformed equation is $aX^2 + 2hXY + bY^2 + (gx_1 + fy_1 + c) = 0$
  - To remove the first degree terms from $ax^2 + by^2 + 2gx + 2fy + c = 0$, the origin is to be shifted to the point $\left(\frac{-g}{a}, \frac{-f}{b}\right)$. In this case, the transformed equation is $aX^2 + bY^2 + \left(\frac{-g^2}{a} + \frac{-f^2}{b} + c\right) = 0$
  - To remove the first degree terms from $2hxy + 2gx + 2fy + c = 0$, the origin is to be shifted to the point $\left(\frac{-f}{h}, \frac{-g}{h}\right)$. In this case, the transformed equation is $2hXY + c = 0$.
- The xy term is removed from $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ by rotation of axes through an angle $\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$
- The condition that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, to take the form $aX^2 + 2hXY + bY^2 = 0$, when the axes are translated is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
- The equation $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ has transformed to $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$, when the origin is shifted to (l, m) then $A = a$; $B = b$; $H = h$;

$$2G = \left(\frac{\partial S}{\partial x}\right)_{(l,m)}$$

$$2F = \left(\frac{\partial S}{\partial y}\right)_{(l,m)} \qquad C = S(l, m)$$

Note : 1) If the rotation is in clockwise direction then replace θ by $-\theta$.

2) On translation or rotation the position of the point, length of line segment, area, perimeter, angles are not changed. But the coordinates and equations change.

LEVEL - I

1. The coordinates of the point (4,5) in the new system, when its origin is shifted to (3,7) are
   1) (1, 2) 2) (-1, 2)
   3) (-1, -2) 4) (1, -2)
2. When the origin is shifted to a point P, the point (2, 0) is transformed to (0, 4) then the co-ordinates of P are
   1) (2, -4) 2) (-2, 4) 3) (-2, -4) 4) (2, 4)
3. The coordinates of the point (3, 7) in the new system when the origin is shifted to (5, -1) are
   1) (8, 6) 2) (2, -8)
   3) (-2, 8) 4) (4, 3)
4. If (2, 3) are the coordinates of a point P in the new system when the origin is shifted to (-3, 7) then the original coordinates of P are
   1) (-1, 10) 2) (5, -4)
   3) (-5, 4) 4) (-1, 5)
5. If the point (5, 7) is transformed to (-1, 2) when the origin is shifted to A, then A =
   1) (4, 9) 2) (6, 5)
   3) (-6, -5) 4) (2, 4)
6. If the axes are rotated through an angle 30^o, the coordinates of $(2\sqrt{3}, -3)$ in the new system are
   1) $\left(\frac{3}{2}, \frac{-5}{2}\right)$ 2) $\left(\frac{-\sqrt{3}}{2}, \frac{5}{2}\right)$
   3) $\left(\frac{3}{2}, \frac{-5\sqrt{3}}{2}\right)$ 4) $\left(3\sqrt{2}, \frac{-5\sqrt{3}}{2}\right)$
7. If the co-ordinates of P are transformed to $(6, 2\sqrt{3})$ when the axes are rotated through an angle 60^o then P is
   1) $(0, -4\sqrt{3})$ 2) $(0, 4\sqrt{3})$
   3) $(4\sqrt{3}, 0)$ 4) $(-4\sqrt{3}, 0)$
8. If the axes are rotated through an angle 30^o in the clockwise direction, the point $(4, 2\sqrt{3})$ in the new system is
   1) (2, 3) 2) $(2, \sqrt{3})$ 3) $(\sqrt{3}, 2)$ 4) $(\sqrt{3}, 5)$
9. If the coordinates of a point P are transformed to $\left(4\sqrt{3}, 2\right)$ when the axes are rotated through an angle 60^o then P =
   1) $(3\sqrt{3}, -5)$ 2) (-1, -5)
   3) $(5\sqrt{3}, -7)$ 4) $(7, -\sqrt{3})$
10. If the axes are translated to the point (-2, -3), then the equation $x^2+3y^2+4x+18y+30=0$ transforms to
    1) $x^2 + y^2 = 4$ 2) $x^2 + 3y^2 = 1$
    3) $x^2 - y^2 = 4$ 4) $x^2 - 3y^2 = 1$
11. If the origin is shifted to the point (-1, 2) without changing the direction of axes, the equation $x^2 - y^2 + 2x + 4y = 0$ becomes
    1) $x^2 + y^2 + 3 = 0$ 2) $x^2 + y^2 - 3 = 0$
    3) $x^2 - y^2 + 3 = 0$ 4) $x^2 - y^2 - 3 = 0$
12. When the axes are translated to the point $(5, -2)$, then the transformed form of the equation $xy + 2x - 5y - 11 = 0$ is
    1) $\frac{x}{y} = 1$ 2) $\frac{y}{x} = 1$ 3) $xy = 1$ 4) $xy^2 = 2$
13. When the axes are translated to the point (1, ½), the equation $5x^2+4xy+8y^2-12x-12y = 0$ transforms to
    1) $5x^2 + 4xy + 8y^2 = 9$ 2) $2x^2 + 3xy + 4y^2 = 0$
    3) $x^2 + 2y^2 - 3y = 0$ 4) $x^2 - 7xy + 8y^2 = 0$
14. In order to make the first degree terms missing in the equation $2x^2 + 7y^2 + 8x - 14y + 15 = 0$, the origin should be shifted to the point
    1) (1, -2) 2) (-2, -1)
    3) (2, 1) 4) (-2, 1)
15. The origin is shifted to (1, 2), the equation $y^2 - 8x - 4y + 12=0$ changes to $y^2 + 4ax = 0$ then a =
    1) 2 2) -2 3) 1 4) -1

16. By translating the axes the equation xy - 2x - 3y - 4 = 0 has changed to XY = k, then k =

1) -10 2) 10 3) 4 4) - 4

17. The point to which the origin should be shifted in order to eliminate x and y terms in the equation $x^2 + 3y^2 - 2x + 12y + 1 = 0$ is

1) (1, -2) 2) (1, 3) 3) (-4, 3) 4) (-1, 2)

18. The point to which the origin should be shifted in order to remove the x and y terms in the equation $14x^2 - 4xy + 11y^2 - 36x + 48y + 41 = 0$ is

1) (1, -2) 2) (-2, 1) 3) (-1, 2) 4) (2, -1)

19. The point to which origin is shifted in order to miss the 1st degree terms in
$2x^2 + 5xy + 3y^2 + 6x + 7y + 1 = 0$ is

1) (2, 1) 2) (1, -2) 3) (2, -1) 4) (1, 2)

20. To which point the origin is to be shifted in order to miss the first degree terms in the equation $2x^2 - 3y^2 - 12x + 18y - 4 = 0$

1) (-3, -3) 2) (3, 3) 3) (-3, 3) 4) (3, -3)

21. The transformed equation of $x\cos\alpha + y\sin\alpha = P$ when the axes are rotated through an angle α is

1) X = P 2) X + P = 0
3) Y = P 4) Y + P = 0

22. The transformed equation of $7x^2 + 2\sqrt{3}xy + 9y^2 = 8$ when the axes are rotated through an angle 60° is

1) $5X^2 + 3Y^2 = 15$ 2) $5X^2 + 3Y^2 = 30$
3) $5X^2 + 3Y^2 = 5$ 4) $5X^2 + 3Y^2 = 3$

23. The transformed equation of $4xy - 3x^2 = 10$ when the axes are rotated through an angle whose tangent is '2' is

1) $X^2 - 4Y^2 = 10$ 2) $4X^2 - Y^2 = 10$
3) XY - 10 = 0 4) $2X^2 - Y^2 + 10 = 0$

24. If the transformed equation of a curve is $17X^2 - 16XY + 17Y^2 = 225$ when the axes are rotated through an angle 45°, then the original equation of the curve is

1) $25x^2 + 9y^2 = 225$ 2) $9x^2 + 25y^2 = 225$
3) $25x^2 - 9y^2 = 225$ 4) $9x^2 - 25y^2 = 225$

25. The angle of rotation of the axes so that the equation $\sqrt{3}x - y + 5 = 0$ may be reduced to the form Y = k, where k is a constant is

1) $\pi/6$ 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/12$

26. The coordinate axes are rotated about the origin 'o' in the counter clockwise direction through an angle 60°. If a and b are the intercepts made on the new axes by a straight line whose equation referred to the original axes is 3x + 4y-5=0 then $\frac{1}{a^2} + \frac{1}{b^2} =$

1) 1/25 2) 1/9 3) 1/16 4) 1

27. If the axes are rotated through an angle 180° the equation 2x - 3y + 4=0 becomes

1) 2X - 3Y - 4 = 0 2) 2X + 3Y - 4 = 0
3) 3X - 2Y + 4 = 0 4) 3X + 2Y + 4 = 0

28. When the axes are rotated through an angle 90° the equation 5x - 2y + 7 = 0 transforms to

1) 2X - 5Y + 7 = 0 2) 2X + 5Y - 7 = 0
3) 2X - 5Y - 7 = 0 4) 2X + 5Y + 7 = 0

29. If the axes are rotated through an angle 60°, then the transformed equation of
$x^2 + y^2 = 25$ is

1) $X^2 + Y^2 = 1$ 2) $X^2 + Y^2 = 9$
3) $X^2 + Y^2 = 16$ 4) $X^2 + Y^2 = 25$

30. If the axes are rotated through an angle $\frac{\pi}{2}$ in anti-clock wise direction the transformed equation of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1) $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$ 2) $\frac{X^2}{b^2} + \frac{Y^2}{a^2} = 1$

3) $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$ 4) $\frac{X^2}{b^2} - \frac{Y^2}{a^2} = 1$

31. The angle of rotation of axes in order to eliminate xy term in the equation $xy = c^2$ is

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

32. If the equation $4x^2 + 2\sqrt{3}xy + 2y^2 - 1 = 0$ becomes $5X^2 + Y^2 = 1$, when the axes are rotated through an angle θ, then θ is

1) 15° 2) 30° 3) 45° 4) 60°

33. The angle of rotation of axes in order to eliminate xy term in the equation
$x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ is

1) $\pi/6$ 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/2$

34. The angle of rotation of the axes so that the equation ax + by + c = 0 may be reduced to X = p is

1) $\tan^{-1}\frac{b}{a}$ 2) $\tan^{-1}\frac{a}{b}$

3) $\frac{\pi}{2}$ 4) 0°

35. The line joining two points A (2,0) and B(3,1) is rotated about A in anticlockwise direction through an angle 15^o. If B goes to C, C =

1) $\left(\frac{4+\sqrt{2}}{2}, \sqrt{6}\right)$ 2) $\left(\frac{6+\sqrt{2}}{2}, \frac{\sqrt{6}}{2}\right)$

3) $\left(\frac{2+\sqrt{2}}{2}, \frac{\sqrt{6}}{2}\right)$ 4) $\left(\frac{4+\sqrt{2}}{2}, \frac{\sqrt{6}}{2}\right)$

36. The line passing through (7,3), (5,1) meets the x-axis at P. If the line is rotated through an angle 30^o in the anti clockwise direction about P then the slope of its new position is

1) $\sqrt{3}$ 2) $1/\sqrt{3}$ 3) $2+\sqrt{3}$ 4) $2-\sqrt{3}$

37. On shifting the origin to a particular point, the equation $x^2 + y^2 - 4x - 6y - 12 = 0$ transforms to $X^2 + Y^2 = K$. Then K =

1) 12 2) 25 3) 24 4) 5

KEY

1.4	2.1	3.3	4.1	5.2
6.3	7.2	8.4	9.1	10.2
11.3	12.3	13.1	14.4	15.2
16.2	17.1	18.1	19.2	20.2
21.1	22.3	23.1	24.1	25.3
26.4	27.1	28.2	29.4	30.2
31.4	32.1	33.1	34.1	35.2
36.3	37.2			

LEVEL II

1. The point (4,3) is translated to the point (3,1) and then axes are rotated through 30^o about the origin, then the new position of the point is

1) $\left(\frac{2\sqrt{3}+1}{2}, \frac{\sqrt{3}-2}{2}\right)$ 2) $\left(\frac{\sqrt{3}+1}{2}, \frac{2\sqrt{3}+1}{2}\right)$

3) $\left(\frac{\sqrt{3}+2}{2}, \frac{2\sqrt{3}-1}{2}\right)$ 4) $\left(\frac{\sqrt{3}-2}{2}, \frac{\sqrt{3}+1}{2}\right)$

2. The condition that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, can take the form $ax^2 + 2hxy + by^2 = 0$ by translating the origin to a suitable point is

1) $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

2) $2fgh - af^2 - bg^2 - ch^2 = 0$

3) $abc - af^2 - bg^2 - ch^2 = 0$

4) $abc + 2fgh = 0$

3. A line 'L' has intercepts 'a' and 'b' on the coordinate axes, without changing the origin, the axes are rotated through an angle 'α'. If the same line has intercepts 'p' and 'q' on the new axes, then we have

1) $a^2 + b^2 = p^2 + q^2$ 2) $a^2 + p^2 = b^2 + q^2$

3) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

4) $\frac{1}{a^2}+\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{q^2}$

4. When the origin is shifted to a suitable point, the equation $2x^2 + y^2 - 4x + 4y = 0$ transformed as $2x^2 + y^2 - 8x + 8y + 18 = 0$. The point to which origin was shifted is

1) (1, 2) 2) (1, -2) 3) (-1, 2) 4) (-1, -2)

5. If the distance between the two given points is 2 units and the points are transferred by shifting the origin to (2, 2), then the distance between the points in their new position is

1) 2 2) 5 3) 6 4) 7

6. If by any change of axes without shifting the origin the expression $ax^2 + 2hxy + by^2$ changes to $a^1X^2 + 2hXY + b^1Y^2$, then

1) $a + a^1 = b + b^1$ 2) $a + b = a^1 + b^1$
3) $a a^1 = bb^1$ 4) $a b^1 = a^1 b$

7. If the area of a triangle is 5 sq.units, then the area of the triangle when the origin is shifted to (1, 2) is

1) 2 sq.u 2) 3 sq.u 3) 4 sq.u 4) 5 sq.u

8. The point (4, 1) undergoes the following three transformations successively

i) Reflection about the line y = x

ii) Transformation through a distance of 2 units along the +ve direction of the x-axis

iii) Rotation through an angle $\frac{\pi}{4}$ about the origin in the anticlockwise direction. The final position of the point is given by the co-ordinates

1) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ 2) $\left(-2, 7\sqrt{2}\right)$

3) $\left(\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ 4) (7, 1)

9. If the origin is shifted to (1, 1) then the new coordinates of P are $(\cos\alpha, \cos\beta)$ then the original coordinates of P are

1) $(2\cos^2\alpha/2, 2\cos^2\beta/2)$

2) $(2\cos^2\alpha/2, 2\sin^2\beta/2)$

3) $(2\sin^2\alpha/2, 2\cos^2\beta/2)$

4) $(-2\cos^2\alpha/2, -2\cos^2\beta/2)$

KEY

1.3	2.1	3.3	4.3	5.1
6.2	7.4	8.3	9.1	

New Pattern Questions

1. To remove the first degree terms in the following equations origin should be shifted to the another point then calculate the new origins for List - II

List - I	**List - II**
(a) $x^2 - y^2 + 2x + 4y = 0$	(1) (5, -7)
(b) $4x^2 + 9y^2 - 8x + 36y + 4 = 0$	(2) (1, -2)
(c) $x^2 + 3y^2 - 2x + 12y + 1 = 0$	(3) (-1, 2)
(d) $2(x - 5)^2 + 3(y + 7)^2 = 10$	(4) (-1, -2)
	(5) (-5, 7)

The correct matching is

	A	B	C	D
1)	4	2	2	5
2)	5	3	3	5
3)	3	2	2	1
4)	4	3	3	1

2. Let us suppose that origin is shifted to (1, 2) then observe the following lists

List - I	**List - II**
(a) (7,5) changes to	(1) (-4, -1)
(b) (-3, 1) changes to	(2) (-2, -4)
(c) (0, 5) changes to	(3) (6, 3)
(d) (-1, -2) changes to	(4) (0, 0)
	(5) (-1, 3)

The correct matching is

	A	**B**	**C**	**D**
1)	3	1	5	4
2)	3	1	5	2
3)	1	5	2	4
4)	4	3	2	1

3. Statement I : The point to which the origin has to be shifted to eliminate x and y term in the equation $a(x+\alpha)^2 + b(y+\beta)^2 = c$ is $(-\alpha, -\beta)$

Statement II : The point to which the origin has to be shifted to eliminate x and y terms in $ax^2 + by^2 + 2gx + 2fy + c = 0$ is $\left(\frac{-g}{a}, \frac{-f}{b}\right)$

Which of the above statement is true :

1) only I 2) only II
3) Both I and II 4) Neither I nor II

4. Statement I : If f(x,y) = 0 is the equation of a curve then its transformed equation (by shifting the origin) is f(x-h, y-k) = 0

Statement II : If the first degree terms of the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are missing when it is translated to the point (x_1, y_1) then the new equation is $ax^2 + 2hxy + by^2 + 2gx_1 + 2fy_1 + c = 0$ which of the above statement is true :

1) only I 2) only II
3) Both I and II 4) Neither I nor II

5. Arrangement of the following equations in ascending order of the angles of rotation of axes so that they reduced to the form X = constant

(A) $\sqrt{3}x + y - 3 = 0$ (B) $x + \sqrt{3}y + 5 = 0$

(C) $x + y + 7 = 0$

1) A, C, B 2) B, A, C
3) B, C, A 4) C, A, B

KEY

1.3	2.2	3.3	4.4	5.1

PREVIOUS EAMCET QUESTIONS

1. The transformed equation of $x^2 + 2\sqrt{3}xy - y^2 - 8 = 0$, when the axes are rotated through an angle $\frac{\pi}{6}$ is **(EAMCET-1997)**

1) $x^2 - y^2 = 0$ 2) $x^2 - y^2 = 4$
3) $x^2 - y^2 = 2$ 4) $x^2 + y^2 = 4$

2. The angle through which the axes are to be rotated to remove the 'xy' term in the equation $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ is **(EAMCET-1998)**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$

3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

3. If the axes are rotated through an angle 30^0 about the origin then the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ is **(EAMCET-1999)**

1) $x^2 + y^2 - a^2 = 0$ 2) $x^2 - y^2 = a^2$

3) $x^2 + y^2 = 2a^2$ 4) $x^2 - y^2 = 2a^2$

4. The coordinate axes are rotated about the origin 0 is counter clockwise direction through an angle of 60^0. If p and q are intercepts made on new axes by a straight line whose equation referred to the original axes is $x + y = 1$, then $\frac{1}{p^2} + \frac{1}{q^2} =$

(EAMCET-2000)

1) 2 2) 4 3) 6 4) 8

5. When axes are rotated through an angle of 45^0 in positive direction without changing origin coordinates of $\left(\sqrt{2},4\right)$ in old system are

(EAMCET-2002)

1) $\left(1-2\sqrt{2},1+2\sqrt{2}\right)$ 2) $\left(1+2\sqrt{2},1-2\sqrt{2}\right)$

3) $\left(2\sqrt{2},\sqrt{2}\right)$ 4) $\left(2,\sqrt{2}\right)$

6. When axes are rotated by an angle of 135^0 initial coordinates of (4, -3) are

(EAMCET-2003)

(1) $\left(\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$ (2) $\left(\frac{1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$

(3) $\left(\frac{-1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$ (4) $\left(\frac{-1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$

KEY

1.2 2.1 3.2 4.1 5.1
6.4